A watermarking algorithm satisfying topological chaos properties

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Abstract

A new watermarking algorithm is given, it is based on the so-called chaotic iterations and on the choice of some coefficients which are deduced from the description of the carrier medium. After defining these coefficients, chaotic discrete iterations are used to encrypt the watermark and to embed it in the carrier medium. This procedure generates a topological chaos and ensures that the required properties of a watermarking algorithm are satisfied.

Key-words: Watermarking, Encryption, Chaotic iterations, Topological chaos, Information hiding

I.   Introduction

Information hiding has recently become a major information security technology, especially with the increasing importance and widespread distribution of digital media through internet. Its aim is to embed a piece of information into digital documents, like pictures or movies, for a large panel of reasons, such as copyright protection, control utilization, data description, content authentication, and data integrity. Digital watermarking is one the techniques used in information hiding. It must offer a lot of desirable characteristics, including security, imperceptibility and robustness. To do so, many different watermarking schemes have been proposed in recent years, which can be classified into two categories: spatial domain, and frequency domain watermarking. In spatial domain watermarking, a large number of bits can be embedded without incurring noticeable visual artifacts; whereas, frequency domain watermarking has been shown to be quite robust against JPEG compression, filtering, noise pollution and so on. More recently, chaotic methods have been proposed to encrypt the watermark before embedding it in carrier image, for security reasons. This methods are usually based on three fundamental chaotic maps: Chebychev, logistic and Arnold's cat maps.

I In this paper, a new watermarking algorithm is given, it is based on the so-called chaotic iterations and on the choice of some coefficients which are deduced from the description of the carrier medium. After defining these coefficients, chaotic discrete iterations are used to encrypt the watermark and to embed it in the carrier medium.

The new algorithm consists in two stages, an encryption stage which encompasses many encryption algorithms and makes them more secure and an embedding stage, also based on chaotic iterations. An authentication of the relevant information carried by the support, can be done during each of this two stages.

The algorithm generates a topological chaos in the sense of Devaney and ensures by this way that the required properties of a watermarking algorithm are satisfied.

After introducing the new algorithm and explaining its theoretical foundations, a case study allows us to evaluated it.

The rest of this paper is organized as follows: first, some basic definitions concerning chaotic iterations are recalled. Then, the new chaos-based watermarking algorithm is introduced. The next section is devoted to the related works and to the contribution of our results in light of existing ones. Section 5 is devoted to the evaluation of our algorithm. A case study is presented and some classical attacks are executed, the results are presented and commented. The paper ends by a conclusion section where the contributions of the paper are summarized and the planned future work are given.

II.   Chaotic iterations: basic recalls

In the sequel $[|1; N|]$ means $\{1, 2, \ldots, N\}$, s^n denotes the nth term of a sequence $s = (s^1, s^2, \ldots)$, V_i denotes the ith component of a vector $V = (V_1, V_2, \ldots)$, and f^k denotes the kth composition of a function f, $f^k = \underbrace{f \circ f \circ \ldots \circ f}_{k\ times}$.

Let us consider a *system* of a finite number N of *cells* so that each cell has a boolean *state*. Then a sequence of length N of boolean states of the cells corresponds to a particular *state of the system*.

Definition 1 A *chaotic strategy* corresponds to a sequence of $[|1; N|]$. The set of all chaotic strategies is denoted by $\mathcal{S}$.

Definition 2 $\mathbb{B}$ denoting $\{0, 1\}$, let $f : \mathbb{B}^N \longrightarrow \mathbb{B}^N$ be an iteration function and $S \in \mathcal{S}$ be a chaotic strategy. The so called *chaotic iterations* are defined by (see [7])

$$\begin{cases} x^0 \in \mathbb{B}^N \\ \forall n \in \mathbb{N}^*, \forall i \in [|1; N|], x_i^n = \begin{cases} x_i^{n-1} & \text{if } S^n \neq i \\ f(x^n)_{S^n} & \text{if } S^n = i. \end{cases} \end{cases} \tag{1}$$

In other words, at the nth iteration, only the S^n-th cell is "iterated". Note that in a more general formulation, $f(x^n)_{S^n}$ can be replaced by $f(x^k)_{S^n}$, where $k \leqslant n$, modelizing for example delays between cells (see *e.g.* [1]).

Chaotic iterations generate a set of vectors (boolean vector in this paper), they are defined by an initial state x^0, an iteration function f and a chaotic strategy S.

III.   A new chaos-based watermarking algorithm

1   Most and least significant coefficients

Let us first introduce the definition of most and least significant coefficient of an image.

Definition 3 For a given image, most significant coefficients (in short MSCs), are coefficients that allow the description of the relevant part of the medium, *i.e.* its most rich part (in terms of embedding informations), through a sequence of bits.

For example, in a spatial description of a grayscale image, a definition of MSCs can be the sequence constituted by the three first bits of each pixel. In a frequency discrete cosine domain description, each 8×8 block of the carrier image is mapped to a list of 64 coefficients; the energy of the image is contained in the firsts of them, so the first fourth coefficients of all the blocks can be a good sequence of MSCs, after binary conversion.

Definition 4 By least significant coefficients (LSCs), we mean a translation of some unsignificant parts of a medium in a sequence of bits (unsignificant can be understand as: "which can be altered without sensitive damages").

This LSCs can be, for example, the last three bits of the gray level of each pixel, in case of spatial domain watermarking.
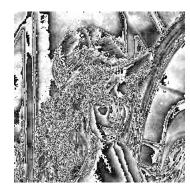

Fig 1a. MSCs of Lena

Fig 1b. LSCs of Lena,
pixel value increased by a factor 10

Discrete cosine, Fourier and wavelet transform can be used to generate LSCs and MSCs, in case of frequency domain watermarking, but other choices are possible.

LSCs are used during the stage of embedding: some of the least significant coefficients of the carrier image will be chaotically chosen, and replaced by the bits of the (possibly encrypted) watermark. With a tall number of LSCs, the watermark can be inserted more than one time, and thus the embedding will be more secure and robust, but more detectable.

MSCs are only useful in case of authentication: thus, encryption and embedding will depend on them. As a consequence, a coefficient should not be defined, at the same time, as a MSC and a LSC: the latter can be altered, while the former are needed to extract the watermark (in case of authentication).

2   Stages of the algorithm

Our watermarking scheme consists in two stages: encryption of the watermark and its embedding.

2.1   Watermark encryption

For security reasons, the watermark can be encrypted before its embedding into the image. A common way to achieve this stage is to use the bitwise exclusive or (XOR), for example between the watermark and a logistic map[1]. In this document, we will introduce a new encryption scheme, based on chaotic iterations.

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[1]The Logistic map is defined by

$$U^0 \in]0, 1[, \ \mu \in [3.57; 4],$$
$$U^{n+1} = \mu \, U^n \, (1 - U^n).$$

Its chaotic strategy will be high sensitive to the MSCs, in case of an authenticated watermark. For more precision see 1.2 in section V. below.

2.2 Watermark embedding

Some LSCs will be substituted by the bits of the possibly encrypted watermark. To choose the sequence of LSCs to be replaced, a chaotic sequence $(U^k)_k$ of integers, lower than the number N of LSCs, is generated from the chaotic strategy used in the encryption stage. Thus, the $U^k - th$ least significant coefficient of the carrier image is substituted by the kth bit of the possibly encrypted watermark. In case of authentication, such a procedure conducts to a choice of the LSCs highly dependant to the MSCs. See 1.3 in section V. for more details.

3 Extraction

The chaotic strategy can be regenerated, even in the case of an authenticated watermarking: the MSCs have not been changed during the stage of embedding watermark. Thus, the few altered LSCs can be found, and the encrypted watermark can be rebuilt, and decrypted.

If the watermarked image is attacked, then the MSCs will change. Consequently, *in case of authentication* and due to the high sensitivity of the embedding sequence, the LSCs designed to receive the watermark will be completely different. Hence, the result of the decrypting stage of the extracted bits will have no similarity with the original watermark.

4 The general chaos-based watermarking algorithm

The different stages of our method can be described in a more general framework. First of all, the representation domain of the carrier image has to be decided (spatial, DCT, DWT, *etc.*) As explained above, this choice will affect the following MSCs and LSCs (most and least significant coefficients). Then, the question of the encryption of the watermark must be asked, and if an encryption is needed, then a cipher method will be chosen. In this paper, the watermark has been encrypted with chaotic iterations, but other methods can be chosen, such as making the bitwise exclusive or with some other boolean maps.

For the next stage, we need to know whether the embedding has to be authenticated. If so, the significant information contained in the carrier image must be summarized in a selection of relevant MSCs: there are lot of possibilities, depending on the representation domain. Then, in case of authentication, the MSCs must be associated, on the one hand, to the encryption of the watermark, and on the other hand, to the embedding of the possibly encrypted watermark.

Last, LSCs must be defined, to receive the watermark, and the embedding method must be chosen in order to alter a few LSCs with the watermark: each of this stages can be achieved in a lot of different ways.

Let us summarize, in a scheme, the above watermarking method.

To obtain a bit sequence X, the following proceeding can be applied: if $U^k < 0.5$, then $X^k = 0$, else $X^k = 1$.

Fig 10. The chaos-based watermarking algorithm
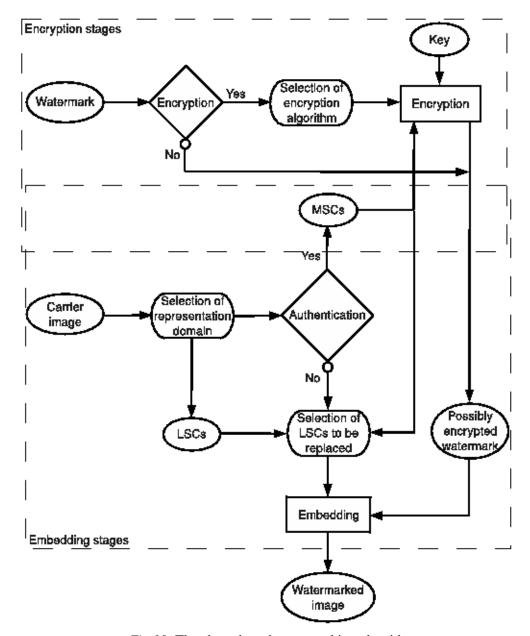
IV.   Related works and contributions

In [9] and [8], a new chaotic watermarking method is presented, starting by the encryption W_e of a watermark W, thanks to the formula: $W_e = W \otimes X$, where $\otimes$ denotes the bitwise exclusive or, and X is the bitwise logistic map. Then, for each bit b of the encrypted watermark, a pixel of the carrier image is chosen

to embed b, with 2-D Arnold cat map[2], and the logistic map is used another time, translated as above into a sequence of $\{4, 5, 6, 7\}$, to choose which least significant bit (LSB) of this pixel will be replaced by b. The watermark extraction is just the inverse process of the embedding algorithm.

Other schemes for the choice of the LSBs have been proposed, the goal is mostly to change the robustness against attacks and to make the detection of the watermark more difficult. To carry out that hash function or digital signature were used. The paper [6] is an example where the weakness of the embedding process is needed. Other embedding domains were also explored. DFT, DCT and DWT are three frequently used transformations. For example, in [10], logistic and Arnold cat maps are used with neural networks to encrypt the watermark which is embedded in the wavelet domain. Other examples of such embedding, using DWT and chaotic map, can be found in [3], or in [4].

In the above mentioned papers, chaos is frequently employed in order to ensure the strong authentication and secure encryption and uniform embedding of the watermark. However, the question of how and what chaotic properties are really necessary to achieve these goals is never elucidated. Moreover, authentication and encryption are not always proposed.

In this study we try to ensure these objectives using a global and comprehensive approach. It should be noticed that chaotic iterations, on which our method is based, can be written in the field of discrete dynamic systems:

$$\begin{cases} x^0 \in \mathcal{X} \\ x^{n+1} = f(x^n) \end{cases}$$

where $(\mathcal{X}, d)$ is a metric space (for a distance to be defined), and f is a continuous function (see [2]). Thus, it becomes possible to study the topological behavior of those chaotic iterations.

Precisely, we have proved in [2] that, if the iterate function f is suitably chosen, then chaotic iterations generate a chaos in the meaning of Devaney (its definition can be found in the Appendix, or in [5]). Therefore, chaotic iterations as Devaney's topological chaos satisfy sensitive dependence to the initial condition, unpredictability, indecomposability and uniform repartition. Sensitivity to initial conditions is used in authentication: in this case, watermark encryption and LSCs are highly dependent to any changes of the carrier image. Unpredictability make it impossible to determine whose LSCs have been altered. Last, the watermark cannot be removed, even by cropping carrier image: it is theoretically possible to make the authentication of a watermarked image, even by studying just a part of it (Devaney's chaos definition contain the so called transitivity property: the watermark can be found in any part of the carrier image).

V.   A case study

In this section, an application example of the above chaotic watermarking method is given, and its robustness through attacks is studied.

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[2]2-D Arnold cat map is defined by

$$\begin{cases} X^{n+1} &= X^n + Y^n \pmod 1 \\ Y^{n+1} &= X^n + 2Y^n \pmod 1 \end{cases} \tag{2}$$

1 Stages and details

1.1 Images description

Carrier image is the so famous Lena, which is a 256 grayscale image, and the watermark is the following 64×64 pixels binary image:

Fig 2. Lena (*scale* 0.5)   *Fig 3.* Watermark

The embedding domain will be the spatial domain: selected MSCs are the four most significant bits of each pixel, and LSCs are the three following bits (a given pixel will at most be modified of four levels of gray, see *Fig. 1a, 1b*).

1.2 Encryption of the watermark

Let us explain how to encrypt the watermark by using chaotic iterations defined by definition 2 in section II.. The initial state x^0 of the system is constituted by the watermark, considered as a boolean vector. The iteration function is the vectorial logical negation[3] f_0, and the chaotic strategy $(S^k)_{k \in \mathbb{N}}$ will depends on whether an authenticated watermarking method is desired or not, as follows.

A chaotic boolean vector is generated by a number T of iterations of a logistic map $((\mu, U_0)$ parameters will constitute the private key). Then, in case of unauthenticated watermarking, the bits of the chaotic boolean vector are grouped six by six, to obtain a sequence of integers lower than 64, which will constitute the chaotic strategy. In case of authentication, the bitwise exclusive or (XOR) is made between the chaotic boolean vector and the MSCs, and the result is converted into a chaotic strategy by joining its bits as above. Thus, the encrypted watermark is the last boolean vector generated by the chaotic iterations.

Let us give some examples of such an encryption ($T = 5000$ iterations, $\mu = 4$, $X_0 = 0.61$):

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[3]The *vectorial logical negation is defined by*

$$f_0 : \quad \begin{array}{ccc} \mathbb{B}^\mathsf{N} & \longrightarrow & \mathbb{B}^\mathsf{N} \\ (x_1, \ldots, x_\mathsf{N}) & \longmapsto & (\overline{x_1}, \ldots, \overline{x_\mathsf{N}}) \end{array}$$

Fig 4a. Without authentication   *Fig 4b.* With authentication   *Fig 4c.* Differences
between $4a$ and $4b$: 49,27 %

1.3 Embedding of the watermark

To embed the watermark, the sequence $(U^k)_{k \in \mathbb{N}}$ of altered bits taken from the LSCs must be defined. To do so, the strategy $(S^k)_{k \in \mathbb{N}}$ defined in the encryption stage is used as follows

$$\begin{cases} U^0 & = & S^0 \\ U^{n+1} & = & S^{n+1} + 2 \times U^n + n \end{cases} \tag{3}$$

Fig 5a. Watermarked Lena
(encryption, no authentication)
PSNR = 54.78

Fig 5b. Differences between original
and watermarked Lena
(pixel values increased by a factor 42)

Remark that the map $\theta \mapsto 2\theta$ of the torus, which is a famous example of topological Devaney's chaos [5], has been chosen to make $(U^k)_{k \in \mathbb{N}}$ high sensitive to the chaotic strategy. As a consequence, $(U^k)_{k \in \mathbb{N}}$ is high sensitive to the alteration of the MSCs: in case of authentication, any significant modification of the watermarked image will conduct to a completely different extracted watermark.

2 Simulation results

In what follows, as in subsection V. 1.2 the embedding domain is the spatial domain, logistic map with parameters ($\mu = 4$, $X_0 = 0.61$) has been used to encrypt the watermark, MSCs are the four firsts bits of each pixel (useful only in case of authentication), and LSCs are the three next bits.

To prove the efficiency and the robustness of the proposed algorithm, some attacks are applied to our chaotic watermarked image. For each attack, a similarity percentage with the watermark is computed: this percentage is the number of equal bits between the original and the extracted watermark, shown as a percentage. Let us notice that a result less than or equal to 50% implies that the image has probably not been watermarked.

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2.1 Geometric attacks

Cropping attack. In this kind of attack, a watermarked image is cropped, such as:

Fig 6. Crop by 100×100 pixels.

In this case, the following results have been obtained...

UNAUTHENTICATION		AUTHENTICATION	
Size (pixels)	Similarity percentage	Size (pixels)	Similarity percentage
10	99.08%	10	89.81%
50	97.31%	50	54.54%
100	92.43%	100	52.24%
200	70.75%	200	51.87%

In what follows, the decrypted watermarks are shown after a crop of 50 pixels, and after a crop of 10 pixels, in the authentication case.

50×50 pixels $\quad$ 10×10 pixels $\quad$ 50×50 pixels
Unauthentication $\qquad$ Authentication
Fig 7. Extracted watermark after a cropping attack.

In case of unauthentication, the watermark still remain after a cropping attack: the desired robustness is reached. It can be noticed that cropping sizes and percentages are rather proportional.

In case of authentication, even a small change of the carrier image (a crop by 10×10 pixels) conduct to a really different extracted watermark. In this case, any attempt to alter the carrier image will be signaled, image is well authenticated.

Rotation attack. Let r_θ be the rotation around the center 128×128 of the carrier image, by angle θ. So, the transformation $r_{-\theta} \circ r_\theta$ is applied to the watermarked image, which is altered as follows

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Fig 8. After $r_{-25°} \circ r_{25°}$

In this case, the following results are obtained.

UNAUTHENTICATION		AUTHENTICATION	
Angle (degree)	Similarity percentage	Angle (degree)	Similarity percentage
2	97.51%	2	70.01%
5	94.67%	5	59.47%
10	91.30%	10	54.51%
25	80.85%	25	50,21%

Decryption of the extracted watermark, after a rotation by $5°$:

Fig 9a. Unauthentication   *Fig 9b.* Authentication

The same conclusion as above can be declaimed: this watermarking method satisfies the desired properties.

2.2 Other attacks

JPEG compression.   A JPEG compression is applied to the watermarked image, depending on a compression level. Let us notice that this attack conducts to a change of the working domain (from spatial to DCT domain). In this case, the following results have been obtained:

UNAUTHENTICATION		AUTHENTICATION	
Compression level	Similarity percentage	Compression level	Similarity percentage
2	82.95%	2	54.39%
5	65.23%	5	53.46%
10	60.22%	10	50.14%
20	53.17%	25	48.80%

A very good authentication through JPEG attack is obtained. As for the unauthentication case, the watermark still remain after a compression level equal to 10: this is a good result for a spatial embedding.

Gaussian noise. Watermarked image can be attacked too by the addition of a Gaussian noise, depending on a standard deviation. In this case, the following results have been obtained:

UNAUTHENTICATION		AUTHENTICATION	
Standard deviation	Similarity percentage	Standard deviation	Similarity percentage
1	74.26%	1	52.05%
2	63.33%	2	50.95%
3	57.44%	3	49.65%
5	51.26%	5	49.43%

We obtain the same conclusion as above.

VI.   Discussion and future work

In this paper, a new way to generate watermarking methods is proposed. The new procedure depends on a general description of the carrier medium to watermark in terms of the significance of some coefficients we are called MSC and LSC in definition 3. We proposed a general and comprehensive algorithm which recalls the different choices and possibilities that our approach offers.

This encryption and the selection of coefficients to alter are based on chaotic iterations [2] which generate a topological chaos in the sense of Devaney [5]. So, the properties of topological chaos are satisfied by our algorithm, such as sensitivity to initial conditions, uniform repartition (thanks to the transitivity) and unpredictability. Thus, the proposed algorithm possesses the desirable properties expected in good watermarking algorithms. For example, a strong authentication of the carrier image is satisfied thanks to the sensitivity, the unpredictability property implies security, and the resistance against attacks and discretion of the watermark are consequences of the transitivity property.

The algorithm has been evaluated through attacks, and all of the expected results have been experimentally obtained. Choices that have been made in this first study are simple: spatial domain, negation function for the iteration function. The aim was not to find the best watermarking method generated by our general algorithm, but to give a simple illustration example, to prove its feasibility.

In future work, in order to increase authentication and resistance to attacks, other choices of the iteration function and the chaotic strategy will be studied and compared. In addition, frequency domain representations will be used to select the MSCs and LSCs. Moreover others properties ensured by chaotic iterations, such as entropy, will be shown and their role in watermarking algorithms will be explained..

References

[1] J. M. Bahi. Boolean totally asynchronous iterations. *Int. Journal of Mathematical Algorithms*, 1:331–346, 2000.

[2] J. M. Bahi and C. Guyeux. Chaotic iterations and topological chaos. 2008, arXiv:0810.3154.

[3] J. Cong, Y. Jiang, Z. Qu, and Z. Zhang. A wavelet packets watermarking algorithm based on chaos encryption. *LNCS*, 3980:921–928, 2006.

[4] Z. Dawei, C. Guanrong, and L. Wenbo. A chaos-based robust wavelet-domain watermarking algorithm. *Chaos, Solitons and Fractals*, 22:47–54, 2004.

[5] R. L. Devaney. *An Introduction to Chaotic Dynamical Systems*. Redwood City: Addison-Wesley, 2. edition, 1989.

[6] S.-H. Liu, H.-X. Yao, W. Gao, and Y.-L. Liu. An image fragile watermark scheme based on chaotic image pattern and pixel-pairs. *Applied Mathematics and Computation*, 185:869–882, 2007.

[7] F. Robert. *Discrete Iterations: A Metric Study*, volume 6 of *Springer Series in Computational Mathematics*. 1986.

[8] X. Wu and Z.-H. Guan. A novel digital watermark algorithm based on chaotic maps. *Physical Letters A*, 365:403–406²², 2007.

[9] X. Wu, Z.-H. Guan, and Z. Wu. A chaos based robust spatial domain watermarking algorithm. *LNCS*, 4492:113–119, 2007.

[10] J. Zhao, M. Zhou, H. Xie, J. Peng, and X. Zhou. A novel wavelet image watermarking scheme combined with chaos sequence and neural network. *LNCS*, 3174:663–668, 2004.

Appendix: Devaney's chaotic dynamic systems

Consider a metric space $(\mathcal{X}, d)$, and a continuous function $f : \mathcal{X} \longrightarrow \mathcal{X}$.

Definition 5 f is said to be *topologically transitive* if, for any pair of open sets $U, V \subset \mathcal{X}$, there exists $k > 0$ such that $f^k(U) \cap V \neq \varnothing$.

Definition 6 An element (a point) x is a periodic element (point) for f of period $n \in \mathbb{N}$, if $f^n(x) = x$. The set of periodic points of f is denoted $Per(f)$.

Definition 7 $(\mathcal{X}, f)$ is said to be *regular* if the set of periodic points is dense in $\mathcal{X}$,

$$\forall x \in \mathcal{X}, \forall \varepsilon > 0, \exists p \in Per(f), d(x, p) \leqslant \varepsilon.$$

Definition 8 f has *sensitive dependence on initial conditions* if there exists $\delta > 0$ such that, for any $x \in \mathcal{X}$ and any neighborhood V of x, there exists $y \in V$ and $n \geqslant 0$ such that $|f^n(x) - f^n(y)| > \delta$.
δ is called the *constant of sensitivity* of f.

Let us recall the definition of a chaotic topological system, in sense of Devaney [5]:

Definition 9 $f : \mathcal{X} \longrightarrow \mathcal{X}$ is said to be *chaotic* on $\mathcal{X}$ if, and only if

1. f has sensitive dependence on initial conditions,

2. f is topologically transitive,

3. $(\mathcal{X}, f)$ is regular.

Watermark → Encryption — No → Watermark possibly encrypted

Encryption — Yes → Selection of encryption algorithm

Watermark → Encryption — No — Authentication — No → Watermark possibly encrypted

Encryption — Yes → Selection of encryption algorithm

Yes → Selection of association

Authentication — No

Authentication — Yes → Selection of association

Carrier image → Selection of image representation → Selection of MSC

Selection of MSC

Carrier image → Selection of image representation → Selection of MSC

Watermark to be insered

Watermark

Watermark → Encryption — No → Watermark possibly encrypted

Encryption — Yes → Selection of encryption algorithm

Authentication — No

Authentication — Yes → Selection of association → Selection of MSC

Carrier Image → Selection of image representation → Selection of MSC

Watermark to be insered

Selection of LSC → Selection of insertion method → Watermark to be insered

Selection of insertion method → Watermark to be insered