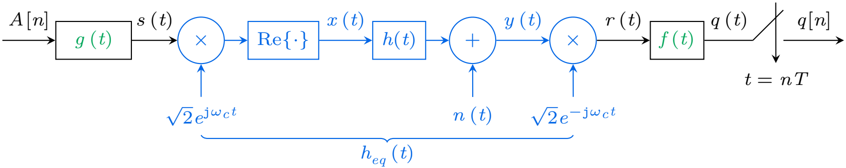
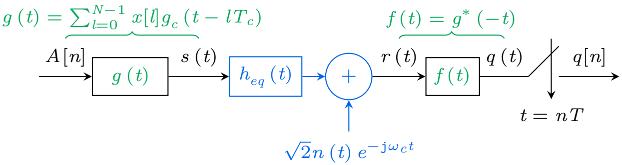
Direct-sequence Spread Spectrum

- Direct-sequence Spread Spectrum (DS-SS) modulation modify the shaping filter at the transmitter $g(t)$ and shaping filter at the receiver $f(t)$:

$$g(t) = \sum_{m=0}^{N-1} x[m] g_c(t - mT_c)$$



- Baseband equivalent model:



Direct-sequence Spread Spectrum

- The baseband DS-SS signal $s(t)$ is given by

$$
\begin{aligned}
s(t) = \sum_n A[n] g(t - nT) &= \sum_n A[n] \sum_{l=0}^{N-1} x[l] g_c(t - nT - lT_c) \\
&\underset{m=l+nN}{=} \sum_n A[n] \sum_{m=nN}^{nN+N-1} x[m-nN] g_c(t - mT_c) \\
&= \sum_n A[n] \sum_m \widetilde{x}[m] w_N[m-nN] g_c(t - mT_c) \\
&= \sum_m \underbrace{\widetilde{x}[m] \sum_n A[n] w_N[m-nN]}_{s[m]} g_c(t - mT_c) \\
&= \sum_m s[m] g_c(t - mT_c)
\end{aligned}
$$

where:

- $\widetilde{x}[m]$: is the spreading sequence made periodical.
- $w_N[m]$: causal window of length N samples.

:

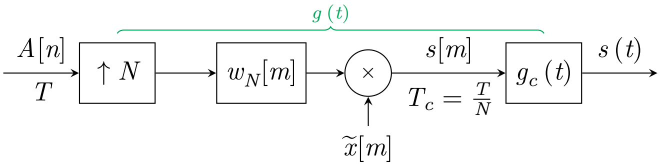
Equivalent model for the DS-SS transmitter

- Digital implementation of a DS-SS baseband transmitted signal

$$s\left(t\right) = \sum_{n} A[n] g\left(t - nT\right) = \sum_{m} s[m] g_c\left(t - mT_c\right)$$

with

$$s[m] = \widetilde{x}[m] \times \sum_{n} A[n] w_N[m - nN]$$



Power spectral density of a DS-SS signal

- The bandpass transmitted DS-SS signal $x(t)$ has a psd:

$$S_X(\mathrm{j}\omega) = \frac{1}{2}\left(S_S(\mathrm{j}\omega - \mathrm{j}\omega_c) + S_S^*(-(\mathrm{j}\omega + \mathrm{j}\omega_c))\right)$$

where $S_S(\mathrm{j}\omega) = \frac{1}{T}\, S_A(e^{\mathrm{j}\omega T})\, |G(\mathrm{j}\omega)|^2$ and now:

  - $g(t) = \sum_{l=0}^{N-1} x[l] g_c(t - lT_c)$ and,
  - $G(\mathrm{j}\omega) = G_c(\mathrm{j}\omega) \sum_{l=0}^{N-1} x[l] e^{-\mathrm{j}\omega m T_c} = G_c(\mathrm{j}\omega)\, X\left(e^{-\mathrm{j}\omega T_c}\right)$

- Then, the baseband transmitted DS-SS signal $s(t)$ has a psd:

$$S_S(\mathrm{j}\omega) = \frac{1}{T}\, S_A(e^{\mathrm{j}\omega T})\, \left|X\left(e^{-\mathrm{j}\omega T_c}\right)\right|^2 \left|G_c(\mathrm{j}\omega)\right|^2$$

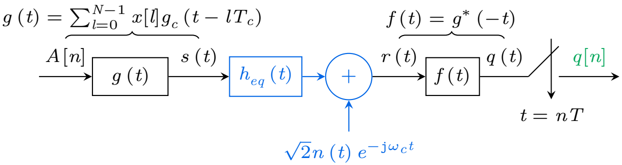
where $G_c(\mathrm{j}\omega)$ can be chosen as a SR-RCF.

The bandwidth of the bandpass DS-SS signal $x(t)$ is given by:

$$W = \frac{2\pi}{T_c}(1+\alpha) = \frac{2\pi N}{T}(1+\alpha)\text{(rad/s)}$$

DS-SS receiver

- The rx is a MF to the tx: $g^*(-t) = \sum_{m=0}^{N-1} x^*[m] g_c(-t - mT_c)$.



- The received samples $q[n]$ are then given by:

$$
\begin{aligned}
q[n] = \left(r(t) * g^*(-t)\right)\Big|_{t=nT} &= \sum_{m=0}^{N-1} x^*[m] \left(r(t) * g_c(-t - mT_c)\right)\Big|_{t=nT} \\
&= \sum_{m=0}^{N-1} x^*[m] \left(\underbrace{r(t) * g_c(-t)}_{v(t)}\right)\Bigg|_{t=nT+mT_c} \\
&= \sum_{m=0}^{N-1} x^*[m] v[nN + m] = \left(v[m] \times \widetilde{x}^*[m]\right) * w_N[-m - nN]
\end{aligned}
$$

  – where $v[m] = v(mT_c) = \left(r(t) * g_c(-t)\right)\big|_{t=mT_c}$

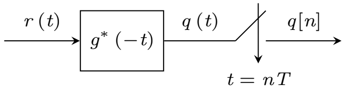
Equivalent model for the DS-SS receiver

- Digital implementation of a DS-SS baseband receiver

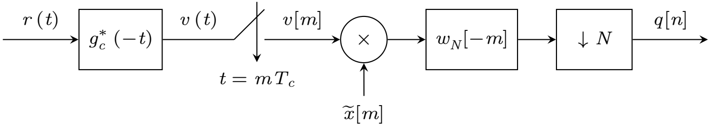
$$q[n] = \sum_{m=0}^{N-1} x^*[m]v[nN+m] = (v[m] \times \widetilde{x}^*[m]) * w_N[-m-nN]$$

where $v[m] = v(mT_c) = \left.(r(t) * g_c(-t))\right|_{t=mT_c}$

- We have that:

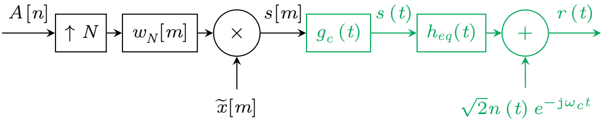


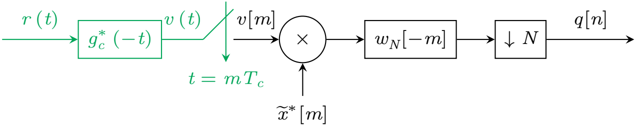
- ...has a digital implementation as:



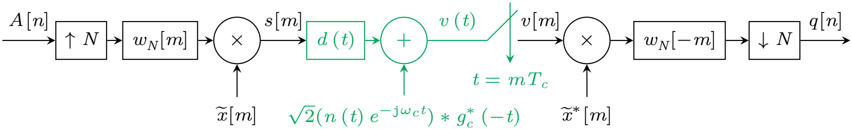
Equivalent channel for the DS-SS

- For the tx-channel-rx chain:



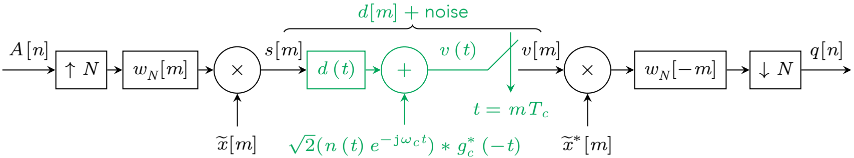


- ...we define the equivalent discrete channel at chip time $d[m] = d(mT_c) = \big(g_c(t) * h_{eq}(t) * g_c^*(-t)\big)\,|_{t=mT_c}$

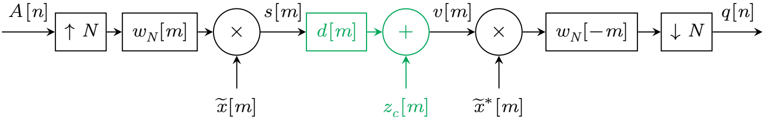


Equivalent channel for the DS-SS @T_c

- ...we define the equivalent discrete channel at chip time
$d[m] = d\left(mT_c\right) = \left(g_c\left(t\right) * h_{eq}\left(t\right) * g_c^*\left(-t\right)\right)|_{t=mT_c}$



– ...where the noise @T_c: $z_c[m] = \sqrt{2}\left(\left(n\left(t\right) e^{-j\omega_c t}\right) * g_c^*\left(-t\right)\right)|_{t=mT_c}$



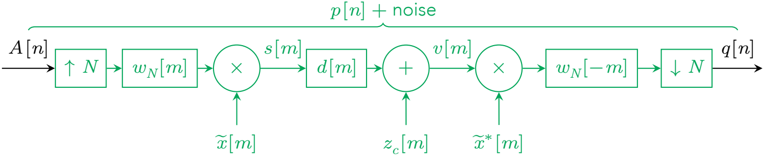
The equivalent channel @T_c that relates $s[m]$ with $v[m]$ is $d[m]$

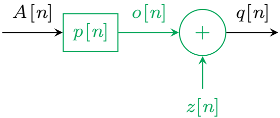
$$v[m] = s[m] * d[m] + z_c[m]$$

Equivalent channel for the DS-SS @T

- ...we define the equivalent discrete channel at symbol time:

$$p[n] = \sum_{m=0}^{N-1} \sum_{l=0}^{N-1} x[m]x^*[l]d[nN+l-m]$$



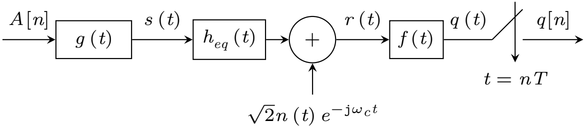


The equivalent channel @T that relates $A[n]$ with $q[n]$ is $p[n]$

$$q[n] = A[n] * p[n] + z[n]$$

Equivalent channel for the DS-SS @T

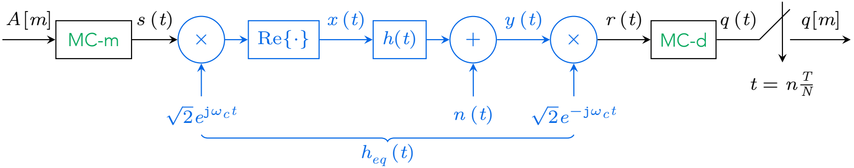
- ...we define the equivalent discrete channel @T for $g(t) = \sum_{m=0}^{N-1} x[m] g_c(t - mT_c)$ and $f(t) = g_c(-t)$:



$$
\begin{aligned}
p[n] &= \left(g(t) * h_{eq}(t) * g(-t)\right)\Big|_{t=nT} \\
&= \sum_{m=0}^{N-1}\sum_{l=0}^{N-1} x[m]x^*[l] \left(g_c(t - mT_c) * h_{eq}(t) * g_c^*(-t - lT_c)\right)\Big|_{t=nT} \\
&= \sum_{m=0}^{N-1}\sum_{l=0}^{N-1} x[m]x^*[l] \left(\underbrace{g_c(t) * h_{eq}(t) * g_c^*(-t)}_{d(t)}\right)\Bigg|_{t=\underbrace{nT + lT_c - mT_c}_{(nN+l-m)T_c}} \\
&= \sum_{m=0}^{N-1}\sum_{l=0}^{N-1} x[m]x^*[l]\; d(t)\big|_{t=\underbrace{nT + lT_c - mT_c}_{(nN+l-m)T_c}} = \sum_{m=0}^{N-1}\sum_{l=0}^{N-1} x[m]x^*[l]\, d[nN + l - m]
\end{aligned}
$$

Multicarrier modulations

- Bandpass modulation where we split the bandwidth in N sub-channels and transmit one sequence in each of them
- We design the MC modulator MC-m and demodulator MC-d at base band

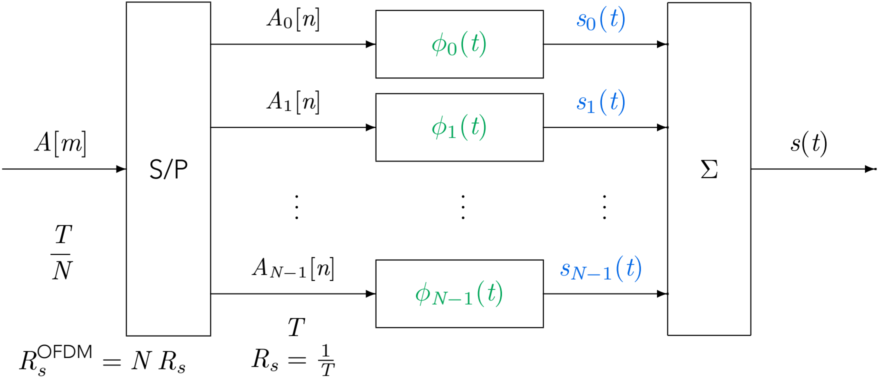


Multicarrier modulations

- Design of the transmitter @baseband, also called OFDM (orthogonal frequency division multiplex)

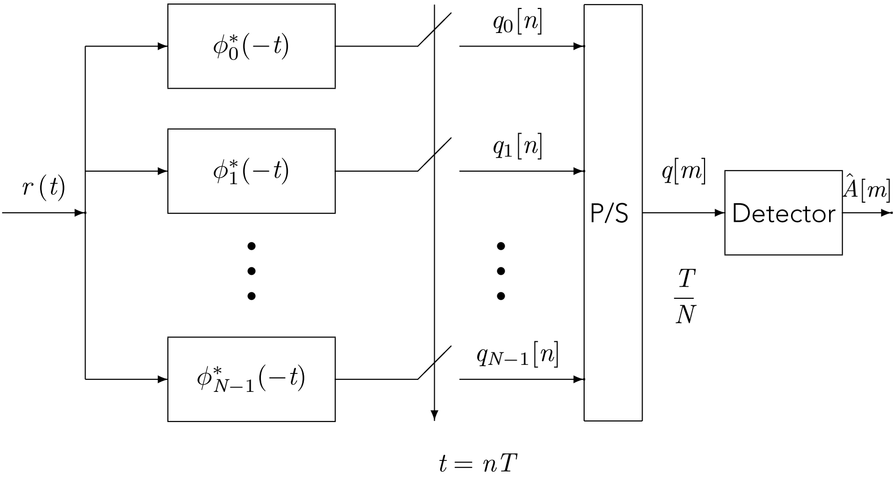
$$\phi_k(t) = \frac{1}{\sqrt{T}} e^{\mathrm{j}\frac{2\pi k}{T}t} w_T(t)$$

$$s_k(t) = \sum_n A_k[n]\phi_k(t - nT) = \frac{1}{\sqrt{T}} \sum_n A_k[n] e^{\mathrm{j}\frac{2\pi k}{T}(t-nT)} w_T(t - nT)$$



Multicarrier modulations

- Design of the receiver at baseband: we use a matched filter $\phi_k^*(-t)$ to each of the transmitted filters.



MC spectrum

- The signal $s(t)$ is the sum of the signals transmitted in each of the subchannels:

$$s(t) = \sum_{k=0}^{N-1} s_k(t) = \sum_{k=0}^{N-1} \sum_{n} A_k[n]\phi_k(t - nT)$$

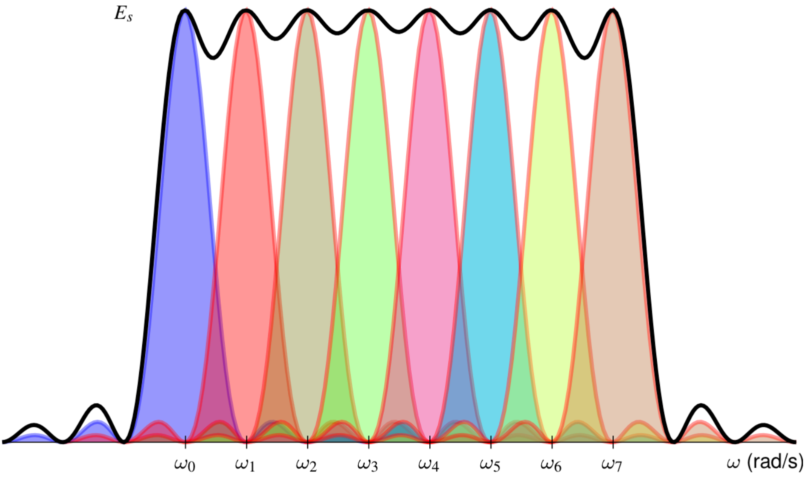
- If the sequences $A_k[n]$ are uncorrelated and white the spectrum of $s(t)$ is the sum of the individual spectrums

$$S(\mathrm{j}\omega) = \sum_{k=0}^{N-1} S_k(\mathrm{j}\omega) = \frac{1}{T} \sum_{k=0}^{N-1} \mathcal{E}_k |\Phi_k(\mathrm{j}\omega)|^2$$

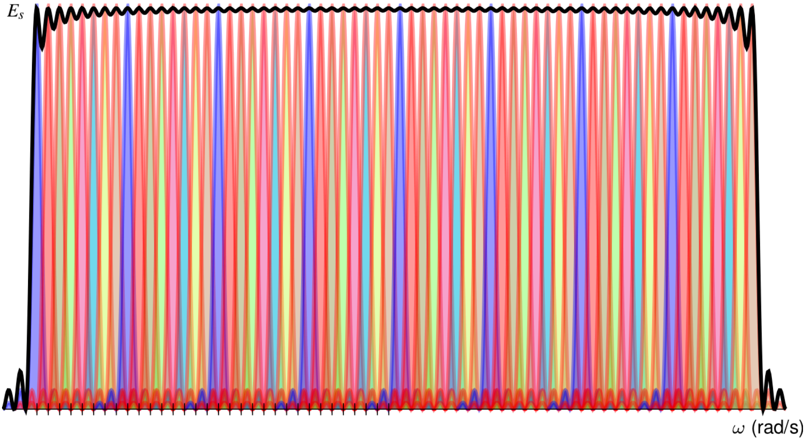
- Frequency response of the pulses $\phi_k(t)$, with $k = 0, 1, \ldots, N-1$

$$|\Phi_k(\mathrm{j}\omega)|^2 = T \operatorname{sinc}^2\left(\frac{(\omega - \frac{2\pi k}{T})T}{2\pi}\right)$$

Example spectrum OFDM $N = 8$



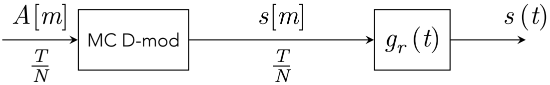
Example spectrum OFDM $N = 64$



MC modulator discrete model (MC D-mod)

- Generation of the BB OFDM signal from the samples $s[m]$.

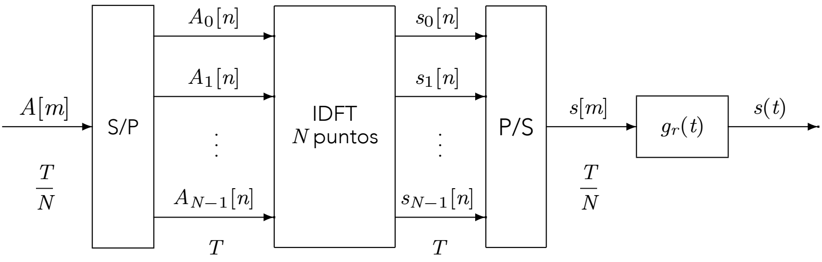
$$s(t) = \sum_m s[m] g_r\left(t - m\frac{T}{N}\right)$$



- Digital implementation of a MC baseband transmitted signal

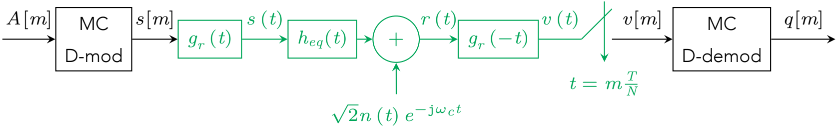
$$\begin{aligned}
s[m] = s\left(m\frac{T}{N}\right) &= \sum_{k=0}^{N-1} s_k(t) = \sum_{k=0}^{N-1} \sum_n A_k[n] \phi_k\left(m\frac{T}{N} - nT\right) \\
&= \frac{1}{\sqrt{T}} \sum_n \sum_{k=0}^{N-1} A_k[n] e^{\mathrm{j}\frac{2\pi k}{T}\left(m\frac{T}{N} - nT\right)} w_T\left(m\frac{T}{N} - nT\right) \\
&= \frac{1}{\sqrt{T}} \sum_n \sum_{k=0}^{N-1} A_k[n] e^{\mathrm{j}\frac{2\pi k}{N}(m-nN)} w_N[m-nN] \\
&= \frac{1}{\sqrt{T}} \sum_n \underbrace{\sum_{k=0}^{N-1} A_k[n] e^{\mathrm{j}\frac{2\pi k}{N} m}}_{s_{m-nN}[n]} w_N[m-nN]
\end{aligned}$$

MC modulator discrete model (MC D-mod)

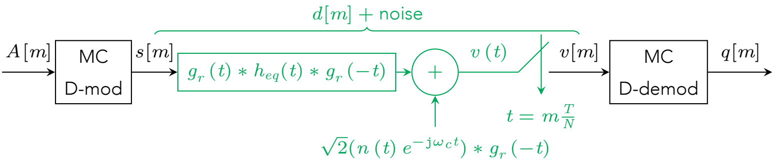


Equivalent channel for MC

- For the tx-channel-rx chain:

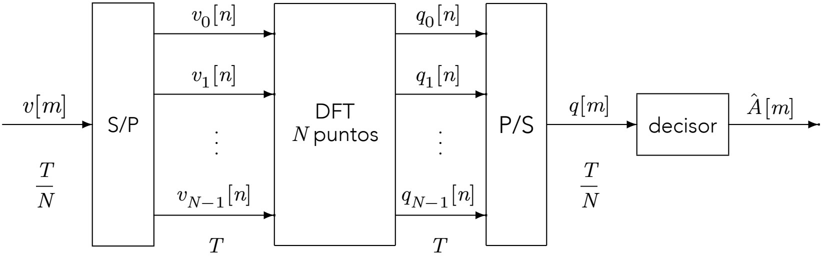


- ...we define the equivalent discrete channel
  $d[m] = d\left(m\frac{T}{N}\right) = \left(g_r(t) * h_{eq}(t) * g_r(-t)\right)|_{t=m\frac{T}{N}}$



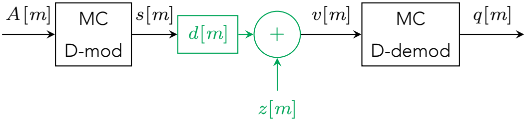
Digital implementation of the MC receiver

- Implementation of the MC D-demod:



Equivalent channel @T

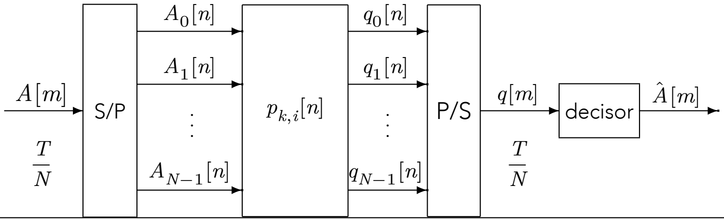
- From the equivalent channel @$\frac{T}{N}$ $d[m]$:



- ...we can define the equivalent channels @T

$$p_{k,i}[n] = \frac{1}{T} \sum_{m=0}^{N-1} \sum_{l=0}^{N-1} e^{-\mathrm{j}\frac{2\pi k}{N} l} e^{\mathrm{j}\frac{2\pi i}{N} m} d[nN + l - m]$$

  – characterizes the interference of $A_i[n]$ that shows up in $q_k[n]$



ISI and ICI

- Given the equivalent channels @T

$$p_{k,i}[n] = \frac{1}{T} \sum_{m=0}^{N-1} \sum_{l=0}^{N-1} e^{-\mathrm{j}\frac{2\pi k}{N} l} e^{\mathrm{j}\frac{2\pi i}{N} m} d[nN + l - m]$$

  – ISI: interference of sequence $A_i[n]$ in $q_i[n]$

$$p_{i,i}[n] = \frac{1}{T} \sum_{m=0}^{N-1} \sum_{m=0}^{N-1} d[nN + l - m]$$

  – ICI: interference of sequence $A_i[n]$ in $q_k[n]$

- We can overcome ICI and ISI if we use a cyclic prefix

$$p_{k,i}[n] = \frac{N}{T} \delta[n] \delta[k - i] D[k]$$

  – $D[k]$ is the DFT of $d[m]$