

COMMUNICATIONS THEORY  
QUESTIONS FOR LAB SESSION 3: COMMUNICATIONS THEORY  
ACADEMIC YEAR 2023/2024

Student 1: .....	Grade		
Student 2: .....	<table border="1" style="display: inline-table; border-collapse: collapse;"><tr><td style="width: 20px; height: 20px; text-align: center; vertical-align: middle;">T</td><td style="width: 40px; height: 20px;"></td></tr></table>	T	
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## 1. Estimation of quantitative metrics of information

This section will present the results obtained in estimating several quantitative information metrics from the realization of variables  $X$  and  $Y$  available in the file

**datosVariablesXY.mat**

### 1.1. Estimation of entropies

1. Find the alphabet and estimate the probability distribution of  $X$ , and afterwards plot the latter,  $p_X(x_i)$

2. Estimate the entropy of random variable  $X$  from realizations thereof

◦ Estimated value for  $H(X)$  :

3. Compare this value against the maximum possible for a random variable with the same alphabet. Explain how you obtained the maximum.

- Find the alphabet and estimate the probability distribution of  $Y$ , and afterwards plot the latter,  $p_Y(x_j)$

- Estimate the entropy of random variable  $Y$  from realizations thereof

◦ Estimated value for  $H(Y)$  :

- Compare this value against the maximum possible for a random variable with the same alphabet. Explain how you obtained the maximum.

## 1.2. Estimation of joint and conditional entropies

- Estimate the joint entropy of random variables  $X$  and  $Y$  from realizations thereof.

◦ Estimated value for  $H(X, Y)$  :

- Find the alphabet and the conditional distribution of  $X$  when  $Y = 1$ ,  $p_{X|Y}(x_i|1)$ , and plot it

- Estimate the conditional entropy of  $X$  given  $Y$

◦ Estimated value for  $H(X|Y)$  :

4. Estimate the conditional entropy of  $Y$  given  $X$

◦ Estimated value for  $H(Y|X)$  :

### 1.3. Estimation of the mutual information between random variables

1. Find the mutual information between  $X$  and  $Y$

◦ Estimated value for  $I(X, Y)$  :

2. Find the mutual information between  $X$  and  $X$

3. Estime la información mutua entre  $X$  y  $X$

◦ Estimated value for  $I(X, X)$  :

4. Find the mutual information between  $Y$  and  $Y$

◦ Estimated value for  $I(Y, Y)$  :

5. Explain the connection between these two values and the entropies obtained in the previous sections.

## 2. Numerical estimation of the capacity of discrete channels (optional)

1. Find the capacity of a channel whose transition probability matrix is

$$\mathbf{P}(Y|X) = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 \\ 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 1/2 & 1/2 \\ 1/2 & 0 & 0 & 1/2 \end{bmatrix}$$

◦ Estimated value for  $C$  :

◦ Probability distribution for which it is attained

2. Find the capacity of a channel whose transition probability matrix is

$$\mathbf{P}(Y|X) = \begin{bmatrix} 0.84 & 0.1 & 0.05 & 0.01 \\ 0.1 & 0.75 & 0.1 & 0.05 \\ 0.05 & 0.1 & 0.75 & 0.1 \\ 0.01 & 0.05 & 0.1 & 0.84 \end{bmatrix}$$

o Estimated value for  $C$  :

o Probability distribution for which it is attained