Engineering mathematics (2022)

Q1. Choose the correct option of the following:

(i) The value of $\begin{vmatrix} a+b & b+c & c+a \\ c & a & b \\ 1 & 1 & 1 \end{vmatrix}$ is equal to

(a) $a+b+c$ (b) 1

(c) 0 (d) None

Ans. C

(ii) If $\log_2(x+6) = 4$, then the value of x is

(a) 12 (b) 10

(c) 8 (d) None

Ans.

(iii) The number of 4-digit numbers that can be formed using the digit 1, 2, 3, 5, 6 if no digit is used more than once in a number is equal to

(a) 60 (b) 30

(c) 120 (d) None

Ans. Out of Syllabus

(iv) Number of middle terms in the expansion of $\left(x - \frac{2}{x^2}\right)^{11}$ is.

(a) 1 (b) 2

(c) 3 (d) None

Ans.

(v) If the angles of a triangle are in the ratio of 3:4:5, then the greatest angle in radians is

(a) $\frac{\pi}{12}$ (b) $\frac{\pi}{3}$

(c) $\frac{5\pi}{12}$ (d) None

Ans.

(vi) The principal value of $\text{Sec}^{1}(2)$ is equal to

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$

(c) π (d) None

Ans.

(vii) Two vertices of a triangle are $(-5, 4)$ and $(3, 7)$, if the centroid is $(1, 2)$. then the third ..... is

(a) $(5, 17)$ (b) $(-5, 17)$

(c) $(5, -17)$ (d) None

tually perpendicular, then the value of λ is

(a) 1 (b) 2

(c) 3 (d) None

Ans. Out of Syllabus

Q2. (a) Resolve into Partial fraction: $\frac{5x+1}{x^2+x-2}$

Ans. $\frac{5x+1}{x^2+x-2} = \frac{5x+1}{(x+2)(x-1)}$

$$\frac{5x+1}{(x+2)(x-1)} = \frac{A}{(x+2)} + \frac{B}{(x-1)}$$

$$5x+1 = A(x-1) + B(x+2)$$

Put $x = 1$ in equation (i)

$$6 = B(3)$$

$$B = 2$$

Put $x = -2$ in equation (ii)

$$-10 + 1 = A(-3)$$

$$A = 3$$

Then,

$$\frac{5x+1}{(x+2)(x-1)} = \frac{3}{(x+2)} + \frac{2}{(x-1)}$$

Q2.(b) If $\frac{\log x}{q-r} = \frac{\log y}{r-p} = \frac{\log z}{p-q}$, then prove that

$$x^{q+r} y^{r+p} z^{p+q} = x^p y^q z^r$$

Ans. Given,

$$\frac{\log x}{q-r} = \frac{\log y}{r-p} = \frac{\log z}{p-q} = k \text{ (lets say).}$$

$$\log x = k(q-r),\ \log y = k(r-p)$$

and $\log z = k(p-q)$

$$\text{LHS} \Rightarrow x^{q+r} \cdot y^{r+p} \cdot z^{p+q} = A$$

$$\Rightarrow \log x^{q+r} + \log y^{r+p} + \log z^{p+q} = \log A$$

$$\Rightarrow (q+r)k(q-r) + (r+p)k(r-p) + (p+q)k(p-q) = \log A$$

$$\Rightarrow k\left[q^2 - r^2 + r^2 - p^2 + p^2 - q^2\right] = \log A$$

$$\Rightarrow 0 = \log A \Rightarrow A = e^0 = 1$$

$$\text{RHS} \Rightarrow x^p \cdot y^q \cdot z^r = B$$

$$\Rightarrow \log x^p + \log y^q + \log z^r = \log B$$

$$\Rightarrow pk(q-r) + qk(r-p) + rk(p-q) = \log B$$

$$\Rightarrow \quad 0 = \log B \Rightarrow B = e^0 = 1$$

$$\therefore \quad LHS = RHS = A = B = 1$$

$$\therefore \quad x^{q+r} \cdot y^{r+p} \cdot z^{p+q} = x^r \cdot y^q \cdot z^r$$

Hence proved.

Q3.(a) Find the term independent of x in the expansion of $\left(x^2 - \frac{2}{x^3}\right)^5$ and find its value.

Ans. We know that the $(r+1)^{th}$ term in binomial expansion of $(x+y)^n$ is given by

$$T_{r+1} = {}^nC_r x^{n-r} y^r$$

The $(r+1)^{th}$ term in binomial expansion of given expression is

$$T_{r+1} = {}^5C_r (x^2)^{5-r} \left(\frac{-2}{x^3}\right)^r$$

$$= {}^5C_r (-2)^r x^{10-2r-3r}$$

$$= {}^5C_r (-2)^r x^{10-5r}$$

For the term independent of x, we have

$$10 - 5r = 0$$

$$r = 2$$

Hence, the term independent of x in binomial expansion of given expression is

$$T_3 = {}^5C_2 (-2)^2$$

$$= \frac{120}{2 \times 6} \times 4$$

$$= 40$$

Q3.(b) Prove that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

Ans. $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

Solving deteminant, we have

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

Applying $c_2 \to c_2 \to -c_1$ and $c_3 \to c_3 \to -c_1$

$$= \begin{vmatrix} 1 & & \\ a & b-a & c-a \\ a^2 & b^2-a^2 & c^2-a^2 \end{vmatrix}$$

Taking (c − a) and (b − a) as common from c_3 and c
respectively.

$$= (b-a)(c-a)\begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^2 & b+a & c-a \end{vmatrix}$$

Expanding the determinant along R_1, we hav

$$= (b-a)(c-a)[1(c+a-b-a) - 0 + 0]$$

$$= (b-a)(c-a)(c-b)$$

$$= (a-b)(b-c)(c-a) = RHS$$

Hence, Proved.

Q4. (a) If $A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, find A^2-9A+14I.

Ans. We have,

$$A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$$

Then,

$$A^2 - 9A + 141$$

$$= \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} - 9\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} + 14\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 16+6 & 12+15 \\ 8+10 & 6+25 \end{bmatrix} - 9\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & 27 \\ 18 & 31 \end{bmatrix} - \begin{bmatrix} 36 & 27 \\ 18 & 45 \end{bmatrix} + \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 22-36+14 & 27-27+0 \\ 18-18+0 & 31-45+14 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Q4.(b) Find the inverse of the matrix $A = \begin{bmatrix} 3 & 5 & 7 \\ 2 & -3 & 1 \\ 1 & 1 & 2 \end{bmatrix}$.

Ans. *Let* $A = \begin{bmatrix} 3 & 5 & 7 \\ 2 & -3 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

$$|A| = 3(-7) - 5(3) + 7(5)$$
$$= -21 - 15 + 35$$
$$-36 + 35 = -1$$

$|A| \neq 0$, *thus A is non-singular and* A^{-1} *exists.*

Then,

Cofactor :

$$A_{11} = (-1)^{1+1}\begin{vmatrix} -3 & 1 \\ 1 & 2 \end{vmatrix} = (-1)(-6-1) = -7$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = (-1)^3(4-1) = -3$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} = 1(2+3) = 5$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} 5 & 7 \\ 1 & 2 \end{vmatrix} = (-1)^3(10-7) = -3$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 3 & 7 \\ 1 & 2 \end{vmatrix} = (1)(6-7) = -1$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 3 & 5 \\ 1 & 1 \end{vmatrix} = (-1)(3-5) = 2$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} 5 & 7 \\ -3 & 1 \end{vmatrix} = (1)(5+21) = 26$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 3 & 7 \\ 2 & 1 \end{vmatrix} = (-1)(3-14) = 11$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 3 & 5 \\ 2 & -3 \end{vmatrix} = (1)(-9-10) = -19$$

$$\text{adj}A = C^T = \begin{bmatrix} -7 & -3 & 5 \\ -3 & -1 & 2 \\ 26 & 11 & -19 \end{bmatrix}^T$$

$$= \begin{bmatrix} -7 & -3 & 26 \\ -3 & -1 & 11 \\ 5 & 2 & -19 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det A}\text{adj}A = \frac{1}{-1}\begin{bmatrix} -7 & -3 & 26 \\ -3 & -1 & 11 \\ 5 & 2 & -19 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 7 & 3 & -26 \\ 3 & 1 & -11 \\ -5 & -2 & 19 \end{bmatrix}$$

Q5.(a) Find the value of Cos 220° + Cos 100° + Cos20°

Ans. $= \cos 220^\circ + \cos 100^\circ + \cos 20^\circ$

$$= 2\cos 160^\circ \cdot \cos 60^\circ + \cos 20^\circ$$

$$\left(\begin{aligned} &\ldots \cos C + \cos D \\ &= 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right) \end{aligned}\right)$$

$$= 2 \times \frac{1}{2}\cos 160^\circ + \cos 20^\circ$$
$$= \cos 160^\circ + \cos 20^\circ$$
$$= 2\cos$$
$$= \cos 220^\circ + \cos 100^\circ + \cos 20^\circ$$
$$= 2\cos\left(\frac{220+100}{2}\right)\cos\left(\frac{220-100}{2}\right) + \cos 20^\circ$$
$$= 2\cos 160^\circ \cos 60^\circ + \cos 20^\circ$$
$$= 2 \times \frac{1}{2} \times \cos 160^\circ + \cos 20^\circ$$
$$= \cos 160^\circ + \cos 20^\circ$$
$$= 2\cos\left(\frac{160+20}{2}\right)\cos\left(\frac{160+20}{2}\right)$$
$$= 2\cos 90^\circ \cos 70^\circ$$
$$= 0 \qquad (\ldots \cos 90^\circ = 0)$$

Q5.(b) Prove that $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$

Ans. *Let* $\theta = 18^\circ$. *Then,*

$$5\theta = 90^\circ$$
$$\Rightarrow 2\theta + 3\theta = 90^\circ$$
$$\Rightarrow 2\theta = 90^\circ - 3\theta$$
$$\Rightarrow \sin 2\theta = \sin(90^\circ - 3\theta)$$
$$\Rightarrow \sin 2\theta = \cos 3\theta$$
$$\Rightarrow 2\sin\theta\cos\theta = 4\cos^3\theta - 3\cos\theta$$
$$\Rightarrow \cos\theta(2\sin\theta - 4\cos^2\theta + 3) = 0$$
$$\Rightarrow 2\sin\theta - 4\cos^2\theta + 3 = 0 \qquad [\because \cos\theta = \cos 18^\circ \neq 0]$$

$$\Rightarrow \quad 2\sin\theta - 4\cos^2\theta + 3 = 0$$

$$\Rightarrow \quad 4\sin^2\theta + 2\sin\theta - 1 = 0$$

$$\Rightarrow \quad \sin\theta = \frac{-2 \pm \sqrt{4+16}}{8}$$

$$\Rightarrow \quad \sin\theta = \frac{-1 \pm \sqrt{5}}{4}$$

$$\Rightarrow \quad \sin\theta = \frac{-1+\sqrt{5}}{4} = \frac{\sqrt{5}-1}{4}$$

[$\because \theta$ lies in Ist quadrant. $\therefore \sin\theta > 0$]

Hence, $\sin 18^\circ = \dfrac{\sqrt{5}-1}{4}$

Q6. (a) Find the value of x, If $\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$.

Ans. $\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$.

$$\Rightarrow \tan^{-1}\left(\frac{2x+3x}{1-2x\times 3x}\right) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{5x}{1-6x^2}\right) = \frac{\pi}{4}$$

$$\Rightarrow \frac{5x}{1-6x^2} = \tan\frac{\pi}{4}$$

$$\Rightarrow \frac{5x}{1-6x^2} = 1$$

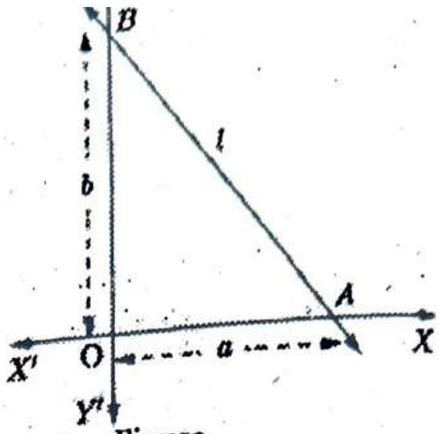
$$6x^2 + 5x - 1 = 0$$

$$x = -1, \frac{1}{6}$$

for positive value x = 1/6

Q6.(b) Prove that $\frac{x}{a} + \frac{y}{b} = 1$ where a and b are intercepts of the from x-axis and y-axis respectively.

Ans. Let the straight line, say l, cut off intercepts a and b on the x-axis and y-axis respectively and let it meet the axes in points A, B (shown in figure 15.19), then the coordinates of points A and B are (a, 0) and (0, b).



Figure

Using two-point form, the equation of the line l is

$$y - 0 = \frac{b-0}{0-a}(x-a)$$

$$y = -\frac{b}{a}(x-a)$$

$$bx + ay - ab = 0$$

$$\frac{x}{a} + \frac{y}{b} = 1$$

which is the required equation of the line cutting off in tercepts a and b on the axes. This is known as intercep form.

Q7.(a) Find the distance between two parallel straig lines 4x 3y 9 = 0 and 4x 3y 24 = 0

Ans. We have,

$$4x - 3y - 9 = 0 \qquad \ldots (i)$$

and $4x - 3y - 24 = 0 \qquad \ldots$ (ii)

Putting x = 0 in equation (i), we have

$$3y + 9 = 0$$

$$y = -\tfrac{9}{3} = -3$$

Thus (0, −3) is a point on (i)

The distance between (i) and (ii)

d = ⊥ distance from (0, −3) on (ii)

$$= \frac{|4\times 0 - 3\times -3 - 24|}{\sqrt{16+9}}$$

$$= \frac{15}{5} = 3$$

Q7.(b) Find the ratio in which the line joining (4, 1) and (2, 3) divides the line joining (1, 2) and (4, 3)

Ans. Let $P(2,3), Q(4,1), R(1,2), S(4,3)$ be the points.

Equation of PQ