

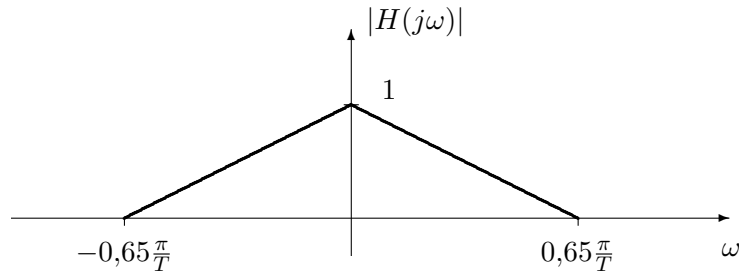
# Exercises for Chapter 2

1. A baseband PAM system uses a transmitter and a receiver such that the joint response transmitter-receiver,  $p(t)$ , is the following triangle pulse<sup>1</sup>

$$p(t) = \begin{cases} 1 - \frac{|t|}{T}, & |t| \leq T \\ 0, & |t| > T \end{cases},$$

where  $T$  is the symbol period. The distortion and noise introduced by the channel can be considered negligible.

- a) Demonstrate that  $p(t)$  fulfills the Nyquist ISI criterion.
  - b) Represent the spectrum (by means of the PSD) of the signal at the output of the receiver filter when  $A[n]$  is white.
2. A baseband PAM communication system working at symbol time  $T$  sec. uses a squared root-raised cosine filter with roll-off  $\alpha = 0,35$  as transmitter filter. The receiver uses a filter that is matched to the joint response of transmitter and channel, i.e.,  $f(t) = g_h(-t)$ , with  $g_h(t) = g(t) * h(t)$ . The channel has the following frequency response<sup>2</sup>



If the noise is white, Gaussian, with power spectral density  $N_0/2$  W/Hz, represent the power spectral density of the noise at the output of the sampler at the receiver.

<sup>1</sup>Problem 5.1 of the book: A. Artés, *et al.*: Comunicaciones Digitales. Pearson Educación, 2007.

<sup>2</sup>Problem 5.3 of the book: A. Artés, *et al.*: Comunicaciones Digitales. Pearson Educación, 2007.

3. A 2-PAM constellation is transmitted through the equivalent discrete channel  $p[n] = \delta[n] + 0,25 \cdot \delta[n-1]$ . The noise is white, Gaussian, with variance  $\sigma_z^2$ . Calculate the exact expression of the probability of error with a memoryless symbol-by-symbol detector<sup>3</sup>
4. A communications system uses a PAM modulation. The information sequence  $A_w[n]$  is a white sequence. To introduce some correlation in the transmitted sequence,  $A_w[n]$  is filtered with a filter with response  $c[n]$ , producing the sequence to be transmitted,  $A[n]$ , i.e.,  $A[n] = A_w[n] * c[n]$ . The transmission filter is a raised cosine filter with roll-off factor  $\alpha = 0$ , i.e., a *sinc* function.<sup>4</sup>
  - a) Write the expression of  $S_A(e^{j\omega})$  as a function of  $S_{A_w}(e^{j\omega})$  and  $C(e^{j\omega})$ .
  - b) Plot the power spectral density of the baseband PAM signal,  $S_S(j\omega)$ , in the following cases:
    - (I)  $c[n] = \delta[n] - \delta[n-1]$
    - (II)  $c[n] = \delta[n] + \delta[n-1]$
    - (III)  $c[n] = \delta[n] - \delta[n-2]$
  - c) Obtain the transmitted power in all of these cases.
5. Figure 16 illustrates four different cases of scattering diagram of a QPSK modulation at output of the receiver. The combined response of the filters in the transmitter and receiver is a raised-cosine filter. Explain the reason of the “dispersion” for each case in Figure 16.<sup>5</sup>

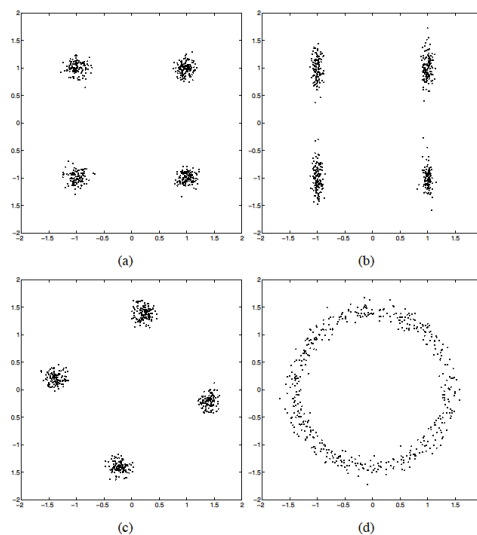


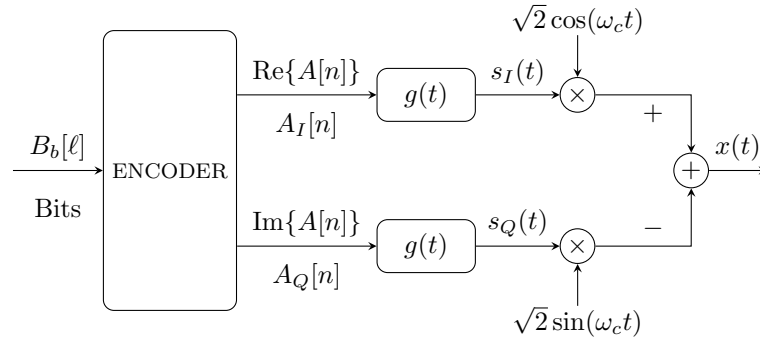
Figure 16: Different examples of dispersion in the scattering diagram.

<sup>3</sup>Problem 5.4 of the book: A. Artés, *et al.*: Comunicaciones Digitales. Pearson Educación, 2007.

<sup>4</sup>Problem 5.6 of the book: A. Artés, *et al.*: Comunicaciones Digitales. Pearson Educación, 2007.

<sup>5</sup>Problem 5.12 of the book: A. Artés, *et al.*: Comunicaciones Digitales. Pearson Educación, 2007.

6. A digital communication system uses the following modulator:



The system works in 1-4 kHz bandwidth. In this bandwidth the channel behaves like a AWGN. Design the transmitter: symbol encoder (using a QAM constellation), shaping filter at the transmitter  $g(t)$  and carrier frequency  $\omega_c$  to make a transmission without ISI to a binary rate of 9600 bits/s and using the whole bandwidth.

7. A baseband communication system uses a BPSK constellation,  $A[n] \in [\pm 1]$ , and the following shaping filter

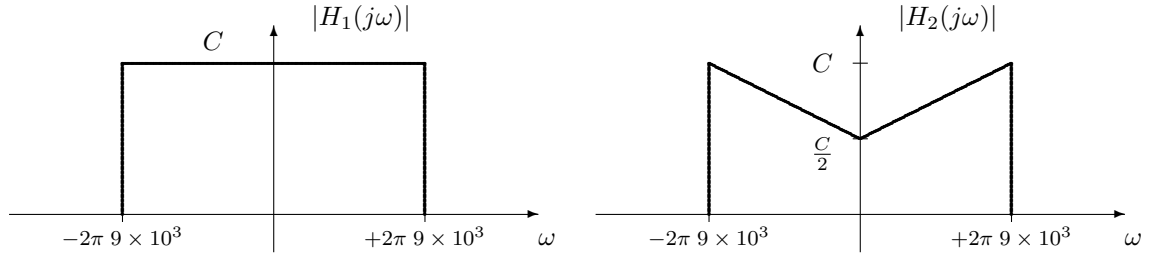
$$g(t) = \begin{cases} \frac{1}{\sqrt{T}}, & -\frac{T}{2} < t < 0 \\ -\frac{1}{\sqrt{T}}, & 0 \leq t < \frac{T}{2} \\ 0 & |t| \geq \frac{T}{2} \end{cases}$$

The modulated signal is transmitted through a linear channel with impulse response

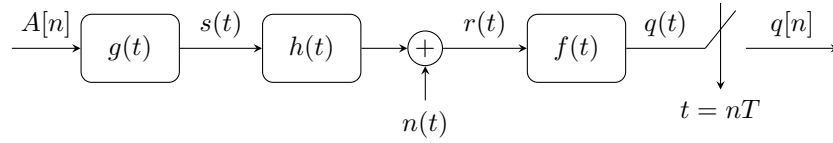
$$h(t) = \delta(t) + \delta(t - T/2),$$

and the receiver uses a matched filter. Noise at the input of the receiver is white, Gaussian, with power spectral density  $N_0/2$  W/Hz.

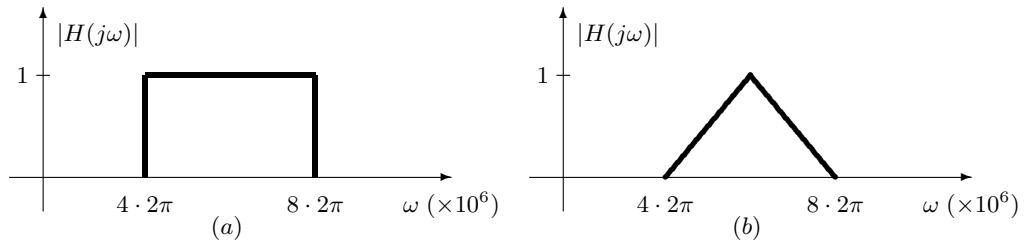
- a) Calculate the equivalent discrete channel  $p[n]$ .
  - b) Obtain the power spectral density of the discrete time noise  $z[n]$  present at the output of the sampler at the receiver, explaining the procedure to obtain the result.
  - c) Calculate the probability of error  $P_e$
8. A baseband transmission system sends the modulated signal through one of these channels. The receiver filter will be matched to the transmitter filter.
- a) Design for Canal 1 the shaping filters at the transmitter and receiver in order to get no ISI and the noise at the output of the sampler be white.
  - b) Design for Canal 2 the shaping filters at the transmitter and receiver in order to get no ISI.



- c) Design for Canal 2 the shaping filters at the transmitter and receiver in order to get white noise at the output of the sampler.
- d) Get the maximum symbol rate in both channels.
9. Next figure shows the block diagram for a baseband PAM system. There,  $A[n]$  is the sequence of transmitted symbols, ( $A[n] \in \{\pm 1\}$ ),  $g(t)$  is a squared-root raised cosine filter,  $h(t)$  is the channel impulse response,  $n(t)$  is the AWGN with PSD  $N_0/2$ ,  $f(t)$  is the receiver filter,  $T$  is the symbol period and  $q[n]$  are the samples at the output of the receiver.



- a) If  $f(t)$  is designed for  $k(t) = g(t) * f(t)$  to match Nyquist criteria, get  $f(t)$  as a function of  $g(t)$  and get the PSD for the discrete filtered noise  $z[n]$ .
- b) If  $h(t) = \delta(t) - \frac{1}{10}\delta(t - 2T)$ , get the impulse response of the discrete equivalent channel  $p[n]$ .
- c) Under (b) conditions, is there ISI in the system?
- d) Assuming  $g(t) = f(t) = 1/\sqrt{T}$  if  $|t| \leq T/2$  and  $f(t) = g(t) = 0$  otherwise (that is,  $f(t)$  and  $g(t)$  are identical normalized squared pulses defined in  $[-T/2, T/2]$ ), get the eye diagram in the absence of noise.
10. Consider the following frequency response of two different communication channels.



- a) Show if it is possible, using a QAM modulation a transmission without ISI and with white noise at the output of the sampler if we use in the receiver a matched filter to

- the transmitter. If your answer is positive, get the maximum transmission rate and get the shaping pulses that you would use on channel in fig. (a) and on channel in fig. (b).
- b) You would like to transmit to a 10 Mbits/s bit rate using a PSK modulation over channel in fig. (a) with squared-root raised cosine filters in the transmitter and receiver. Get the minimum number of symbols needed  $M$  in the PSK modulation and the obtained symbol rate.
  - c) Given the constellation of previous section, obtain the feasible range of values for the roll-off factor  $\alpha$  of the shaping filters taking into account the available bandwidth and from the range of  $\alpha$  values get the one minimizing the effect of deviations from optimal sampling instants at the receiver.
11. A communication system uses a squared-root raised cosine filter in the transmitter for a baseband PAM modulation with roll-off factor  $\alpha$ . In the receiver there is a matched filter to the transmitter. Assume that the channel is AWGN with an impulse response  $h(t)$  and noise PSD  $N_0/2$ . The channel bandwidth is 4 kHz.
- a) Show if the sampled noise at the output of the matched filter is white.
  - b) Get the maximum symbol transmission rate without ISI and get the roll-off factor needed for this rate.
  - c) For the roll-off factor obtained in previous item, draw the PSD of the transmitted signal in these two cases:
    - i) Sequence  $A[n]$  is white with mean symbol energy  $E_s$ .
    - ii) Sequence  $A[n]$  has a PSD  $S_A(e^{j\omega}) = 1 + 1 \cdot \cos(\omega)$ .
  - d) If the roll-off factor used is  $\alpha = 0,25$  transmitting at the maximum symbol rate possible without ISI, get the number of symbols  $M$  needed to get a binary rate of 19200 bits per second.
12. A digital communications system uses as transmitter filter  $g(t)$  a squared root-raised cosine pulse with roll-off factor  $\alpha$ . The receiver employs a matched filter.
- a) If the transmission is performed through the linear channel with response  $h(t) = \delta(t) + \frac{1}{4} \cdot \delta(t - 2T)$ , calculate the equivalent discrete channel.
  - b) If the channel is a baseband channel with bandwidth  $B = 10$  kHz, and the desired binary rate is 54 kbits/s, using a baseband  $M$ -PAM
    - (I) Calculate the minimum order of the constellation (number of symbols  $M$ ) allowing to achieve the desired rate.
    - (II) Calculate the symbol rate,  $R_s$ , which is necessary to obtain such binary rate with this constellation.

- (III) Calculate, for this  $M$ , the value of  $\alpha$  that allows to completely fill the available bandwidth.
- c) Repeat the previous question if the channel is a bandpass channel and the modulation is a bandpass PAM using a  $M$ -QAM constellation.
13. A linear baseband modulation uses a normalized rectangular pulse of duration  $T$ . This modulation is transmitted through a linear channel with impulse response  $h(t) = \delta(t) - 0,5\delta(t - \frac{T}{2})$ .

In the receiver, we consider two different scenarios. In the first case, the receiver employs a matched filter to the transmitter. In the second case, the receiver employs a matched filter to the rectangular pulse shown in Figure 17.

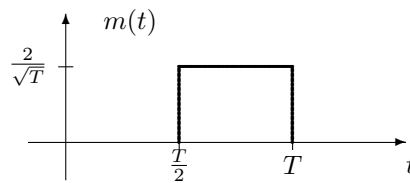


Figura 17: Rectangular pulse corresponding to exercise number 13.

- a) Calculate the equivalent discrete channel in both cases.
- b) In the second scenario, analyze if the sampled noise at the output of the (second) matched filter is white.
- c) Explain, from the point of view of the ISI and sampled noise at the output of the receiver, what is the best option for the receiver.
14. A digital communication system has assigned to its use the frequency range between 30 and 40 MHz. A  $M$ -QAM modulation will be used. Both transmitter and receiver will employ root-raised cosine filters with roll-off factor  $\alpha$ .
- a) Obtain the maximum symbol rate allowing to transmit without intersymbol interference (ISI), and determine the value for  $\alpha$  that is used to achieve such maximum rate.
- b) If a transmission rate of 36 Mbits/s is desired, obtain the minimum constellation order  $M$  (number of symbols in the  $M$ -QAM constellation) that is required.
- c) When transmitting at the maximum symbol rate without ISI, plot the power spectral density of the transmitted signal in two cases:
- (I) Sequence of data,  $A[n]$ , is white.
  - (II) Sequence of data,  $A[n]$ , has the following autocorrelation function

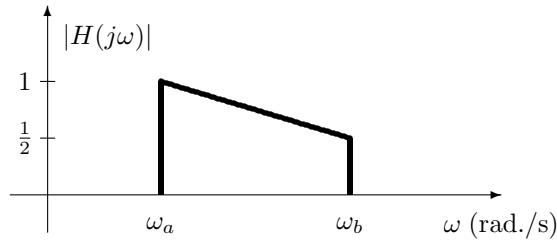
$$R_A[k] = 2 \cdot \delta[k] + \delta[k - 1] + \delta[k + 1].$$

15. A digital communication system has been assigned the frequency range of 10 - 15 MHz. The modulation that will be used is a 16-QAM.

a) If the transmitter uses a square-root raised-cosine (SRRC) shaping pulse with a roll-off factor of  $\alpha = 0,25$ , the receiver is a matched filter to the transmitter and assuming that the channel frequency response is flat in the range of frequencies used for the transmission:

- I) Get the maximum symbol rate and the maximum binary rate without ISI.
- II) Get the power spectral density of the modulated signal  $x(t)$  if the information sequence  $A[n]$  is white.

b) If in the range of frequencies assigned the channel behaves as in next figure (with  $\omega_a = 2\pi \times 10 \times 10^6$  and  $\omega_b = 2\pi \times 15 \times 10^6$ ) and the transmit and receive filter are as defined before:



- I) Show if it is possible or not the transmission without ISI.
  - II) Discuss if the discrete noise at the output of the receiver  $z[n]$  is white. Explain your answer.
- c) For the channel of previous section and still assuming that the receiver is a matched filter to the transmitter:
- I) Get the transmitter filter so that there is no ISI. The filter can be given in the time domain  $g(t)$  or in the frequency domain  $G(j\omega)$ .
  - II) Discuss if in this case the discrete noise at the output of the receiver is white or not.

16. A digital communication system uses a causal square pulse of length  $T$  that is normalized in energy. The receiver uses a matched filter (matched to  $g(t)$ ). The modulated signal is transmitted through a channel whose complex equivalent baseband response is:

$$h_{eq}(t) = \delta(t) + j\delta\left(t - \frac{T}{2}\right).$$

a) Without taking into account the channel effect (i.e.,  $h_{eq}(t) = \delta(t)$ ), do the selected transmitter and receiver filters fulfill the ISI Nyquist criterion?

- b) Obtain the equivalent discrete channel and the constellation at the receiver when the transmitted constellation is an orthogonal constellation with symbols  $A[n] \in \{+1, +j\}$ .
- c) Repeat the previous section if now

$$h_{eq}(t) = j\delta(t - T).$$

Explain if in that case ISI will be present or not.

17. Two digital communication systems are available. The first one is a baseband system and the second one is a bandpass system. The available range of frequencies for the first system is between 0 and 20 kHz, and the constellation is a  $M$ -PAM. The second system has been allotted the frequency range between 20 and 40 kHz, and uses a  $M$ -QAM constellation. In both systems, transmitter and receiver filters will be matched, and the transmitter filter is a root-raised cosine filter with roll-off factor  $\alpha$ .
  - a) Obtain the maximum symbol rate that can be achieved in a transmission without inter-symbol interference (ISI) if the channel has an ideal behavior in its specified frequency band. Indicate the value or set of values of  $\alpha$  that can be used to obtain such maximum rate:
    - i) In the baseband system.
    - ii) In the bandpass system.
  - b) If a roll-off factor  $\alpha = 0,25$  is used, represent the power spectral density of the transmitted signal, properly labeling each axis of the picture:
    - i) In the baseband system, using a 2-PAM constellation.
    - ii) In the bandpass system, using a 4-QAM constellation.
  - c) In the bandpass system, if you pretend an ISI free transmission at binary rate of 64 kbits/s:
    - i) Select the carrier frequency,  $\omega_c$ , that you would use for transmission.
    - ii) Obtain the minimum required constellation order (number of symbols,  $M$ , in the constellation) that allows to transmit at the specified binary rate.
    - iii) Obtain the symbol rate used to transmit at the required binary rate when the constellation obtained in the previous section is used.
18. A digital communication system has assigned for its use the band of frequencies (channel) between 800 MHz and 950 MHz. In this frequency band, the behavior of the channel is considered ideal (it does not introduce linear distortion, but only introduces white, Gaussian additive noise with power spectral density  $N_0/2$ ). The transmitted modulation is a 16-QAM with normalized levels, and sequence  $A[n]$  is white. The transmission filter



is a square-root raised cosine with roll-off factor  $\alpha$ . For the reception filter there are two possibilities:

- Filter  $f_a(t)$ , a square-root raised cosine with roll-off factor  $\alpha$ .
  - Filter  $f_b(t)$  given by  $f_b(t) = \frac{1}{\sqrt{T}}$  for  $|t| \leq T/2$  and  $f_b(t) = 0$  for  $|t| > T/2$ .
- a) For  $\alpha = 0,25$  and receiver filter  $f(t) = f_a(t)$ , obtain the maximum symbol rate and maximum bit rate, indicating the value for the carrier frequency  $\omega_c$  that has to be used to achieve these rates transmitting in the specified frequency band.
  - b) For  $\alpha = 0,25$ , plot the power spectral density of the modulated transmitted signal  $x(t)$ . Proper labels, with numerical values, have to be included in both axes, and only the positive range of frequencies ( $\omega \geq 0$  rad/s.) has to be plotted.
  - c) For  $\alpha = 0$ , demonstrate on one hand if there is or not intersymbol interference (ISI) in the transmission, and on the other hand if the sampled noise at the receiver,  $z[n]$ , is or not white, in the following cases:
    - I) Receiver filter is  $f(t) = f_a(t)$ .
    - II) Receiver filter is  $f(t) = f_b(t)$ .
19. A baseband PAM modulation transmits a sequence of data  $A[n] \in \{0, A\}$  with probabilities  $P(A[n] = 0) = 1 - p$  and  $P(A[n] = A) = p$ . Autocorrelation function is given by:

$$R_A[k] = \mathbb{E} \{A[n]A[n+k]\} = A^2p(1-p) \cdot \delta[k] + A^2p^2$$

- a) Calculate the mean energy per symbol of the transmitted sequence.
- b) Obtain the power spectral density of PAM signal, and plot it with proper axis labels, if the shaping filter is a normalized root-raised cosine filter with roll-off factor  $\alpha = 0,25$ , and sequence is transmitted at symbol rate  $R_s = 1$  kbauds. Provide also the bandwidth of the PAM signal.

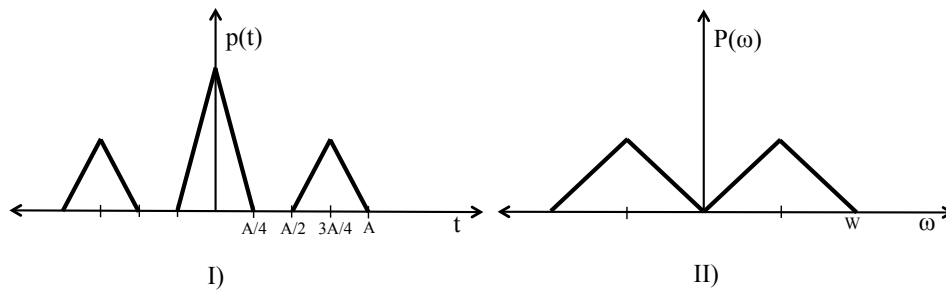
NOTE: Some transformations

$$x[n] = \delta[n] \xleftrightarrow{TF} X(e^{j\omega}) = 1$$

$$x[n] = 1, \forall n \xleftrightarrow{TF} X(e^{j\omega}) = 2\pi \sum_k \delta(\omega + 2\pi k)$$

A communication system transmits in an available bandwidth of 5 kHz. The transmitter is using a square-root raised cosine filter with a roll-off factor of  $\alpha$ . The receiver is using a matched filter to the transmitter.

- a) If a baseband transmission is used, and the channel is an additive white Gaussian channel with power spectral density of  $N_0/2$ , show if the system has ISI or not and if the noise sampled at the output of the receiver  $z[n]$ , is white. Justify your answer.



- b) Get the maximum symbol rate and bit rate that can be used without ISI. Identify the value of  $\alpha$  used for this purpose in these scenarios:
- A baseband transmission is used with a normalized 4-PAM constellation.
  - A bandpass transmission is used with a normalized 16-QAM modulation.
- c) If the transmission is baseband, the roll-off factor  $\alpha = 0$ , and the transmitted sequence,  $A[n]$ , has the following autocorrelation function

$$R_A[k] = \delta[k] - \frac{1}{2}\delta[k-2] - \frac{1}{2}\delta[k+2],$$

get and represent the power spectral density of the transmitted signal, labeling the axes correctly.

20. A baseband communication system uses in the receiver a matched filter to the transmitter. The continuous time equivalent channel  $p(t)$  and the equivalent channel in the frequency domain  $P(\omega)$  are respectively given by figure I) and figure II).
- Show for each of the scenarios if there is or there is not ISI. Explain carefully the criteria used to determine the presence of ISI. In those scenarios without ISI get the relationship between  $p(t)$  and/or  $P(\omega)$  parameters ( $A$  and/or  $W$ ) and the symbol time.
  - If the channel is  $h(t) = \delta(t)$ , get the shaping filter at the transmitter,  $g(t)$ , for the scenario in I).
  - If the channel is  $h(t) = \delta(t)$ , for scenario II) show if the sampled noise at the output of the matched filter is white or not. Explain carefully your answer.