Communication Theory
Lab 1 Report

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2. Analytical Description of a stochastic process

Procedure

The random process was simulated by first generating 100 realizations of the random variables X and Y from a normal distribution with mean $\mu = 0$ and variance $\sigma^2 = 1/2$ using the `normrnd` function. They were stored in column vectors `X` and `Y`, white the time instants to be simulated were stored in a row vector `t=0:0.5:20`.

Then, they were combned in pairs to form realizations of the random process $Z(t) = X(t)\cos(2\pi f_0 t) + Y(t)\sin(2\pi f_0 t)$, where each X-Y pair resulted in one realization of $Z(t)$. Each realization of the process was stored in a row of the vector `Z`, where each column contained the value at the instant corresponding to that column of the time vector.

Then, the mean of all realizations was computed by using the `mean` function in the first dimension, yielding a row vector `mZ` containing the mean of all realizations at each time instant. The average value of this vector was computed by using the `mean` function once again, and the resulting average value was stored in `mZavg`.

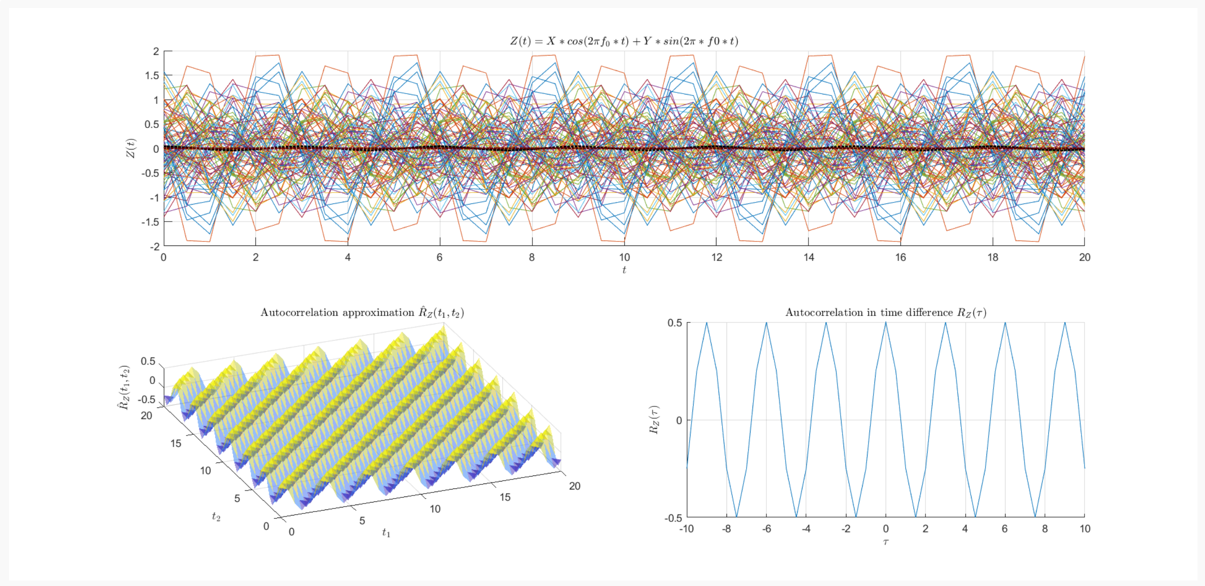
The correlation function $R_Z(t_1, t_2)$ was computed by creating a copy of the `Z` matrix in the third dimension using the `permute` function, and then performing element-wise multiplication between the original and this copy. The resulting 3D matrix was then averaged in the first dimension with `mean` and permuted back into dimensions 1 & 2 (from 2 & 3), storing the result in `RZavg`.

Finally, the analytical $R_Z(\tau)$ was generated. First, the time difference vector `tau` was created using `-10:0.5:10`, and then the values were computed using $R_Z = \sigma^2 \cos(2\pi f_0 \tau)$

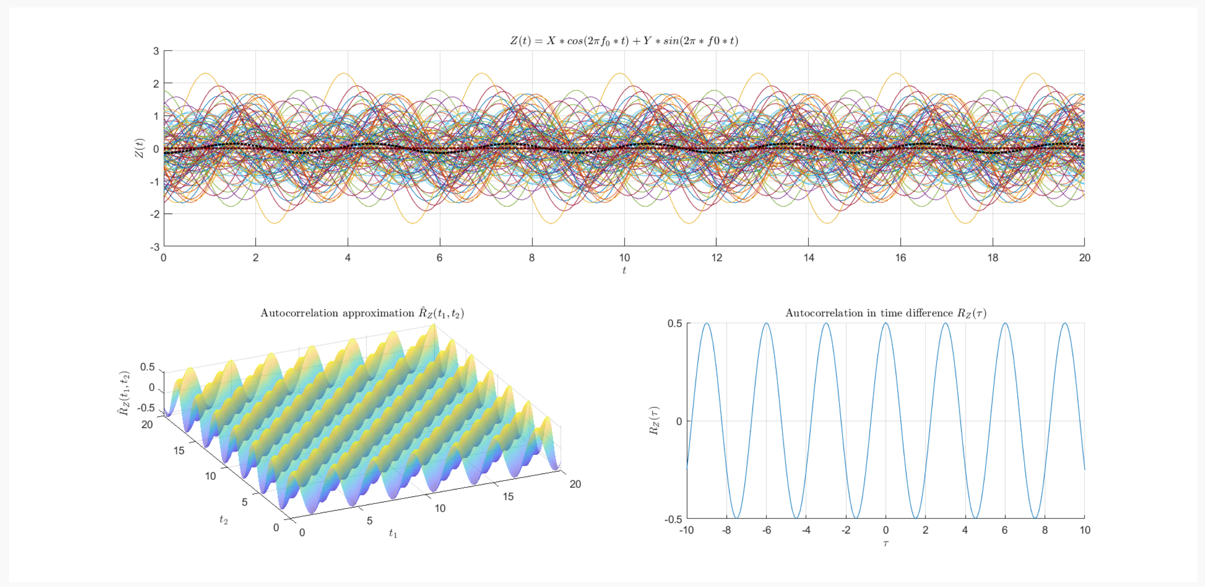
Then, the plots were generated in a 2x2 grid using `plot` and `surf`, using the first row for the simulation and average time-domain plots, and the bottom left and right plots for the 3D and 2D autocorrelation plots, respectively. The plots' parameters were adjsuted for aesthetics. The titles were written using LaTeX syntax, which had to be set up for interpretation.

Additional plots were generated using a 0.1 time step to increase the resolution of the plots, since 0.5 was too coarse.

Results and plots



Requested plot with 0.5 time step



Plot with 0.1 time step

Code

```
%% Clear
set(groot,'defaulttextinterpreter','latex');
set(groot, 'defaultLegendInterpreter', 'latex');

%% Distribution and simulation parameters
avg = 0;
var = 1/2;
stdev = sqrt(var);
f0 = 1/3; % frequency

tstep = 0.5;
t = (0:tstep:20);
N = 100; % realizations


%% Generate normal distribution
X = normrnd(avg, stdev, [N, 1]);
Y = normrnd(avg, stdev, [N, 1]);
Z = X.*cos(2*pi*f0*t) + Y.*sin(2*pi*f0*t);


%% Numerical computations
mZ_ = mean(Z, 1); % Using _ to indicate numerical approximation
mZ = repmat(avg, size(t)); % Not using _ to indicate analytical expression

RZ_ = permute(mean(Z.*permute(Z, [1, 3, 2])), [2,3,1]);

tau = linspace(-10, 10, size(t, 2));
RZtau = var * cos(2 * pi * f0 * tau); % No _ because this uses the analytical
expression


%% Plots
figure(1);
legend('show');
subplot(2, 2, 1:2, 'replace'); grid on; hold on;
title('$Z(t) = X*cos(2 \pi f_0*t) + Y*sin(2 \pi*f0*t)$', Interpreter='latex');
for i = 1:N
    plot(t, Z(i, :), HandleVisibility='off', LineWidth=0.1);
end
plot(t, mZ_, Color='black', LineStyle=':', DisplayName='$\hat{m}_Z(t)$',
LineWidth=1.8);
plot(t, mZ, Color='#660000', LineStyle=':', DisplayName='$m_Z(t)$',
LineWidth=1.8);
xlabel('$t$'); ylabel('$Z(t)$');

subplot(2, 2, 3, 'replace');
[t1_, t2_] = meshgrid(t);
surf(t1_, t2_, RZ_, FaceAlpha=0.5, EdgeColor='none');
title('Autocorrelation approximation $\hat{R}_Z(t_1, t_2)$',
Interpreter='latex');
xlabel('$t_1$'); ylabel('$t_2$'); zlabel('$\hat{R}_Z(t_1, t_2)$');

subplot(2, 2, 4, 'replace'); grid on; hold on;
plot(tau, RZtau);
title('Autocorrelation in time difference $R_Z(\tau)$', Interpreter='latex');
xlabel('$\tau$'); ylabel('$R_Z(\tau)$');
```

3. Statistical Description of a stochastic process

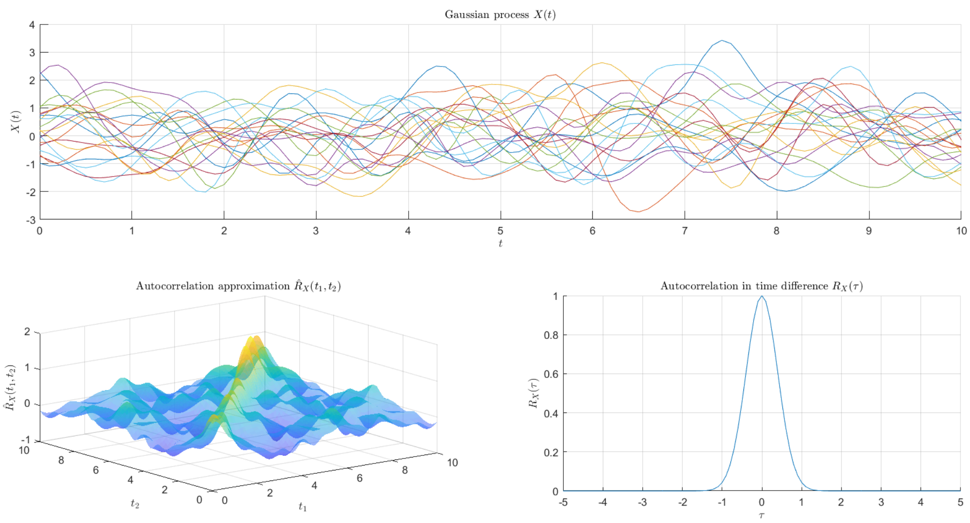
Procedure

The stochastic process was simulated using the given parameters passed to the `mvrnd` function (multivariate normal distribution), and the covariance matrix was computed usin the same method as in section 2. The 2D covariance was computed using the following equation for Gaussian processes and the given expressions, using the substitution $t_1 - t_j = \tau$:

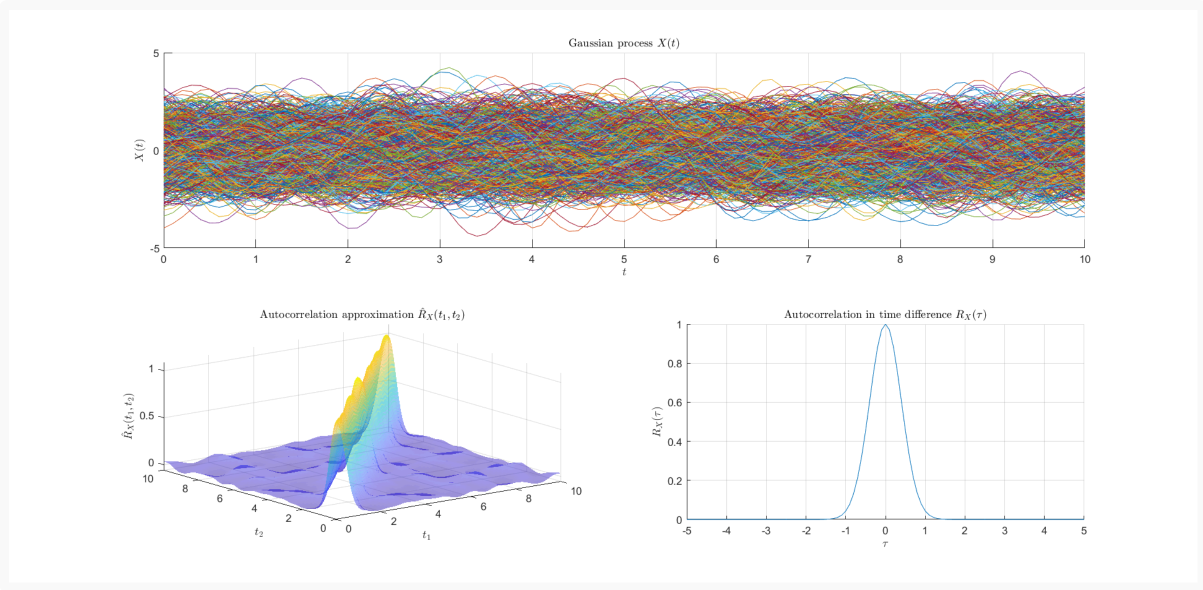
$$C_{ij} = R_X(t_i, t_j) - \mu_X(t_i)\mu_X(t_j) = R_X(t_i, t_j)$$

After generating these samples, the experiment was repeated using 2000 iterations instead of 20, in order to confirm that the noise in the covariance matrix was only the result of the numerical approximation with a low number of samples.

Results and plots



Requested plot using 20 samples



Plot using 2000 samples

Code

```matlab
%% Setup
set(groot,'defaulttextinterpreter','latex');,
set(groot, 'defaultLegendInterpreter', 'latex');


%% Parameters

tstep = 0.1;
t = 0:tstep:10;
T = length(t);
N = 20;

stdev = 1/sqrt(2*pi); % Sigma
var = stdev^2; % Sigma squared

mX = zeros(size(t)); % Mu
CovX = exp(-(t-t').^2/(2*var));


%% Generate samples

X = mvnrnd(mX, CovX, N);
RX_ = permute(mean(X.*permute(X, [1, 3, 2])), [2,3,1]);

tau = -5:tstep:5;
RXtau = exp(-(tau).^2/(2*var));


%% Plot

figure(1);
subplot(2, 2, 1:2, 'replace'); grid on; hold on;
for i = 1:N
    plot(t, X(i, :), HandleVisibility='off', LineWidth=0.1);
end

title('Gaussian process $X(t)$', Interpreter='latex');
xlabel('$t$'); ylabel('$X(t)$');

subplot(2, 2, 3, 'replace');
[t1_, t2_] = meshgrid(t);
surf(t1_, t2_, RX_, FaceAlpha=0.5, EdgeColor='none'); hold on;

title('Autocorrelation approximation $\hat{R}_X(t_1, t_2)$',
Interpreter='latex');
xlabel('$t_1$'); ylabel('$t_2$'); zlabel('$\hat{R}_X(t_1, t_2)$');


subplot(2, 2, 4, 'replace'); grid on; hold on;
plot(tau, RXtau);

title('Autocorrelation in time difference $R_X(\tau)$', Interpreter='latex');
xlabel('$\tau$'); ylabel('$R_X(\tau)$');
```