Regression via sketch&solve (subspace embeddings)

ℓ_2-regression

$$\min \|Ax-b\|^2$$

Sketch $\bar{A} = [A \;\; b] = U \Sigma V^T$

Find S s.t.

$$\forall z \quad \|SUz\|^2 \overset{1\pm\varepsilon}{=} \|Uz\|^2$$

$$\Downarrow \quad (\forall x) \quad Ax-b \in col(U)$$

$$(\forall x) \quad \|Ax-b\|^2 \overset{1\pm\varepsilon}{=} \|S(Ax-b)\|^2$$