Detection in digital communication systems

Digital Communications

Year 2024 – 2025

Laboratory exercise 1

Objectives

The transmission of information in digital communications systems, both baseband and passband, can be affected by various types of distortion. Some sources of distortion, such as noise or linear channel distortion effect, are inherent to the transmission itself, while others may be due to errors in some module of the communications system.

In this laboratory exercise, which will be carried out in the first two practical sessions of the course, some of the distortion effects that occur during transmission will be simulated in Matlab, so that we learn to simulate at a basic level a digital communication system, and to analyze the consequences of some of its parameters through the simulations performed.

The materials to be delivered and the deadlines for delivery will be specified in the Aula Global platform.

Matlab functions

Communications library

Some functions available in the Matlab communications library (*Communications Toolbox*) will be used for the realization of the laboratory exercise. The most important functions are listed below.

Name	**Breaf description**
awgn	Adds white Gaussian noise to a vector signal
qammod	Modulates an input signal using a QAM constellation with modulation order M
qamdemod	Demodulates a QAM signal with modulation order M
biterr	Compare two binary vectors and calculate the number of errors and the bit error rate (BER).
scatterplot	Plots a scatter diagram

To find more information about these functions or any other function, you can use the `help` command in Matlab.

The file "DemoP1.m", available from Aula Global, performs a basic simulation of a digital communications system (simulated in discrete time using the equivalent discrete channel model).

Other Matlab functions of interest

Some other Matlab functions may be interesting for the development of the laboratory exercise. Some of them are described below.

Name	Brief description
help	Provide information about any function (help function_name)
rand	Generates random numbers with uniform distribution
randn	Generates random numbers with Gaussian distribution
randi	Generates random integers in a specified range
plot	Generates random integers in a specified range
stem	Represents a discrete function in time
semilogy	Represents a function with a logarithmic scale on the ordinate axis.
find	Finds the indices of the elements of a vector that satisfy a condition.
conv	Performs the convolution of two discrete sequences
print	Allows to save a figure in different graphic formats
inv	Calculates the inverse of a matrix
pinv	Calculates the Moore-Penrose pseudo-inverse of a matrix
transpose	Calculates the transpose of a matrix or vector
ctranspose	Performs the conjugate transpose of a matrix (Hermitian operator)
xlabel	Put a label on the abscissa axis of a figure
ylabel	Put a label on the ordinate axis of a figure
title	Put a title on a figure
legend	Labels each of the curves of a figure with a legend

To find more information about these functions or any other function, you can use the help command in Matlab.

1. Noise effect - Signal-to-noise-ratio (SNR) or E_b/N_0

Noise is one of the main factors that degrade performance in a communication system. The magnitude of the degradation depends on the statistics and distribution of the noise signal, the power of the noise signal and the power of the transmitted signal. As for the statistics and distribution of the noise, it is most common to model the noise $n(t)$ that is added to the modulated signal as a stationary random process, white, Gaussian, with power spectral density $N_0/2$. When a filter $f(t)$ is such that $|F(j\omega)|^2$ meets the Nyquist criterion for the absence of ISI at symbol time T, then the sampled noise at the output of the demodulator, $z[n]$, is also white and Gaussian, and its power spectral density takes a constant value, $S_z\left(e^{j\omega}\right) = \sigma_z^2$, which coincides with the variance of the Gaussian distribution and with the power of the signal, being $\sigma_z^2 = \frac{N_0}{2}$ for baseband transmissions and $\sigma_z^2 = N_0$ for passband transmissions[1].

As for the power of the transmitted signal and the noise power, the relationship between the two is quantified with different measures. When working in discrete time, one of the most common is the so-called *energy per bit/* N_0 ratio, or E_b/N_0 ratio. This ratio is is defined as the ratio between the average energy per transmitted bit and the value of N_0, which in this case is

$$\frac{E_b}{N_0} = \frac{E_s}{m\,N_0},$$

where $E_s = E[|A[n]|^2]$ is the average energy per symbol of the transmitted constellation, and given that the constellation has M symbols, $m = \log_2(M)$ is the number of bits per symbol. Typically

[1]In this case the noise is a complex process $z[n] = z_I[n] + jz_Q[n]$, with the real part modeling the noise in the in-phase component, $z_I[n]$, and the imaginary part modeling the noise in the quadrature component, $z_Q[n]$, and that both noise components are independent and with variance $\sigma_{z_I}^2 = \sigma_{z_Q}^2 = \frac{N_0}{2}$.

this ratio is expressed in decibels, then:

$$\frac{E_b}{N_0}(dB) = 10 \log_{10} \frac{E_b}{N_0}.$$

As the power of the noise increases, the probability of making errors when transmitting increases, since the observation received at the output of the demodulator, $q[n]$, is further away on average from the value of the symbol that has been transmitted. The existing noise level, in bandpass communications systems, can be visualized by means of scatter plots, which represent the in-phase component observation, $q_I[n]$, versus the quadrature component observation, $q_Q[n]$.

The dispersion diagram for a 16-QAM modulation transmitted over a Gaussian channel, without linear distortion, shall be plotted for various values of the ratio E_b/N_0, in particular for the following values

- 20 dB
- 15 dB
- 10 dB
- 5 dB

For each of the E_b/N_0 values, besides, N_0 value should be provided.

2. Effects of the Inter-Symbol-Interference (ISI)

One of the most common effects in digital communications systems is intersymbol interference (ISI), which occurs due to the linear distortion introduced by the communication channel. It is well known that the main effect of intersymbol interference is that a distorted constellation, different from the transmitted symbol constellation, shows up at the receiver. This effect can be visualized in bandpass communication systems also by means of scatter plots.

The degradation introduced by ISI in a communication system depends on several factors, such as the length of the equivalent discrete channel, the relative amplitude of its terms with respect to the main term, and the order of the constellation being transmitted. In this section, the effect of these factors will be analyzed by plotting scatter plots obtained in the transmission on different equivalent discrete channels.

Obtain the scatter diagram of a 4-QAM when we transmit with a $E_b/N_0 = 40$ dB through the following discrete channel with length $L_p + 1 = 2$ ($L_p = 1$)

$$p[n] = \delta[n] + a\,\delta[n-1],$$

for different values of a, in particular $a \in \{\frac{1}{16}, \frac{1}{8}, \frac{1}{4}\}$. Explain the effect of increasing a.

Next, repeat the previous section with a 16-QAM modulation and explain the difference observed with respect to the previous case and the conclusions drawn in this case.

Finally, the effect of increasing the length of the equivalent discrete channel will be tested, for which the previous sections will be repeated with the channel

$$p[n] = \delta[n] + a\,\delta[n-1] + \frac{a}{4}\,\delta[n-2].$$

3. Effect of the noise in the probability of error

Noise affects constellations differently depending on their size, since for the same ratio E_b/N_0 the relative distance between symbols (or the noise level if this distance is kept fixed) changes. Furthermore, at the bit level, the type of binary assignment used is relevant. In this section we will analyze the existing differences. With this purpose, the bit error rate (BER) will be represented as a function of the ratio E_b/N_0 in dB for different constellations using a Gray encoding for the binary assignment.[2]

Plot the bit error probability for E_b/N_0 values in the range between 0 and 20 dB in steps of 1 dB using a Gray coding for the following cases

- 4-QAM, 16-QAM, y 64-QAM modulations in bandpass (plot the in the same figure to facilitate comparison).

Given the exponential behavior of the probability of error, it may be useful to represent the BER with Matlab command **semilogy**, that gives a logarithmic representation of the ordinate axes.

It is also interesting to see how the relationship between the probability of symbol error, P_e, and the probability of bit error, BER, evolves as the signal-to-noise ratio increases. To do this, the student will represent this relationship, as a function of E_b/N_0, for the different QAM modulations.

4. Effect of the ISI in the probability of error

Intersymbol interference also affects the performance of a system. When analyzing its effect, the relative values of the equivalent discrete channel coefficients and the most appropriate delay for the decision are relevant.

To check this, the error probability for a 16-QAM modulation transmitting on the next equivalent discrete channel will be calculated.

$$p[n] = -\frac{b}{2}\,\delta[n] + b\,\delta[n-1] + a\,\delta[n-2] + b\,\delta[n-3] - \frac{b}{2}\,\delta[n-4]$$

In particular:

1. For $a = 1$ y $b = 1/16$, with $E_b/N_0 = 15$ dB, represent BER and the probability of symbol error as a function of the decision delay d, for $d \in \{0, 1, 2, 3, 4\}$. Justify the result obtained comparing with the scenarios where there is no ISI.
2. Repeat previous section for $b = 1/4$.
3. For $a = 1/2$ y $b = 1/32$, with $E_b/N_0 = 21$ dB, repeat previous section. For the system to obtain the best performance in this case, indicate what needs to be done now at the communication receiver different from the case where $a = 1$. Explain the results obtained, discussing the effect of the values of a, b, and the relationship E_b/N_0.

5. Error probabilities with non-coherent receivers (Optional)

When in a digital bandpass communication system the phases of the carrier in receiver are synchronized (with the same phase) as that of the carriers used in transmission for modulation, the

[2] In order for the error probability value obtained to be relatively reliable, a minimum number of 100 errors must be observed when evaluating performance. You do not need to consider error probabilities less than 10^{-6}.

receiver is said to be a coherent receiver. When there is a certain phase difference, the receiver is said to be non-coherent. The effect of the phase difference in a non-coherent receiver is that the received constellation has a twist with an angle equal to the phase. This naturally results in a degradation of performance in the receiver.

To observe this effect, for a 16-QAM with $E_b/N_0 = 12$ dB, represent the BER as a function of the phase difference between carriers, giving values to this difference in the range between $-180°$ y $180°$ in $5°$ steps.

6. Channel equalizer (Optional)

When the complexity of a maximum likelihood sequence detector in a communications system makes its implementation unfeasible, a sub-optimal solution with an acceptable complexity is the use of a channel equalizer. In this section we will test the operation of the system with a channel equalizer.

Given the channel

$$p[n] = 0.47\,\delta[n] + \delta[n-1] + 0.47\,\delta[n-2],$$

design an equalizer with 11 coefficients ($L_w = 10$), and a delay $d = 5$ using the following design criteria:

- Zero forcing
- MMSE [3]

The performance of both solutions for a 16-QAM modulation will be evaluated as a function of the E_b/N_0 ratio for a range of the same between 0 and 25 dB (5 dB steps), and compared with those obtained in the case without intersymbolic interference.
It is interesting to see how the coefficients of the MMSE equalizer (and of the joint channel-equalizer response) evolve with the E_b/N_0 ratio and compare them with those of the ZF equalizer, which is independent of noise. Compare the ZF equalizer response with the MMSE equalizer responses for E_b/N_0 ratios of 0 dB, 15 dB and 25 dB.

Bibliography

- *Comunicaciones Digitales.* A. Artés, F. Pérez González, J. Cid Sueiro, R. López Valcarce, C. Mosquera Nartallo y F. Pérez Cruz. Ed. Pearson Educación. 2007.
- *Communication Systems Engineering.* J. G. Proakis y M. Salehi. Prentice-Hall. 1994.

[3] Note that in this case the solution will be different for each E_b/N_0 value.