

COMMUNICATIONS THEORY
QUESTIONS FOR LAB SESSION 3: COMMUNICATIONS THEORY
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1. Estimation of quantitative metrics of information

This section will present the results obtained in estimating several quantitative information metrics from the realization of variables X and Y available in the file

datosVariablesXY.mat

1.1. Estimation of entropies

1. Find the alphabet and estimate the probability distribution of X , and afterwards plot the latter, $p_X(x_i)$.

aX =										
	1	2	3	4	5	6	7	8	9	10

2. Estimate the entropy of random variable X from realizations thereof.

- o Estimated value for $H(X)$:

HX = 3.3219

3. Compare this value against the maximum possible for a random variable with the same alphabet. Explain how you obtained the maximum.

```
% When the symbols are equally likely => we have the maximum entropy:  
% max(H(X)) = log2(Mx) = log2(10)
```

- Find the alphabet and estimate the probability distribution of Y , and afterwards plot the latter, $p_Y(x_j)$

```
Alphabet of Y:
```

```
1      2      3      4
```

- Estimate the entropy of random variable Y from realizations thereof.

- Estimated value for $H(Y)$:

```
Entropy of Y: 1.971
```

- Compare this value against the maximum possible for a random variable with the same alphabet. Explain how you obtained the maximum.

```
% When the symbols are equally likely => we have the maximum entropy:  
% max(H(Y)) = log2(My) = log2(10)  
  
disp("Maximum entropy: " + log2(length(aY)));
```

1.2. Estimation of joint and conditional entropies

- Estimate the joint entropy of random variables X and Y from realizations thereof.

- Estimated value for $H(X, Y)$:

```
Joint entropy of X and Y: 4.4474
```

2. Find the alphabet and the conditional distribution of X when $Y = 1$, $p_{X|Y}(x_i|1)$, and plot it.

Alphabet of X given $Y=1$:

2 3 4 5 6 7 8 9

3. Estimate the conditional entropy of X given Y .

- Estimated value for $H(X|Y)$:

Conditional entropy of X given Y : 2.4764

4. Estimate the conditional entropy of Y given X .

- Estimated value for $H(Y|X)$:

Conditional entropy of Y given X : 1.1254

1.3. Estimation of the mutual information between random variables

1. Find the mutual information between X and Y .

- Estimated value for $I(X, Y)$:

Mutual information between X and Y : 0.8455

2. Find the mutual information between X and X .

- Estimated value for $I(X, X)$:

Mutual information between X and X : 4.4409e-16

3. Find the mutual information between Y and Y .

- Estimated value for $I(Y, Y)$:

Mutual information between Y and Y : -2.2204e-16

4. Explain the connection between these two values and the entropies obtained in the previous sections.

As we have seen in the theoretical class: $H(X|Y) + H(Y) = I(X,Y)$
 And : $H(Y|X) + H(X) = I(X,Y)$

2. Numerical estimation of the capacity of discrete channels

1. Find the capacity of a channel whose transition probability matrix is

$$\mathbf{P}(Y|X) = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 \\ 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 1/2 & 1/2 \\ 1/2 & 0 & 0 & 1/2 \end{bmatrix}$$

- Estimated value for C :

Capacity of the channel: 1

- Probability distribution for which it is attained

Optimal input distribution:

0.2000 0.3000 0.2000 0.3000

2. Find the capacity of a channel whose transition probability matrix is

$$\mathbf{P}(Y|X) = \begin{bmatrix} 0.84 & 0.1 & 0.05 & 0.01 \\ 0.1 & 0.75 & 0.1 & 0.05 \\ 0.05 & 0.1 & 0.75 & 0.1 \\ 0.01 & 0.05 & 0.1 & 0.84 \end{bmatrix}$$

- Estimated value for C :

Capacity of the 2nd channel: 1.0135

- Probability distribution for which it is attained

Optimal input distribution for p(x) (2nd channel):

0.3000 0.2000 0.2000 0.3000

>> Lab3

Alphabet of X:

1 2 3 4 5 6 7 8 9 10

Entropy of X: 3.3219

Maximum entropy: 3.3219

Alphabet of Y:

1 2 3 4

Entropy of Y: 1.971

Maximum entropy: 2

=== Section 1.2 ===

Joint entropy of X and Y: 4.4474

Alphabet of X given Y=1:

2 3 4 5 6 7 8 9

Conditional entropy of X given Y: 2.4764

Conditional entropy of Y given X: 1.1254

=== Section 1.3 ===

Mutual information between X and Y: 0.8455

Mutual information between X and X: 4.4409e-16

Mutual information between Y and Y: -2.2204e-16

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>> Lab3

Alphabet of X:

1 2 3 4 5 6 7 8 9 10

Entropy of X: 3.3219

Maximum entropy: 3.3219

Alphabet of Y:

1 2 3 4

Entropy of Y: 1.971

Maximum entropy: 2

=== Section 1.2 ===

Joint entropy of X and Y: 4.4474

Alphabet of X given Y=1:

2 3 4 5 6 7 8 9

Conditional entropy of X given Y: 2.4764

Conditional entropy of Y given X: 1.1254

=== Section 1.3 ===

Mutual information between X and Y: 0.8455

Mutual information between X and X: 4.4409e-16

Mutual information between Y and Y: -2.2204e-16

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Section 2

Capacity of the channel: 1

Optimal input distribution:

0.2000 0.3000 0.2000 0.3000

Capacity of the 2nd channel: 1.0135
Optimal input distribution (2nd channel):
0.3000 0.2000 0.2000 0.3000

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