Application of the Computational Theory of Perceptions to Human Gait Pattern Recognition

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Abstract

This paper aims to contribute to the field of human gait pattern recognition by providing a solution based on the Computational Theory of Perceptions. Our model differs significantly from others, e.g., based on machine learning techniques, because we use a linguistic model to represent the subjective designer's perceptions of the human gait process. This model is easily understood and provides good results. We include a practical demonstration with an Equal Error Rate of 3 %.

Key words: Gait recognition, Fuzzy Logic, Fuzzy Finite State Machine, Computational Theory of Perceptions, Authentication.

1. Introduction

Currently, industry demands new techniques for user authentication. Authentication based on biometrics is one area that has grown over the last few years. There are two types of biometric characteristics that are useful in this field: physiological characteristics, such as fingerprints [1] or DNA, and behavioral characteristics like signature [2], voice [3] or gait.

Gait analysis has been explored thoroughly during the last decade as a behavioral biometric measurement. Some areas of application include: access control, surveillance, activity monitoring and clinical analysis.

Most research is based on computer vision [4, 5, 6, 7, 8, 9, 10]. The main advantage of this approach is that there is no need to wear sensors, therefore allowing identification from a distance. For some applications, the main drawbacks of these methods are: dependence on illumination, misinterpretations due to shadows, need of a complex system for capturing images and its computational cost.

Nevertheless, solutions based on accelerometers [11, 12, 13, 14, 15], provide a smart solution to the problem of capturing the signal and its practical implementation as a commercial product. They can be used in the dark and provide 3-D data whereas computer vision systems produce 2-D projections. However the user must wear sensors and this makes the solution invalid for certain applications.

Regarding algorithms used for gait pattern recognition, the most frequent are based on Neural Networks [16] or Hidden Markov Models [17, 18]. However, the published results do not yet demonstrate the availability of a sufficiently robust method for a marketable product.

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In this paper, we make emphasis in modeling the knowledge acquired by a human observer of the system. For example, it is interesting to consider how a human observer has not difficulties recognizing the gait as a quasi-periodic process, i.e., the signal evolves in time approximately repeating its shape and period. Moreover, a human observer is clearly able to separate the relevant from the irrelevant features in the observed signal.

Although, we consider that both procedures are complementary, our approach is based on modeling the designer's perception of the system in contrast with a procedure based on machine learning.

We aim to contribute to the pattern recognition field by providing a technique for modeling this type of human perceptions. We present a new method for human gait recognition involving analysis of the accelerations produced during a complete gait cycle. We used a Fuzzy Finite State Machine (FFSM) [19] to model the perception of the signal evolution, where each state was established using our knowledge about the physiological phases of the human gait.

The model was implemented using fuzzy linguistic variables and rules to describe a set of states that the signal undergoes during its evolution in time. This type of model provides sufficient flexibility to represent the variations in both, signal amplitude and states time span. The model is expressed using linguistic terms that make its interpretation easier with a low computational cost.

Once the model was available, we used three relevant features of the human gait (homogeneity, symmetry and the relation weight/legs length) to recognize the gait style corresponding to a specific person. In the demonstration, we explain how to solve the problem of authentication of one person among 11 people with an Equal Error Rate (EER) of 3 %.

We have limited the scope of this paper to the case of using accelerometers to obtain the signal, but also the described method could be applied to signals that were obtained by computer vision.

2. Computing with the meaning of human perceptions

The Computational Theory of Perceptions (CTP) was outlined in the Zadeh's seminal paper "From computing with numbers to computing with words - from manipulation of measurements to manipulation of perceptions" [20] and further developed in subsequent papers [21]. The general goal of CTP is to develop computational systems with the capability of computing with the meaning of Natural Language (NL) expressions, i.e., with the capability of computing with imprecise descriptions of the world in a similar way that humans do it.

In this section, we introduce a set of definitions including and developing ideas taken from CTP. We focus our effort on exploring the possibilities of this theory in the field of pattern recognition.

2.1. Perceptions

In CTP, the computational model of a physical model is based on the subjective perceptions of a person that we will call the *designer*.

A perception (p) is a unit of information acquired by the *designer* about different parts of the system and its environment.

The *designer*'s perceptions are described using granules. A granule is a clump of elements which are drawn together by indistinguishability, similarity, proximity or functionality [22].

In CTP, the boundary of a granule is fuzzy. Fuzziness of granules allows us to model the way in which human concepts are formed, organized and manipulated in an environment of imprecision, uncertainty and partial truth [23].

The concept of linguistic variable is essential in the formal description of perceptions. Informally, a linguistic variable is a variable whose values are words or sentences in a NL [24]. For example, the linguistic variable *Age*, with possible values {*very old*, *old*, *quite new*, *new*}, can be used to describe a subjective perception of the age of an automobile.

The attributes of a perception are linguistic variables with values defined using fuzzy sets. The *designer* describes his/her perceptions using constraints [25], i.e., defining a set of relevant attributes and the sets of their possible linguistic values.

2.2. *First-order Perceptions*

The *designer* uses first-order perceptions to define the maximum level of granularity in a model. Typically, the *designer* obtains a first-order perception (p^1) using data provided by a sensor. There are two forms of representing (p^1):
The linguistic representation, e.g.:

p^1: "The Temperature is High"

And the formal representation:

$$p^1\colon\ T = \mu_{A_i}(z)$$

where:

- T is a linguistic variable (e.g.: Temperature).
- A_i is a linguistic term belonging to the set of possible linguistic values of T (e.g.: {Low, Warm, High}).
- $\mu_{A_i}(z)$ is the membership function associated with the linguistic term A_i.
- z is a numerical value obtained from the sensor (e.g.: 45$^{\text{O}}$C).

2.3. *Second-order perceptions*

The concept of granularity allows the *designer* to create a hierarchy of perceptions. In this structure, the *designer* uses a set of lower order perceptions to *explain* a higher order perception.

For example, two first-order perceptions:

p_1^1: "The Temperature is Warm".

p_2^1: "The Humidity is Medium".

could be used to *explain* the perception of Comfort in a room:

p^2: "The Room is Comfortable".

Typically this *explanation* has the form of a set of fuzzy rules such as:

$$\text{IF } p_1^1 \text{ AND } p_2^1 \text{ THEN } p^2$$

The network in this example can be extended easily by considering additional perceptions like "Acoustic noise" or "Number of persons in the room" to *explain* the perception of "Comfort". Furthermore, this perception can be used to *explain* a higher order perception, e.g., "Efficiency" of an air conditioning system, (see Fig. 1).

A granular network represents the *explanation* of a perception with certain level of granularity. For example, we summarize the description of an object by hiding the irrelevant granules and remarking the relevant ones. In CTP, the model of a generic perception is called a Protoform [25]. Here, the value of the attributes of a second-order perception changes dynamically when the first-order perceptions change, e.g., when the values provided by the sensors change.

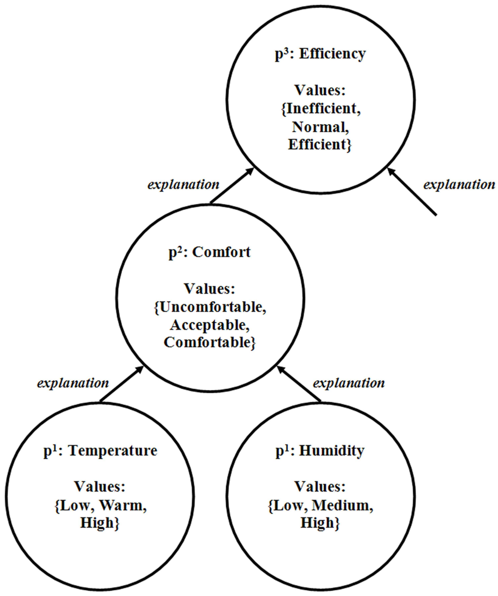

Figure 1: Granular network corresponding to a higher-order perception.

2.4. Perception of a system evolving in time

The most of practical applications concern with the perception of systems that evolve in time. The model of the perception of a system evolving in time is a protoform that describes how the system changes between the different states.

If, for the sake of simplicity, we restrict our attention to time invariant discrete-time systems, the equations that represent the evolution of a system in time are:

$$\begin{cases} x[t+1] = f(x[t], u[t]) \\ y[t] = g(x[t], u[t]) \end{cases}$$

where

- $x[t]$ is a vector that represents the state of the system at time t as it is perceived by the *designer*.
- $y[t]$ is a vector that represents the system output as it is perceived by the *designer*.
- $u[t]$ is a vector that represents the system input as it is perceived by the *designer*.
- $f(x[t], u[t])$ is an *explanation* of how the system state evolves in time.
- $g(x[t], u[t])$ is an *explanation* of how the system output evolves in time.

We say that the system described by these equations is a Fuzzy System when at least one of the variables is fuzzy [23]. It is important to remark that, from the point of view of our approach, the model of a system consists of a particular description of the *designer*'s perception of the system. This description contains, in certain degree of detail, an *explanation* of how the perceived outputs could be caused by the perceived inputs.

3. Fuzzy Finite State Machine

In the rest of this paper, we focus our attention on a specific type of computational models that provide a linguistic summary of data obtained by sensors. Here the *designer* must:

- Define a set of first-order perceptions ($u[t]$) using values provided by sensors.
- Define a second-order perception ($y[t]$) of this information with a level of granularity suitable for the final user purposes.
- Design an *explanation* of the system evolution consisting of a set of fuzzy rules that allows obtaining the values of the output linguistic variables in function of the inputs. This *explanation* includes a set of intermediate order perceptions $x[t]$ representing the relevant internal states of the system.

In a preliminary research, we have learnt that Fuzzy Finite State Machines are suitable tools for modeling signals which evolve approximately following a repetitive pattern [26, 27, 28]. We will see that Finite State Machines provide an interesting paradigm to design the sets of fuzzy rules that allow us to implement the functions $f(x[t], u[t])$ and $g(x[t], u[t])$ for modeling this type of signals.

A Fuzzy Finite State Machine (FFSM) is a tuple:

$$\{X, U, Y, f, g, X_0\}$$

where:

- X is the set of states $\{x_1, x_2, ..., x_{n_x}\}$. Every state represents the pattern of a repetitive situation. We say that, the system is in a specific state, when the current input variables and the previous state activations fulfil certain conditions. The activation of a state is a matter of degree, i.e., the FFSM could be partially in several states simultaneously. We will denote $\mathcal{X}_i \epsilon [0, 1]$ the degree of activation of a state. Defining the states includes determining their temporal order, i.e., the sequence with which the system follows the different relevant states.
- U is the input vector $\{u_1, u_2, ..., u_{n_u}\}$. U is a set of first-order perceptions where each variable u_i takes its value in a domain defined with a set of linguistic labels $\{A_{i1}, A_{i2}, ..., A_{in}\}$. Fig. 3 shows an example of these linguistic labels over the vertical axis.
- Y is the output vector $\{y_1, y_2, ..., y_{n_y}\}$. Y represents a summary of the values taken by the inputs while staying in a specific state.
- f is the state transition function $X[t+1] = f(U[t], X[t])$. This function can be implemented using a set of fuzzy rules:
  - Rules that constrain the signal amplitude. We distinguish between rules to stay in a state x_i (R_{ii}) and rules to change from the state x_i to the state x_j (R_{ij}):
    $R_{ii}:$ IF $\mathcal{X}_i(t) \vee (u$ is $C_i)$ THEN $\mathcal{X}_i(t+1)$
    $R_{ij}:$ IF $\mathcal{X}_i(t) \wedge (u$ is $C_j)$ THEN $\mathcal{X}_j(t+1)$
    where C_i and C_j represent the conditions of amplitude for the state x_i and x_j respectively.

    We used $\vee$ in R_{ii} to introduce an inertia to change of state. This makes the FFSM more robust against spurious in the input. This OR is typically implemented using the Maximum operator.

    We used $\wedge$ in R_{ij} to define the conditions to change more sharply. This AND is typically implemented using the Minimum operator.
  - Rules that constrain the signal time span. We did this using two additional linguistic labels (see Fig. 4):
    T_{stay_i}: is the maximum time that the signal is expected to remain in state x_i.
    $T_{change_{ij}}$: is the minimum time that the signal is expected to remain in state x_i before changing to state x_j.
    Therefore, adding the temporal conditions:
    $R_{ii}:$ IF $\mathcal{X}_i(t) \vee (u = C_i) \vee (d_i = T_{stay_i})$ THEN $\mathcal{X}_i(t+1)$
    $R_{ij}:$ IF $\mathcal{X}_i(t) \wedge (u = C_j) \wedge (d_i = T_{change_{ij}})$ THEN $\mathcal{X}_j(t+1)$
    where d_i is the duration of the state x_i.

- g is the output function $Y[t] = g(U[t], X[t])$. The output variables are obtained as a summary of the values of the inputs while the system remained in the considered state, e.g., using the average or the standard deviation (see an example in the next section).
- X_0 is the initial state.

4. Human Gait Pattern Recognition

The following sections describe how to apply the introduced above concepts in the field of human gait pattern recognition.

4.1. Linguistic terms in the application domain

Before embarking on a description of the different phases of the human gait, it is needed to introduce a small set of terms belonging to the domain of language.

- Reference foot: One foot.
- Opposite foot: The other foot.
- Stance phase: It begins when the heel contacts the ground and ends when the toes rise off the ground.
- Swing phase: It covers the period when the foot is not in contact with the ground.

The human gait is a quasi-periodic process with peculiarities that allow identifying a specific person. We used three characteristics to distinguish among different human gait styles:

- Symmetry: The degree with which the movement of a leg is similar to the other one.
- Homogeneity: The degree with which the whole gait profile repeats in time.
- The estimated proportion between legs length and weight.

We have designed a model of the human gait, i.e., a protoform where these characteristics appear remarked whereas the irrelevant aspects remain hidden.

4.2. Input variables

We attached a sensor in the belt, centered in the back, that provided measurements of three orthogonal accelerations every 100 millisecond. We programmed a PDA to receive the data via a Bluetooth connection and to record them with a timestamp. Every record contained the following information:

$$(Timestamp, a_x, a_y, a_z)$$

where:

- a_x is the vertical acceleration.
- a_y is the lateral acceleration.
- a_z is the acceleration in the progress direction.

During a first analysis of data, we realized that a_x and a_y were indicative for the states we wanted to distinguish. a_z was more difficult to use because it has to do with the walking speed and this speed can vary for the same person. Therefore, we used the two first accelerations as input to the fuzzification process.

4.2.1. Normalization

As an initial step, we normalized the signals. First, we subtracted the average making them to be centered on zero. Then, we rescaled them in the range given by their standard deviations. This allowed us to perform the analysis at the scale that gives us more information about the signal changes. Fig. 2 shows an example of the evolution of these two accelerations during one cycle and a half.

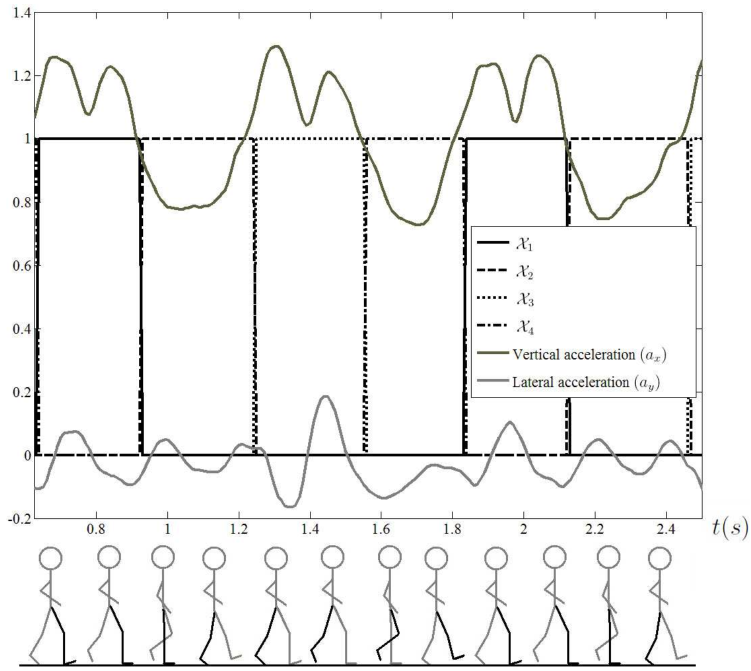

Figure 2: Vertical and lateral acceleration in g units during the four states of the human gait cycle.

4.2.2. Fuzzification

This step allows defining the first-order perceptions. In this level of granularity, the linguistic variables take a value belonging to the set {Negative, Zero, Positive}. Fig. 3 shows the drawing of these trapezoidal linguistic labels over the vertical axis. Note that, thanks to the normalization step, each trapezoidal linguistic label covers one third of the total amplitude.

4.3. Set of rules

The states were defined as follows:

- x_1: Reference foot is in stance phase and opposite foot is in stance phase (double limb support).
- x_2: Reference foot is in stance phase and opposite foot is in swing phase (reference limb single support).
- x_3: Reference foot is in stance phase and opposite foot is in stance phase (double limb support but different of x_1 because the feet position).
- x_4: Reference foot is in swing phase and opposite foot is in stance phase (opposite limb single support).

We used a set of fuzzy rules to explain the signal evolution between the different states. In contrast to machine learning techniques, we derived the rules from the *designer*'s perceptions about the human gait acceleration signals. The use of linguistic rules allows the *designer* to include his/her experience about the human gait in a easy way.

The model is able to synchronize without the need of doing previous segmentation of the signal. We chose the initial state $X_0 = \{x_1\}$ ($\mathcal{X}_1 = 1$ and $\mathcal{X}_i = 0\ \forall i \neq 1$), i.e., the FFSM synchronizes with the signal when the conditions of x_1 are fulfilled.

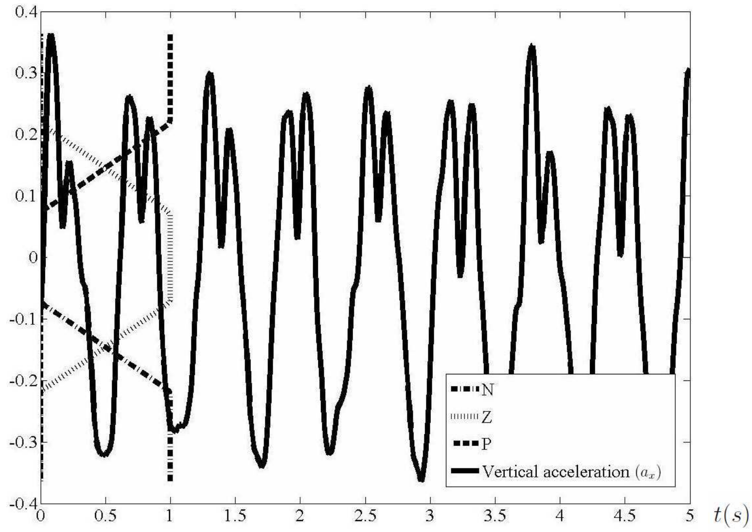

Figure 3: Trapezoidal linguistic labels for the normalized vertical acceleration.

4.3.1. Conditions of amplitude

We defined the conditions of amplitude to remain in a state or to change between states by combining the information obtained from the sensors and the available generic knowledge about the human gait. We defined 8 rules (4 to remain in each state and 4 to change between states).

We aggregated the conditions to remain in a specific state using OR to make the system more robust against spurious in the input. For example, the lateral acceleration (a_y) can fluctuate in the state x_3 while the vertical acceleration (a_x) matches its condition.

We aggregated the conditions to change between states using AND, trying to make the changes of state as sharp as possible.

- R_{11} : IF $\mathcal{X}_1(t) \vee (a_x = P) \vee (a_y = P)$ THEN $\mathcal{X}_1(t+1)$
- R_{22} : IF $\mathcal{X}_2(t) \vee (a_x = N) \vee (a_y = Z)$ THEN $\mathcal{X}_2(t+1)$
- R_{33} : IF $\mathcal{X}_3(t) \vee (a_x = P) \vee (a_y = N)$ THEN $\mathcal{X}_3(t+1)$
- R_{44} : IF $\mathcal{X}_4(t) \vee (a_x = N) \vee (a_y = Z)$ THEN $\mathcal{X}_4(t+1)$
- R_{12} : IF $\mathcal{X}_1(t) \wedge (a_x = N) \wedge (a_y = Z)$ THEN $\mathcal{X}_2(t+1)$
- R_{23} : IF $\mathcal{X}_2(t) \wedge (a_x = P) \wedge (a_y = N)$ THEN $\mathcal{X}_3(t+1)$
- R_{34} : IF $\mathcal{X}_3(t) \wedge (a_x = N) \wedge (a_y = Z)$ THEN $\mathcal{X}_4(t+1)$
- R_{41} : IF $\mathcal{X}_4(t) \wedge (a_x = P) \wedge (a_y = P)$ THEN $\mathcal{X}_1(t+1)$

4.3.2. Temporal conditions

We applied self-correlation analysis to the vertical acceleration to obtain an approximation of the signal period (T). In agreement with our knowledge about the typical human gait cycle, we assigned to each state approximately a 25 % of T. Fig. 4 shows the linguistic labels T_{stay} and T_{change} used to define the temporal constraints. T_{stay} was tuned manually with the conservative criteria of ensuring that the state will span at least the 25 %. T_{change} was designed to allow the change of state as soon as the conditions for changing are fulfilled.

Adding these temporal conditions, the rules were formulated as follows:

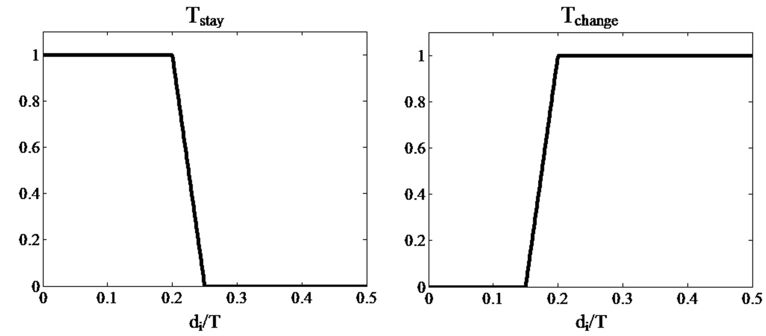

Figure 4: Temporal conditions for the state x_i.

- R_{11} : IF $\mathcal{X}_1(t) \vee (a_x = P) \vee (a_y = P) \vee (d_1 = T_{stay})$ THEN $\mathcal{X}_1(t+1)$
- R_{22} : IF $\mathcal{X}_2(t) \vee (a_x = N) \vee (a_y = Z) \vee (d_2 = T_{stay})$ THEN $\mathcal{X}_2(t+1)$
- R_{33} : IF $\mathcal{X}_3(t) \vee (a_x = P) \vee (a_y = N) \vee (d_3 = T_{stay})$ THEN $\mathcal{X}_3(t+1)$
- R_{44} : IF $\mathcal{X}_4(t) \vee (a_x = N) \vee (a_y = Z) \vee (d_4 = T_{stay})$ THEN $\mathcal{X}_4(t+1)$
- R_{12} : IF $\mathcal{X}_1(t) \wedge (a_x = N) \wedge (a_y = Z) \wedge (d_1 = T_{change})$ THEN $\mathcal{X}_2(t+1)$
- R_{23} : IF $\mathcal{X}_2(t) \wedge (a_x = P) \wedge (a_y = N) \wedge (d_2 = T_{change})$ THEN $\mathcal{X}_3(t+1)$
- R_{34} : IF $\mathcal{X}_3(t) \wedge (a_x = N) \wedge (a_y = Z) \wedge (d_3 = T_{change})$ THEN $\mathcal{X}_4(t+1)$
- R_{41} : IF $\mathcal{X}_4(t) \wedge (a_x = P) \wedge (a_y = P) \wedge (d_4 = T_{change})$ THEN $\mathcal{X}_1(t+1)$

where d_i is the duration of the state x_i.

Fig. 2 represents the degree of activation $\mathcal{X}_i$ of the four states of the FFSM following the evolution of the human gait. It shows how this set of fuzzy rules is able to separate efficiently the four phases of the human gait.

4.4. Output variables

After some experimentation, we realized that, although a_y is useful to distinguish between the states x_1 and x_3, it does not provide relevant information for our purpose. Also, as mentioned above, the acceleration in the direction of march a_z depends on the person's walking speed.

Indeed, the use of these variables or other additional signals, e.g. gyroscopes, could be considered. However the algorithms would grow up in complexity and we should lose the advantage of the simplicity.

Therefore, once identified the four phases in the signal, we focused on the characteristics of the vertical acceleration a_x. This acceleration provided sufficient information for our purpose. Here in after, for simplicity, we denote the vertical acceleration as a.

Fig. 5 shows the evolution of a along the four phases. The four rectangles represent graphically the relevant characteristics of each cycle of a specific gait. The dimensions of every rectangle summarize the values of the acceleration while staying in each state, i.e. they are a graphical representation of a protoform of the human gait. Therefore, the output of the FFSM is a vector:

$$y_i = (\overline{t_i},\, \overline{a_i},\, \sigma_{t_i},\, \sigma_{a_i})$$

The elements of this vector are:

- $\overline{t_i}$: The horizontal coordinate of the center of each rectangle is the temporal "center of mass" of the vertical acceleration in the state x_i. Note that the "mass" in every instant t is calculated as the vertical acceleration $a(t)$ weighted by the degree of activation $\mathcal{X}_i(t)$ of the state x_i.

$$\overline{t_i} = \frac{\sum_{t=0}^{T} t \cdot a(t) \cdot \mathcal{X}_i(t)}{\sum_{t=0}^{T} a(t) \cdot \mathcal{X}_i(t)}$$

- $\overline{a_i}$: The vertical coordinate of the center of each rectangle is the average of the vertical acceleration during the state x_i.

$$\overline{a_i} = \frac{\sum_{t=0}^{T} a(t) \cdot \mathcal{X}_i(t)}{\sum_{t=0}^{T} \mathcal{X}_i(t)}$$

- σ_{t_i}: The width of each rectangle is the standard deviation of the temporal distribution of the vertical acceleration weighted by the degree of activation $\mathcal{X}_i(t)$ of the state x_i.

$$\sigma_{t_i}^2 = \frac{\sum_{t=0}^{T} (t - \overline{t_i})^2 \cdot a(t) \cdot \mathcal{X}_i(t)}{\sum_{t=0}^{T} a(t) \cdot \mathcal{X}_i(t)}$$

- σ_{a_i}: The height of each rectangle is the standard deviation of the vertical acceleration during the state x_i.

$$\sigma_{a_i}^2 = \frac{\sum_{t=0}^{T} (a(t) - \overline{a_i})^2 \cdot \mathcal{X}_i(t)}{\sum_{t=0}^{T} \mathcal{X}_i(t)}$$

where:

- $a(t)$ is the vertical acceleration at the instant t.
- $\mathcal{X}_i(t)$ is the degree of activation of the state x_i at the instant t.
- T is the duration of a complete cycle.

5. Relevant characteristics

Using this model, we were able to analyze the differences among gaits of different people. We used the areas of the rectangles to distinguish the peculiarities of each specific gait.

In agreement with the section above, each cycle was modeled using four rectangles (states) and each rectangle was represented by the vector $y_i = (\overline{t_i}, \overline{a_i}, \sigma_{t_i}, \sigma_{a_i})$.

As mentioned above, we used three characteristics of the human gait that are useful to recognize the gait style corresponding to a specific person, namely, homogeneity, symmetry and the relation between the weight and the length of legs.

5.1. Homogeneity

The Homogeneity ($\mathcal{H}$) was obtained by comparing a gait with itself. The Homogeneity is based on the standard deviation of the sequence of rectangles of each state.

The Homogeneity of the state x_i ($\mathcal{H}_i$) was formulated as follows:

$$\mathcal{H}_i = \begin{cases} \frac{\overline{\mathcal{A}_i} - std(\mathcal{A}_i)}{\overline{\mathcal{A}_i}} & \text{if} \quad std(\mathcal{A}_i) < \overline{\mathcal{A}_i} \\ 0 & \text{if} \quad std(\mathcal{A}_i) \geq \overline{\mathcal{A}_i} \end{cases}$$

where:

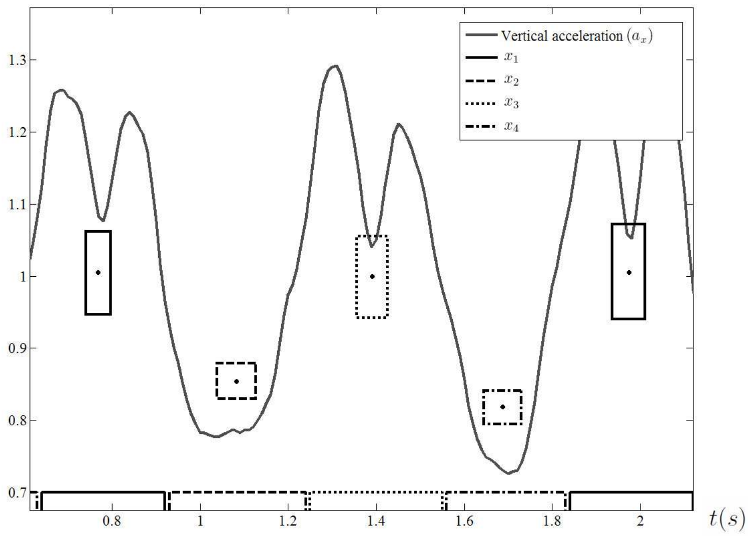

Figure 5: Characteristic rectangles and vertical acceleration in g units during the human gait cycle.

- $\mathcal{A}_i$ are the areas of the rectangles corresponding to the state x_i in the total number of available cycles.
- $\overline{\mathcal{A}_i}$ is the mean of this sequence of areas.
- $std(\mathcal{A}_i)$ is the standard deviation of this sequence of areas.

This equation provides a value $\mathcal{H}_i \in [0, 1]$. A low standard deviation indicates similar areas, i.e., Homogeneity close to 1. A high standard deviation indicates differences, i.e., Homogeneity close to 0.

The total Homogeneity ($\mathcal{H}$) summarizes the homogeneities of the four states as follows:

$$\mathcal{H} = \frac{1}{4}\sum_{i=1}^{4} \mathcal{H}_i$$

5.2. Symmetry

Symmetry ($\mathcal{S}$) was obtained by comparing the movement of both legs. Symmetry is based on comparing the areas of the states x_1 and x_2 (stance and swing phase of the reference foot) versus the areas of x_3 and x_4 (stance and swing phase of the opposite foot). A gait will be symmetric if the areas of the states x_1 and x_2 are similar to the areas of the states x_3 and x_4. The Symmetry in a cycle j ($\mathcal{S}_j \in [0, 1]$) was formulated as follows:

$$\mathcal{S}_j = \begin{cases} \frac{\mathcal{A}_{j3}+\mathcal{A}_{j4}}{\mathcal{A}_{j1}+\mathcal{A}_{j2}} & \text{if} \quad \mathcal{A}_{j1} + \mathcal{A}_{j2} \geq \mathcal{A}_{j3} + \mathcal{A}_{4j} \\ \frac{\mathcal{A}_{j1}+\mathcal{A}_{j2}}{\mathcal{A}_{j3}+\mathcal{A}_{j4}} & \text{if} \quad \mathcal{A}_{j1} + \mathcal{A}_{j2} < \mathcal{A}_{j3} + \mathcal{A}_{j4} \end{cases}$$

where:
$\mathcal{A}_{j1}$, $\mathcal{A}_{j2}$, $\mathcal{A}_{j3}$, $\mathcal{A}_{j4}$ are the areas of the rectangles corresponding to states x_1, x_2, x_3, x_4 in the cycle j.

The total Symmetry ($\mathcal{S}$) was calculated as the average of the symmetries of the sampled cycles M:

$$\mathcal{S} = \frac{1}{M}\sum_{j=1}^{M} \mathcal{S}_j$$

5.3. The fourth root model

The "fourth root model" is an empiric model for estimating the distance covered with a number of steps measuring the vertical acceleration [29]. This model includes an experimental parameter ($\mathcal{C}$) that is practically invariant for a certain person, but that varies significantly among different people. $\mathcal{C}$ is related with the proportion between the legs length and the weight. It is calculated experimentally making a person to walk a known distance. The stride length is given by the formula:

$$Step = \mathcal{C} \cdot (a_{max} - a_{min})^{1/4}$$

where:

- $Step$ is the stride length.
- $\mathcal{C}$ is the experimental parameter.
- a_{max} is the maximum of the vertical acceleration.
- a_{min} is the minimum of the vertical acceleration.

The stride length can also be calculated as the product between the walking speed of the person (V) and the period of the corresponding cycle (T):

$$\left.\begin{array}{l} Step = \mathcal{C} \cdot (a_{max} - a_{min})^{1/4} \\ Step = V \cdot T \end{array}\right\} \Rightarrow \frac{C}{V} = \frac{T}{(a_{max}-a_{min})^{1/4}}$$

We considered that V is approximately constant for each person as a function of his/her selection (see section 6). Therefore, we assumed that $\mathcal{C}/V$ was a constant $\mathcal{K}$:

$$\mathcal{K} = \frac{T}{(a_{max}-a_{min})^{1/4}}$$

where:

- T is the mean of the period during various cycles.
- a_{max} is the mean of the maximum vertical accelerations during various cycles.
- a_{min} is the mean of the minimum vertical accelerations during various cycles.

We used this constant $\mathcal{K}$ as a third invariant characteristic of the human gait.

5.4. Authentication

We applied these formulas to obtain a vector of characteristics ($\mathcal{H}$, $\mathcal{S}$, $\mathcal{K}$) for each person in a database.Empirically, we tested that this vector provided sufficient separation among the gaits of different persons whereas the samples of the same person were distributed randomly around a center of gravity. Therefore, we used Gaussian membership functions to represent the distribution of values of these variables on the axes of a three-dimensional domain.

These membership functions were formulated as follows:

$$\mu_{\mathcal{H}} = e^{-\frac{(\mathcal{H}-\overline{\mathcal{H}})^2}{2\cdot\sigma_{\mathcal{H}}^2}}$$
$$\mu_{\mathcal{S}} = e^{-\frac{(\mathcal{S}-\overline{\mathcal{S}})^2}{2\cdot\sigma_{\mathcal{S}}^2}}$$
$$\mu_{\mathcal{K}} = e^{-\frac{(\mathcal{K}-\overline{\mathcal{K}})^2}{2\cdot\sigma_{\mathcal{K}}^2}}$$

where:

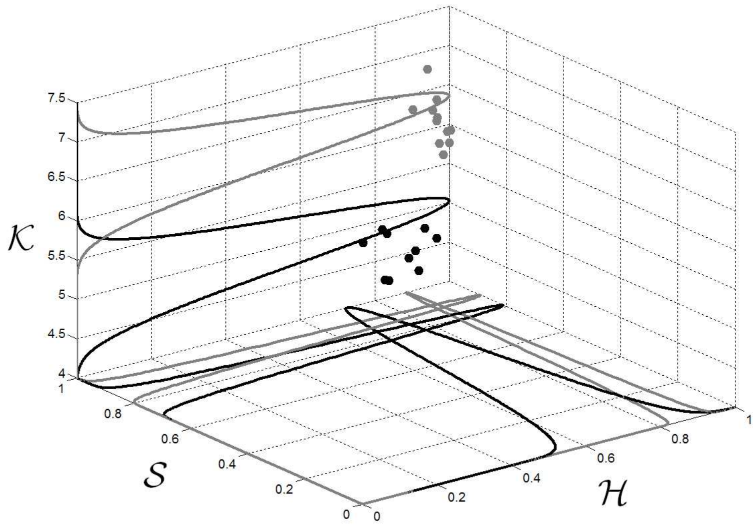

Figure 6: Gaussian membership functions for two different people.

- $\overline{\mathcal{H}}$ is of the mean the Homogeneity values of a person and $\sigma_{\mathcal{H}}$ the standard deviation.
- $\overline{\mathcal{S}}$ is the mean of the Symmetry values of a person and $\sigma_{\mathcal{S}}$ the standard deviation.
- $\overline{\mathcal{K}}$ is the mean of the $\mathcal{K}$ values of a person and $\sigma_{\mathcal{K}}$ the standard deviation.

Fig. 6 shows an example of two clusters with their respective membership functions.

The process of authentication of a person was performed using a sample of his/her gait ($\mathcal{H}^*$, $\mathcal{S}^*$, $\mathcal{K}^*$). We calculated the membership values ($\mu_{\mathcal{H}^*}$, $\mu_{\mathcal{S}^*}$, $\mu_{\mathcal{K}^*}$). And then, the intersection of these conditions was formulated as follows:

$$Score = \min(\mu_{\mathcal{H}^*}, \mu_{\mathcal{S}^*}, \mu_{\mathcal{K}^*})$$

where *Score* represents the degree of membership of this sample to the cluster associated to the authenticated person. If $Score > \lambda$ the human gait is accepted, being λ a threshold that depends on the application.

In summary, Fig. 7 shows a granular network that *explains* the perception that allows the *designer* to identify a person using the human gait.

6. Experimentation

We tested this pattern recognition method by authenticating one person among eleven people. Subjects were instructed to walk at a self-selected, comfortable walking speed. Each subject walked 20 samples of 10 steps each, so we obtained a total of 220 samples (20 genuine and 200 impostors).

Each sample was tested using the leave-one-out cross validation (K-fold cross-validation with K being equal to the number of observations in the original sample) against the remaining training data.

The Equal Error Rate (EER) is obtained from the intersection between the False Acceptance Rate (FAR) and the False Rejection Rate (FRR) versus the threshold λ (see Fig. 8). With our experimental data, we obtained the values: $\lambda = 0.0118$ and EER = 3 %.

In order to show the advantage of our approach, we studied the results obtained by other researchers (See table 1). It is worth to remark that these Equal Error Rates are relatively comparable. These works use different number of subjects, different sensor configuration and different methods to analyze the signal.

We have summarized the differences and similarities of these works as follows:

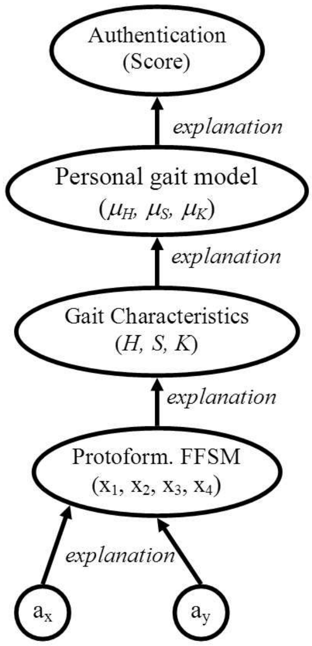

Figure 7: The granular network explaining the n'th-order perception needed for authenticate a gait.

Ailisto et al. [11] authenticate users of portable devices from the accelerations obtained by a three-axis accelerometer placed on the belt, at back. They divide the signal into one step long parts using the maximum and the minimum of the signal. They assume that the right and left steps are not necessarily symmetrical. They used 36 subjects that walked in their normal, fast and slow speed. They perform three different analysis: correlation, frequency domain and they use two variants of data distribution statistics method. They obtaining an EER of 7 %, 10 %, 18 % and 19 % respectively.

Gafurov et al. [12] authenticate 22 subjects walking in their normal speed wearing a three-axial accelerometer in their hip. They normalize each gait cycle in time. They use a cycle length analysis method to obtain an EER of 16 %. In a subsequent paper [13] they authenticate 21 subjects with the accelerometer fixed on the ankle. They use the same on time-normalized cycle length method and histogram similarity analysis to obtain an EER of 9 % and 5 % respectively. Finally, in [14] they authenticate 50 subjects with the accelerometer placed in the trousers pocket. They perform the same preprocessing and use four methods: absolute distance, correlation, histogram and higher order moments. The best result (with the absolute distance) is an EER of 7 %.

Rong et al. [15] use a three-axis accelerometer fixed on the user's waist to obtain the signal of 21 subjects. They perform a preprocessing of the signal that includes wavelet denoising, gait cycles dividing and dynamic time warping. They analyze the signal in time and frequency domains obtaining an EER of 5.6 % and 21.1 % respectively.

Table 1: Summary of accelerometer-based Gait Pattern Recognition Works.

Work	# Subjects	EER %
Ailisto et al. [11]	36	7, 10, 18, 19
Gafurov et al. [12]	22	16
Gafurov et al. [13]	21	5, 9
Gafurov et al. [14]	50	7.3, 9.2, 14, 20
Rong et al. [15]	21	5.6, 21.1
This paper	11	3

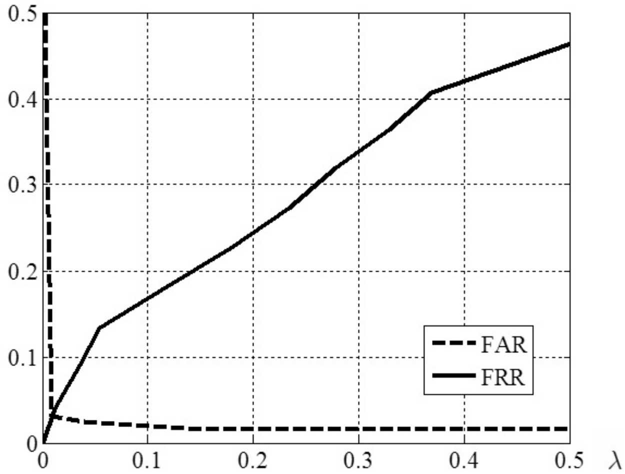

Figure 8: FAR and FRR versus the threshold λ

In our approach, the segmentation or preprocessing of the signal was not needed. The FFSM was able to divide the acceleration signals into cycles and to synchronize with the signal automatically.

The main difference of our contribution is the use of a fuzzy linguistic model for describing the human gait. The flexibility of this paradigm allows the *designer* to focus the model on the relevant characteristics of the signal for an specific purpose

7. Conclusions

In this paper, we have contributed to the field of pattern recognition presenting a new method of signal analysis based on the CTP. We have explored the possibility of using a model of the perceptions of a human observer as a complement to the well established automatic machine learning procedures.

We have focused our effort on modeling the perception of a quasi-periodic signal. Specifically, we have proposed a flexible model of the human gait that allows representing linguistically the relative variations of period and amplitude of this type of signal.

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