Engineering mathematics (2012)

[GROUP-A]

Q1. It contains (10) parts each of 2 marks :

(i) The value of $\left(\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{2}{3}\right)$ *is*

(a) $\frac{6}{17}$ (b) $\frac{17}{5}$ (c) $\frac{17}{6}$ (d) None

Ans.(c)

(ii) The value of middle term in the expansion of

$\left(\sqrt{\frac{x}{3}}+\frac{3}{2x^2}\right)^{12}$ *will be*

(a) $\frac{6567}{16x^9}$ (b) $\frac{6237}{2^4x^9}$ (c) $\frac{3762}{8x^9}$ (d) None

Ans.(b)

(iii) The logarithm of 144 of the base $2\sqrt{3}$ *is*

(a) 5 (b) 6 (c) 7 (d) None

Ans.(d)

(iv) The value of sin 15° is

(a) $\frac{2\sqrt{2}}{1-\sqrt{3}}$ (b) 6 (c) 7 (d) None

Ans.(c)

(v) If the points (1, 4), (k, –2) and (–3, 16) are collinear, then the value of K must be equal to

(a) 5 (b) 7 (c) 3 (d) None

Ans.(c)

(vi) The general solution of the equation $\sin\theta=\frac{\sqrt{3}}{2}$ *is*

(a) $n\pi+(-1)^n\pi/4$ (b) $n\pi+(-1)^n\pi/3$

(c) $2n\pi+(-1)^n\pi/4$ (d) None

Ans.(b)

(vii) The centre and radius of the circle $x^2+(y-1)^2=2$ *is represented by*

(a) $(1,-1),\sqrt{2}$ (b) $(1,0),\sqrt{2}$

(c) $(0,1),\sqrt{2}$ (d) None

Ans.(c)

(viii) If the vectors $2\vec{i}+\lambda\vec{j}+\vec{k}$ *and* $\vec{i}-2\vec{j}+3\vec{k}$ *are*

(a) 5/2 (b) 3/2 (c) 0 (d) None

Ans.(a)

(ix) The workdone by the force $\vec{F}=5\vec{i}-3\vec{j}+2\vec{k}$ *acting on a partical to displace it from (2, 1, 3) to (4, -1, 5) is*

(a) 26 units (b) 38 units (c) 20 units (d) None

Ans.(c)

(x) If the matrix $A=\begin{bmatrix}1&2&3\\2&3&4\\1&1&2\end{bmatrix}$, *the value of* $A^2-4A+5I$ is.....

Ans.

[GROUP-B]

Q2. If $a^2+b^3=34ab$, *prove that*

Ans $\log\left(\frac{a+b}{6}\right)=\frac{1}{2}(\log a+\log b)$ ...(i)

$\Rightarrow a^2+b^2+2ab=34ab+2ab$

$\Rightarrow (a+b)^2+36ab$

$\Rightarrow \left(\frac{a+b}{6}\right)^2=(ab)$ ...(ii)

From equation (ii) taking log both sides,

$\Rightarrow \log\left(\frac{a+b}{6}\right)^2=\log(ab)$

$\Rightarrow \log\left(\frac{a+b}{6}\right)=\frac{1}{2}[\log a+\log b]$ **Proved.**

Q3. Prove that $\begin{vmatrix}x&a&a\\a&x&a\\a&a&x\end{vmatrix}=(x-a)^2(x+2a)$

Ans. $R_1\to R_1-R_2=\begin{vmatrix}x-a&a-x&0\\a&x&a\\a&a&x\end{vmatrix}$

$C_1\to C_1+C_2=\begin{vmatrix}x-a&0&0\\a&a+x&a\\a&2a&x\end{vmatrix}$

$=\begin{vmatrix}1&0&0\\a&a+x&a\\a&2a&x\end{vmatrix}=(x-a)$

$=(x-a)\begin{vmatrix}a+x&a\\2a&x\end{vmatrix}$

$=(x-a)[x(a+x)-2a^2]$

$=(x-a)[xa+x^2-2a^2]$

$=(x-a)[x^2+2ax-ax-2a^2]$

$=(x-a)[x(x+2a)-a(x+2a)]$

$=(x-a)(x-a)(x+2a)$

$=(x-a)^2(x+2a)$ Proved.

Q4. *If* $\begin{bmatrix}1 & -2 & 3\\ -4 & 2 & 5\end{bmatrix}$ *and* $B=\begin{bmatrix}1 & 3\\ -1 & 0\\ 2 & 4\end{bmatrix}$, *verify that*

$(AB)'=B'A'$.

Ans. $AB=\begin{bmatrix}1 & -2 & 3\\ -4 & 2 & 5\end{bmatrix}\begin{bmatrix}1 & 3\\ -1 & 0\\ 2 & 4\end{bmatrix}$

$\Rightarrow AB=\begin{bmatrix}1+2+6 & 3+0+12\\ -4-2+10 & -12+0+20\end{bmatrix}$

$\Rightarrow AB=\begin{bmatrix}9 & 15\\ 4 & 8\end{bmatrix} \Rightarrow AB'=\begin{bmatrix}9 & 4\\ 15 & 8\end{bmatrix}$

hence, $(AB')=B'A'$ provide

Q5. *If* x^p *occurs in the expansion of* $\left(x^2+\frac{1}{x}\right)^{2n}$, *then prove that its coefficient is*

$$\frac{\lfloor 2n}{\lfloor\frac{1}{3}(4n-p)\lfloor\frac{1}{3}(2n+p)}$$

Ans. $t_{r+1}={}^{2n}C_r\, x^{2(2n-r)}\left(\frac{1}{x}\right)^r={}^{2n}C_r\, x^{4n-3r}$ Co-eff. of

x^p, $4n-3r=p$, $4n-p=3r$, $r=\frac{1}{3}(4n-p)$

$={}^{2n}C_{\frac{1}{3}(4n-p)}=\frac{(2n)!}{\left[2n-\frac{1}{3}(4n-p)\right]!\frac{1}{3}(4n-p)!}$

$=\frac{(2n)!}{\left[\frac{1}{3}(2n-p)\right]!\left[\frac{1}{3}(4n-p)\right]}$

$=\frac{\lfloor 2n}{\lfloor\frac{1}{3}(2n-p)\lfloor\frac{1}{3}(4n-p)}$

Q6. ***Prove that radian is a Constant angle.***

Ans. Let O be the centre of a circle and radius OR = r. If we take an produced to meet the circle at the point of C. Then the of the arc ABC half the circumference and AOC, the a the centre subtended by the arc = a straight angle = tw angles.

Now if we take the ratio of two arc and that of the two we have

$\frac{arc\ AB}{arc\ ABC}=\frac{r}{(1/2\times 2\pi r)}=\frac{1}{\pi}$

$\Rightarrow \frac{AOB}{AOC}=\frac{\perp radian}{2\times\frac{\pi}{2}}=\frac{1}{\pi}$

So, $\perp radian=\frac{\pi}{2}$ right angles.

Here, π is constant so radian is also constant.

Q7. *If* $\tan\alpha=\frac{1}{2}$, $\tan\beta=\frac{1}{3}$, *prove that* $\alpha+\beta=\frac{\pi}{4}$.

Ans. $\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{-\tan\alpha\cdot\tan\beta}$

$=\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\cdot\frac{1}{3}}=1$

$\Rightarrow (\alpha+\beta)=\tan^{-1}(1)=\frac{\pi}{4}$ Proved.

Q8. *Prove that* $\cos\tan^{-1}\sin\cot^{-1}x=\sqrt{\frac{x^2+1}{x^2+2}}$

Ans. $\cos\tan^{-1}\sin\cot^{-1}\frac{x}{1}\cos\tan^{-1}\sin\left(\sin^{-1}\frac{1}{\sqrt{1+x^2}}\right)$

$=\cos\tan^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)=\cos\left(\cos^{-1}\left(\frac{\sqrt{1+x^2}}{\sqrt{1+x^2+1}}\right)\right)$

$=\frac{\sqrt{1+x^2}}{\sqrt{x^2+2}}$ **Proved.**

Q9. ***Show that the points (3, 3), (h, 0), & (0, k) are collin***

$\frac{1}{h}+\frac{1}{k}=\frac{1}{3}$

Ans. This three points are collinear,

Area Δ ABC = 0

$$\Rightarrow \begin{vmatrix} 3 & 3 & 1 \\ h & 0 & 1 \\ 0 & k & 1 \end{vmatrix} = 0 \Rightarrow 3(0-k) - 3(h-0) + 1(hk) = 0$$

$$\Rightarrow -3k - 3h + hk = 0$$

Q10. Find the equation of the straight line which passage through the point (2, 3) whose intercept on the y-axis s thrice that on the x-axis.

Ans. Let, x-intercept $= a$

then, y-intercept $= 3a$

we know, equation of intercept,

$$\frac{x}{a} + \frac{y}{b} = 1 \Rightarrow \frac{x}{a} + \frac{y}{3a} = 1$$

We know, point 'A' is passing through equation

$$\Rightarrow \frac{2}{a} + \frac{3}{3a} = 1 \Rightarrow 2 + 1 = a,\ a = 3$$

$$\Rightarrow \frac{x}{3} + \frac{y}{9} = 1$$

Q11. A force $\vec{F} = 3\vec{i} + 2\vec{j} - 4\vec{k}$ is acting of the point (1, –1, 2). Find the moment of $\vec{F}$ about the point (2, –1, 3).

Ans. $\overline{AB} = \overline{B} - \overline{A} = (3\hat{i} + 2\hat{j} - 4\hat{k}) - (\hat{i} - \hat{j} + 2\hat{k})$

$$= (\hat{i} + \hat{k})$$

$$\overline{M} = \overline{AB} \times \overline{F} = (\hat{i} + \hat{k}) \times (3\hat{i} + 2\hat{j} - 4\hat{k})$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 1 \\ 3 & 2 & -4 \end{vmatrix} = \hat{i}(0-2) - \hat{j}(-4-3) + \hat{k}(2)$$

$$\overline{M} = (-2\hat{i} + 7\hat{j} + 2\hat{k})$$ **Proved.**

[GROUP-C]

Q12. Solve the following system of equation by matrix inversion

$$2x - y + 3z = 9$$
$$x + y + z = 6$$
$$x - y + z = 2$$

Ans. $2x - y + 3z = 9$

$x + y + z = 6$

$x - y + z = 2$

$$\begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ 2 \end{bmatrix}$$

$AX = B,\ A^{-1} = AX = A^{-1}B,\ X = A^{-1}B$ ....(i)

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$$

$$Adj(A) = \begin{bmatrix} 2 & 0 & -2 \\ -2 & -1 & 1 \\ -4 & 1 & 3 \end{bmatrix}^{t}$$

$$|A| = 2(2) - 1(0) + 3(-2) = 4 - 6 = -2$$

$$-2 \begin{bmatrix} \\ \\ -2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -1 & +1 & +2 \\ 0 & 1/2 & 1/2 \\ +1 & -1/2 & 3/2 \end{bmatrix}$$

$$X = \begin{bmatrix} -1 & 1 & 2 \\ 0 & 1/2 & -1/2 \\ 1 & -1/2 & 3/2 \end{bmatrix} \begin{bmatrix} 9 \\ 6 \\ 2 \end{bmatrix} = \begin{bmatrix} -9+6+4 \\ 0+3-1 \\ 9-3+3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix},\ (x, y, z) = (1, 2, 3)$$ **Proved.**

Q13. ***Resolve into partial fraction*** $\frac{x^4}{x^3+1}$

Ans. $$\frac{x}{x^3+1} = \frac{x}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{Bx+c}{x^2-x+1}$$

$$\Rightarrow \frac{x}{(x+1)(x^2-x+1)} = \frac{A(x^2-x+1) + (Bx+C)(x+1)}{(x+1)(x^2-x+1)}$$

$$\Rightarrow x = A(x^2-x+1) + (Bx+C)(x+1)$$

put $x = -1$

$$\Rightarrow -1 = A(1+1+1) + 0,\ A = -\frac{1}{3}$$

put $x = 0$

$0 = A(1) + C(1)$

$$C = -A = \frac{1}{3}$$

put $x = 1$

$1 = A(1) + (B+C)(2)$

$\Rightarrow 1 - A = 2(B+C)$

$$\Rightarrow 1 + \frac{1}{3} = 2(B+C),\ B+C = \frac{4}{6},\ B = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

now,

$$AB = \sqrt{(6-4)^2 + (0+1)^2} = \sqrt{4+1} = \sqrt{5}$$

$$BC = \sqrt{(6-7)^2 + (0-2)^2} = \sqrt{1+4} = \sqrt{5}$$

now,

$$AC = \sqrt{(7-4)^2 + (2+1)^2} = \sqrt{9+9} = 3\sqrt{2}$$

$$BD = \sqrt{(6-5)^2 + (0+1)^2} = \sqrt{1+1} = \sqrt{2}$$

Here, the diagonals of ABCD is not equal to it can be not a square.

Q18. If Prove that $\tan\frac{\theta}{2} = \sqrt{\frac{1-e}{1+e}}\tan\phi$, ***Prove that***

$$\cos\phi = \frac{\cos\theta - c}{1 - e\cos\theta}$$

Ans. We know,

$$\cos\phi = \frac{1-\tan^2\frac{\phi}{2}}{1+\tan^2\frac{\phi}{2}} = \frac{1-\frac{1+e}{1-e}\tan^2\frac{0}{2}}{1+\frac{1+e}{1-e}\tan^2\frac{\theta}{2}}$$

$$= \frac{1-e-(1+e)\tan^2\frac{\theta}{2}}{(1-e)} \times \frac{(1-e)}{1-e+(1+e)\tan^2\frac{\theta}{2}}$$

$$= \frac{(1-e)-(1+e)\tan^2\theta/2}{(1-e)+(1+e)\tan^2 0/2}$$

$$= \frac{1-\tan^2\theta/2 - e - e\tan^2\theta/2}{1+\tan^2\theta/2 - e + e\tan^2\theta/2}$$

$$= \frac{1-\tan^2\theta/2 - e(1+\tan^2\theta/2)}{1+\tan^2\theta/2 - e(1-\tan^2\theta/2)}$$

$$= \frac{\frac{1-\tan^2\theta/2}{1+\tan^2\theta/2}}{1-e\frac{(1-\tan^2\theta/2)}{1+\tan^2\theta/2}} = \frac{\cos\theta - e}{1-e\cos\theta}$$ **Proved.**

Q19. For the matrices $A = \begin{bmatrix} 3 & 5 & 7 \\ 2 & -3 & 1 \\ 1 & 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 6 & 7 & 9 \end{bmatrix}$

Find AB and A^{-1}.

Ans. $AB = \begin{bmatrix} 3 & 5 & 7 \\ 2 & -3 & 1 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 6 & 7 & 9 \end{bmatrix}$

$$= \begin{bmatrix} 3+15+42 & 6+20+49 & 9+25+63 \\ 2-9+6 & 4-12+7 & 6-15+9 \\ 1+3+12 & 2+4+14 & 3+5+18 \end{bmatrix}$$

$$= \begin{bmatrix} 60 & 75 & 97 \\ -1 & -1 & 0 \\ 16 & 20 & 26 \end{bmatrix}$$

$$A^{-1} = \frac{Adj.(A)}{|A|}$$

$|A| = 3(-6-1)-5(4-1)+7(2+3)$
$= -21-15+35 = -1$

$C_{11} = +(-6-1) = -7$; $C_{12} = -(4-1) = -3$
$C_{13} = +(2+3) = 5$; $C_{21} = -(10-7) = -3$
$C_{22} = +(6-7) = -1$; $C_{23} = -(3-5) = 2$
$C_{31} = +(5+21) = 26$; $C_{32} = -(3-14) = 11$
$C_{33} = +(9-10) = -19$;

$$A^{-1} = -1\begin{bmatrix} -7 & -3 & 5 \\ -3 & -1 & -2 \\ 26 & 11 & 19 \end{bmatrix} = \begin{bmatrix} 7 & 3 & -26 \\ 3 & 1 & -11 \\ -5 & 2 & 19 \end{bmatrix}$$ **Ans.**