Engineering mathematics (2014)

Q1. *Select the correct answer :*

(i) If $A = \tan^{-1}$***, then the value of sin 2A is equal to***

(a) $\dfrac{2x}{\sqrt{1-x^2}}$ (b) $\dfrac{2x}{\sqrt{1+x^2}}$ (c) $\dfrac{2x}{1+x^2}$ (d) None

Ans.(c)

Explⁿ.: Given: $A = \sin^{-1} x$; $x = \tan A$

$$\sin 2A = \frac{2\tan A}{1+\tan^2 A} = 2x/1+x^2.$$

(ii) If $A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$***, then the value of A² is equal to***

(a) $\begin{bmatrix} 22 & -27 \\ 18 & 16 \end{bmatrix}$ (b) $\begin{bmatrix} 20 & 15 \\ 16 & 14 \end{bmatrix}$

(c) $\begin{bmatrix} 22 & 27 \\ 18 & 31 \end{bmatrix}$ (d) None

Ans.(c)

Explⁿ.: $A \times A \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} \times \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 22 & 27 \\ 18 & 31 \end{bmatrix}$

(iii) If $^n p_r = 2520$ ***and*** $^n c_r = 21$***, then the value of r.***

(a) 5 (b) 3 (c) 4 (d) None

Ans.(c)

Explⁿ.: $^n p_r = r!\,^n c_r$

$$r! = \frac{^n p_r}{^n c_r} = \frac{2520}{21} = 120 \qquad \therefore r! = 5! \text{ i.e. } r = 5$$

(iv) Centre and radius of the circle

$x^2 + y^2 + 8x - 6y - 11 = 0$ is

(a) $(-4, 3), 6$ (b) $(4, -3), -6$

(c) $(-4, -3), 6$ (d) None

Ans.(a)

Explⁿ.: Given: $x^2 + y^2 + 8x - 6y - 11 = 0$

Centre $(-g, -f)$ $r = \sqrt{g^2 + f^2 - c}$

$2g - 8$ $2f = -6$ $\& C = -11$

$g = 4$ $f = -3$

Centre $= (-4, 3)$ $r = \sqrt{16 + 9 + 11} = \sqrt{36} = 6.$

(v) The co-efficient of x^7 ***in the expansion of*** $\left(3x^2 + \frac{1}{5x}\right)^{11}$ ***is***

(a) $^{11}C_4$ (b) $^{11}C_5 \frac{3^6}{5^5}$ (c) $^{11}C_5 \frac{5^5}{3^6}$ (d) None

Ans.(b)

Explⁿ.: $T_{r+1} = {}^{11}C_r (3x^2)^{11-r} \left(\frac{1}{5x}\right)^r$

$$= {}^{11}C_r . 3^{11-r} . x^{22-2r} . \frac{1}{5r} . x^{-r}$$

$$= {}^{11}C_r . \frac{3^{11-r}}{5r} . x^{22-3r}$$

But $x^7 = x^{22-3r}$

$\Rightarrow 22 - 3r = 7$

$3r = 22 - 7 = 15$

$r = 5$

$\therefore$ the co--efficient of $x^7 = {}^{11}C_5 . \frac{3^6}{5^5}$

(vi) If characteristics of logx is – 2, the number of zero after decimal 10 and before first significant number in x is

(a) 3 (b) 2 (c) 1 (d) None

Ans.(c)

(vii) The centroid of a triangle whose vertices are (4 –3), (–9, 7) & (8, 8) is

(a) (2, 3) (b) (1, 4) (c) (1, 6) (d) None

Ans.(b)

Explⁿ.: Centroid of a $\Delta = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

$$= \left(\frac{4 - 9 + 8}{3}, \frac{-3 + 7 + 8}{3}\right) = (1, 4)$$

(viii) What is x in $2\log_8 x = \log_8 16$***?***

(a) $x = 8$ (b) $x = 16$ (c) $x = 4$ (d) None

Ans.(a)

Explⁿ.: $\log_8 x^2 = \log_8 16$

$x^2 = 16$; $x = 8$

(ix) Slope of a line joining the points (1, 1) and (2, 4) is

(a) 1 (b) 2 (c) 3 (d) None

Ans.(c)

Explⁿ.: Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4-1}{2-1} = \dfrac{3}{1} = 3$

(x) If $\vec{a} = 3\hat{i} - 2\hat{j} - 2\hat{k}$ ***and*** $\vec{b} = 2\hat{i} + 3\hat{j} + \hat{k}$ ***then the vector product of*** $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$ ***is equal to***

(a) $8\hat{i}-14\hat{j}+26\hat{k}$ (b) $-8i+14j-26k$

(c) $\hat{i}+3\hat{j}-\hat{k}$ (d) None of these

Ans.(b)

Expl.: $(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})=(5\hat{i}+\hat{j}-\hat{k})\times(\hat{i}-5\hat{j}-3\hat{k})$

$$=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 5 & 1 & -1\\ 1 & -5 & -3\end{vmatrix}$$

$$=(-8)\hat{i}-(-14)\hat{j}+(-26)\hat{k}=-8\hat{i}+14\hat{j}-26\hat{k}$$

Q2.(a) ***Resolve the expression*** $\dfrac{1+3x+2x^2}{(1-2x)(1-x^2)}$ ***into partial fraction.***

Ans. The degree of numerator is less than denominator.

$$\frac{1+3x+2x^2}{(1-2x)(1-x)(1+x)}=\frac{(1+x)(1+2x)}{(1-2x)(1-x)(1+x)}$$

$$=\frac{1+2x}{(1-2x)(1-x)}$$

$$\frac{1+2x}{(1-2x)(1-x)}=\frac{A}{1-2x}+\frac{B}{1-x}$$

$$1+2x=A(1-x)+B(1-2x)$$

Euating the co-efficient of x and constant terms, we get

$2=-A-2B$ ......(1)

$1=A+B$ ......(2)

Solving (1) and (2)

$$A+B=1$$
$$\underline{-A-2B=2}$$
$$-B=3$$

$B=3$

$A=1+3=4$

$$\therefore \frac{1+3x+2x^2}{(1-2x)(1-x^2)}=\frac{1+2x}{(1-2x)(1-x)}=\frac{4}{1-2x}-\frac{3}{1-x}$$

2.(b) Show that $\begin{vmatrix}1-x & x & x^2\\ 1-y & y & y^2\\ 1-z & z & z^2\end{vmatrix}=(x-y)(y-z)(z-x)$

Ans. $C_1\to C_1+C_2$

$$\begin{vmatrix}1 & x & x^2\\ 1 & y & y^2\\ 1 & z & z^2\end{vmatrix}$$

$R_2\to R_2-R_1;\ R_3\to R_3-R_1$

$$\begin{vmatrix}1 & x & x^2\\ 0 & y-x & y^2-x^2\\ 0 & z-x & z^2-x^2\end{vmatrix}$$

$$=(y-x)(z-x)\begin{vmatrix}1 & x & x\\ 0 & 1 & y+x\\ 0 & 1 & z+x\end{vmatrix}$$

$$=(y-x)(z-x)\{1(z+x-y-x)\}$$

$$=(y-x)(z-x)(z-y)$$

$$=(x-y)(y-z)(z-x)\ \text{Proved.}$$

Q3.(a) Show that the middle term in the expansion of $\left(x-\dfrac{1}{x}\right)^{2n}$ ***is*** $\dfrac{1.3.5.7......(2n-1).(-2)^n}{n!}$.

Ans. Middle term in the expansion of $(x-1/x)^{2n}$

Here exponent = 2n (even)

No. of term = 2n + 1(odd)

$\therefore$ there will be only one middle term

i.e. $\left(\dfrac{2n}{2}+1\right)^{th}$

i.e. $(n+1)^{th}$ term

i.e. $T_{n+1}={}^{2n}C_n.x^{2n-r}.(-1/x)^n$

$$=\frac{2n!}{(2n-n)!\,n!}.x^n.\frac{1}{x^n}(-1)^n$$

$$=\frac{2n.(2n-1)(2n-2)......4.2}{n!\times n!}.(-1)^2$$

$$=\frac{2^n.\{1.2.3.4.....(2n-1)\}\ (-1)^n}{n!\times n!}$$

$$=\frac{(-2)^n.\{1.3.5.7.....(2n-1)\}}{n!}\ \text{Proved.}$$

Q3.(b) Find the term independent of x in the expansion of $\left(2x^2-\dfrac{3}{x^3}\right)^{25}$.

Ans. The term independent of x i.e. x^0 in $\left(2x^2-\dfrac{3}{x^3}\right)^{25}$

let it be in (r + 1)th term,

$$T_{r+1}={}^{25}C_r.(2x^2)^{25-r}\left(\frac{-3}{x^3}\right)^r 6$$

$$={}^{25}C_r(2)^{25-r}.x^{50-2r}.(-3)^r.x^{-3r}$$

$$=(-1)^r.{}^{25}C_r.2^{25-r}.x^{50-5r}.(-3)^r$$

Put $50-5r=0$

i.e. 11th term is independent of x.

$$T_{10+1} = {}^{25}C_{10}.2^{15}.(-3)^{10}.x^0$$

$$= 2^{15}.3^{10}.{}^{25}C_{10}.$$

Q4. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$. ***Show that*** $A^2 = A^{-1}$.

Ans. Given

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & -1 & 0 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2+1 & -1+1+0 & 1+0+0 \\ 2-2+0 & -2+1+0 & 2+0+0 \\ 1+0+0 & -1+0+0 & 1+0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & +1 \end{bmatrix}$$

A

Determinant of matrix A -

$$|A| = \begin{vmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{vmatrix} = (0+1) = 1 \neq 0$$

Co-factors of A : Co-factors of Ist row

$$C_{11} = (-1)^{1+1} \begin{vmatrix} -1 & 0 \\ 0 & 0 \end{vmatrix} = 0$$

$$C_{12} = (-1)^{1+2} \begin{vmatrix} 2 & 0 \\ 1 & 0 \end{vmatrix} = 0$$

$$C_{13} = (-1)^{1+3} \begin{vmatrix} 2 & -1 \\ 1 & 0 \end{vmatrix} = 1$$

Co-factors of 2nd row:

$$C_{21} = (-1)^{2+1} \begin{vmatrix} -1 & 1 \\ 0 & 0 \end{vmatrix} =$$

$$C_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = -1$$

$$C_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 1 \\ 2 & 0 \end{vmatrix} = 2$$

$$C_{33} = (-1)^{3+3} \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} = 1$$

Co-factor matrix :

$$B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & -1 \\ 1 & 2 & 1 \end{bmatrix}$$

Adj. of A = B'

$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{\text{Adj. of A}}{|A|}; \quad A^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$$

$\therefore A^2 = A^{-1}$ Hence proved.

Q5.(a) If $\frac{\log x}{q-r} = \frac{\log y}{r-p} = \frac{\log z}{p-q}$, ***then prove that***

$x^{q+r}.y^{r+p}.z^{p+q} = x^p.y^2.z^r$.

Ans. Given :

$$\frac{\log x}{q-r} = \frac{\log y}{r-p} = \frac{\log z}{p-q} = k$$

$\therefore$ Ist and the last

$$\log x = k.(q-r)$$

$$x = 10^{k(q-r)} \qquad \text{..........(i)}$$

Similarly,

$$y = 10^{k(r-p)} \qquad \text{..........(ii)}$$

and $z = 10^{k(p-q)}$ ....(iii)

Now, taking powers $(q+r)$, $(r+p)$, $(p+q)$ on both sides of (i), (ii) and (iii) then multiplying, we get

$$x^{q+r}.y^{r+p}.z^{p+q} = 10^{k(q-r)(q+r)}.10^{k(r-p)(r+p)}$$

$$= 10^{k(q^2-r^2+r^2-p^2+p^2)}\,10^{k(p-q)(p+q)}$$

$$= 10^{k(0)}$$

$$= 10^0 = 1 \qquad \text{..........(iv)}$$

Again, taking powers p, q, r on both sides of (i), (ii) and we got

$= 10^{0} - 1$ ....(v)

$x^{q+r}.y^{r+p}.z^{p+q} = x^{p}.y^{q}.z^{r}$ from (iv) and (v) **Proved.**

Q5.(b) Prove that $\log_a m^n = n \log_a m$

Ans. Let $\log_a m^n = x$

than $a^x = m^n$ ....(i)

Again let, $\log_a m = y$

$\Rightarrow a^y = m$ ....(ii)

from (i) and (ii)

$a^x = (a^y)^n$; $x = ny$

$\log_a m^n = n \log_a m$ Hence proved.

Q6.(a) Prove that :

$$\sin^{-1} x + \sin^{-1} y = \sin^{-1}\left[x.\sqrt{1-y^2} + y\sqrt{1-x^2}\right]$$

Ans. Let $\sin^{-1} x = \alpha$ $\Rightarrow x = \sin\alpha$

$\sin^{-1} y = \beta$ $\Rightarrow y = \sin\beta$

Now, $\sin(\alpha + \beta + \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$

$$= x.\sqrt{1-\sin^2\beta} + y.\sqrt{1-\sin^2 x}$$

$$= x.\sqrt{1-y^2} + y.\sqrt{1-x^2}$$

$$\therefore \alpha + \beta = \sin^{-1}[x.\sqrt{1-y^2} + y.\sqrt{1-x^2}]$$

i.e. $\sin^{-1} x + \sin^{-1} y = \sin^{-1}\left[x.\sqrt{1-y^2} + y.\sqrt{1-x^2}\right]$

Hence proved.

Q6.(b) Prove that $2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} = \pi/4$.

Ans. $2\tan^{-1}\frac{1}{3} = \tan^{-1}\frac{2/3}{1-1/3^2}$

$$= \tan^{-1}\frac{2}{3}\times\frac{9}{8} = \tan^{-1}\frac{3}{4}$$

$$\therefore 2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\times\frac{1}{7}} = \tan^{-1}\frac{25/28}{25/28}$$

$$= \tan^{-1} 1 = \pi/4.$$

i.e. $2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} = \pi/4$ Proved.

Q7. (a) If $A + B = 45^\circ$, show that $(1 + \tan A)(1 + \tan B) = 2$.

Ans. Given : $A + B = 45^0$

$\tan(A + B) = \tan 45^0$

$$\frac{\tan A + \tan B}{1 - \tan A.\tan B}$$

$\tan A + \tan B = 1 - \tan A.\tan B$

$\tan A + \tan B + \tan A.\tan B = 1$

Adding 1 on both sides, we get

$1 + \tan A + \tan B + \tan A.\tan B = 1 + 1 = 2$

$(1 + \tan A) + \tan B(1 + \tan A) = 2$

$(1 + \tan A)(1 + \tan B) = 2$ Hence proved.

Q7. (b) Prove that :

$$\sin 12^0.\sin 48^0.\sin 54^0 = \frac{1}{8}.$$

Ans. L.H.S. $= \frac{1}{2}\{2\sin 48^0.\sin 12^0\}.\sin 54^0$

$$= \frac{1}{2}\{\cos(48-12) - \cos(48+12)\}.\sin 54^0$$

$$= \frac{1}{2}\{\cos 36^0 - \cos 60^0\}.\sin 54^0$$

$$= \frac{1}{2}\{\cos 36^0 - \frac{1}{2}\}.\cos(90 - 54^0)$$

$$= \frac{1}{2}\{\cos 36^0 - \frac{1}{2}\}.\cos 36^0\}$$

$$= \frac{1}{2}\left[\left(\frac{\sqrt{5}+1}{4}\right) - \frac{1}{2}\right]\frac{\sqrt{5}-1}{4}$$

$$= \frac{1}{8}\left(\sqrt{5}+1-2\right)\left(\frac{\sqrt{5}+1}{4}\right)$$

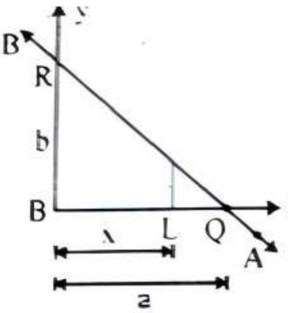
$$= \frac{1}{8}\left\{\frac{(\sqrt{5}-1)(\sqrt{5}+1)}{4}\right\}$$

$$= \frac{1}{8}\left\{\frac{5-1}{4}\right\} = \frac{1}{8}$$ Proved.

Q8.(a) Show that the eqn. of the line making intercept a and b on the co-ordinate axes is $\frac{x}{a} + \frac{y}{b} = 1$.

Ans. Let AB be a line which cuts of intercepts and b on x-axis and y-axis respectively.

Let line AB cuts x-axis at Q and Y-axis at R i.e. OQ = a and OR = b



Let P(x, y) be any point on line AB, then OL = x and PL = y.

Now, from Δ' QLP and ΔQOR, we have

$$\frac{QL}{QO} = \frac{PL}{OR}$$

or, $\frac{a-x}{a} = \frac{y}{b}$ or, $1-\frac{x}{a} = \frac{y}{b}$

or, $\boxed{\frac{x}{a}+\frac{y}{b}=1}$ ....(i)

Since (i) will be satisfied by the co-ordinates of all points on line AB and will not be satisfied by the co-ordinates of any point which does not lie on AB and hence (i) is the required equation of line AB.

Q8.(b) In what ratio is the line joining the points (2, 3) and (4, –5) divided by the line passing through the points (6, 8) and (–3, –2)?

Ans. Let AB and CD be the intersecting line joining the points (2, 3) A(2, 3) and B(4, –5) and C(6, 8) and D(–3, –2).

Let P(x, y) be the point of intersection. It divides AB in the ratio AP = PB = m : n.

Eqn. AB :

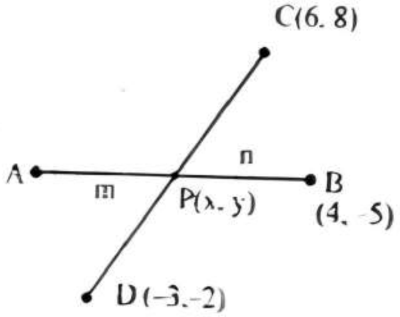
$$y-3 = \frac{-5-3}{4-2}(x-2)$$

$$y-3 = \frac{-8}{2}(x-2)$$

or $y-3 = (-4)(x-2)$

or $y-3 = -4x+8$

or $4x+y = 11$ ....(1)



eqn. CD :

$$y-8 = \frac{-2-8}{-3-6}(x-6)$$

$$y-8 = \frac{-10}{-9}(x-6)$$

$10x-9y+12=0$ ....(2)

For point P :

Solving (1) and (2)

$4x+y=11$ ......(1) × 9

$10x-9y=-12$ ......(2) × 1

$$\begin{array}{r} 36x+9y=99 \\ 10x-9y=-12 \\ \hline 46x=+87 \end{array}$$

$$x = \frac{87}{46}$$

Substituting $x = \frac{87}{46}$ in eqn.(i); we get

$$4\times\frac{87}{46}+y=11$$

$$y = 11-4\times\frac{87}{46}$$

$$= 11-\frac{174}{23} = \frac{253-174}{23} = \frac{79}{23}$$

Co-ordinates of $P(x, y) = \left(\frac{87}{46}, \frac{79}{23}\right)$

using section formula

$$\left(\frac{87}{46}, \frac{79}{23}\right) = \left\{\frac{m.4+2n}{m+n}, \frac{-5m+3n}{m+n}\right\}$$

$$\frac{87}{46} = \frac{4m+2n}{m+n}$$

$87(m+n) = 46(4m+2n)$

$87m+87n = 184m+92n$

$87m-184m = 92n-87n$

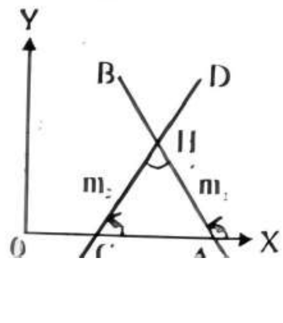
$-97m = 5n$

$\frac{m}{n} = \frac{5}{-97}$ i.e., 5 : (–97).

Q9. (a) Find the angle between two lines whose equation given by $y = m_1x+C_1$ & $y = m_2x+C_2$ respectively.

Ans. Let AB and CD be two given lines having slopes m_1 a respectively.

Let AB and CD make angles θ_1 and θ_2 respectively wi positive direction of x-axis.



... it is clear that

$\theta = \theta_1 - \theta_2$

$\tan\theta = \tan(\theta_1 - \theta_2)$

$= \dfrac{\tan\theta_1 - \tan\theta_2}{1 + \tan_1 - \tan\theta_2}$

$$\tan\theta = \frac{m_1 - m_2}{1 + m_2.m_2} \qquad \text{.........(1)}$$

Again $\alpha = \pi - \theta$

$\tan\alpha = \tan(\pi - \theta) = -\tan\theta$

$= \dfrac{m_1 - m_2}{1 + m_1 m_2}$

$$\tan\alpha = -\frac{m_1 - m_2}{1 + m_1.m_2} \qquad \text{.........(2)}$$

$\therefore$ from (1) and (2), it is clear that the two angles between lines AB and CD are given by

$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1.m_2} \qquad \text{.........(3)}$$

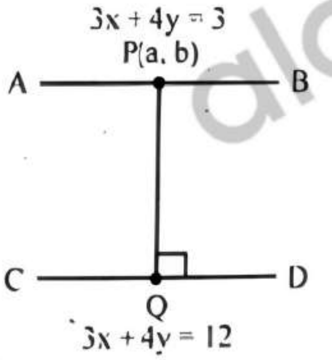
Acute angle θ between lines AB & CD is given by

$$\boxed{\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1.m_2}\right|}$$

Q9. (b) Find the distance between the parallel lines $3x + 4y = 3$ and $3x + 4y = 12$.

Ans. The eqn. of the given st. lines

$3x + 4y\ 3$



$3x + 4y = 3$ ....(1)

& $3x + 4y = 12$ ....(2)

Clearly, they are parallel.

Distance between parallel straight lines (1) and (2) = the length of the perpendicular drawn from

$= P(\alpha, \beta)$ upon line (2).

$$= \left|\frac{12 - 3}{\sqrt{3^2 + 4^2}}\right| = \left|\frac{9}{5}\right| = \frac{9}{5}.$$

Q10.(a) Find the equation to the circle describes on the line joining the points (x_1, y_1) as diameter.

Ans. Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be the extremitities of a diameter and but $P(\alpha, \beta)$ be any point on the circle.

Now slope of $AP = \dfrac{\beta - y_1}{\alpha - x_1} = m_1$ (say)

and slope of $PB = \dfrac{\beta - y_2}{\alpha - x_1} = m_2$ (say)

Now, $\angle APB = 90^0$

$\therefore m_1.m_2 = -1$

i.e. $\dfrac{\beta - y_1}{\alpha - x_1} \times \dfrac{\beta - y_2}{\alpha - x_1} = -1$

$(\beta - y_1) \times (\beta - y_2) = -(\alpha - x_1)(\alpha - x_2)$

$(\alpha - x_1)(\alpha - x_2) + (\beta - y_1)(\beta - y_2) = 0$ ....(1)

Eqn., (1) is also valid when $P(\alpha, \beta)$ coincides with $A(x_1, y_1)$ or $P(\alpha, \beta)$ coincides with B. Thus eqn.(1) is valid for any arbitrary point $P(\alpha, \beta)$ on the circle.

Hence, equation of the circle which is the locus of point $P(\alpha, \beta)$.

$$\boxed{(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0}$$

10.(b) Find the equation of the circle with the circle $2x^2 + 2y^2 + 8x + 12y - 25 = 0$ and having its circumference equal to 6π units.

Ans. Given equation of circle

$2x^2 + 2y^2 + 8x + 12y - 25 = 0$

$2g = 4$; $g = 2$; $2f = 12$; $f = 6$

$\therefore$ centre $(-g, -f) = (-2, -6)$

Circumference of required circle

$\pi 6 = 2\pi r$; $2r = 6$

$r = 3$

$\therefore$ eqn., of required concentric circle having centre $(-2, -6)$ and $r = 3$.

i.e. $(x + 2) + (y + 6)^2 = 3^2$

$x^2 + 4x + 4 + y^2 + 12y + 36 = 9$

i.e. $x^2 + y^2 + 4x + 12y + 31 = 0$.

Q11.(a) If $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$. Calculate the angle between the vectors $2\vec{a} + \vec{b}$ and $\vec{a} + 2\vec{b}$.

Ans. Given :

$\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$, $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$

then, $2\vec{a} + \vec{b} = 2(\hat{i} + 2\hat{j} - 3\hat{k}) + (3\hat{i} + 2\hat{j} - 3\hat{k})$

$= 5\hat{i} + 6\hat{j} - 9\hat{k}$

and $\vec{a} + 2\vec{b} = (\hat{i} + 2\hat{j} - 3\hat{k}) + 2(3\hat{i} - \hat{j} + 2\hat{k})$

$= 7\hat{i} + \hat{k}$

Let 'θ' be the angle between $2\vec{a} + \vec{b}$ & $\vec{b} + 2\vec{a}$