Engineering mathematics (2021)

1. Choose the correct answer from given choices .

(i) The value of ${}^nC_r + {}^nC_{r-1}$ is equal to

(a) ${}^{n+1}C_r$ (b) ${}^{n+1}C_{r+1}$

(c) ${}^nC_{r+1}$ (d) None of these

Ans.(a)

(ii) The value of $\log_{2\sqrt{3}}144$ is

(a) 3 (b) 5

(c) 4 (d) None of these

Ans.(c)

(iii) The points (2,3), (5, k) and (6,7) are collinear. Then the value of k will be

(a) 5 (b) 7

(c) 6 (d) None of these

Ans.(c)

(iv) If $A=\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$ and $B=\begin{bmatrix}-1 & 3\\ 2 & 5\end{bmatrix}$, then the transpose of (AB) is

(a) $\begin{bmatrix}4 & 21\\ 3 & 13\end{bmatrix}$ (b) $\begin{bmatrix}2 & 15\\ 3 & 12\end{bmatrix}$

(c) $\begin{bmatrix}4 & 3\\ 21 & 13\end{bmatrix}$ (d) None of these

Ans.(c)

(v) For the circle $x^2+y^2-6x-2y-6=0$, the radius and centre is

(a) 3; (0,0) (b) 5; (3,4)

(c) 4; (3,1) (d) None of these

Ans.Out of Syllabus

(vi) The value of $\sin^{-1}x+\cos^{-1}x$ is equal to

(a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$ (d) None of these

Ans.(c)

(vii) If $A=\begin{bmatrix}4 & 7 & 3\\ 6 & 8 & 9\\ 8 & 14 & 6\end{bmatrix}$, then the value of A is

(a) 1 (b) 2

(c) 0 (d) – 1

Ans.(c)

(viii) If $a=3i-4j+5k$, then modulus of $|a|$ will be

(a) $4\sqrt{5}$ (b) $5\sqrt{2}$

(c) $3\sqrt{2}$ (d) None of these

Ans.Out of Syllabus

Q2. (a) Resolve into partial fraction : $\frac{2x+3}{(x-3)(x+1)}$

Ans. $\frac{2x+3}{(x-3)(x+1)}$

Let $\frac{2x+3}{(x-3)(x+1)}=\frac{A}{x-3}+\frac{B}{x+1}$ ... (i)

Multiplying both sides by $(x-3)(x+1)$, we get

$2x+3=A(x+1)+B(x-3)$ ... (ii)

Or $2x+3=Ax+A+Bx-3B$

Or $2x+3=x(A+B)+A-3B$

Comparing the coefficient of like terms on both sid

$A+B=2$

$A-3B=3$

Solving these equation, we get

$A=\frac{9}{4},\quad B=-\frac{1}{4}$

Substituting the values of A and B in equation (i), we

$$\frac{2x+3}{(x-3)(x+1)}=\frac{9}{4(x-3)}-\frac{1}{4(x+1)}$$

Q2.(b) If $\frac{\log a}{b-c}=\frac{\log b}{c-a}=\frac{\log c}{a-b}$, then prove t

$a^{b+c}.b^{c+a}.c^{a+b}=1.$

Ans. Let $\frac{\log a}{b-c}=k\Rightarrow \log a=k(b-c)$ ...(i)

$\frac{\log b}{c-a}=k\Rightarrow \log b=k(c-a)$ ... (ii)

$\frac{\log c}{a-b}=k\Rightarrow \log c=k(a-b)$ ... (iii)

Adding eqn (i), (ii) and (iii)

$\log a+\log b+\log c=k(b-c+c-a+a-b)$

$\log abc=0$

$abc=1$

$(abc)^{a+b+c}=1$

$$a^{a+b+c}.b^{a+b+c}.c^{a+b+c}=1$$

$$a^{a}.a^{b+c}.b^{b}.b^{c+a}.c^{c}.c^{a+b}=1$$

$$a^{a}.b^{b}.c^{c}\left[a^{b+c}.b^{c+a}.c^{a+b}\right]=1 \qquad \text{... (iv)}$$

Now,

Multiplying 'a' by equation (i), 'b' by equation (ii) and 'c' by eqnuation (ii).

We get

$$\log a^{a}=k(ab-ac) \qquad \text{... (v)}$$

$$\log b^{b}=k(bc-ab) \qquad \text{... (vi)}$$

$$\log c^{c}=k(ca-bc) \qquad \text{... (vii)}$$

Adding equation (v), (vi) and (vii) we get,

$$\log a^{a}+\log b^{b}+\log c^{c}=0$$

$$\log a^{a}.b^{b}.c^{c}=0$$

$$a^{a}b^{c}.c^{c}=1$$

Then, putting these value to eqn (iv), we get

$a^{b+c}.b^{c+a}.c^{a+b}=1$ Proved.

Q3. (a) Prove that $\begin{vmatrix}1 & a & a^2\\ 1 & b & b^2\\ 1 & c & c^2\end{vmatrix}=(a-b)(b-c)(c-a).$

Ans. *Let Δ be the given determinant. Applying $R_2 \to R_2 \;\; R_1$ and $R_3 \to R_3 \;\; R_1$, we get*

$$\Delta=\begin{vmatrix}1 & a & a^2\\ 0 & b-a & b^2-a^2\\ 0 & c-a & c^2-a^2\end{vmatrix}$$

$$\Rightarrow \Delta=(b-a)(c-a)\begin{vmatrix}1 & a & a^2\\ 0 & 1 & b+a\\ 0 & 1 & c+a\end{vmatrix}$$ *[Taking out (b a) common from R_2 & (c a) from R_3]*

$$\Rightarrow \Delta=(b-a)(c-a)\begin{vmatrix}1 & a & a^2\\ 0 & 1 & b+a\\ 0 & 0 & c-b\end{vmatrix}$$

[Applying $R_3 \to R_3 \;\; R_2$]

$$\Rightarrow \Delta=(b-a)(c-a)(c-b)\begin{vmatrix}1 & a & a^2\\ 0 & 1 & b+a\\ 0 & 0 & 1\end{vmatrix}$$

[Taking out (c b) common from R_3]

$$\Rightarrow \Delta=(b-a)(c-a)(c-b)\times 1\times\begin{vmatrix}1 & b+a\\ 0 & 1\end{vmatrix}$$

[Expanding along C_1]

$$\Rightarrow \Delta=(b-a)(c-a)(c-b)\times 1=(a-b)(b-c)(c-a)$$

Q3.(b) Show that the middle term in the expansion of $(1+x)^{2n}$ is $\frac{1.3.5.7.....(2n-1)}{\text{l}n}.2^{n}x^{2}$, where n is a positive integer.

Ans. We know that the $(r+1)^{th}$ term in binomial expansion of $(x+y)^m$ is given by

$$T_{r+1}={}^{m}C_r x^{m-r}y^{r}$$

Now, given expression is $(1+x)^{2n}$.

The $(r+1)^{th}$ term in binomial expansion of given expression is

$$T_{r+1}={}^{2n}C_r(1)^{2n-r}(x)^{r}$$

$$={}^{2n}C_r x^{r}$$

Here, m = 2n, which is even.

So, the middle term of the expansion is $\left(\frac{m+2}{2}\right)^{th}$ term,

i.e., $(n+1)^{th}$ term.

For $(n+1)^{th}$ term, we have

$$r+1=n+1 \Rightarrow r=n$$

Hence, the middle term in binomial expansion of given expression is

$$T_n={}^{2n}C_n x^{n}=\frac{(2n)!}{n!n!}x^{n}$$

$$=\frac{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\ldots(2n-1)(2n)}{n!n!}x^{n}$$

$$=\frac{[1\cdot 3\cdot 5....(2n-1)][2\cdot 4\cdot 6...(2n)]}{n!n!}x^{n}$$

$$=\frac{[1\cdot 3\cdot 5....(2n-1)]2^{n}[1\cdot 2\cdot 3...n]}{n!n!}x^{n}$$

$$=\frac{[1\cdot 3\cdot 5....(2n-1)]2^{n}(n!)}{n!n!}x^{n}$$

$$=\frac{[1\cdot 3\cdot 5....(2n-1)]2^{n}}{n!}x^{n}$$

Hence, the middle term in the expansion of $(1+x)^{2n}$ is $=\frac{1\cdot 3\cdot 5....(2n-1)}{n!}2^{n}x^{n}$.

Q4. (a) Prove that $2\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}=\frac{\pi}{4}$.

Ans.Same as 2016, Q.no. 6(b).

Q4.(b) Prove that $\dfrac{\sin^3 A+\cos^3 A}{\sin A+\cos A}+\dfrac{\sin^3-\cos^3 A}{\sin A-\cos A}=2.$

Ans. $$\text{L.H.S.}=\frac{\sin^3 A+\cos^3 A}{\sin A+\cos A}+\frac{\sin^3 A-\cos^3 A}{\sin A-\cos A}$$

$$=\frac{(\sin A+\cos A)(\sin^2 A-\sin A+\cos^2 A)}{(\sin A+\cos A)}$$

$$+\frac{(\sin A-\cos A)(\sin^2 A+\sin A\cos A+\cos^2 A}{(\sin A-\cos A)}$$

$$=(1-\sin A\cos A)+(1+\sin A\cos A)$$

$$[....\sin^2 A+\cos^2 A=1]$$

$$=1-\sin A\cos A+1+\sin A\cdot\cos A=2=\text{R.H.S}\ \textbf{Proved.}$$

Q5. (a) For the two vectors $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ **and** $\vec{b}=3\hat{i}-\hat{j}+2\hat{k}$ **find the dot product** $(a+b).(a-b)$.

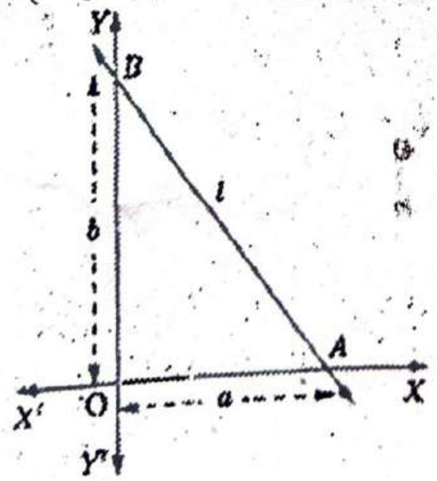
Ans.Out of Syllabus

Q5.(b)A particle acted on by forces $4\hat{i}+\hat{j}-3\hat{k}$ **and** $3\hat{i}+\hat{j}-\hat{k}$**, is displaced from the point** $\hat{i}+2\hat{j}+3\hat{k}$ **to the point** $5\hat{i}+4\hat{j}+\hat{k}$**. Find the total work done.**

Ans. Out of Syllabus

Q6.(a)Find the equation of a straight line which makes intercepts of a and b with x -axis and y-axis respectively.

Ans. Let the straight line, say l, cut off intercepts a and b on the x-axis and y-axis respectively and let it meet the axes in points A, B (shown in figure 15.19), then the co-ordinates of points A and B are (a, 0) and (0, b).



Using two-point form, the equation of the line l is

$$y-0=\frac{b-0}{0-a}(x-a)$$

$$y=-\frac{b}{a}(x-a)$$

$$bx+ay-ab=0$$

$$\frac{x}{a}+\frac{y}{b}=1$$

which is the required equation of the line cutting off intercepts a and b on the axes. This is known as intercept form.

Q6.(b) Find the distance between the parallel lines 3x+4y-5=0 and 6x+8y-45 =0.

Ans. We have $3x+4y-5=0$ ...(1)

and $6x+8y-45=0$ ... (2)

Putting $x=0$ in (1), we have

$$4y-5=0$$

or $$y=\frac{5}{4}$$

Thus $\left(0,\frac{5}{4}\right)$ is point on (1).

The distance between (1) and (2)

$$d=\perp \text{ distance from}\left(0,\frac{5}{4}\right)\text{ on (1)}$$

$$=\frac{\left|6\times 0+8\times\frac{5}{4}-45\right|}{\sqrt{36+64}}$$

$$=\frac{|10-45|}{10}=\frac{|-35|}{10}=3.5$$

Q7. (a) Find the inverse of the matrix $A=\begin{bmatrix}3&5&7\\2&-3&1\\1&1&2\end{bmatrix}$.

Ans. *Let* $$A=\begin{bmatrix}3&5&7\\2&-3&1\\1&1&2\end{bmatrix}$$

$$|A|=3(-6-1)-5(4-1)+7(2-(-3))$$
$$=3(-7)-5(3)+7(5)$$
$$=-21-15+35$$

$-36+35=-1$

$|A| \neq 0$, *thus A is non-singular and* A^{-1} *exists.*

Then,

Cofactor :

$$A_{11}=(-1)^{1+1}\begin{vmatrix}-3 & 1\\ 1 & 2\end{vmatrix}=(-1)(-6-1)=-7$$

$$A_{12}=(-1)^{1+2}\begin{vmatrix}2 & 1\\ 1 & 2\end{vmatrix}=(-1)^3(4-1)=-3$$

$$A_{13}=(-1)^{1+3}\begin{vmatrix}2 & -3\\ 1 & 1\end{vmatrix}=1(2+3)=5$$

$$A_{21}=(-1)^{2+1}\begin{vmatrix}5 & 7\\ 1 & 2\end{vmatrix}=(-1)^3(10-7)=-3$$

$$A_{22}=(-1)^{2+2}\begin{vmatrix}3 & 7\\ 1 & 2\end{vmatrix}=(1)(6-7)=-1$$

$$A_{23}=(-1)^{2+3}\begin{vmatrix}3 & 5\\ 1 & 1\end{vmatrix}=(-1)(3-5)=2$$

$$A_{31}=(-1)^{3+1}\begin{vmatrix}5 & 7\\ -3 & 1\end{vmatrix}=(1)(5+21)=26$$

$$A_{32}=(-1)^{3+2}\begin{vmatrix}3 & 7\\ 2 & 1\end{vmatrix}=(-1)(3-14)=11$$

$$A_{33}=(-1)^{3+3}\begin{vmatrix}3 & 5\\ 2 & -3\end{vmatrix}=(1)(-9-10)=-19$$

$$\text{adj}A=C^T=\begin{bmatrix}-7 & -3 & 5\\ -3 & -1 & 2\\ 26 & 11 & -19\end{bmatrix}^T$$

$$=\begin{bmatrix}-7 & -3 & 26\\ -3 & -1 & 11\\ 5 & 2 & -19\end{bmatrix}$$

$$A^{-1}=\frac{1}{\det A}\text{adj}A=\frac{1}{-1}\begin{bmatrix}-7 & -3 & 26\\ -3 & -1 & 11\\ 5 & 2 & -19\end{bmatrix}$$

$$A^{-1}=\begin{bmatrix}7 & 3 & -26\\ 3 & 1 & -11\\ -5 & -2 & 19\end{bmatrix}$$

Q7.(b) If $A=\begin{bmatrix}1 & -2 & 3\\ -4 & 2 & 5\end{bmatrix}$ and $B=\begin{bmatrix}2 & 3\\ 4 & 5\\ 2 & 1\end{bmatrix}$, then find AB and BA if possible and show that $AB \neq BA$.

Ans. $A=\begin{bmatrix}1 & -2 & 3\\ -4 & 2 & 5\end{bmatrix}$ and $B=\begin{bmatrix}2 & 3\\ 4 & 5\\ 2 & 1\end{bmatrix}$

Since, order of A is 2×3 and order of B is 3×2, AB and BA both exist.

Also,

$$AB=\begin{bmatrix}1 & -2 & 3\\ -4 & 2 & 5\end{bmatrix}\begin{bmatrix}2 & 3\\ 4 & 5\\ 2 & 1\end{bmatrix}$$

$$=\begin{bmatrix}2-8+6 & 3-10+3\\ -8+8+10 & -12+10+5\end{bmatrix}$$

$$AB=\begin{bmatrix}0 & -4\\ 10 & 3\end{bmatrix}$$

$$BA=\begin{bmatrix}2 & 3\\ 4 & 5\\ 2 & 1\end{bmatrix}\begin{bmatrix}1 & -2 & 3\\ -4 & 2 & 5\end{bmatrix}$$

$$BA=\begin{bmatrix}2-12 & -4+6 & 6+15\\ 4-20 & -8+10 & 12+25\\ 2-4 & -4+2 & 6+5\end{bmatrix}=\begin{bmatrix}-10 & 2 & 21\\ -16 & 2 & 37\\ -2 & -2 & 11\end{bmatrix}$$

Q8.(a)If the sides of ΔABC are 3, 4 and 5 unit respectively, then find the smallest angle of the triangle.

Ans.Out of Syllabus

Q8.(b)Find the equation of the circle whose centre is (2,-3) and which passes through the point of intersection of 3x -2 y = 1 and 4x + y = 27.

Ans.Out of Syllabus

Q9.(a)Find the 6th term of $(a+2b)^8$.

Ans. Because the formula is for the $(r+1)^{th}$ term, r is one less than the number of the term you need.

So, to find the sixth term in this binomial expansion, use,

$r=5, n=8, x=a$ and $y=2b$

Then,

The 6^{th} term is

$$^nC_r x^{n-r} y^r = 8C_5 a^{8-5}(2b)^5$$

$$= 56 \cdot a^3 (2b)^5$$

$$= 56(2)^5 a^3 b^5$$

$$= 1792\, a^3 b^5$$

Q9.(b)Prove $\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \frac{1}{16}$.

Ans.Same as 2015, Q.no. 4(a).

Q10.(a)If the co-ordinates of end point of a diameter of circle be (x_1, y_1) and (x_2, y_2), find the equation of the circle.

Ans.Out of Syllabus

Q10.(b)Find the angle between the lines $2x \;\; y + 3 = 0$ and $x + y \;\; 2 = 0$

Soln.

We have $\quad 2x - y + 3 = 0 \quad$ ...(1)

and $\quad x + y - 2 = 0 \quad$ ...(2)

Slope of teh first lien $2x - y + 3 = 0$ is

$$m_1 = -\frac{1}{1} = -1$$

If θ is the angle between the lines, them

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$$

$$= \left|\frac{-1-2}{1+(2)(-1)}\right| = 3$$

Thus $\quad \theta = \tan^{-1}(3)$