COMMUNICATIONS THEORY
QUESTIONS FOR LAB SESSION 3: COMMUNICATIONS THEORY
ACADEMIC YEAR 2023/2024

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1. Estimation of quantitative metrics of information

This section will present the results obtained in estimating several quantitative information metrics from the realization of variables X and Y available in the file

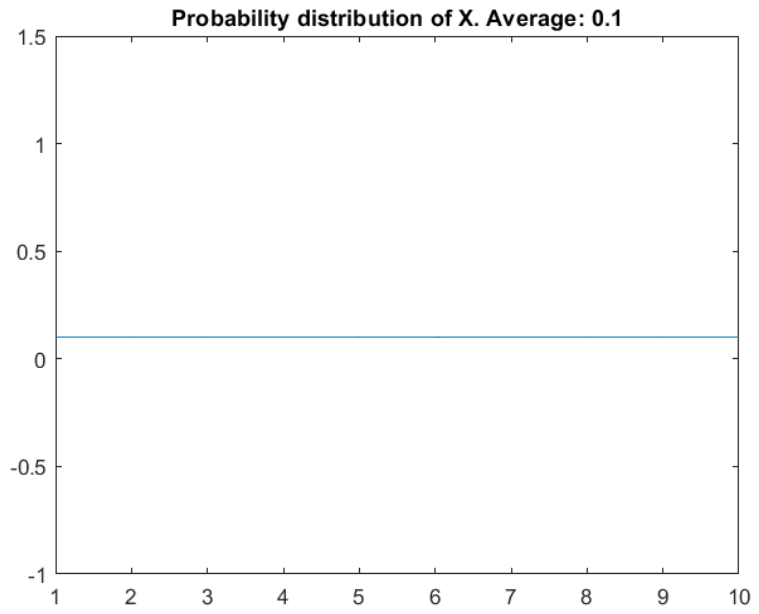
datosVariablesXY.mat

1.1. Estimation of entropies

1. Find the alphabet and estimate the probability distribution of X, and afterwards plot the latter, $p_X(x_i)$.

```
aX =

     1     2     3     4     5     6     7     8     9    10
```



2. Estimate the entropy of random variable X from realizations thereof.

- Estimated value for $H(X)$:

```
HX =

    3.3219
```

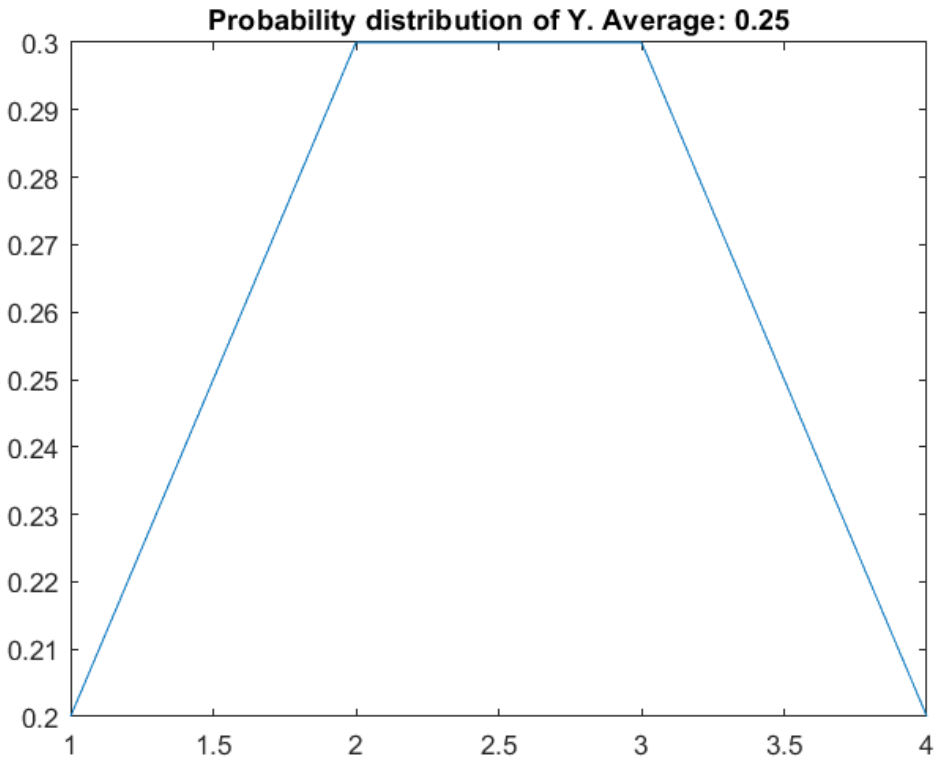
3. Compare this value against the maximum possible for a random variable with the same alphabet. Explain how you obtained the maximum.

```
% When the symbols are equally likely => we have the maximun entropy:
% max(H(X)) = log2(Mx) = log2(10)
```

Maximum entropy = 3.3219

4. Find the alphabet and estimate the probability distribution of Y, and afterwards plot the latter, $p_Y(x_j)$

```
Alphabet of Y:
     1     2     3     4
```



5. Estimate the entropy of random variable Y from realizations thereof.

- Estimated value for $H(Y)$:

```
Entropy of Y: 1.971
```

6. Compare this value against the maximum possible for a random variable with the same alphabet. Explain how you obtained the maximum.

```
% When the symbols are equally likely => we have the maximun entropy:
% max(H(Y)) = log2(My) = log2(10)

disp("Maximum entropy: " + log2(length(aY)));
```

Maximum entropy: 2

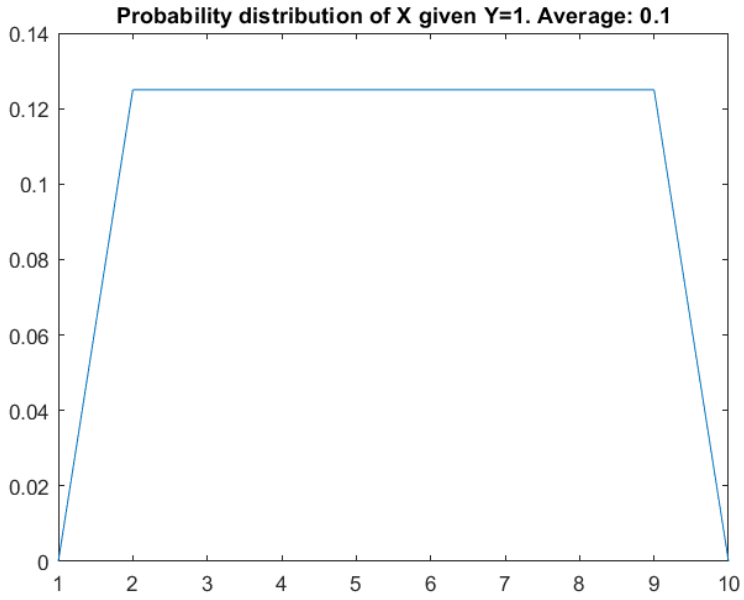
1.2. Estimation of joint and conditional entropies

1. Estimate the joint entropy of random variables X and Y from realizations thereof.

   - Estimated value for $H(X,Y)$: 4.4474

2. Find the alphabet and the conditional distribution of X when $Y = 1$, $p_{X|Y}(x_i|1)$, and plot it.

```
Alphabet of X given Y=1:
     2     3     4     5     6     7     8     9
```



3. Estimate the conditional entropy of X given Y.

   - Estimated value for $H(X|Y)$: 2.4764

4. Estimate the conditional entropy of Y given X.

   - Estimated value for $H(Y|X)$: 1.1254

1.3. Estimation of the mutual information between random variables

1. Find the mutual information between X and Y.

   - Estimated value for $I(X,Y)$: 0.8455

2. Find the mutual information between X and X.

   - Estimated value for $I(X,X)$: 4.4409e-16

3. Find the mutual information between Y and Y.

   - Estimated value for $I(Y,Y)$:

```
: -2.2204e-16
```

4. Explain the connection between these two values and the entropies obtained in the previous sections.

As we have seen in the theoretical class: H(X|Y) + H(Y) = I(X,Y)
And : H(Y|X) + H(X) = I(X,Y)

H(X) = I(X,Y) + H(X | Y) = 0.8455 + 2.4764 = 3.3219
H(Y) = I(X,Y) + H(Y | X) = 0.8455 + 1.1254 = 1.971

The mutual information between a variable and itself is essentially 0

2. Numerical estimation of the capacity of discrete channels

1. Find the capacity of a channel whose transition probability matrix is

$$\mathbf{P}(Y|X) = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 \\ 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 1/2 & 1/2 \\ 1/2 & 0 & 0 & 1/2 \end{bmatrix}$$

   - Estimated value for C :

```
: 1
```

   - Probability distribution for which it is attained

```
Optimal input distribution:
    0.2000    0.3000    0.2000    0.3000
```

2. Find the capacity of a channel whose transition probability matrix is

$$\mathbf{P}(Y|X) = \begin{bmatrix} 0.84 & 0.1 & 0.05 & 0.01 \\ 0.1 & 0.75 & 0.1 & 0.05 \\ 0.05 & 0.1 & 0.75 & 0.1 \\ 0.01 & 0.05 & 0.1 & 0.84 \end{bmatrix}$$

   - Estimated value for C :

Type equation here.
1.0135

- Probability distribution for which it is attained

```
Optimal input distribution for p(x) (2nd channel):
    0.3000    0.2000    0.2000    0.3000
```

Results in the command window:

```
>> Lab3
Alphabet of X:
    1     2     3     4     5     6     7     8     9    10

Entropy of X: 3.3219
Maximum entropy: 3.3219
Alphabet of Y:
    1     2     3     4

Entropy of Y: 1.971
Maximum entropy: 2

=== Section 1.2 ===

Joint entropy of X and Y: 4.4474
Alphabet of X given Y=1:
    2     3     4     5     6     7     8     9

Conditional entropy of X given Y: 2.4764
Conditional entropy of Y given X: 1.1254

=== Section 1.3 ===

Mutual information between X and Y: 0.8455
Mutual information between X and X: 4.4409e-16
Mutual information between Y and Y: -2.2204e-16
:)
>> Lab3
Alphabet of X:
    1     2     3     4     5     6     7     8     9    10

Entropy of X: 3.3219
Maximum entropy: 3.3219
Alphabet of Y:
    1     2     3     4

Entropy of Y: 1.971
Maximum entropy: 2

=== Section 1.2 ===

Joint entropy of X and Y: 4.4474
Alphabet of X given Y=1:
    2     3     4     5     6     7     8     9

Conditional entropy of X given Y: 2.4764
Conditional entropy of Y given X: 1.1254

=== Section 1.3 ===

Mutual information between X and Y: 0.8455
Mutual information between X and X: 4.4409e-16
Mutual information between Y and Y: -2.2204e-16

#### Section 2 ####

Capacity of the channel: 1
Optimal input distribution:
    0.2000    0.3000    0.2000    0.3000

Capacity of the 2nd channel: 1.0135
```

```
Optimal input distribution (2nd channel):
    0.3000    0.2000    0.2000    0.3000

>>
```