Engineering mathematics (2019)

Q.1. Answer the following:

(i) A.M between (x - y) and (x + y) is equal to:

(a) 2x (b) 2y (c) x (d) y

Ans.

(ii) The value of $\log_a 1$ is equal to

(a) 1 (b) 0 (c) a (d) None

Ans.

(iii) ^nC_r is equal to

(a) $^nC_{n-r}$ (b) $^nC_{n-1}$ (c) $^nC_{r-n}$ (d) None

Ans.

(iv) The value of cosec 270° is equal to

(a) 1 (b) 0 (c) -1 (d) None

Ans.

(v) The value of $\begin{vmatrix} 3 & 5 & 8 \\ 1 & 1 & 2 \\ 2 & 3 & 5 \end{vmatrix}$ *is equal to*

(a) 5 (b) 0 (c) 8 (d) None

Ans.

(vi) The principal value of $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$ *is equal to*

(a) $\frac{3\lambda}{4}$ (b) $\frac{\lambda}{4}$ (c) $\frac{-\lambda}{4}$ (d) None

Ans.

(vii) Two non vertical lines with slopes m_1 and m_2 are perpendicular if and only if

(a) $m_1 = m_2$ (b) $m_1.m_2 = 1$

(c) $m_1.m_2 = -1$ (d) None of these

Ans.

(viii) If the equation $x^2 + y^2 + 4x + 6y + 7 = 0$ represents a circle, then its centre will be

(a) (2, 3) (b) (–2, –3)

(c) (2, 7) (d) None of these

Ans.

Q2.(a) If $x = \log_a(bc)$, $y = \log_b(ca)$, $z = \log_c(ab)$. Prove that

$$\frac{1}{x+1}+\frac{1}{y+1}+\frac{1}{z+1}=1.$$

Ans. LHS $\frac{1}{x+1}+\frac{1}{y+1}+\frac{1}{z+1}$

$$=\frac{1}{\log_a(bc)+\log_a(a)}+\frac{1}{\log_b(ca)+\log_b(b)}+\frac{1}{\log_c(ab)+\log_c(c)}$$

$$=\frac{1}{\log_a(abc)}+\frac{1}{\log_b(abc)}+\frac{1}{\log_c(abc)}$$

$$=\log_{abc}(a)+\log_{abc}(b)+\log_{abc}(c)$$

$$=\log_{abc}(abc)=1=\text{RHS.}$$ Proved.

Q2.(b) If a, b, c are in G.P. prove that $\log_a x, \log_b x, \log_c x$ ***are in H.P.***

Ans. Given: a, b, c are in G.P. Then, $\frac{b}{a}=\frac{c}{b}$.

If $\log_a x$, $\log_b x$ and $\log_c x$ are in HP

Then, $\frac{1}{\log_a x}, \frac{1}{\log_b x}$ and $\frac{1}{\log_c x}$ are in AP.

Now,

$$\frac{1}{\log_b x}-\frac{1}{\log_a x}=\frac{1}{\log_c x}-\frac{1}{\log_b x}$$

$$\log_x b-\log_x a=\log_x c-\log_x b$$

$$\log_x\frac{b}{a}=\log_x\frac{c}{b}$$

$$\therefore \frac{b}{a}=\frac{c}{b}=1$$ Hence Proved.

Q3.(a) Resolve into partial fraction $\frac{x-1}{(x+1)(x-2)}$.

Ans. $$\frac{x-1}{(x+1)(x-2)}=\frac{A}{x+1}+\frac{B}{x-2}$$

Multiplying both sides by (x + 1) (x – 2), we get

$x-1=A(x-2)+B(x+1)$

$\Rightarrow x-1=Ax-2A+Bx+B$

$\Rightarrow x-1=(A+B)x+B-2A$

$A+B=1$ .... (i) and $B-2A=-1$ ..... (ii)

Putting the value of A in eqn. (ii)

$B-2(1-B)=-1$

$B-2+2B=-1$

$3B=1$

$B=\frac{1}{3}$ and $A=1-\frac{1}{3}=\frac{2}{3}$

$$\therefore \frac{x-1}{(x+1)(x-2)}=\frac{2}{3(x+1)}+\frac{1}{3(x-2)}$$ Ans.

Q3.(b) Prove that $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

Ans.

Q4.(a) Find the coefficient of x7 in the expansion of $\left(x^2+\dfrac{1}{x}\right)$.

Ans. Binomial expansion of $\left(x^2+\dfrac{1}{x}\right)^{11}$

$$= {}^{11}C_r (x^2)^{11-r}\left(\frac{1}{x}\right)^r$$

$$= {}^{11}C_r\, x^{22-2r}.x^{-r}$$

$$= {}^{11}C_r\, x^{22-3r}$$

For the term containing x^7, we have

$22 - 3r = 7$

$3r = 15$

$r = 15$

So, the term containing x^7 in binomial expansion of given expression is

$$= {}^{11}C_5\, x^{22-3\times 5}$$

$$= {}^{11}C_5\, x^7$$

$$= \frac{11!}{5!\,(11-5)!}x^7$$

$$= \frac{11\times 10\times 9\times 3\times 7}{5\times 4\times 3\times 2}x^7$$

$$= 11\times 3\times 2\times 7\, x^7$$

$$= 462x^7$$

Hence, the coefficient of x^7 is 462.

Q4.(b) Find the middle term in the expansion of $\left(1-\dfrac{x^2}{2}\right)$.

Ans. Binomial expension of $\left(1-\dfrac{x^2}{2}\right)^{14}$

$$= {}^{14}C_r (1)^{14-r}\left(\frac{-x^2}{2}\right)^r$$

Hence, n = 14, which is even

So, the middle term of the expansion is $\left(\dfrac{n+2}{2}\right)^{th}$ term,

i.e., 8^{th} terms

For 8th term, ...

$r + 1 = 8$

$r = 7$

Hence, the middle term in binomial expansion of given expression is

$$T_8 = {}^{14}C_7 (1)^{14-7}\left(\frac{-x^2}{2}\right)^7$$

$$= \left(\frac{-1}{2}\right)^7 {}^{14}C_7\, x^{14}$$

$$= -\frac{1}{128}\times\frac{14\times 13\times 12\times 11\times 10\times 9\times 8}{7\times 6\times 5\times 4\times 3\times 2\times 1}x^{14}$$

$$= \frac{-429}{16}x^{14} \quad \text{Ans.}$$

Q5.(a) If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$***, show that $A^2 = 5A + 7 = 0$.***

Ans. Given:

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$-5A = -5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -15 & -5 \\ 5 & -10 \end{bmatrix}$$

$$7I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

Now,

$$\text{LHS} = A^2 - 5A + 7 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} + \begin{bmatrix} -15 & -5 \\ 5 & -10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix} = 0 = \text{RHS}$$

Hence Proved.

Q5.(b) Solve the following by matrix inversion method.

$$\left.\begin{aligned} x - y + z &= 1 \\ 2x + y - z &= 2 \\ x - 2y - z &= 4 \end{aligned}\right\}$$

Ans. Given:

$$x - y + z = 1$$
$$2x + y - z = 2$$
$$x - 2y - z = 4$$

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ 1 & -2 & -1 \end{bmatrix} \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

$AX = B$
$X = A^{-1}B$

$$\text{adj } A = \begin{bmatrix} (-1-2) & (-2+1) & (-4-1) \\ (1+2) & (-1-1) & (-2+1) \\ (1-1) & (-1-2) & (1+2) \end{bmatrix}^T$$

$$= \begin{bmatrix} -3 & -1 & -5 \\ 3 & -2 & -1 \\ 0 & -3 & 3 \end{bmatrix}^T$$

$$\text{adj } A = \begin{bmatrix} -3 & 3 & 0 \\ -1 & -2 & -3 \\ -5 & -1 & 3 \end{bmatrix}$$

$$|A| = 1(-1-2) + 1(-2+1) + 1(-4-1)$$
$$= -3 - 1 - 5 = -9$$
$$\therefore |A| = 9$$

$$A^{-1} = \frac{1}{|A|} \text{adj} A = \frac{1}{9}\begin{bmatrix} -3 & 3 & 0 \\ -1 & -2 & -3 \\ -5 & -1 & 3 \end{bmatrix}$$

$$X = \frac{1}{9}\begin{bmatrix} -3 & 3 & 0 \\ -1 & -2 & -3 \\ -5 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} -3+6+0 \\ -1-4-12 \\ -5-2+12 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 3 \\ -17 \\ 5 \end{bmatrix}$$

$$X = \begin{bmatrix} 1/3 \\ -17/9 \\ 5/9 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1/3 \\ -17/9 \\ 5/9 \end{bmatrix}$$

$$x = \frac{1}{3};\ y = \frac{-17}{9};\ z = \frac{5}{9} \quad \text{Ans.}$$

Q6.(a) Prove that $2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} = \frac{\pi}{4}$.

Ans. ***Same as 2016, Q.no. 6(b).***

Q6.(b) If $\tan A = \frac{5}{6}$ ***and*** $\tan B = \frac{1}{11}$***, prove that*** $A + B = 45^\circ$.

Ans. Given:

$$\tan A = \frac{5}{6} \text{ and } \tan B = \frac{1}{11}$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A.\tan B}$$

$$= \frac{\frac{5}{6} + \frac{1}{11}}{1 - \frac{5}{6} \times \frac{1}{11}}$$

$$= \frac{\frac{55+6}{66}}{\frac{66-5}{66}} = \frac{\frac{61}{66}}{\frac{61}{66}}$$

$\tan(A + B) = 1$
$(A + B) = \tan^{-1}(1)$

$$= \frac{\pi}{4} = \text{RHS}$$ Hence proved.

Q7.(a) In any ΔABC***, if*** a^2, b^2, c^2 ***are in A.P. prove that CotA, CotB, CotC are in A.P.***

Ans. Given: a^2, b^2, c^2 are in A.P

$$\Rightarrow b^2 - a^2 = c^2 - b^2$$
$$\Rightarrow K^2\sin^2 B - K^2\sin^2 A = K^2\sin^2 C - K^2\sin^2 B$$
$$\Rightarrow \sin(B+A).\sin(B-A) = \sin(C+B).\sin(C-B)$$
$$\Rightarrow \sin C.\sin(B-A) = \sin A.\sin(C-B)$$
$$(\because A + B + C = \pi)$$

$$\Rightarrow \frac{\sin(B-A)}{\sin A} = \frac{\sin(C-B)}{\sin C}$$

$$\Rightarrow \frac{\sin B.\cos A - \cos B.\sin A}{\sin A.\sin B}$$

$$= \frac{\sin C.\cos B - \cos C.\sin B}{\sin C.\sin B}$$

$$\Rightarrow \cot A - \cot B = \cot B - \cot C$$

Hence, cotA, cotB, cotC are in A.P.

Q7.(b) Prove that $\text{Sin}\,75^\circ = \dfrac{\sqrt{6}+\sqrt{2}}{4}$.

Ans. $\sin 75^\circ = \sin(45^\circ + 30^\circ)$

$$= \sin 45^\circ.\cos 30^\circ + \cos 45^\circ.\sin 30^\circ$$

$$(\sin(A+B) = \sin A.\cos B + \cos A.\sin B)$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

Multiplying $\sqrt{2}$ in numerator and denomenator

$$= \frac{\sqrt{3}+1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{4} = \text{RHS}$$ Hence Proved.

Q8.(a) Find the equation of a line passing through the points. (–1, 1) and (2, –4).

Ans. Points are (–1, 1) and (2, –4) equation to the straight line passing the points (–1, 1) and (2, –4) is

$$y - (1) = \frac{-4-1}{2+1}(x+1)$$

$$y - 1 = \frac{-5}{3}(x+1)$$

$$3y - 3 = -5x - 5$$

$$5x - 3y + 2 = 0$$

Q8.(b) Find the equation of line passing through the point (3, –2) and perpendicular to the line x – 3y + 720.

Ans. The equation of any line perpendicular to

$x - 3y + 7 = 0$ is

$$-3x + y + \lambda = 0$$

If it passes through the point (3, –2), then

$$-3 \times 3 + (-2) + \lambda = 0$$

$$-9 - 2 + \lambda = 0$$

$$\lambda = 11$$

Thus equation of required line is $-3x + y + 11 = 0$.

Q9.(a) Find the equation of the circle whose centre is (2, –5) and which passes through the point (3, 2).

Ans. The general equation for a circle with center (a, b) and radius r is

$$(x-a)^2 + (y-b)^2 = r^2$$

we are given,

$a = 2$ and $b = -5$

Given the center (2, –5) and a point on the circumference radius (3, 2), we can evaluate the radius by pythagorean theorem

$$r^2 = (2-3)^2 + (-5-2)^2$$

$$r^2 = 1 + 49$$

$$r^2 = 50$$

Therefore the equation of the circle is

$$(x-2)^2 + (y-(-5))^2 = 50$$

$$(x-2)^2 + (y+5)^2 = 50$$

Q9.(b) Show that the equation $x^2 + y^2 - 6x + 4y - 36 = 0$ represents a circle. Also find its centre and radius.

Ans. Given: The equation is

$$x^2 + y^2 - 6x + 4y - 36 = 0$$

Here $2g = -6$

$g = -3$

$2f = 4$

$f = 2$ and $c = -36$

Thus radius $r = \sqrt{g^2 + f^2 - c}$

$$= \sqrt{(-3)^2 + (2)^2 - (-36)}$$

$$= \sqrt{9 + 4 + 36}$$

$$= \sqrt{49} = 7 > 0$$

As the radius is > 0, so the given equation represent a real circle

Centre is $(-g, -f)$

$= (3, -2)$ and radius = 7.

Q10.(a) Prove that $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{d}) + \vec{c} \times (\vec{a} + \vec{b}) = 0$

Ans. *Same as 2014, Q.12(b)*

Q10.(b) A force $\vec{F} = 3\vec{i} + 2\vec{j} - 4\vec{k}$ acting at a point (1, –1, 2). Find the moment of the force about the point (2, –1, 3).

Ans. $\vec{F} = 3\hat{i} + 2\hat{j} - 4\hat{k}$

P(1, -1, 2) Q(2, -1, 3)

$$\vec{r} = \overrightarrow{PQ} = (2-1)\hat{i} + (-1+1)\hat{j} + (3-2)\hat{k}$$

$$= \hat{i} + \hat{k}$$

$\therefore$ Moment of force $= \vec{r} \times \vec{F}$

$$= (\hat{i} + \hat{k}) \times (3\hat{i} + 2\hat{j} - 4\hat{k})$$

$$\vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 1 \\ 3 & 2 & -4 \end{vmatrix}$$

$$= \{(0 \times -4) - (2 \times 1)\}\,\hat{i} + (3 \times 1 + 4)\,\hat{j} + (2 - 0)\,\hat{k}$$

$$\vec{r} \times \vec{F} = -2\hat{i} + 7\hat{j} + 2\hat{k}$$