

Engineering mathematics (2017)

Q1. Choose the correct options of the following :

(i) The value of $\log_4 3$ is

- (a) 1 (b) $\frac{1}{3}$
(c) $\frac{1}{4}$ (d) None of these

Sol.(c)

(ii) The value of Determinant $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ is

- (a) 0 (b) $(a+b+c)$
(c) $1+a+b+c$ (d) None of these

Sol.(d)

(iii) A square matrix A is said to be non-singular if

- (a) $|A| = 0$ (b) $|A| \neq 0$
(c) $|A| = 1$ (d) None of these

Sol.(b)

(iv) In the expansion of $(a+x)^n$, total number of terms is

- (a) n (b) n+1 (c) n-1 (d) None of these

Sol.(b)

(v) The value of $\sin^4 \theta + \cos^4 \theta + 3 \sin^2 \theta \cdot \cos^2 \theta$ is equal to

- (a) 0 (b) 1 (c) 2 (d) 3

Sol.

(vi) The value of $\sin^2 75^\circ - \sin^2 15^\circ$ is

- (a) $\frac{1}{2}$ (b) 1 (c) $\frac{\sqrt{3}}{2}$ (d) 0

Sol.(c)

(vii) If $\cos A = \frac{1}{2}$ then the value of $\cos 3A$ is equal to

- (a) 0 (b) -1 (c) 1 (d) None of these

Sol.(b)

(viii) $\sin^{-1} x + \cos^{-1} x =$

- (a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{2}$ (c) 1 (d) None of these

Sol.(a)

(ix) Two vectors \vec{a} and \vec{b} are perpendicular to each other if

- (a) $\vec{a} \cdot \vec{b} = 0$ (b) $\vec{a} \times \vec{b} = 0$
(c) $\vec{a} \cdot \vec{b} = 1$ (d) None of these

Sol.(d) None of these

x) Two straight lines having slopes m_1 and m_2 are parallel if

- a) $m_1 \times m_2 = 0$
 c) $m_1 \cdot m_2 = -1$
 Sol. (b)

- (b) $m_1 = m_2$
 (d) None of these

Group - B

Answer any five questions :

Q2(a) Show that $\frac{1}{\log_{ab} abc} + \frac{1}{\log_{bc} abc} + \frac{1}{\log_{ca} abc} = 2$

Sol. L.H.S = $\frac{1}{\log_{ab} abc} + \frac{1}{\log_{bc} abc} + \frac{1}{\log_{ca} abc}$
 $\log_{abc} ab + \log_{abc} bc + \log_{abc} ca$
 $= \log_{abc} (ab \times bc \times ca)$
 $= \log_{abc} (abc)^2$
 $= 2 \log_{abc} abc$
 $= 2 \times 1 = \text{R.H.S Proved.}$

Q2.(b) Resolve into partial fraction $\frac{x+4}{x(x+1)(x+2)}$

Sol. Let $\frac{x+4}{x(x+1)(x+2)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x+2}$

Multiplying both sides by $x(x+1)(x+2)$, we get
 $x+4 = A(x+1)(x+2) + Bx(x+2) + Cx(x+1)$
 Which is true for all values of x .

Putting $x = -2$ in above relation, we get

$2 = -2 \times C \times -1$
 $C = 1$

Putting $x = -1$

$3 = Bx - 1 \times 1$
 $B = -3$

Putting $x = 0$

$4 = 2A$
 $A = 2$

Thus, the required partial fractions are :

$\frac{x+4}{x(x+1)(x+2)} = \frac{2}{x} - \frac{3}{x+1} + \frac{1}{x+2} \text{ Ans.}$

Q3.(a) Show that $\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ y+z & z+x & x+y \end{vmatrix} = (x+y+z)(x-y)(y-z)(z-x)$

Sol. L.H.S $\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ y+z & z+x & x+y \end{vmatrix}$

Applying $R_2 \rightarrow R_1 + R_3$, we get

$$= \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ x^2 & y^2 & z^2 \\ y+z & z+x & x+y \end{vmatrix}$$

$$= (x+y+z) \begin{vmatrix} 1 & 1 & 1 \\ x^2 & y^2 & z^2 \\ y+z & z+x & x+y \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 - C_1$, $C_3 \rightarrow C_3 - C_1$, we get,

$$\Delta = (x+y+z) \begin{vmatrix} 1 & 0 & 0 \\ x^2 & y^2 - x^2 & z^2 - x^2 \\ y+z & x-y & x-z \end{vmatrix}$$

$$\Delta = (x+y+z) \begin{vmatrix} y^2 - x^2 & z^2 - x^2 \\ x-y & x-z \end{vmatrix}$$

$$= (x+y+z)(x-y)(z-x) \begin{vmatrix} y-x & z-x \\ 1 & -1 \end{vmatrix}$$

$$= (x+y+z)(x-y)(z-x)(x-y-z-x)$$

$$= (x+y+z)(x-y)(z-x)(y-z) - \text{R.H.S}$$

Q3.(b) If $f(x) = x^2 - 5x + 7$ and $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$; Find $f(A)$.

Sol. $f(x) = x^2 - 5x + 7$

$$f(A) = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} + \begin{bmatrix} -15 & 15 \\ 5 & -10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} + \begin{bmatrix} -15 & -5 \\ 5 & -10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

$$\therefore f(A) = 0$$

Q4. Solve the following system of equations by using matrix inversion method

$2x + y - z = 1, x - y + z = 2, 3x + y - 2z = -1.$

Sol. Set of equation are

$$2x + y - z = 1$$

$$x - y + z = 2$$

$$3x + y - 2z = -1$$

writing them in matrix form, we get

where Let

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & -1 & 1 \\ 3 & 1 & -2 \end{bmatrix} = \text{Matrix of coefficients.}$$

$$\begin{aligned} |A| &= 2(2-1) - 1(-2-3) + 1(1+3) \\ &= 2 + 5 + 4 \\ &= 11 \neq 0 \end{aligned}$$

$\therefore A^{-1}$ exists.

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \text{Matrix of unknowns}$$

$$B = \begin{bmatrix} -3 \\ 10 \\ -3 \end{bmatrix} = \text{Matrix of constants.}$$

To Find A^{-1} :

$$\text{Let } |A| = \begin{vmatrix} 2 & 1 & -1 \\ 1 & -1 & 1 \\ 3 & 1 & -2 \end{vmatrix} = \begin{vmatrix} a^1 & b^1 & c^1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = \text{say}$$

If A_1, B_1, C_1, \dots are cofactors of elements a_1, b_1, c_1, \dots then we have.

$$A_1 = + \begin{vmatrix} -1 & 2 \\ 1 & -2 \end{vmatrix} = (2-1) = 1$$

$$B_1 = - \begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} = -(-2-1) = 3$$

$$C_1 = + \begin{vmatrix} 1 & -1 \\ 3 & 1 \end{vmatrix} = 1+3 = 4$$

$$A_2 = - \begin{vmatrix} 1 & -1 \\ 1 & -1 \end{vmatrix} = -(-2+1) = 1$$

$$B_2 = + \begin{vmatrix} 2 & -1 \\ 3 & -1 \end{vmatrix} = (-4+3) = -1$$

$$C_2 = - \begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} = -(2-3) = 1$$

$$A_3 = + \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} = (1-1) = 0$$

$$B_3 = - \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = -(2+1) = -3$$

$$C_3 = + \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = -2-1 = -3$$

The matrix of cofactor

$$= \begin{bmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{bmatrix} = \begin{bmatrix} 1 & 5 & 4 \\ 1 & -1 & 1 \\ 0 & -3 & -3 \end{bmatrix}$$

$\therefore \text{ad } A = \text{transpose of matrix of cofactors.}$

$$= \begin{bmatrix} 1 & 5 & 4 \\ 5 & -1 & -3 \\ 4 & 1 & -3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \cdot \text{ad } A$$

$$= \frac{1}{11} \begin{bmatrix} 1 & 5 & 4 \\ 5 & -1 & -3 \\ 4 & 1 & -3 \end{bmatrix}$$

Next $X = A^{-1} \cdot B$ gives.

$$= \frac{1}{11} \begin{bmatrix} -3+10 \\ -15-10+10 \\ -12+10+9 \end{bmatrix}$$

$$= \frac{1}{11} \begin{bmatrix} 7 \\ -16 \\ 7 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7/11 \\ -16/11 \\ 7/11 \end{bmatrix}$$

By equality of matrix, $x = 7/11$, $y = -16/11$, $z = 7/11$.
This is the required solution.

Q5.(a) Prove that ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

Sol. Same as 2015 Q.1 (i).

Q5.(b) Find the Coefficient of x^5 in the expansion $(1+2x)^6(1-x)^7$.

Sol. The given expression is $(1+2x)^6(1-x)^7$

$$\begin{aligned} &= [1 + {}^6C_1 1^5(2x) + {}^6C_2 1^4(2x)^2 + {}^6C_3 1^3(2x)^3 + {}^6C_4 1^2(2x)^4 \\ &\quad + {}^6C_5(2x)^5 + {}^6C_6(2x)^6] [1 + {}^7C_1 1^6(-x) + {}^7C_2 1^5(-x)^2 \\ &\quad + {}^7C_3 1^4(-x)^3 + {}^7C_4 1^3(-x)^4 + {}^7C_5 1^2(-x)^5 \end{aligned}$$

$$\begin{aligned}
& + {}^7C_6(1)(-x)^6 + {}^7C_7(-x)^7 \\
& = [1 + {}^6C_1(2x) + 4 \times {}^6C_2x^2 + 8 {}^6C_3x^3 + 16 {}^6C_4x^4 + 32 {}^6C_5x^5 \\
& + 64 {}^6C_6x^6 + (1 - {}^7C_1x + {}^7C_2x^2 - {}^7C_3x^3 + {}^7C_4x^4 \\
& - {}^7C_5x^5 + {}^7C_6x^6 - {}^7C_7x^7)] \\
& = -{}^7C_1 + 2 \times {}^6C_1 \times {}^7C_4 - 4 \times {}^6C_2 \times {}^7C_3 + 8 \times {}^6C_3 \times {}^7C_2 \\
& - 16 \times {}^6C_4 \times {}^7C_1 + 32 {}^6C_5 \\
& = -\frac{7!}{5! \times 2!} + 2 \times \frac{6!}{1! \times 5!} \times \frac{7!}{4! \times 3!} - 4 \times \frac{6!}{2! \times 4!} \times \frac{7!}{3! \times 4!} \\
& + \frac{8 \times 6!}{3! \times 3!} \times \frac{7!}{2! \times 5!} \\
& - 16 \times \frac{6!}{4! \times 2!} \times \frac{7!}{1! \times 6!} + 32 \times \frac{6!}{5! \times 1!} \\
& = -\frac{7 \times 6}{2} + \frac{2 \times 6 \times 7 \times 6 \times 5}{6} - \frac{4 \times 6 \times 5 \times 7 \times 6 \times 5}{2 \times 6} \\
& + \frac{8 \times 6 \times 5 \times 4}{3 \times 2} \times \frac{7 \times 6}{2} - 16 \times \frac{6 \times 5}{2} \times \frac{7}{1} + 32 \times 6 \\
& = -21 + 420 - 2100 + 3360 - 1680 + 192 \\
& = 3972 - 3801 \\
& = 171
\end{aligned}$$

Hence the coefficient of x^5 is 171 Ans

Q6.(a) Prove that $\sin 10^\circ \cdot \sin 30^\circ \cdot \sin 50^\circ \cdot \sin 70^\circ = \frac{1}{16}$

Sol. LHS = $\sin 10^\circ \cdot \sin 30^\circ \cdot \sin 50^\circ \cdot \sin 70^\circ = \sin 30^\circ (\sin 10^\circ \sin 50^\circ \sin 70^\circ)$

$$= \frac{1}{2} (\sin 10^\circ \sin 50^\circ) \sin 70^\circ = \frac{1}{2} \left(\frac{1}{2} 2 \sin 10^\circ \sin 50^\circ \right) \sin 70^\circ$$

$$= \frac{1}{4} \{ \cos(10^\circ - 50^\circ) - \cos(10^\circ + 50^\circ) \} \sin 70^\circ$$

$$= \frac{1}{4} (\cos 40^\circ - \cos 60^\circ) \sin 70^\circ = \frac{1}{4} \left(\cos 40^\circ - \frac{1}{2} \right) \sin 70^\circ$$

$$= \frac{1}{4} \left[\frac{1}{2} (\sin 110^\circ + \sin 30^\circ) - \frac{1}{2} \sin 70^\circ \right]$$

$$= \frac{1}{8} \left[\sin 110^\circ + \frac{1}{2} - \sin 70^\circ \right]$$

$$= \frac{1}{8} \left[\sin(180^\circ - 70^\circ) + \frac{1}{2} - \sin 70^\circ \right]$$

$$= \frac{1}{8} \left[\sin 70^\circ + \frac{1}{2} - \sin 70^\circ \right]$$

$$= \frac{1}{8} \times \frac{1}{2} = \frac{1}{16} = R.H.S$$

Q6.(b) Show that $\sin 18^\circ = \left(\frac{\sqrt{5}-1}{4} \right)$

Sol.

$$\text{Let } \theta = 18^\circ$$

$$\text{Now, } 2\theta + 3\theta = 5\theta$$

$$\therefore 2\theta = 90^\circ - 3\theta$$

Taking sine ratio of both sides, we get,

$$\sin 2\theta = \sin(90^\circ - 3\theta)$$

$$\therefore \sin 2\theta = \cos 3\theta$$

$$\therefore 2 \sin \theta \cos \theta = 4 \cos^3 \theta - 3 \cos \theta$$

$$\therefore -2 \sin \theta \cos \theta = \cos \theta (4 \cos^2 \theta - 3)$$

As $\theta = 18^\circ$, $\cos \theta \neq 0$, dividing throughout by $\cos \theta$, we get

$$2 \sin \theta = 4 \cos^2 \theta - 3$$

$$2 \sin \theta = 4(1 - \sin^2 \theta) - 3$$

$$2 \sin \theta = 4 - 4 \sin^2 \theta - 3$$

$$4 \sin^2 \theta + 2 \sin \theta - 1 \dots \text{quadratic in } \sin \theta \text{ where } a = 4, b = 2, c = -1$$

Using quadratic formula,

$$\sin \theta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{4 - 4(4)(-1)}}{2 \times 4}$$

$$= \frac{-2 \pm 2\sqrt{5}}{2 \times 4}$$

$$= \frac{2(-1 \pm \sqrt{5})}{2 \times 4}$$

$$= \frac{-1 \pm \sqrt{5}}{4}$$

$$\therefore \sin \theta = \frac{-1 + \sqrt{5}}{4} \text{ OR } \frac{-1 - \sqrt{5}}{4}$$

But $\theta = 18^\circ < 90^\circ \therefore \sin \theta$ lies in first quadrant

$$\therefore \sin \theta = +ve$$

$$\therefore \sin \theta = \frac{-1 + \sqrt{5}}{4} \text{ is correct}$$

$$\text{Thus, } \sin 18^\circ = \frac{\sqrt{5}-1}{4} \quad (i)$$

Q7. (a) Prove that $\tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{7} + \tan^{-1} \frac{1}{8}$

Sol. L.H.S $\tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{7} + \tan^{-1} \frac{1}{8}$

$$= \tan^{-1} \left[\frac{\frac{1}{3} + \frac{1}{5}}{1 - (\frac{1}{3})(\frac{1}{5})} \right] + \tan^{-1} \left[\frac{\frac{1}{7} + \frac{1}{8}}{1 - (\frac{1}{7})(\frac{1}{8})} \right]$$

$$\left(\because \tan^{-1} x + \tan^{-1} y = \tan^{-1} \left[\frac{x+y}{1-xy} \right] \right)$$

$$= \tan^{-1} \left[\frac{\frac{8}{15}}{\frac{14}{15}} \right] + \tan^{-1} \left[\frac{\frac{15}{56}}{\frac{55}{56}} \right]$$

$$= \tan^{-1} \left[\frac{8}{14} \right] + \tan^{-1} \left[\frac{15}{55} \right]$$

$$= \tan^{-1} \left[\frac{4}{7} \right] + \tan^{-1} \left[\frac{3}{11} \right]$$

$$= \tan^{-1} \left[\frac{\frac{4}{7} + \frac{3}{11}}{1 - \frac{4}{7} \times \frac{3}{11}} \right] = \tan^{-1} \left[\frac{\frac{44+21}{77}}{\frac{65}{77}} \right] = \tan^{-1} [1]$$

$= \frac{\pi}{4}$ R.H.S Proved.

Q7. (b) Show that Points A (a,0), B (0,b) and C (3a,-2b) are Collinear, Also find the equation containing them.

Sol. The given three points are collinear if and only if

$$D = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Let

$$D = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} a & 0 & 1 \\ 0 & b & 1 \\ 3a & -2b & 1 \end{vmatrix}$$

Now expanding the determinant first row wise, we get

$$D = a(b + 2b) - 0 + 1(0 - 3ab) \\ D = 3ab - 3ab$$

the condition for collinear points is satisfied.

\therefore The points A, B, C are collinear.

Q8. (a) Find the equation of the line passing through the point of intersection of the lines $4x + 7y - 3 = 0$ and $2x - 3y + 1 = 0$, which has equal intercepts on the axes.

Sol. We have

$$4x + 7y - 3 = 0 \dots (i)$$

$$2x - 3y + 1 = 0 \dots (ii)$$

For point of intersection of (i) and (ii)

$$4x + 7y = 3 \dots (i) \times 1$$

$$2x - 3y = -1 \dots (ii) \times 2$$

$$4x + 7y = 3$$

$$4x - 6y = -2$$

$$\begin{array}{r} - \\ + \\ + \\ \hline -13y = 5 \end{array}$$

$$y = \frac{5}{13}$$

Putting the value of y in eqⁿ (ii)

$$2x - \frac{3 \times 5}{13} = -1$$

$$2x - \frac{15}{13} = -1$$

$$2x = \frac{15}{13} - 1$$

$$2x = \frac{15-13}{13} = \frac{2}{13}$$

$$x = \frac{1}{13}$$

Now

Let the equation of the line in the intercept form

$$\text{be } \frac{x}{a} + \frac{y}{b} = 1$$

It is given that a = b

Hence the equation of the line becomes

$$\frac{x}{a} + \frac{y}{a} = 1$$

$$x + y = a \dots (i)$$

Since the line passes through the point $\left(\frac{1}{13}, \frac{5}{13} \right)$

$$\text{We have } \frac{1}{13} + \frac{5}{13} = a \Rightarrow a = \frac{6}{13}$$

Hence putting $a = \frac{6}{13}$ in (i),

The required equation of the line is,

$$x + y = \frac{6}{13}$$

Q8.(b) If p_1 and p_2 are lengths of perpendiculars from the origin to the lines $x \sec \theta + y \csc \theta = a$ and $x \cos \theta - y \sin \theta = a \cos 2\theta$ respectively, then prove that $4p_1^2 + p_2^2 = a^2$.

Sol. Hence

$$p_1 = \frac{0 \times \sec \theta + 0 \times \csc \theta - a}{\sqrt{\sec^2 \theta + \csc^2 \theta}}$$

$$= \frac{-a}{\sqrt{\frac{1}{\cos^2 \theta} + \frac{1}{\sin^2 \theta}}}$$

$$= \frac{-a \sin \theta \cos \theta}{\sqrt{\sin^2 \theta + \cos^2 \theta}} = \frac{-a}{2} \sin 2\theta$$

Again, $p_2 = \frac{0 \times \cos \theta - 0 \times \sin \theta - a \cos^2 \theta}{\sqrt{\cos^2 \theta + \sin^2 \theta}}$
 $= -a \cos^2 \theta$

$$\sin^2 \theta = \frac{-2p_1}{a} \text{ and } \cos^2 \theta = \frac{-p_2}{a}$$

Squaring and adding, we get

$$= \frac{4p_1^2}{a^2} + \frac{p_2^2}{a^2}$$

$$\therefore 4p_1^2 + p_2^2 = a^2 \text{ Ans.}$$

Q9.(a) Find the equation of the circle passing through the points (5,7), (6,6) and (2,-2) Find its centre and radius.

Sol. Let the equation of the circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0 \dots (i)$$

Since the point (5,7) lies on the circle therefore

$$5^2 + 7^2 + 2 \times 5 \times g + 2 \times 7 \times f + c = 0$$

$$25 + 49 + 10g + 14f + c = 0$$

$$74 + 10g + 14f + c = 0 \dots (ii)$$

Similarly, Since the point (6,6) lies on the circle

$$6^2 + 6^2 + 2 \times 6 \times g + 2 \times 6 \times f + c = 0$$

$$36 + 36 + 12g + 12f + c = 0$$

$$72 + 12g + 12f + c = 0 \dots (iii)$$

Again, since the point (2,-2) lies on the circle,

$$2^2 + (-2)^2 + 4g - 4f + c = 0$$

$$4 + 4 + 4g - 4f + c = 0$$

$$8 + 4g - 4f + c = 0 \dots (iv)$$

Subtracting eqⁿ(iv) from (iii) and eqⁿ from

$$64 + 8g + 16f = 0$$

$$8g + 16f = -64 \dots (v)$$

$$-2 + 2g - 2f = 0$$

$$2g - 2f = 2 \dots (vi)$$

Equating equation (v) and (vi)

$$8g + 16f = -16$$

$$8g - 8f = 8$$

$$24f = -72$$

$$f = -6$$

$$g = \frac{-64 + 16 \times 6}{8}$$

$$g = \frac{32}{8}$$

Putting $f = -6$, $g = 4$ in eqⁿ(iv)

$$8 + 4 \times 4 - 4 \times -6 + c = 0$$

$$8 + 16 + 24 + c = 0$$

$$c = -48$$

Therefore, the required equation is

$$x^2 + y^2 + 8x - 12y - 48 = 0$$

Centre is the point (-4, 6)

and whose radius is

$$= \sqrt{4^2 + 6^2 + 48}$$

$$= \sqrt{16 + 36 + 48}$$

$$= 10 \text{ Ans.}$$

Q9.(b) Find the equation of a circle, the end points of one of whose diameters are A (2,3) and B (-3,5).

$$\text{Sol. } x_1 = 2, y_1 = 3; x_2 = -3, y_2 = 5,$$

The required equation of the circle is.

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

$$\Rightarrow (x - 2)(x + 3) + (y - 3)(y - 5) = 0$$

$$\Rightarrow (x^2 + 3x - 2x - 6) + (y^2 - 5y + 3y - 15) = 0$$

$$\Rightarrow (x^2 + x - 6) + (y^2 - 2y - 15) = 0$$