# Engineering mathematics (2013)

*Q1. Choose the correct options.*

**(i) In a $\Delta ABC$ $\cos B$ is equal to**

(a) $\frac{a^2+c^2-b^2}{2ac}$ (b) $\frac{b^2+c^2-a^2}{2bc}$

(c) $\frac{a^2+b^2-c^2}{2ab}$ (d) None of these

**Ans.** (a)

**(ii)** $\left[\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{2}{3}\right]$ **is equal to**

(a) $\frac{6}{17}$ (b) $\frac{5}{17}$ (c) $\frac{17}{6}$ (d) $\frac{17}{5}$

**Ans.** (c)

**Ex :** Given $\left[\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{2}{3}\right]$

We know that $\cos^{-1}x=\tan^{-1}\frac{\sqrt{1-x^2}}{x}$

$$\therefore \cos^{-1}\frac{4}{5}=\tan^{-1}\frac{\sqrt{1-\left(\frac{4}{5}\right)^2}}{\frac{4}{5}}=\tan^{-1}\frac{3/5}{4/5}=\tan^{-1}\frac{3}{4}$$

$$\therefore \tan\left[\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{2}{3}\right]=\tan\left[\tan^{-1}\frac{3/4+2/3}{1-\frac{3}{4}\times\frac{2}{3}}\right]$$

$$=\tan\left[\tan^{-1}\frac{(9+8)/12}{6/12}\right]=\tan[\tan^{-1}17/6]=\frac{17}{6}$$

***(iii) The centroid of the $\Delta$ whose vertices are (4, –3), (–9, 7) and (8, 8) is***

(a) (1, 4) (b) (2, 3) (c) (3, 2) (d) (6, 1)

**Sol.** (a)

**Ex :** Centroid of a $\Delta=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$

$$=\left(\frac{4-9+8}{3},\frac{-3+7+8}{3}\right)=(1,4)$$

***(iv) The radius of the circle $x^2+y^2-4x-6y-12=0$ is***

(a) 1 (b) 2 (c) 3 (d) 5

**Ans.** (d) Given equn. of circle

$$x^2+y^2-4x-6y-12=0$$

$g=-2;\ f=-3;\ c=-12$

$$\text{radius}=\sqrt{g^2+f^2-c}=\sqrt{4+9+12}=\sqrt{25}=5$$

***(v) If $\vec{a}=3\hat{i}+2\hat{j}+9\hat{k}$ and $\vec{b}=\hat{i}+x\hat{j}+3\hat{k}$ are parallel then the value of x is equal to***

**Ex :** Given $\vec{a}=3\hat{i}+2\hat{j}+9\hat{k}$

$$\vec{b}=\hat{i}+x\hat{j}+3\hat{k}$$

Condition of two vectors being parallel

$$\frac{a_1}{b_1}=\frac{a_2}{b_2}=\frac{a_3}{b_3};\quad \frac{3}{1}=\frac{2}{x}=\frac{9}{3}$$

from the 1st two ratios, we get

$$\frac{3}{1}=\frac{2}{x};\ x=\frac{2}{3}$$

***(vi) $\log_e x+\log_e y+\log_e z$ is equal to***

(a) $\log_e(x+y+z)$ (b) $\log_e(xyz)$

(c) $xyz$ (d) None

**Ans.** (b)

***(vii) The value of*** $\begin{vmatrix}2 & 3 & 4\\ 2x & 3x & 4x\\ 1 & 2 & 3\end{vmatrix}$ ***is equal to***

(a) $x$ (b) $9x$ (c) 0 (d) 1

**Ans.** (c)

**Ex :** $R_2\to\frac{1}{x}R_2$

$$x\begin{vmatrix}2 & 3 & 4\\ 2 & 3 & 4\\ 1 & 2 & 3\end{vmatrix}=0\quad [\because R_1 \text{ and } R_2 \text{ are identical}]$$

***(viii) No. of terms in the expansion of $x^4(1+3x^4)^{15}$ is***

(a) 21 (b) 15 (c) 16 (d) 19

**Ans.** (c)

***(ix) If ${}^{2n}C_3:{}^{n}C_3=11:1$, then the value of n is***

(a) 3 (b) 4 (c) 5 (d) 6

**Ans.** (d)

**Ex :** ${}^{2n}C_3:{}^{n}C_3=11:1$

$$\frac{\lfloor 2n}{\lfloor 3\times\lfloor 2n-3}\times\frac{\lfloor 3\times\lfloor n-3}{\lfloor n}=\frac{11}{1}$$

$$\frac{2n.(2n-1)(2n-2)\lfloor 2n-3}{\lfloor 3\times\lfloor 2n-3}\times\frac{\lfloor 3\times\lfloor n-3}{n.(n-1)(n-2)\lfloor n-3}=\frac{11}{1}$$

**(x) *The degree measure of*** $\left(\frac{7\pi}{12}\right)^c$ ***is***

(a) $105°$ (b) $106°$ (c) $110°$ (d) $102°$

**Ans.**(a)

**Ex.:** $1^c = \frac{180}{\pi} \therefore \left(\frac{7\pi}{12}\right) \times \frac{180}{\pi} = 7\times 15 = 105°$

**[GROUP - B]**

***Q2.(a) Prove that :***

$$\cos 24° + \cos 55° + \cos 125° + \cos 204° + \cos 300° = \frac{1}{2}$$

**Ans.**L.H.S.

$$= \cos 24° + \cos 55° + \cos 125° + \cos 204° + \cos 300°$$
$$= \cos 24° + \cos 204° + \cos 55° + \cos 125° + \cos 300°$$
$$= \cos 24° + \cos(180° + 24°) + \cos 55° + \cos(180° - 55°) + \cos(360° - 60°)$$
$$= \cos 24° - \cos 24° + \cos 55° - \cos 55° + \cos 60°$$
$$= \frac{1}{2} \text{ Proved.}$$

***Q2.(b)*** $\frac{\sin(A-B)}{\sin A.\sin B} + \frac{\sin(B-C)}{\sin B.\sin C} + \frac{\sin(C-A)}{\sin C.\sin A} = 0$

**Ans.**L.H.S. $= \frac{\sin(A-B)}{\sin A.\sin B} + \frac{\sin(B-C)}{\sin B.\sin C} + \frac{\sin(C-A)}{\sin C.\sin A}$

1st term $= \frac{\sin(A-B)}{\sin A.\sin B}$

$$= \frac{\sin A.\cos B - \cos A.\sin B}{\sin A.\sin B} = \cot B - \cot A \quad ...(1)$$

$$= \frac{\sin A.\cos B}{\sin A.\sin B} = \cot B$$

Similarly, $\frac{\sin(B-C)}{\sin B.\sin C}$

$$= \frac{\sin B.\cos C - \cos B.\sin C}{\sin B.\sin C} = \cot C - \cot B \quad ...(2)$$

and $\frac{\sin(C-A)}{\sin C.\sin A}$

$$= \frac{\sin C.\cos A - \cos C.\sin A}{\sin C.\sin A} = \cot A - \cot C \quad ...(3)$$

$$\therefore \frac{\sin(A-B)}{\sin A.\sin B} + \frac{\sin(B-C)}{\sin B.\sin C} + \frac{\sin(C-A)}{\sin C.\sin A}$$

$= \cot B - \cot A + \cot C - \cot B + \cot A - \cot C$

***Q3.(a) If 2 tan α = 3 tan β, show that :***

$$\tan(\alpha - \beta) = \frac{\sin 2\beta}{5 - \cos 2\beta}$$

**Ans.**Given $2\tan\alpha = 3\tan\beta$.

$$\text{R.H.S} = \frac{\sin 2\beta}{5 - \cos 2\beta} = \frac{2\tan\beta / 1 - \tan^2\beta}{5 - \frac{1-\tan^2\beta}{1+\tan^2\beta}}$$

$$= \frac{2.\tan\beta}{5 + 5\tan^2\beta - 1 + \tan^2\beta} = \frac{2\tan\beta}{4 + 6\tan^2\beta}$$

$$= \frac{\tan\beta}{2 + 3\tan^2\beta} = \frac{2 + 3\tan\beta}{2 + 3\tan^2\beta \cdot \tan\beta}$$

$$= \frac{\tan\beta}{2 + 2\cdot\tan\alpha\cdot\tan\beta}$$

$$= \frac{\tan\beta}{2} \times \frac{\tan\beta}{1 + \tan\alpha\cdot\tan\beta} \times \frac{\tan\alpha - \tan\beta}{\tan\alpha - \tan\beta}$$

$$= \frac{\tan\beta}{2} \times (\alpha - \beta) \times \frac{1}{\frac{3}{2}\tan\beta - \tan\beta}$$

$$= \frac{\frac{\tan\beta}{2} \times \tan(\alpha - \beta)}{\frac{\tan\beta}{2}} = \tan(\alpha - \beta)$$

$$\text{i.e., } \tan(\alpha - \beta) = \frac{\sin 2\beta}{5 - \cos 2\beta} \text{ Proved.}$$

***Q3.(b)*** $\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{8}{17} = \sin^{-1}\frac{77}{85}$

**Ans.** We know that,

$$\sin^{-1}x + \sin^{-1}y = \sin^{-1}\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\}$$

$$\therefore \sin^{-1}\frac{3}{5} + \sin^{-1}\frac{8}{17} = \sin^{-1}\left\{\frac{3}{5}\sqrt{1 - \frac{64}{289}} + \frac{8}{17}\sqrt{1 - \frac{9}{25}}\right\}$$

$$= \sin^{-1}\left\{\frac{3}{5}\frac{\sqrt{289-64}}{17} + \frac{8}{17}\frac{\sqrt{25-9}}{5}\right\}$$

$$= \sin^{-1}\left\{\frac{3}{5}\cdot\frac{15}{17} + \frac{8}{17}\times\frac{4}{5}\right\}$$

$$= \sin^{-1}\frac{77}{85} \text{ Proved.}$$

***Q4.(a) Show that the point (1, 5), (2, 4) and (3, 3) are collinear.***

**Ans. 1st Method :**

$$AB=\sqrt{(2-1)^2+(4-5)^2}=\sqrt{2}$$

$$BC=\sqrt{(3-2)^2+(3-4)^2}=\sqrt{2}$$

$$AC=\sqrt{(3-1)^2+(3-5)^2}=\sqrt{2}$$

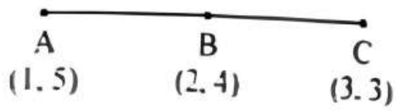
$\therefore$ $AB+BC=AC$

Hence, points are collinear.

**2nd Method :**

Area of of $\Delta = 0$

$$\frac{1}{2}\begin{vmatrix}1 & 2 & 3\\ 5 & 4 & 3\\ 1 & 1 & 1\end{vmatrix}=0$$




On expansion

$\therefore$ points are collinear.

**3rd Method :**

Collinear = slopes of AB
= slope of BC

Slope of AB = Slope of BC.

***Q4.(b) Find the equation of a line passing through the point (1, 1) and perpendicular to the line 3x + 4y = 12.***

**Ans.** Equation of required line passing through (1, 1) and perpendicular to given line $3x+4y=12$

$$4x-3y+K=0$$

Passing through (1, 1)

$$4\times1-3\times1+K=0$$

$$4-3+K=0\quad K=-1$$

$\therefore$ required equation.

$$4x-3y-1=0$$

***Q5.(a) Find the length of the perpendicular drawn from the point (–1, 4) to the line 3x + 4y – 5 = 0.***

**Ans.** Length of perpendicular from (–1, 4) to the line $3x+4y-5=0$

$$p=\pm\frac{3(-1)+4\times4-5}{\sqrt{9+16}}=\pm\frac{-3+16-5}{5}=\pm\frac{8}{5}$$

***Q5.(b) What is the equation of a circle which has the point (5, 7) and (2, 17) as its diameter?***

**Ans.** Equation of circle when extremetities of diameter are given

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

$$(x-5)(x-2)+(y-7)(y-17)=0$$

$$x^2-7x+10+y^2-24y+119=0$$

$$x^2+y^2-7x-24y+129=0$$

***Q6.(a) Find the angle between the vectors $\vec{a}$ and $\vec{b}$ when $\vec{a}=3\hat{i}+2\hat{j}-2\hat{k}$ and $\vec{b}=2\hat{i}-2\hat{j}+4\hat{k}$.***

**Ans.** Given : $\vec{a}=3\hat{i}+2\hat{j}-2\hat{k}$ and $\vec{b}=2\hat{i}-2\hat{j}+4\hat{k}$

Let 'θ' be the angle between vectors $\vec{a}$ and $\vec{b}$ then.

$$\cos\theta=\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}=\frac{(3\hat{i}+2\hat{j}-2\hat{k})\cdot(2\hat{i}-2\hat{j}+4\hat{k})}{\sqrt{3^2+2^2+(-2)^2}\,\sqrt{2^2+(-2)^2+4^2}}$$

$$=\frac{6-4-8}{\sqrt{17}-\sqrt{24}}=\frac{-6}{2\sqrt{17\times6}}=\frac{-3}{\sqrt{102}}$$

$$\theta=\cos^{-1}\left(-\frac{3}{\sqrt{102}}\right)\text{ Ans.}$$

***Q6.(b) Show that the moment of force $\vec{F}=\hat{i}-\hat{j}+\hat{k}$ acting on a point $-A=(4\hat{i}+3\hat{j}-2\hat{k})$ about the point $P=(\hat{i}+\hat{k})$ is $-6\hat{j}-6\hat{k}$.***

**Ans.** Given : $\vec{F}=\hat{i}-\hat{j}+\hat{k}$ ; $\overline{PA}=(3\hat{i}+3\hat{j}-3\hat{k})$

$\therefore$ moment of $\vec{F}$ about P

$$=\overline{PA}\times\overline{F}$$

$$=(3\hat{i}+3\hat{j}-3\hat{k})\times(\hat{i}-\hat{j}+\hat{k})$$

$$=-3\hat{k}+3\hat{j}-3\hat{k}+3\hat{i}-3\hat{j}-3\hat{i}$$

$$=-6\hat{j}-6\hat{k}$$

***Q7.(a) Prove that***

$$x^{\log y-\log z}\;xy^{\log z-\log x}\;xz^{\log x-\log y}=1$$

**Ans.** $\log y-\log z\,\log z-\log x\,\log x-\log y$

Let $P=x\,xy\;xz$

Taking log on both sides we get.

$$\left\{x^{\log y-\log z}\;xy^{\log z-\log x}\;xz^{\log x-\log y}\right\}$$

$$=\log y-\log z\,\log z-\log x\,\log x-\log y$$

$$\log x\quad+\log y\quad+\log z$$

$$=(\log y-\log z)\log x+(\log z-\log x)\log y+(\log x-\log y)\log z$$

$$=\log x\log y-\log x\log z+\log y.\log z-\log y.\log x+\log z.\log x-\log z.\log y=0$$

$\log P=0\qquad\therefore P=1$

$\log z-\log x\qquad \log x-\log y$

*Q7.(b) Solve $2x - y = 17$ and $3x + 5y = 6$ for x and y by cramer's Rule.*

Ans. Given: $2x - y = 17$ .....(1)

$3x + 5y = 6$ .....(2)

$$D = \begin{vmatrix} 2 & -1 \\ 3 & 5 \end{vmatrix} = 10 + 3 = 13 \neq 0$$

$$D = \begin{vmatrix} 17 & -1 \\ 6 & 5 \end{vmatrix} = 85 + 6 = 91$$

$$D_2 = \begin{vmatrix} 2 & 17 \\ 3 & 6 \end{vmatrix} = (12 - 51) = -39$$

$$\therefore x = \frac{D_1}{D} = \frac{91}{13} = 7; \quad y = \frac{D_2}{D} = \frac{-39}{13} = -3$$

$\therefore x = 7 : y = -3$

*Q8. Resolve into partial fractions (any one):*

(a) $\frac{4x+23}{(x+2)(x-3)(x+4)}$ (b) $\frac{x^2+1}{(x+2)^2(x-3)}$

(c) $\frac{3x+9}{(x^2+1)(x-2)}$

Ans.(a) $\frac{4x+23}{(x+2)(x-3)(x+4)}$

Degree of num. < degree of den

$\therefore$ The given fraction is in its lowert term

$\therefore$ Reducing the fraction into potential fraction.

$$\frac{4x+23}{(x+2)(x-3)(x+4)} = \frac{A}{x+2} + \frac{B}{x-3} + \frac{C}{x+4}$$

$4x + 23 = A(x-3)(x+4) + B(x+2)(x+4) + C \quad (x+2)(x+3)$

Equating the co-efficient of $x^2$, $x$ and constant term from both sides we get,

$A + B + C = 0$ .....(1)

$A + 6B - C = 4$ .....(2)

$-12A + 8B - 6C = 23$ .....(3)

By (1) + (2)

$2A + 7B = 4$ .....(4)

By (1) + (3)

$-6A + 14B = 23$ .....(5)

Solving (4) and (5)

$2A + 7B = 4$ .....(4) × 2

$-6A + 14B = 23$ .....(5) × 1

$$\begin{array}{r} 4A + 14B = 8 \\ \underset{+}{-6A} \underset{-}{+14B} = \underset{-}{23} \\ \hline 10A \quad = -15 \end{array}$$

$$A = -\frac{15}{10} = -\frac{3}{2}$$

Substituting $A = -\frac{3}{2}$ in equn. (4)

$$-3 + 7B = 4; \; B = 1$$

Substituting the values of A and B in eqn. (1); we get,

$A + B + C = 0$

$$-\frac{3}{2} + 1 + C; \; C = \frac{3}{2} - 1 = \frac{1}{2}$$

$\therefore$ Required partial fraction

$$\frac{4x+23}{(x+2)(x-3)(x+4)} = \frac{-3}{2} \cdot \frac{1}{x+2} + \frac{1}{x-3} + \frac{1}{2} \cdot \frac{1}{x+4}$$

Ans. (b) $\frac{x^2+1}{(x+2)^2(x-3)}$

Degree of num. < degree of den.

$\therefore$ The given fraction is in its lowert term

$\therefore$ Reducing the fraction into potential fraction

$$\frac{x^2+1}{(x+2)^2(x-3)} = \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{x-3}$$

$x^2 + 1 = A(x+2)(x-3) + B(x-3) + C(x+2)^2$

Equating the co-efficient of $x^2$, $x$ and constant term from sides we get,

$A + C = 1$ .....(1)

$-A + B - 4C = 0$ .....(2)

$-6A - 3B + 4C = 1$ .....(3)

By (1) + (2)

$B + 5C = 1$ .....(4)

By (2) × 6 − (3)

$$\begin{array}{r} -6A + 6B + 24C = 0 \\ \underset{+}{-6A} \underset{+}{-3B} \underset{-}{+4C} = 1 \\ \hline 7B + 20C = -1 \end{array}$$ .....(5)

Solving (4) and (5); we get;

$B + 5C = 1$ .....(4) × 7

$7B + 20C = -1$ .....(5)

$$\begin{array}{r} 7B + 35C = 7 \\ \underset{-}{7B} \underset{-}{+20C} = \underset{+}{-1} \\ \hline +15C = 8 \end{array}$$

$\therefore$ $C = 8/15$

Substituting $C = 8/15$ in eqn (4); we get,

$$B + 5 \times \frac{8}{15} = 1 \; B = 1 - 8/3 = -\frac{5}{3}$$

Substituting the value of C in equn. (1)

$$A + \frac{8}{15} = 1; \; A = \frac{8}{15} - 1 = \frac{-7}{15}$$

$\therefore$ Required partial fraction

$$\frac{x^2+1}{(x+2)^2(x-3)} = -\frac{7}{15} \cdot \frac{1}{(x+2)} - \frac{5}{3} \cdot \frac{1}{(x+2)^2} + \frac{8}{15} \cdot \frac{1}{(x-3)}$$

Ans.(c) $\frac{3x+9}{(x^2+1)(x-2)}$

$\overline{(x^2+1)(x-3)}$

$3x+9=(Ax+B)(x-3)+C(x^2+1)$

Equating the co-efficient of $x^2$, $x$ and constant term from both sides we get.

$A+C=0$ ......(1)

$-2A+B+2C=3$ ......(2)

$-2B+C=9$ ......(3)

By (1) + (2)

$$\begin{array}{r} 2A+2C=0 \\ -2A+3B+2C=3 \\ \hline 4A-B=3 \end{array} \quad ......(4)$$

By (1) − (3)

$A+2B=-9$ ......(5)

Solving (4) and (5); we get.

By (4) × 2 + (5)

$$\begin{array}{r} 8A-2B=6 \\ A+2B=9 \\ \hline 9A\ \ =15 \end{array}; \qquad A=\frac{15}{9}=\frac{5}{3}$$

Substituting the value of A in eqn. (4); we get

$4A-B=3$

$$4\times\frac{5}{3}-B=3$$

$$B=\frac{20}{3}-3=\frac{11}{3}$$

Substituting the value of A in eqn. (1)

$$C=-A=-\frac{6}{3}$$

∴ Required partial fraction

$$\frac{3x+9}{(x^2+1)(x-3)}=\frac{Ax+B}{x^2+1}+\frac{C}{x-3}$$

$$=\frac{\frac{5}{3}x+\frac{11}{3}}{(x^2+1)}+\frac{-5/3}{x-3}=\frac{1}{3}\left[\frac{5x+11}{x^2+1}-\frac{5}{x-3}\right]$$

**Q9. (a)** ***Evaluate $(1.01)^5$ correct to 4 places of decimal using Binomial approximation.***

**Ans.(a)** $(1.01)^5=(1+0.01)^5$

$$=1+5.1^4(0.01)+\frac{5.4}{2}(.01)^2+\frac{5.4.3}{6}.(.01)^3$$

$$+\frac{5.4.3.2}{24}.(.01)^4+\frac{5.4.3.2.1}{120}\times(.01)^5$$

$$=1+5\times(0.01)+10.(0.01)^2+10.(0.01)^3$$
$$+5.(0.01)^4+10.(0.01)^5$$

$=1.0501$ Ans.

**Q9. (b)** ***Find the term independent of $x$ in the expansion of***

$$\left(3x^2+\frac{2}{x^2}\right)$$

**Ans.(b)** Let $(r+1)^{th}$ term be independent of x i.e., exponent of x is zero.

$$T_{r+1}={}^{20}C_r\cdot(3x^2)^{20-r}\left(\frac{2}{x^2}\right)^r$$

$$={}^{20}C_r\cdot3^{20-r}\cdot x^{40-2r}\cdot\frac{2^r}{x^{2r}}$$

$$={}^{20}C_r\cdot3^{20-r}\cdot2^r\cdot x^{40-2r}$$

∴ $40-4r=0$

$10-r=0,\quad 10=r,\qquad r=10$

∴ $T_{10+1}={}^{20}C_{10}\cdot3^{10}\cdot2^{10}$ Ans.

**Q10.(a)** ***Prove that*** $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}=4abc.$

**Ans.(a)** $R_1\to R_1-R_2-R_3$

$$\Delta=\begin{vmatrix} 0 & -2c & -2b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

$R_2\to cR_2-bR_3$

$$\Delta=\frac{1}{c}\begin{vmatrix} 0 & -2c & -2b \\ 0 & c(c+a-b) & b(c-a-b) \\ c & c & a+b \end{vmatrix}$$

$$=\frac{1}{c}c(-2bc)[c-a-b-(c+a-b)]$$

$$=(-2bc)(-2a)=4abc$$

***Q10.(b) Show that*** $A^3-4A^2+A=0$ ***when*** $A=\begin{bmatrix}2 & 3\\1 & 2\end{bmatrix}$

**Ans.(b)** $A=\begin{bmatrix}2 & 3\\1 & 2\end{bmatrix}$

$$A^2=\begin{bmatrix}2 & 3\\1 & 2\end{bmatrix}\begin{bmatrix}2 & 3\\1 & 2\end{bmatrix}=\begin{bmatrix}4+3 & 6+6\\2+2 & 3+4\end{bmatrix}=\begin{bmatrix}7 & 12\\4 & 7\end{bmatrix}$$

$$A^2\cdot A=\begin{bmatrix}7 & 12\\4 & 7\end{bmatrix}\begin{bmatrix}2 & 3\\1 & 2\end{bmatrix}=\begin{bmatrix}14+12 & 21+24\\8+7 & 12+14\end{bmatrix}$$

$$=\begin{bmatrix}26 & 45\\15 & 26\end{bmatrix}$$

$$A^3-4A^2+A=\begin{bmatrix}26 & 45\\15 & 26\end{bmatrix}-4\begin{bmatrix}7 & 12\\4 & 7\end{bmatrix}+\begin{bmatrix}2 & 3\\1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}26 & 45\\15 & 26\end{bmatrix}-\begin{bmatrix}28 & 48\\16 & 28\end{bmatrix}+\begin{bmatrix}2 & 3\\1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}28 & 48\\16 & 28\end{bmatrix}-\begin{bmatrix}28 & 48\\16 & 28\end{bmatrix}+\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}=0$$

i.e., $A^3 - 4A^2 + A = 0$

**Q11.** ***Solve the following system of equations by matrix method.***

$$x + 2y - 3z = -4$$

$$2x + 3y + 2z = 2$$

$$3x - 3y - 4z = 11$$

**Ans.** The given equation can be written as

$$AX = B$$

$$X = A^{-1} . B$$

Where,

$$A = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{vmatrix}; \quad X = \begin{vmatrix} x \\ y \\ z \end{vmatrix} \quad B = \begin{vmatrix} -4 \\ 2 \\ 11 \end{vmatrix}$$

Determinant of A:

$$|A| = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{vmatrix}$$

$$= 1(-12 + 6) - 2(-8 - 6) - 3(-6 - 9)$$

$$= -6 + 28 + 45$$

$$= 22 + 45 = 67 \neq 0$$

Cofactors of A :

Co-factor of 1st row :

$$A_{11} = (-1)^2(-12 + 6) = -6$$

$$A_{12} = (-1)^{1+2}(-8 - 6) = 14$$

$$A_{13} = (-1)^{1+3}(-6 - 9) = -15$$

Co-factor of 2nd row :

$$A_{21} = (-1)^{2+1}(-8 - 9) = 17$$

$$A_{22} = (-1)^{2+2}(-4 + 9) = 5$$

$$A_{23} = (-1)^{2+3}(-3 - 6) = 9$$

Co-factor of 3rd row :

$$A_{31} = (-1)^{3+1}(4 + 9) = 13$$

$$A_{32} = (-1)^{3+2}(2 + 6) = -8$$

$$A_{33} = (-1)^{3+3}(3 - 4) = -1$$

$\therefore$ Co-factor matrix

$$C = \begin{bmatrix} -6 & 14 & -15 \\ 17 & 5 & 9 \\ 13 & -8 & -1 \end{bmatrix} = \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj . of A}}{|A|} = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1}.B = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}\begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$$

$$= \frac{1}{67}\begin{bmatrix} 24 + 34 + 143 \\ -56 + 10 - 88 \\ +60 + 18 - 11 \end{bmatrix} = \frac{1}{67}\begin{bmatrix} 201 \\ -134 \\ +67 \end{bmatrix}$$

$$x = \frac{201}{67}; \quad y = \frac{-134}{67}; \quad z = \frac{+67}{67}$$

$$x = 3; \quad y = -2; \quad z = 1 \quad \left.\begin{matrix} x = 3 \\ y = -2 \\ z - 1 \end{matrix}\right\}$$