Engineering mathematics (2018)

Q1. Answer the following

(i) The value of $\log_2 \log_2 \log_2 16$ is equal to

(a) 1 (b) 2 (c) -1 (d) None of the above

Sol. $\log_2 \log_2 \log_2 16 = \log_2 \log_2 \{\log_2 (2)^4\}$

$$= \log_2 \log_2 \{4 \log_2 2\}$$
$$= \log_2 \log_2 \{4 \times 1\}$$
$$= \log_2 \log_2 4$$
$$= \log_2 \{\log_2 (2)^2\}$$
$$= \log_2 \{2 \times \log_2 2\}$$
$$= \log_2 2 = 1$$

(ii) The number of terms in the expansion of $x^6(1+3x^4)^{15}$ is

(a) 21 (b) 15 (c) 16 (d) 19

Sol. (c)

(iii) The value of sin 18° is equal to

(a) $\frac{\sqrt{5}+1}{4}$ (b) $\frac{1-\sqrt{5}}{4}$

(c) $\frac{\sqrt{5}-1}{4}$ (d) None of the above

Sol. Let $\theta = 18^0$

$$5\theta = 5 \times 18^0 = 90^0$$
$$3\theta + 2\theta = 90^0$$
$$2\theta = 90^0 - 3\theta$$

$\sin 2\theta = \sin(90^0 - 3\theta)$ {Applying sin on both side}

$$\Rightarrow \sin 2\theta = \cos 3\theta$$
$$\Rightarrow 2\sin\theta \cos\theta = 4\cos^3\theta - 3\cos\theta$$
$$\Rightarrow 2\sin\theta \cos\theta = \cos\theta(4\cos^2\theta - 3)$$
$$\Rightarrow 2\sin\theta - 4\cos^2\theta + 3 = 0$$
$$\Rightarrow 2\sin\theta - 4(1-\sin^2\theta) + 3 = 0$$
$$\Rightarrow 4\sin^2\theta + 2\sin\theta - 1 = 0$$

Using form

$$ax^2 + bx + c = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So, $$\sin\theta = \frac{-2 \pm \sqrt{4 - 4 \times 4 \times (-1)}}{2 \times 4}$$

$$= \frac{-1 \pm \sqrt{5}}{4}$$
$$= \frac{-1+\sqrt{5}}{4}, \frac{-1-\sqrt{5}}{4}$$

$1 \le \sin\theta \le 1$

$\therefore$ $$\sin\theta = \frac{-1+\sqrt{5}}{4}$$

$$\sin 18^0 = \frac{\sqrt{5}-1}{4}$$

(iv) The value of $\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$ *is equal to*

(a) $x+y+z$ (b) 0 (c) 1 (d) None of the above

Sol. $\Delta = \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$

$R_1 \to R_1 + R_2$

$\therefore$ $$\Delta = \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$$

Taking x+y+z common from R_1

$\therefore$ $$\Delta = (x+y+z)\begin{vmatrix} 1 & 1 & 1 \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$$

$$= (x+y+z)\{1(x-y) + 1(y-z) + 1(z-x)\}$$
$$= (x+y+z)(x-y+y-z+z-x)$$
$$= 0$$

(v) The Principal value of $\sin^{-1}\left(\frac{-1}{2}\right)$ is equal to

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{-\pi}{6}$ (d) None of the above

Sol. Let $\sin^{-1}\left(\frac{1}{2}\right) = y$

$$\Rightarrow \sin y = -\frac{1}{2}$$
$$\Rightarrow \sin y = \sin\left(\frac{-\pi}{6}\right)$$

Range of principal value of $\sin^{-1}$ is $-\frac{\pi}{2}$ and $\frac{\pi}{2}$

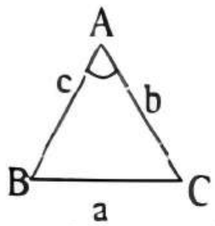
$\therefore$ Principal value is $\left(-\frac{\pi}{6}\right)$

(vi) In a ΔABC, if a, b and c are the sides of the corresponding angles respectively, then cos A is equal to

(a) $\frac{b^2+c^2-a^2}{2bc}$ (b) $\frac{a^2+b^2-c^2}{2ab}$

(c) $\frac{c^2+a^2-b^2}{2ca}$ (d) None of the above

Sol. (a) Using cosine formula -



$$\cos = \frac{b^2+c^2-a^2}{2bc}$$

(vii) If the points (-2, -5), (2,-2) and (8,k) are collinear then the value of k is equal to

(a) 5 (b) 3 (c) $\frac{5}{2}$ (d) None of the above

Sol. (c) A(-2, -5) B(2, -2) C(8, K)

If Point A, B, C are collinear

Then, Slope of line AB = Slope of line AC

$$\therefore \quad \frac{-2+5}{2+2}=\frac{K+5}{8+2}$$

$$\Rightarrow \quad \frac{3}{4}=\frac{K+5}{10}$$

$$\Rightarrow \quad 10\times 3 = 4K+20$$

$$\Rightarrow \quad 4K = 30-20 = 10$$

$$\Rightarrow \quad K = \frac{10}{4} - 2.5 = \frac{5}{2}$$

(viii) The equation of straight line parallel to the line 4x + 7y + 5 = 0 and passing through the point (1, -2) is given by

(a) 4x + 7y + k = 0 (b) 4x - 7y + 10 = 0

(c) 4x + 7y + 10 = 0 (d) None of the above

Sol. (c) Equation of line parallel to 4x+7y+5 = 0 and passing through (1, -2) is

$$4x + 7y + \lambda = 0 \qquad \text{......(i)}$$

Since, (i) passes through (1, -2)

$$\therefore \quad 4\times 1+7\times(-2)+\lambda=0$$

$$\Rightarrow \quad 4-14+\lambda=0$$

$$\Rightarrow \quad \lambda=10$$

$$\therefore \quad 4x+7y+10=0$$

Q2. (a) Prove that $\log_a m = \log_b m \times \log_a b$

Sol. (a) $LHS = \log_a m$

$RHS = \log_b m \times \log_a b$

$$= \frac{\log m}{\log b}\times\frac{\log b}{\log a} \qquad \left\{\because \log_a b = \frac{\log b}{\log a}\right\}$$

$$= \frac{\log m}{\log a}$$

$$= \log_a m$$

Q2.(b) If $\frac{\log a}{b-c}=\frac{\log b}{c-a}=\frac{\log c}{a-b}$ then prove that $a^a.b^b.c^c = 1$.

Sol. Given : $\frac{\log a}{b-c}=\frac{\log b}{c-a}=\frac{\log c}{a-b}=K$

$\log a = K(b-c)$ $\quad \log b = K(c-a)$ $\quad \log c = K(a-b)$

$a\log a = aK(b-c)$ $\quad b\log b = bK(c-a)$ $\quad c\log c = cK(a-b)$

$\log a^a = K(ab-ac)$ $\quad \log b^b = K(bc-ab)$ $\quad \log c^c = K(ac-bc)$

$$\log a^a + \log b^b + \log c^c = K[ab-ac+bc-ab+ac-bc]$$
$$= K\times 0$$

$$\log a^a + \log b^b + \log c^c = 0$$

$$\log(a^a b^b c^c) = \log 1$$

$$a^a b^b c^c = 1$$

Q3. (a) Resolve into partial fractions : $\frac{x^2+1}{x(x^2-1)}$.

Sol. $\frac{x^2+1}{x(x^2-1)}=\frac{A}{x}+\frac{Bx+C}{(x^2-1)}$

$$\frac{x^2+1}{x(x^2-1)}=\frac{A(x^2-1)+x(Bx+C)}{x(x^2-1)}$$

$$x^2+1 = x^2(A+B)+(x-A)$$

Comparing coefficients on both side -

A + B = 1; C = 0; A = -1; B = 2

$$\therefore \quad \frac{x^2+1}{x(x^2-1)}=\frac{-1}{x}+\frac{2x}{(x^2-1)}$$

$$=\frac{2x}{(x^2-1)}-\frac{1}{x}$$

Q3.(b) Prove that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$ $= (a+b+c)^3$

Sol. $\Delta = \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

$R_1 = R_1 + R_2 + R_3$

$$\begin{vmatrix} a-b-c+2b+2c & 2a+b-c-a+2c & 2a+2b+c-a-b \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Taking (a+b+c) common from R1

$$\Delta = (a+b-c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$c_2 \to c_2 - c_1 \qquad c_3 \to c_3 - c_1$

$$\Delta = (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -b-c-a & 0 \\ 2c & 0 & -a-b-c \end{vmatrix}$$

Taking (a+b+c) common from c_2 and c_3

$$\Delta = (a+b+c)^3\begin{vmatrix} 1 & 0 & 0 \\ 2b & -1 & 0 \\ 2c & 0 & -1 \end{vmatrix}$$

$$= (a+b+c)^3 \times 1\{1-0\}$$

$$= (a+b+c)^3$$

Q4. (a) Solve the following by matrix inversion method:

$$x + y + z = 5$$
$$x + 2y + 3z = 12$$
$$2x - y + z = 4$$

Sol. $x + y + z = 5 \qquad x + 2y + 3z = 12$

$2x - y + z = 4$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & -1 & 1 \end{bmatrix} \qquad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad B = \begin{bmatrix} 5 \\ 12 \\ 4 \end{bmatrix}$$

AX = B

$X = A^{-1}B$

$$adj\ A = \begin{bmatrix} (2\times1+3) & (2\times3-1) & (-1-4) \\ (-1-1) & (1-2) & (2+1) \\ (3-2) & (1-3) & (2-1) \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 5 & -5 \\ -2 & -1 & 3 \\ 1 & -2 & 1 \end{bmatrix}$$

$$adj\ A = \begin{bmatrix} 5 & -2 & 1 \\ 5 & -1 & -2 \\ -5 & 3 & 1 \end{bmatrix}$$

$|A| = 1 \times 5 + 1 \times 5 + 1 \times (-5) = 5$

$$\therefore \quad X = \frac{1}{|A|} adj\ A\,B \qquad \left[\because A^{-1} = \frac{1}{|A|} adj\ A\right]$$

$$X = \frac{1}{5}\begin{bmatrix} 5 & -2 & 1 \\ 5 & -1 & -2 \\ -5 & 3 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ 12 \\ 4 \end{bmatrix}$$

$$\Rightarrow \quad X = \frac{1}{5}\begin{bmatrix} 5 \\ 5 \\ 15 \end{bmatrix}$$

$$\therefore \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$$

$\therefore \quad x = 1; y = 1; z = 3$

Q4.(b) Show that $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$ ***satisfies the equation*** $x^2 - 3x - 7 = 0$.

Sol. $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix}$$

$\therefore \quad x^2 - 3x - 7$

$= A^2 - 3A - 7I$

$$= \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix} - 3\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} - 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix} + \begin{bmatrix} -15 & -9 \\ 3 & 6 \end{bmatrix} + \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix}$$

$$= \begin{bmatrix} 22-15-7 & 9-9+0 \\ -3+3+0 & 1+6-7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\therefore \quad A^2 - 3A + 7I = 0$

Q5. (a) Find the middle term in the expansion of $\left(\frac{2x}{3} - \frac{3}{2x}\right)^8$.

Sol. Binomial Expansion of $\left(\frac{2x}{3} - \frac{3}{2x}\right)^8$

$$= {}^8C_r\left(\frac{2x}{3}\right)^{8-r}\left(\frac{-3}{2x}\right)^r$$

$\because \quad n = 8$ is even

$\therefore \quad$ 4th term will be middle term

$$= {}^8C_4\left(\frac{2x}{3}\right)^4\left(\frac{-3}{2x}\right)^4$$

$$= {}^8C_4(-1)^4 \times \left(\frac{2x}{3}\right)^4 \times \left(\frac{3}{2x}\right)^4$$

$$= {}^8C_4$$

$$= \frac{8}{4}\times\frac{7}{3}\times\frac{6}{2}\times 5$$
$$= 70$$

Q5.(b) Find the coefficient of x^2 in the expansion of $\left(3x-\frac{1}{x}\right)^{12}$

Sol. Binomial Expansion of $\left(3x-\frac{1}{x}\right)^{12}$

$$= {}^{12}C_r(3x)^{12-r}\left(\frac{-1}{x}\right)^r$$
$$= (-1)^r\ {}^{12}C_r\,(3)^{12-r}(x)^{12-r-r}$$
$$= (-1)^r(3)^{12-r}\ {}^{12}C_r\,(x)^{12-2r}$$

We have to coefficient of x^2

$\therefore\ 12 - 2r = 2$

$\Rightarrow 2r = 10 \Rightarrow r = 5$

$\therefore$ Coefficient of $x^2 = (-1)^5(3)^7 \times {}^{12}C_5$

$$= = -3^7\times{}^{12}C_5$$

Q6. (a) Prove that $\tan^{-1}\frac{2}{5}+\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{12}=\frac{\pi}{4}$.

Sol. $= \tan^{-1}\left(\frac{2}{5}\right)+\tan^{-1}\left[\frac{\frac{1}{3}+\frac{1}{12}}{1-\left(\frac{1}{3}\times\frac{1}{12}\right)}\right]$

$\therefore\ \tan^{-1}A+\tan^{-1}B=\tan^{-1}\left(\frac{A+B}{1-AB}\right)$

$$= \tan^{-1}\left(\frac{2}{5}\right)+\tan^{-1}\left(\frac{15}{35}\right)$$
$$= \tan^{-1}\left(\frac{2}{5}\right)+\tan^{-1}\left(\frac{3}{7}\right)$$
$$= \tan^{-1}\left(\frac{\frac{2}{5}+\frac{3}{7}}{1-\left(\frac{2}{5}\times\frac{3}{7}\right)}\right)=\tan^{-1}\left(\frac{\frac{29}{35}}{\frac{29}{35}}\right)$$
$$= \tan^{-1}1 = \frac{\pi}{4}$$

Q6.(b) If $A + B = 45^0$ then prove that (cot A -1) (cot B-1)= 2.

Sol. $A+B=45^0$

$(\cot A-1)(\cot B-1)$

$= \cot A\cot B - \cot A - \cot B + 1$

$= (\cot A\cot B - \cot A - \cot B) + 1$ ......(i)

$\cot(A+B)=\cot 45^0$

$\frac{\cot A\cot B-1}{\cot A+\cot B}=1$

$\Rightarrow \cot A\cot B - 1 = \cot A + \cot B$

$\Rightarrow \cot A\cot B - \cot A - \cot B = 1$ ....(ii)

From (i) & (ii)

$(\cot A - 1)(\cot B - 1) = 1 + 1 = 2$

Q7. (a) Prove that in a triangle ABC, $\frac{A-B}{2}=\frac{a-b}{a+b}\cot\frac{C}{2}$.

Sol. In ABC Triangle

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=2R$$
$$\frac{a-b}{a+b}=\frac{2R\sin A-2R\sin B}{2R\sin A+2R\sin B}$$
$$=\frac{\sin A-\sin B}{\sin A+\sin B}$$
$$=\frac{2\cos\frac{A+B}{2}.\sin\frac{A-B}{2}}{2\sin\frac{A+B}{2}.\cos\frac{A-B}{2}}$$
$$=\cot\frac{A+B}{2}.\tan\frac{A-B}{2}$$
$$=\cot\left(\frac{\pi-c}{2}\right)\tan\frac{A-B}{2}\qquad [as\ A+B+C=\pi]$$
$$=\tan\left(\frac{C}{2}\right)\tan\frac{A-B}{2}$$
$$\therefore\quad \tan\frac{A-B}{2}=\frac{a-b}{a+b}\cot\frac{C}{2}$$

Q7.(b) In a triangle ABC, Prove that a(bcos C- c cos B)= b^2 - c^2.

Sol. Using cosine formula

$$a^2+b^2-2ab\cos C=c^2$$
$$a^2+c^2-2ac\cos B=b^2$$

Substracting (i) and (ii)

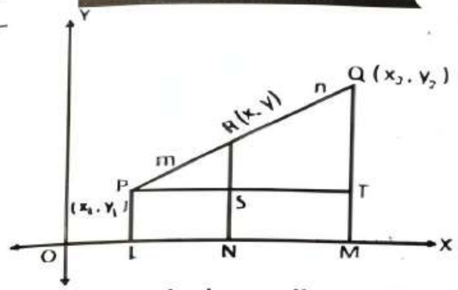
$$\therefore\quad b^2-c^2=(a^2+c^2-2ac\cos B)-(a^2+b^2-2ab(\cos C)$$
$$=c^2-2ac\cos B-b^2+2ab\cos C$$
$$=-(b^2-c^2)-2ac\cos B+2ab\cos C$$
$$\Rightarrow\ 2(b^2-c^2)=2a[-c\cos B+b\cos C]$$
$$\Rightarrow\ (b^2-c^2)=a[-c\cos B+b\cos C]$$

$\therefore$ a [bcosc - acosb] = b^2 - c^2

Q8. (a) Find the co-ordinates of points which divide the given line segments joining the point (x_1, y_1) and (x_2, y_2) internally in the ratio $m_1 : m_2$

Sol. (a) Internal Division of line segment :

Let (x_1, y_1) and (x_2, y_2) be the cartesian co-ordinates of the points P and Q respectively referred to rectangular co-ordinate axes $\overline{OX}$ and $\overline{OY}$ and the point R divides the line segment $\overline{PQ}$ internally in a given ratio m : n (say, i.e. $\overline{PR}:\overline{RQ}$ = m : n. We are to find the co-ordinates of R.



Let, (x, y) be the required co-ordinate of R. From P, Q and R, draw $\overline{PL}, \overline{QM}$ and $\overline{RN}$ perpendiculars on $\overline{OX}$, Again, draw $\overline{PT}$ parallel to $\overline{OX}$ to cut $\overline{RN}$ at S and $\overline{QM}$ at T.

Then,

$\overline{PS} = \overline{LN} = \overline{ON} - \overline{OL} = x - x_1$

$\overline{PT} = \overline{LM} = \overline{OM} - \overline{OL} = x_2 - x_1$

$\overline{RS} = \overline{RN} - \overline{SN} = \overline{RN} - \overline{PL} = y - y_1$

and $\overline{QT} = \overline{QM} - \overline{TM} = \overline{QM} - \overline{PL} = y_2 - y_1$

Again, $\overline{PR}/\overline{RQ} = m/n$

or, $\overline{RQ}/\overline{PR} + 1 = n/m + 1$

or, $(\overline{RQ} + \overline{PR}/\overline{PR}) = (m+n)/m$

or, $\overline{PQ}/\overline{PR} = (m+n)/m$

Now, by construction, the triangles PRS and PQT are similar; hence,

$\overline{PS}/\overline{PT} = \overline{RS}/\overline{QT} = \overline{PR}/\overline{PQ}$

Taking, $\overline{PS}/\overline{PT} = \overline{PR}/\overline{PQ}$ we get,

$(x - x_1)/(x_2 - x_1) = m/(m+n)$

or, $x(m+n) - x_1(m+n) = mx_2 - mx_1$

or, $x(m+n) = mx_2 - mx_1 + mx_1 + nx_1 = mx_2 + nx_1$

Therefore, $x = (mx_1 + nx_1)/(m+n)$

Again, taking $\overline{RS}/\overline{QT} = \overline{PR}/\overline{PQ}$ we get,

$(y - y_1)/(y_2 - y_1) = m/(m+n)$

or, $(m+n)y - (m+n)y_1 = my_2 - my_1$

or, $(m+n) = my_2 - my_1 + my_1 + ny_1 = my_2 + ny_1$

Therefore, $y = my_2 + ny_1/(m+n)$

Therefore, the required co-ordinates of the point R are

$((mx_2 + nx_1)/(m+n), (my_2 + ny_1)/(m+n))$

Q8.(b) Find the angle between the straight lines 2x - 8y = 7 and 6x - y = 12.

Sol. $2x - 8y = 7 \quad 6x - y = 12$

$8y = 2x - 7 \qquad y = 6x - 12$

$\Rightarrow y = \frac{2x}{8} - \frac{7}{8} \qquad m_2 = 6$

$\Rightarrow y = \frac{x}{4} - \frac{7}{8}$

$m_1 = \frac{1}{4}$

Comparing above equations with y = mx + c

$\therefore \quad m_1 = \frac{1}{4} \qquad m_2 = 6$

Angle between lines

$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1 m_2}$$

$$= \pm\frac{\left(\frac{1}{4} - 6\right)}{1 + \left(\frac{1}{4} \times 6\right)}$$

$$= \pm\frac{\left(\frac{-23}{4}\right)}{\frac{10}{4}}$$

$$= \pm\left(\frac{-23}{10}\right)$$

$$\tan\theta = \mp\frac{-23}{10}$$

$$\therefore \quad \theta = \tan^{-1}\left(\mp\frac{-23}{10}\right)$$

Q9. (a) If one end of a diameter of the circle $x^2 + y^2 - 2x - 4y + 1 = 0$ be (1,0). Find other end.

Sol. $x^2 + y^2 - 2x - 4y + 1 = 0$

P (1, 0)

Comparing above equation with

$x^2 + y^2 + 2gx + 2fy + c = 0$

2g = -2 => g = -1

2f = -4 => f = -2

$\therefore$ Centre = (-g, -f) = (1, 2)

Radius $= \sqrt{g^2 + f^2 - c} = \sqrt{1 + 4 - 1} = \sqrt{4} = 2$

Let other end of dia be (x, y)

$\therefore \quad \frac{(x+1)}{2} = 1$

$\Rightarrow x = 1$

$\frac{y+0}{2} = 2 \quad \Rightarrow y = 4$

$\therefore$ Point = (1, 4)

Q9.(b) Find the equation of the circle passing through the points (2,3), (-1,6) and having its centre on the line 2x + 5y + 1 = 0.

Sol. Let equation of circle be -

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

It posses through (2, 3) and (-1, 6)

$2^2+3^2+2g\times 2+2f\times 3+c=0$

$4g + 6f + c = -13$ .....(i)

$1 + 36 + 2g \times(-1) + 2f \times 6 + c = 0$

$12f - 2g + c = -37$ ....(ii)

Substracting (i) and (ii)

$6g - 6t = 24$

$\therefore$ $g - f = 4$ ....(iii)

Since centre (-g, -f) lie on $2x + 5y + 1 = 0$

$\therefore$ $-2g - 5f + 1 = 0$

$2g + 5f + 1$ ....(iv)

On solving (iii) and (iv) we get

$g = 3; f = (-1)$

$\therefore$ $4 \times 3 + 6 \times (-1) + c = -13$

$\therefore$ $c = -19$

$\therefore$ $x^2 + y^2 + 6x - 2y - 19 = 0$

Q10.(a) ***$\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a}\cdot\vec{b} = 4$. Find $|\vec{a}-\vec{b}|$.***

Sol. $|\vec{a}-\vec{b}|^2 = (\vec{a}-\vec{b}).(\vec{a}-\vec{b})$

$$= \vec{a}.\vec{a}-\vec{a}.\vec{b}-\vec{b}.\vec{a}+\vec{b}.\vec{b}$$

$$=|\vec{a}|^2-2\vec{a}.\vec{b}+|\vec{b}|^2$$

$$=2^2-2\times4+3^2$$

$$= 4 - 8 + 9 = 5$$

$\therefore$ $|\vec{a}-\vec{b}| = \pm\sqrt{5}$

Since, magnitude is not negative

$\therefore$ $|\vec{a}-\vec{b}| = \sqrt{5}$

Q10.(b) ***A force $\vec{F} = 2\hat{i} + 3\hat{j} - 5\hat{k}$ is applied at the point (1, -1,2). Find the moment of the force about the point (2, -1, 3).***

Sol. $\vec{F} = 2\hat{i}+3\hat{j}-5\hat{k}$

P(1, -1, 2) Q(2, -1, 3)

$\vec{r} = \overrightarrow{PQ} = (2-1)\hat{i}+(-1+1)\hat{j}+(3-2)\hat{k}$

$= \hat{i}+\hat{k}$

$\therefore$ Moment of force $= \vec{r}\times\vec{F}$

$$= (\hat{i}+\hat{k})\times(2\hat{i}+3\hat{j}-5\hat{k})$$

$$\vec{r}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 1 \\ 2 & 3 & -5 \end{vmatrix}$$

$$= \{(0\times-5) - (3\times1)\}\ \hat{i} + (2\times1+5)\ \hat{j} + (3-0)\ \hat{k}$$

$$\vec{r}\times\vec{F} = -3\hat{i}+7\hat{j}+3\hat{k}$$