Engineering mathematics (2016)

Q1. Choose the correct answer from the following questions

(i) The value of $\log_{2\sqrt{3}} 144$ *is ....*

(a) +4 (b) –4 (c) both (d) None

Sol.(a)

$$\log_{2\sqrt{3}} 144 = x$$

$$(2\sqrt{3})^x = 144$$

$$2^x 3^{x/2} = 2^4.3^2$$

$$x = 4$$

(ii) The value of the determinant $\begin{bmatrix} 18 & 1 & 17 \\ 22 & 3 & 19 \\ 26 & 5 & 21 \end{bmatrix} = ...$

Sol. $18(21\times3-19\times5)-1(22\times21-19\times26)+17(22\times5-26\times3)$

$= 18(63 - 95 - 1(462 - 494 + 17(110 - 78)$

$= -576 + 32 + 544 = 0$

(iii) Find the 5th term in the expansion of $(1 + x)^5$.

Sol. $T_{r+1} = {}^5C_r (1)^{5-r} (x)^r$

$= {}^5C_4 x^5$

(iv) A square matrix A is said to be singular if $|A| = ...$

Sol. One

(v) The value of $\sin\left(\frac{31\pi}{3}\right) = ...$

Sol. $\frac{\sqrt{3}}{2}$

(vi) In ΔABC, *if a + b + c = 2s, then* $\cos\frac{A}{2} = ...$

Sol. cos B is equal to $= \frac{a^2 + c^2 - b^2}{2a}$.

(vii) If two lines are parallel to each other having their slopes m_1 *and* m_2, *then* $m_1 = ...$.

Sol. zero

(viii) Angle between the lines whose slopes are 1/2 and 3 is

Sol.

$$\tan^{-1}\left(\frac{m_2 - m_1}{1 + m_1 m_2}\right) = \tan^{-1}\left(\frac{3 - \frac{1}{2}}{1 + \frac{1}{3} + 3}\right)$$

$$= \tan^{-1}\left(\frac{\frac{5}{2}}{\frac{5}{2}}\right) = \tan^{-1}(1) = 95^0 \frac{\pi}{4}$$

(ix) The radius of the circle $x^2 + y^2 - 4x + 6y - 5 = 0$ is ....

Sol. $2g = -4$; $2f = 6$.

$g = -2$; $f = 3$, $c = -5$.

$$\text{radius} = \sqrt{g^2 + f^2 - c} = \sqrt{4 + 9 + 25} = \sqrt{38}$$

(x) The value of $\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + c \times (\vec{a} + \vec{b}) =$ ......

Sol. zero

Q2.(a) If $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b}$, then prove that $a^{b+c}, b^{c+a}, c^{a+b} = 1$.

Sol. Given, $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b} = k$

Taking 1st and last term

$$\frac{\log a}{b-c} = k$$

$\log a = k(b-c)$

$a = 10^{k(b-c)}$ ....(i)

Similarly, $b = 10^{k(c-a)}$ ....(ii)

$c = 10^{k(a-b)}$ ....(iii)

Now, taking powers $(b+c)$, $(c+a)$, $(a+b)$ on both sides of (i), (ii) and (iii) then multiplying, we get

$$a^{b+c}, b^{c+a}, c^{a+b} = 10^{k(b-c)(b+c)} . 10^{k(c+a)(c-a)} . 10^{k(a-b)(a+b)}$$

$$= 10^{k(b^2 - c^2) + c^2 - a^2 + a^2 - b^2)}$$

$$= 10^{k(0)} = 10^0 = 1$$ Proved.

Q2.(b) Resolve $\frac{x-1}{(x-2)(x-3)}$ into partial fraction.

Sol. $\frac{x-1}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$

Putting $k = 2$ in LHS and then

We get, $x = 3$

$A = -1$ and $B = 2$

Or

$x - 1 = A(x-3) + B(x-2)$

Equating the coefficient of x^2, x and constant term.

$x - 1 = Ax - 3A + Bx - 2B$.

$A + B = 1$ ....(1)

$-3A - 2B = -1$ ....(2)

$$\begin{aligned} 2A + 2B &= 2 \\ 3A + 2B &= 1 \\ \hline -A \quad &= 1 \end{aligned}$$

$A = -1$

$B = 2$

$\therefore$ Required partial fraction

$$= -\frac{1}{x-2} + \frac{2}{x-3}.$$

Q3.(a) Find the coefficient of x7 in the expansion of $\left(x^2 + \frac{1}{x}\right)^{11}$.

Sol. $T_{r+1} = {}^{11}C_r (x^2)^{11-r} . \left(\frac{1}{x}\right)^r$

$= {}^{11}C_r x^{22-2r} . x^{-r}$

$= {}^{11}C_r . x^{22} - 3r$

But $x^7 = x^{22-3r}$

$22 - 3r = 7$

$3x = 15$

$r = 5$

$\therefore$ the Co-efficient of x7 $= {}^{11}C_5$.

Q3.(b) Prove that

$$\begin{vmatrix} x+a & 1 & 1 \\ 1 & x+b & 1 \\ 1 & 1 & x+c \end{vmatrix} = x^2(x+a+b+c).$$

Sol. $\Delta = \begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix}$

$C_1 \to C_1 + C_2 + C_3$

$$\Delta = \begin{vmatrix} x+a+b+c & b & c \\ x+a+b+c & x+b & c \\ x+a+b+c & b & x+c \end{vmatrix}$$

$$= (x+a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & x+b & c \\ 1 & b & x+c \end{vmatrix}$$

$R_2 \to R_2 - R_1; R_3 - R_3 - R_1 1$

$$= (x+a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & x & 0 \\ 0 & 0 & x \end{vmatrix}$$

$$= x^2(x+a+b+c)$$ Proved.

Q4. Solve the following system of equation by matrix method: $2x - y + 3z = 9$; $x + y + z = 6$; $x - y + z = 2$

Sol. Same as 2013, Ques. 4.

Q5. Prove that $\cos x \cos 2x \cos 4x \cos 8x = \dfrac{\sin 16x}{16\sin x}$

Sol. L.H.S $= \cos x \cos 2x \cos 4x \cos 8x$

$$= \frac{(2\sin x\cos x)}{2\sin x}\cos 2x,\ \cos 4x \cos 8x$$

$$= \frac{(2\sin 2x\cos 2x)}{4\sin x}.\cos 4x \cos 8x$$

$$= \frac{(2\sin 4x\cos 4x)}{8\sin x}\cos 8x$$

$$= \frac{2\sin 8x}{16\sin x}\cos 8x$$

$$= \frac{\sin 16x}{16\sin x} = \text{R.H.S.} \qquad \text{Proved.}$$

Q6.(b) ***Prove that*** $2\tan^{-1}\dfrac{1}{3} + \tan^{-1}\dfrac{1}{7} = \dfrac{\pi}{4}$.

Sol. L.H.S. $= 2\tan^{-1}\dfrac{1}{3} + \tan^{-1}\dfrac{1}{7}$

$$= \tan^{-1}\left(\frac{2\times\frac{1}{3}}{1-\frac{1}{9}}\right)\tan^{-1}\left(\frac{1}{7}\right)\left(\because 2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}\right)$$

$$= \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= \tan^{-1}\left(\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\times\frac{1}{7}}\right)$$

$$\left(\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right)$$

$$= \tan^{-1}(1) = \frac{\pi}{4} = \text{R.H.S.} \qquad \text{Proved.}$$

Q7.(a) If the point (h, 0), (a, b) and (0, k) lie on a line, show that $\dfrac{a}{h} + \dfrac{b}{k} = 1$.

Sol. This three points lie on a line i.e. collinear,

Area $\Delta ABC = 0$.

$$\Rightarrow \begin{vmatrix} a & b & 1 \\ h & 0 & 1 \\ 0 & k & 1 \end{vmatrix} = 0$$

$$\Rightarrow a(0-k) - b(h-0) + 1(hk-0) = 0$$

$$-ak - bh + hk = 0$$

$$ak + bh = hk$$

$$\frac{a}{h} + \frac{b}{k} = 1 \qquad \text{Proved.}$$

Q7.(b) Find the equation of the line passing through the point (−2, −4) and perpendicular to the line $3x - y + 5 = 0$.

Sol. Let the required equation of St. line which is perpendicular to the given line and passes through $(-2, -4)$ be $-x + 3y + k = 0$.

Also it passes through $(-2, -4)$

$$2 + 3 \times -4 + k = 0$$

$$k = 10$$

$\therefore$ Required equation $= -x + 3y + 10 = 0$.

Q8.(a) If the straight line $\dfrac{x}{a} + \dfrac{y}{b} = 1$ ***passing through the point (8, −9) and (12, −15), find the value of a and b.***

Sol. Given, $\dfrac{x}{a} + \dfrac{y}{b} = 1$

$$bx + ay = ab \qquad \text{........(i)}$$

It passes through the point $(8, -9)$ and $(12, -15)$

Then, $8b - 9a = ab$ ....(ii)

$12b - 15a = ab$ ....(iii)

Solving (ii) and (iii)

$$8b - 9 = 12b - 15a$$

$$4b = 5a$$

$$b = \frac{3}{2}a$$

ting the value of b in eq.(ii)

$$48 \times \frac{3}{2}a - 9a = a \times \frac{3}{2}a$$

$$12a - 9a = \frac{3}{2}a^2$$

$$3a = \frac{3}{2}a^2$$

$$a = 2;\ b = 3.$$

Q8.(b) Show that the equation $3x^2 + 3y^2 + 12x - 18y - 11 = 0$ ***represent a circle. Also find its centre and radius.***

Sol. Given $3x^2 + 3y^2 + 12x - 18y - 11 = 0$ be a 2nd degree non homogeneous equation in x and y.

i.e., $3x^2 + 12x + 3y^2 - 18y - 11 = 0$

$$x^2 + 4x + y^2 - 6y^2 - \frac{11}{3} = 0$$

$$(x+2)^2 + (y-3)^2 - \frac{11}{3} = 0$$

$$(x-(-2))^2 + (y-(3))^2 = \frac{11}{3} \qquad \text{.....(i)}$$

Co-ordinates of its centre (α, β) and radius is r and we know the general equation of circle.

i.e. $(x-\alpha)^2+(y-\beta)^2=r^2$ ......(ii)

Compare (i) and (ii)

Centre $(-g, -f)$ and radius $= \sqrt{g^2+f^2-c}$

Centre $(-2, 3)$ and radius $= \sqrt{4+9-\frac{11}{3}}$

$$= \sqrt{\frac{12+27-11}{3}} = \sqrt{\frac{28}{3}}.$$

Q9.(a) Find the area of the triangle whose two adjacent sides are determined by the vectors $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}+\hat{j}-\hat{k}$.

Sol. Given: $\vec{F}=\hat{i}+\hat{j}+\hat{k}$

$\vec{r}=\hat{i}+\hat{j}-2\hat{k}$

Moment of force about the given point.

$\therefore$ Moment $= \vec{r}\times\vec{F}$

$$= (\vec{i}+\vec{j}-2\vec{k})\times(\vec{i}+\vec{j}+\vec{k})$$

$$= \vec{k}-\vec{j}-\vec{k}+\vec{i}-2\vec{j}+2\vec{i}$$

$$= 3\vec{i}-3\vec{j}.$$

Q10.(a) If $A=\begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix}$, then find the value of $A^2-4A+5I_3$

Sol. $A^2=\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}\times\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1+4+4 & 2+2+4 & 2+4+2 \\ 2+2+4 & 4+1+4 & 4+2+2 \\ 2+4+2 & 4+2+2 & 4+4+1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$$

$$-4A = -4\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} -4 & -8 & -8 \\ -8 & -4 & -8 \\ -8 & -8 & -4 \end{bmatrix}$$

$$5I_3 = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$4^2-4A+5I_3 = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}+\begin{bmatrix} -4 & -8 & -8 \\ -8 & -4 & -8 \\ -8 & -8 & -4 \end{bmatrix}+\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 9-4+5 & 8-8+0 & 8-8+0 \\ 8-8+0 & 9-4+5 & 8-8+0 \\ 8-8+0 & 8-8+0 & 9-4+4 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix} \textbf{ Ans.}$$

Q10.(b) Find the adjoint of matrix $\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$

Sol.

$C_{11}=(4+6)=10; C_{12}=-(4-9)=5; C_{13}=(-4-6)=-10$

$C_{21}=-(-6+6)=0; C_{22}=(4+9)=13; C_{23}=-(-4+9)=-5$

$C_{31}=(-9-6)=-15; C_{32}=-(6-6)=0; C_{33}=(4+6)=10$

$$\text{Adj. of matrix} = \begin{bmatrix} 10 & 5 & 10 \\ 0 & 13 & -5 \\ -15 & 0 & 10 \end{bmatrix}^T$$

$$= \begin{bmatrix} 10 & 0 & -15 \\ 5 & 13 & 0 \\ -10 & -5 & 10 \end{bmatrix}.$$