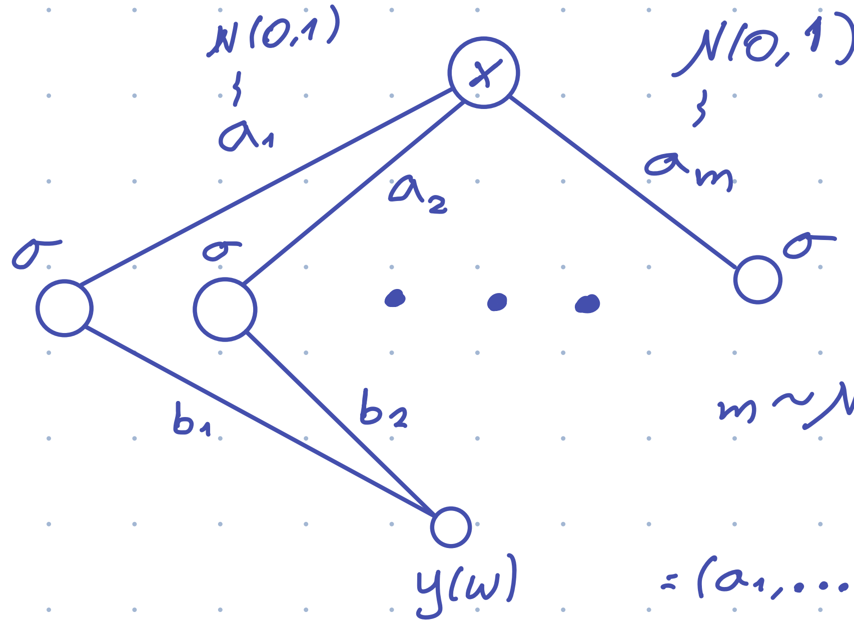
1-dim

$X = \{x\}$

$Y = \{y\}$ $\in \mathbb{R}$



$m \sim N(0, \frac{1}{\sqrt{m}})$

$= (a_1, \dots, a_m, b_1, \dots, b_m)$

$$\hat{y} = y(w) = \sum_{i=1}^{m} b_i \sigma(a_i x)$$

$$L(w) = \frac{1}{2} \| y - y(w) \|^2$$

$$\frac{\partial \hat{y}}{\partial a_i} = b_i \sigma'(a_i x) \cdot x \qquad \frac{\partial^2 \hat{y}}{\partial a_i \partial a_j} = \begin{cases} \underbrace{b_i x^2 \sigma''(a_i x)}_{O(\frac{1}{m})} & i=j \\ 0 & \text{o.w.} \end{cases}$$

$$\frac{\partial \hat{y}}{\partial b_i} = \sigma(a_i x) \qquad \frac{\partial^2 \hat{y}}{\partial b_i \partial b_j} = 0$$

$$\frac{\partial^2 \hat{y}}{\partial a_i \partial b_j} = \begin{cases} \sigma'(a_i x) \cdot x & i=j \\ 0 & \text{o.w.} \end{cases}$$

$O(1)$

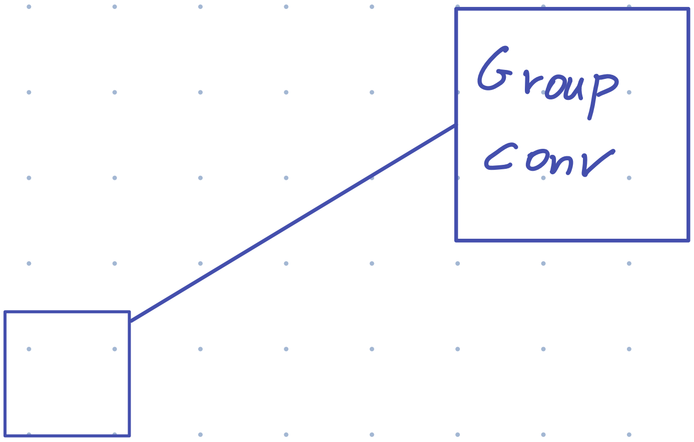
$$\frac{\partial^2 \hat{y}}{\partial b_i a_j} = \begin{cases} \sigma'(a_i x) \cdot a_i & i=j \\ 0 & \text{o.w.} \end{cases}$$

$$\text{Hessian} = \begin{bmatrix} \ddots\; O(1) & \ddots\; O(1) \\ \ddots\; O(\tfrac{1}{m}) & 0 \end{bmatrix}$$

spectral norm $O(1)$

$$\mathbb{E}\|\nabla_w y(w)\|^2 = m$$

$$\mathbb{E}\|\nabla_w^2 y(w)\| = O(1)$$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