2.c. Raised-cosine filtering

The RCF is one the typical shapes used for the joint response of the transmit and receive filters. The impulse response is (see Fig. 14):

$$h_{RC}^{\alpha,T}(t) = \left(\frac{\sin(\pi t/T)}{\pi t/T}\right)\left(\frac{\cos(\alpha\pi t/T)}{1-(2\alpha t/T)^2}\right)$$

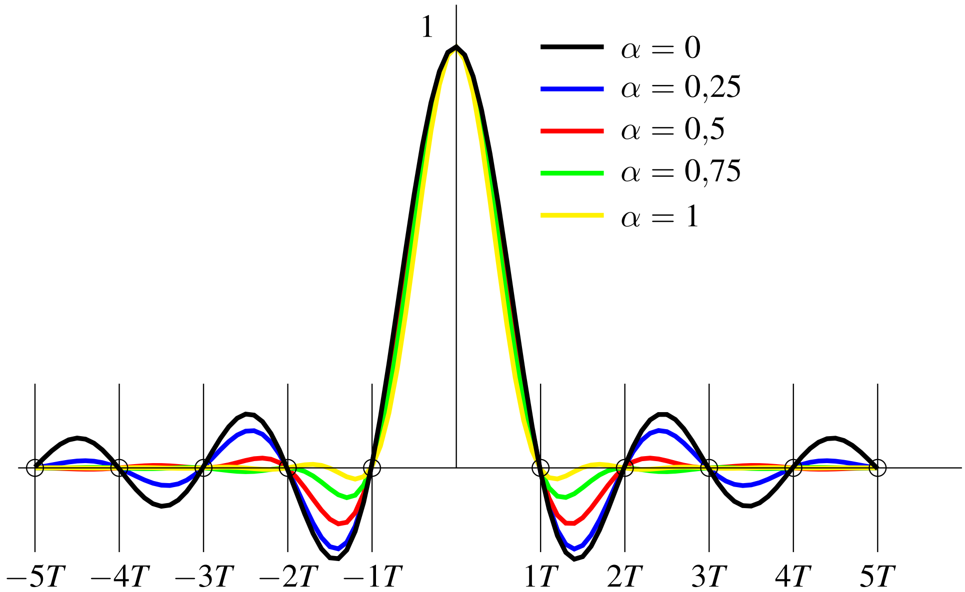

Figura 14: Impulse response of a RCF.

The frequency response is (see Fig. 15)

$$H_{RC}^{\alpha,T}(j\omega) = \begin{cases} T & 0 \leq |\omega| < (1-\alpha)\dfrac{\pi}{T} \\ \dfrac{T}{2}\left[1-\sin\left(\dfrac{T}{2\alpha}\left(|\omega|-\dfrac{\pi}{T}\right)\right)\right] & (1-\alpha)\dfrac{\pi}{T} \leq |\omega| \leq (1+\alpha)\dfrac{\pi}{T} \\ 0 & |\omega| > (1+\alpha)\dfrac{\pi}{T} \end{cases}$$

The shape and bandwidth occupied by the RCF depends on a parameter called roll-off factor $0 \leq \alpha \leq 1$:

$$W = \frac{\pi}{T}(1+\alpha) = \pi R_s (1+\alpha)\ (\mathsf{rad/s})$$

in (Hz)$W = \frac{1}{2T}(1+\alpha) = \frac{R_s}{2}(1+\alpha)$ (Hz).

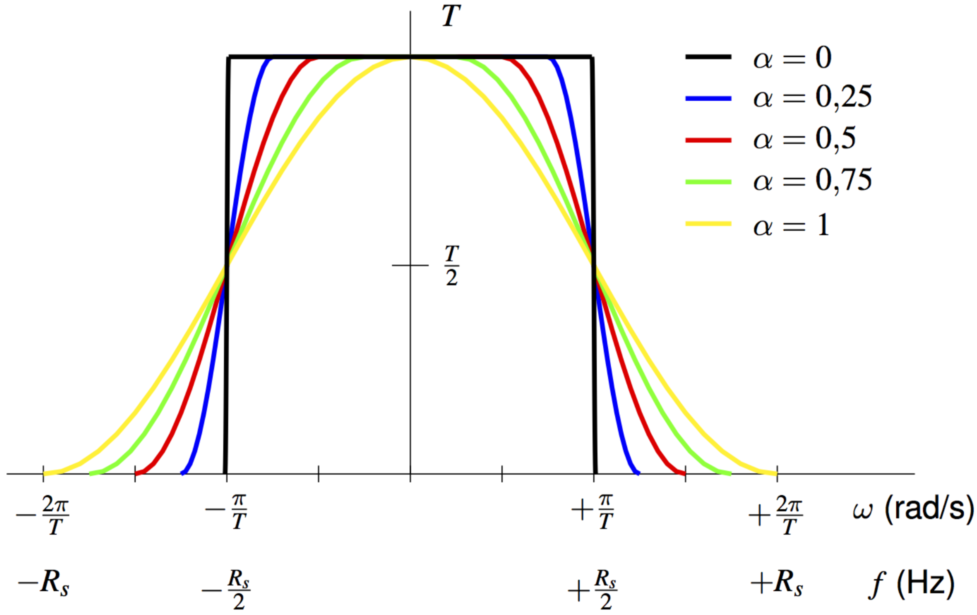

Figura 15: Frequency response of a RCF.

Typically we choose the joint impulse response between $g(t)$ and $f(t)$ as:

$$g(t) * f(t) = h_{RC}^{\alpha,T}(t)$$

and in the spectral domain:

$$G(\mathtt{j}\omega) \times F(\mathtt{j}\omega) = H_{RC}^{\alpha,T}(j\omega)$$

This means that we can make that the transmit filter is a **squared-root RCF** and the receiver being matched to that filter, i.e. $G(\mathtt{j}\omega) = \sqrt{H_{RC}^{\alpha,T}(j\omega)}$ and $F(\mathtt{j}\omega) = G^*(\mathtt{j}\omega) = \sqrt{H_{RC}^{*\alpha,T}(j\omega)}$.