Graph simulation results for 1 Qubit with weak measurements

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1 Intro

In this report we will do a data analysis of the probability distribution of entropy production(Q) in a weak measurement quantum system simulation with 1 qubit [1]. we will first show the regimes of Equation 14 [2] given different values of $\theta = T/\tau$ and map them to the behavior of σ_z. we will then plot the histogram of Q, calculated using the simulation, and a plot of the distribution using Equation 14 for the different θ regimes, as well as growing the number of trajectories. and finally we will plot average Q vs θ.

2 Analysis of equation 14

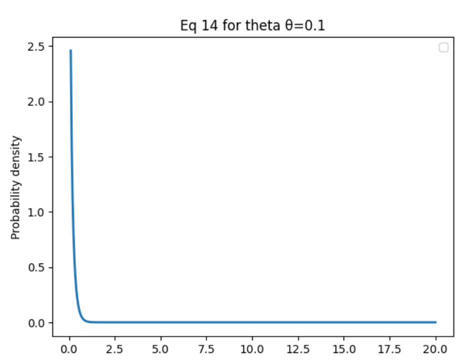

Figure 1: plot of equation 14 for $\theta = 0.1$.

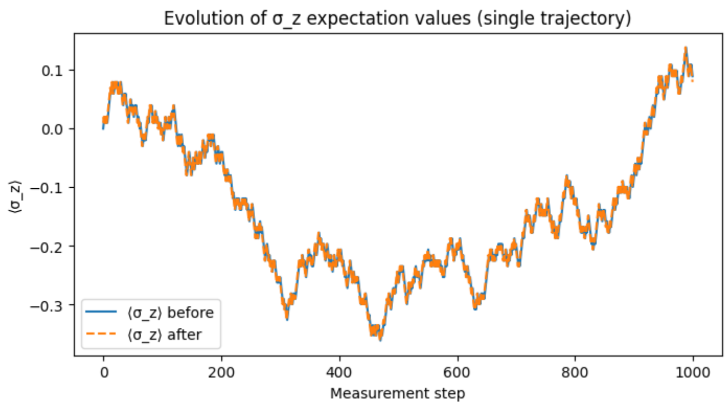

Figure 2: values of σ_z during simulation for $\epsilon = 0.01$ and $n = 1000$. σ_z does not reach a critical point and keeps oscillating

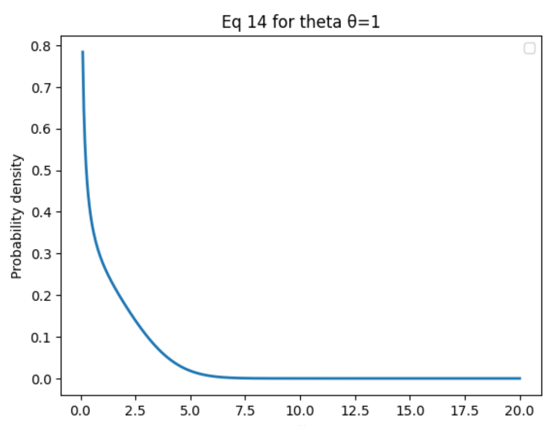

Figure 3: plot of equation 14 for $\theta = 1$.

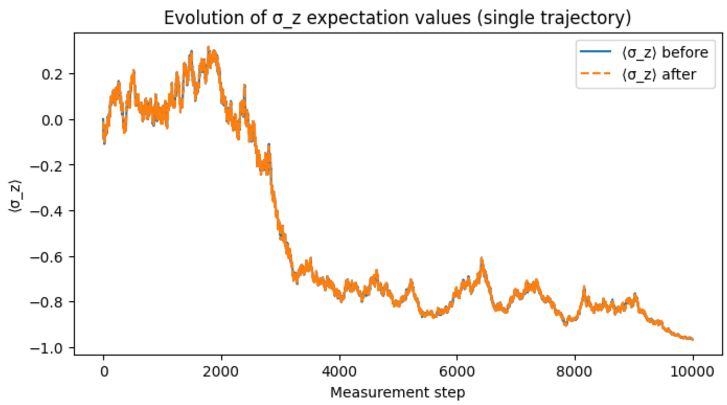

Figure 4: values of σ_z during simulation for $\epsilon = 0.01$ and $n = 10000$. σ_z does not reach a critical point and keeps oscillating but has a harder tendency to a pole with some runs converging to a pole.

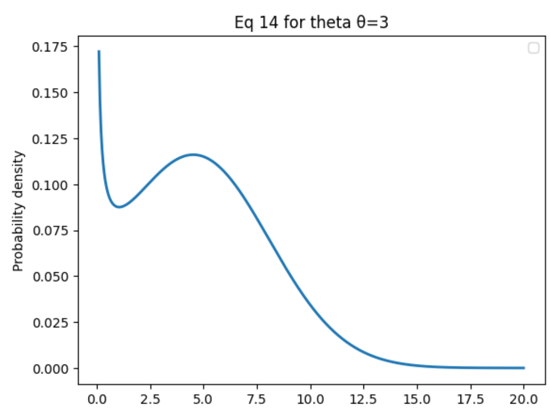

Figure 5: plot of equation 14 for $\theta = 3$.

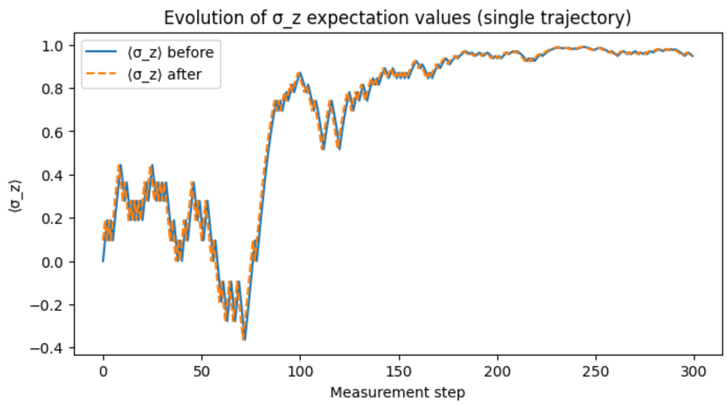

Figure 6: values of σ_z during simulation for $\epsilon = 0.1$ and $n = 300$. σ_z now tends to converge to a pole in most runs.

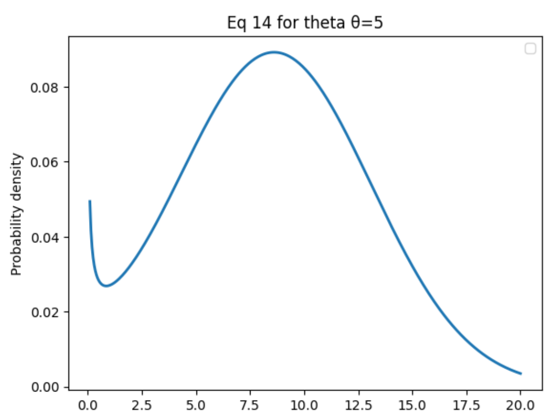

Figure 7: plot of equation 14 for $\theta = 5$.

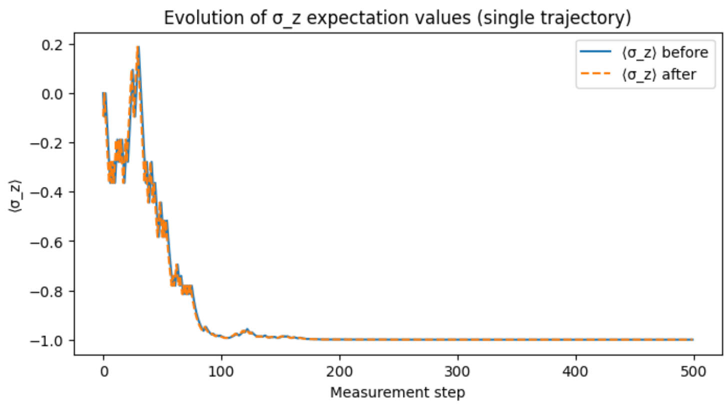

Figure 8: values of σ_z during simulation for $\epsilon = 0.1$ and $n = 500$. σ_z convergence now very common and with strong force, sometimes within the first 100 measurements.

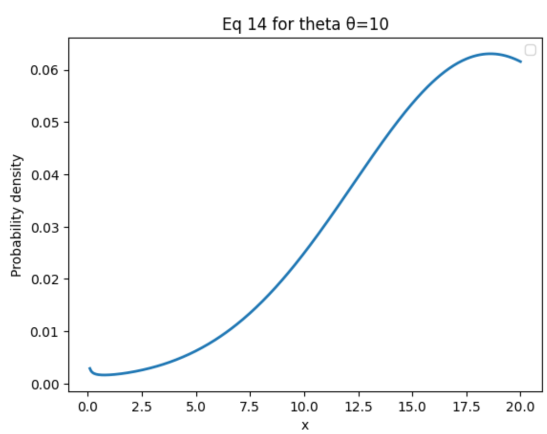

Figure 9: plot of equation 14 for $\theta = 10$.

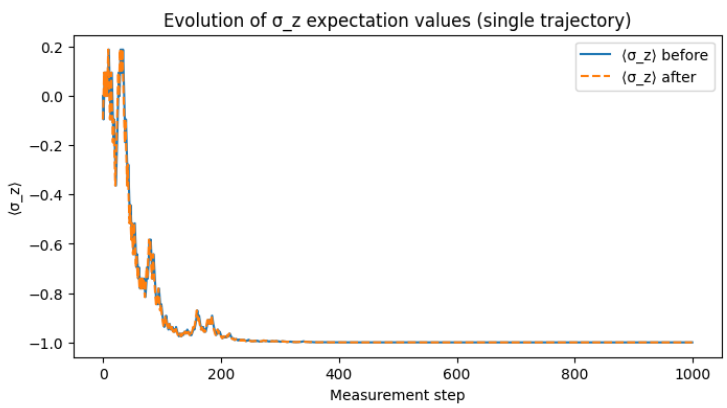

Figure 10: values of σ_z during simulation for $\epsilon = 0.1$ and $n = 1000$. σ_z convergence on every run.

We can see that as theta grows the rate of convergence of σ_z increases. a needed improvement would be to define a convergence meter, say when at least the last 10% of σ_z are at a pole we have convergence, so that we can run the simulation with the parameters multiple times and see the statistics of convergence of a given set of parameters.

3 High Parameter limit: number of trajectories

In this section we used the different ϵ and number of measurements(n) as defined for the different regimes in the previous section and calculated Q for very high number of trajectories (N = 10,000). we then show the histogram of Q and the plot of equation 14 with $\theta = n * \epsilon^2$ to see the correlation between them.

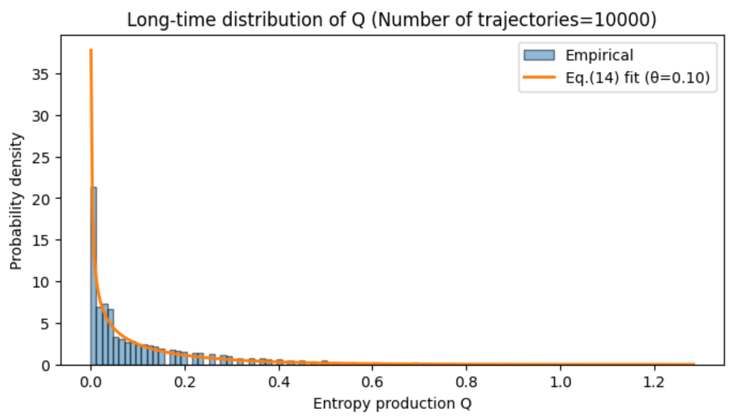

Figure 11: Distribution of the entropy production Q for 10000 trajectories with $\epsilon = 0.01$ and $n = 1000$ (blue histogram). Eq. (14) with $\theta = 0.1$ (red plot).

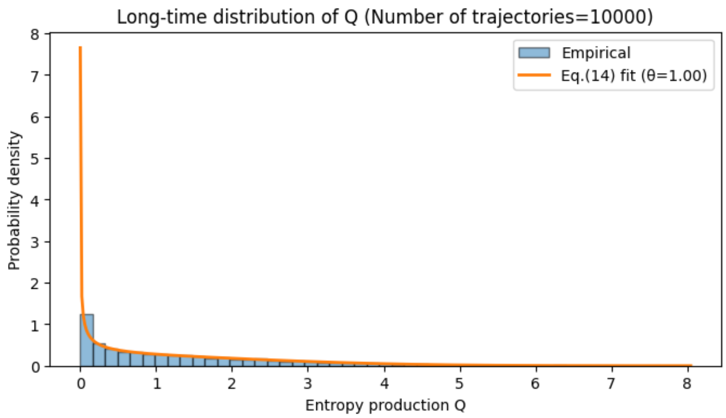

Figure 12: Distribution of the entropy production Q for 10000 trajectories with $\epsilon = 0.01$ and $n = 10000$ (blue histogram). Eq. (14) with $\theta = 1$ (red plot).

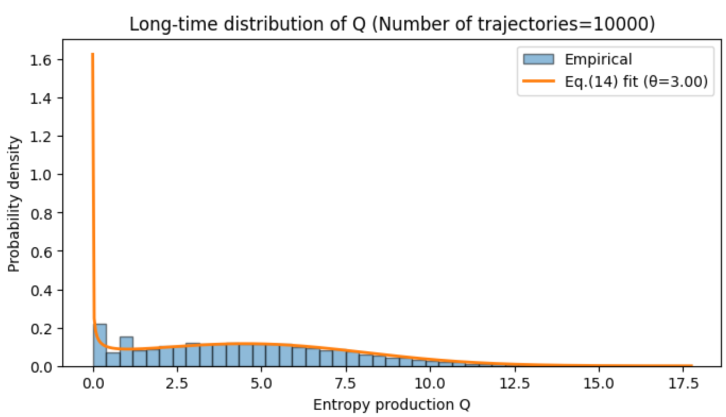

Figure 13: Distribution of the entropy production Q for 10000 trajectories with $\epsilon = 0.1$ and $n = 300$ (blue histogram). Eq. (14) with $\theta = 3$ (red plot).

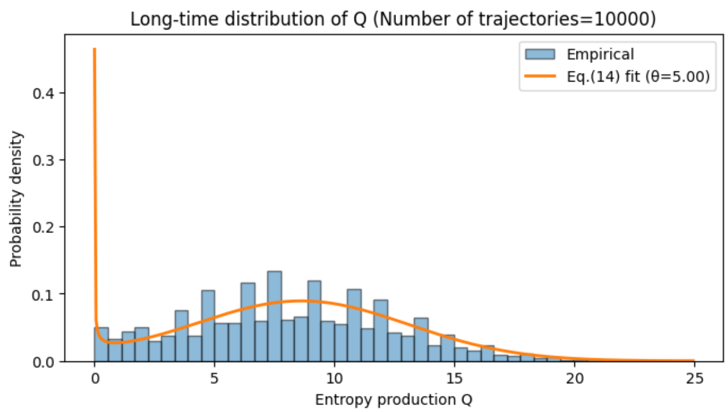

Figure 14: Distribution of the entropy production Q for 10000 trajectories with $\epsilon = 0.1$ and $n = 500$ (blue histogram). Eq. (14) with $\theta = 5$ (red plot).

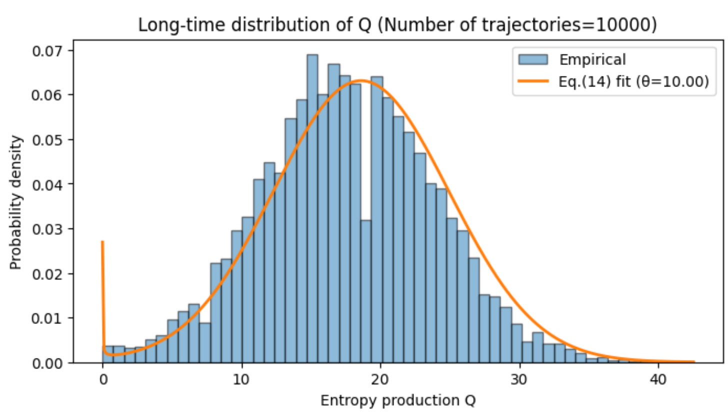

Figure 15: Distribution of the entropy production Q for 10000 trajectories with $\epsilon = 0.1$ and $n = 1000$ (blue histogram). Eq. (14) with $\theta = 10$ (red plot).

The model seems to fit quite well. We have some odd fluctuations with higher θ values and the diverging near zero seems to vanish, and seems like a gaussian distribution.

4 comparison of $< Q >$ to θ

in the final section we go over a set of n and ϵ so that we get a group of theta values:
$\theta = [0.1\ , 3, 5, 7, 10, 13, 15, 18, 20, 25]$
We run the simulation for 10,000 trajectories and calculate the mean Q for each one. Finally we plot them with together.

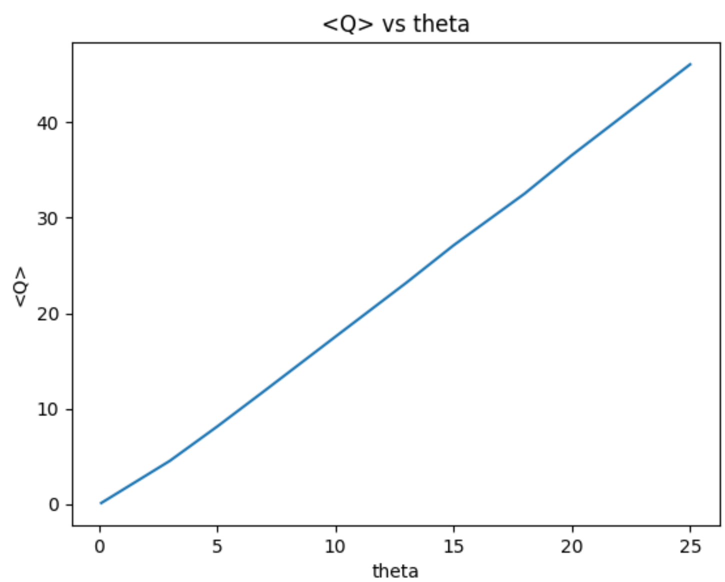

Figure 16: $< Q >$ as a function of θ for 10000 trajectories.

there seems to be a linear relation between them so lets create a fit

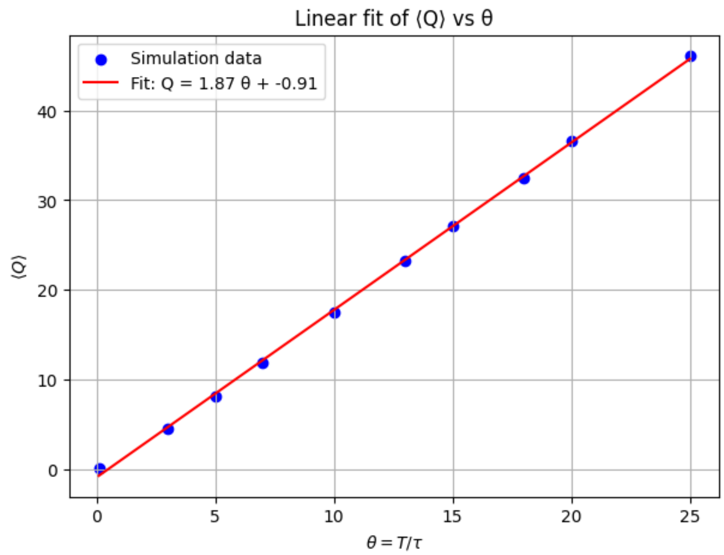

Figure 17: fit of $< Q >$ as a function of θ for 10000 trajectories.

this seems to roughly match equation 5 [2]: $< Q >= 3\theta/2$ although that might not be fully accounting for the fact that this is a no drive scenario.

References

[1] X. Turkeshi, A. Biella, R. Fazio, M. Dalmonte and M. Schirò, *Measurement–Induced Entanglement Transitions in the Quantum Ising Chain: From Infinite to Zero Clicks*, Phys. Rev. B **103**, 224210 (2021).

[2] J. Dressel, A. Chantasri, A.N. Jordan and A.N. Korotkov *Arrow of Time for Continuous Quantum Measurement*, Phys. Rev. A **96**, 062119 (2017), arXiv:1610.03818.