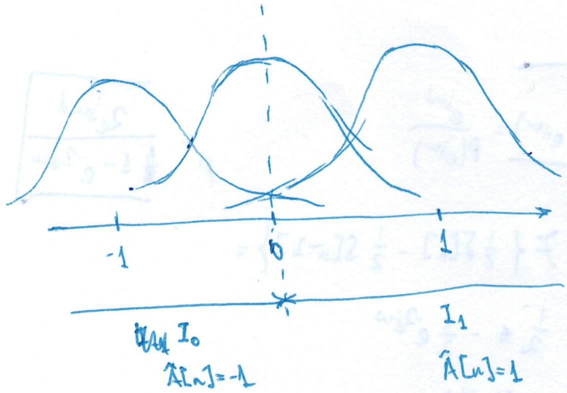
14.

a) SBS detector with AWGN: distance

| A[n] | A[n-2] | o[n] |
|---|---|---|
| 1 | 1 | 0 |
| 1 | -1 | 1 |
| -1 | 1 | -1 |
| -1 | -1 | 0 |




For A[n]=1 : there's a 50-50 chance of getting o[n]=0 or o[n]=1

Same for A[n]=-1 with o[n]=0 or o[n]=-1.

• d=0 :

$$P_e = p(A[n]=1)\cdot P_e\big|_{A[n]=1} + p(A[n]=-1)\cdot P_e\big|_{A[n]=-1}$$

$\downarrow$ symm.

$$= P_e\big|_{A[n]=1}$$

$$= 0.5\cdot Q\left(\frac{1}{\sigma_z}\right) + 0.5\cdot Q(0) = 0.25 + 0.5 Q(3.16) \approx \boxed{0.25}$$

~~b) If unconstrained.~~

$\cancel{W_{ZF}(e^{j\omega}) = \frac{e^{j\omega d}}{P(e^{j\omega})}}$

• For d=1 :

$o[n] = A[n] * p[n] = \frac{1}{2}A[n] - \frac{1}{2}A[n-2]$

o[n] has no information about A[n-1], so the best we can do is guess and get a 0.5 probability of error.

b)

$$\boxed{W_{ZF_u}(e^{j\omega}) = \frac{e^{j\omega d}}{P(e^{j\omega})}} \rightarrow = \boxed{\frac{2e^{j\omega d}}{1-e^{-2j\omega}}}$$

$$P(e^{j\omega}) = \mathcal{F}\left\{\tfrac{1}{2}\delta[n] - \tfrac{1}{2}\delta[n-2]\right\} =$$

$$= \frac{1}{2} - \frac{1}{2}e^{-2j\omega}$$

$$P_e \approx Q\left(\frac{d_{min}^{A[n-d]}}{2\,\sigma_{\tilde{z}}}\right)$$

$$\sigma_{\tilde{z}}^2 = \frac{\sigma_z^2}{2\pi}\int_{-\pi}^{\pi}\frac{d\omega}{|P(e^{j\omega})|^2} = \frac{\sigma_z^2}{2\pi}\int_{-\pi}^{\pi}\frac{d\omega}{\left|\frac{1}{2}(1-\cancel{\sin(2\omega)}\cos(2\omega)+j\sin(2\omega))\right|^2} =$$

$$= 2\frac{\sigma_z^2}{\pi}\int_{-\pi}^{\pi}\frac{d\omega}{(1-\cos(2\omega))^2+\sin^2(2\omega)} = \frac{2\sigma_z^2}{\pi}\int_{-\pi}^{\pi}\frac{d\omega}{1+\cos^2(2\omega)-2\cos(2\omega)+\sin^2(2\omega)} =$$

$$= 2\frac{\sigma_z^2}{\pi}\int_{-\pi}^{\pi}\frac{d\omega}{2-2\cos(2\omega)} = \frac{\sigma_z^2}{\pi}\int_{-\pi}^{\pi}\frac{d\omega}{1-\cos(2\omega)} = \frac{\sigma_z^2}{2\pi}\int_{-\pi}^{\pi}\frac{d\omega}{\sin^2\omega} = -\frac{\sigma_z^2}{2\pi}\cot\omega\Big|_{\omega=-\pi}^{\pi} = \infty$$

$$\boxed{P_e \approx Q\left(\frac{d_{min}^{A[n-d]}}{\infty}\right) = \boxed{0.5}}$$

c)

$$\boxed{W_{MMSE_u}(e^{j\omega}) = \cancel{\ldots}\ \frac{P^*(e^{j\omega})\cdot e^{-j\omega d}}{|P(e^{j\omega})|^2+\frac{\sigma_z^2}{E_s}}} = \boxed{\frac{\frac{1}{2}(1-e^{+2j\omega})}{\sin^2\omega+0.1}}$$

$$\sigma_{\tilde{z}}^2 = \frac{\sigma_z^2}{2\pi}\int_{-\pi}^{\pi}\frac{d\omega}{|P(e^{j\omega})|^2+\frac{\sigma_z^2}{E_s}} = \cancel{0.1+\sin^2\omega}$$

$$= \frac{\sigma_z^2}{2\pi}\int_{-\pi}^{\pi}\frac{d\omega}{0.1+\sin^2\omega} =$$

$$= 18.94$$

$$\boxed{P_e^{MMSE_u} \approx Q\left(\frac{d_{min}^{A[n-d]}}{2\,\sigma_{\tilde{z}}}\right) = Q\left(\frac{1}{\sqrt{18.94}}\right) = Q(0.230) \approx \boxed{0.41}}$$

d)

$$\bar{W} = \begin{bmatrix} w[0] \\ w[1] \\ w[2] \end{bmatrix} = \mathrm{pinv}(\bar{P})\cdot \bar{c}_d = \mathrm{pinv}\underbrace{\begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{2} \end{pmatrix}}_{p[n]\text{ values}} \cdot \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}}_{c[n]=\delta[n-d]}$$