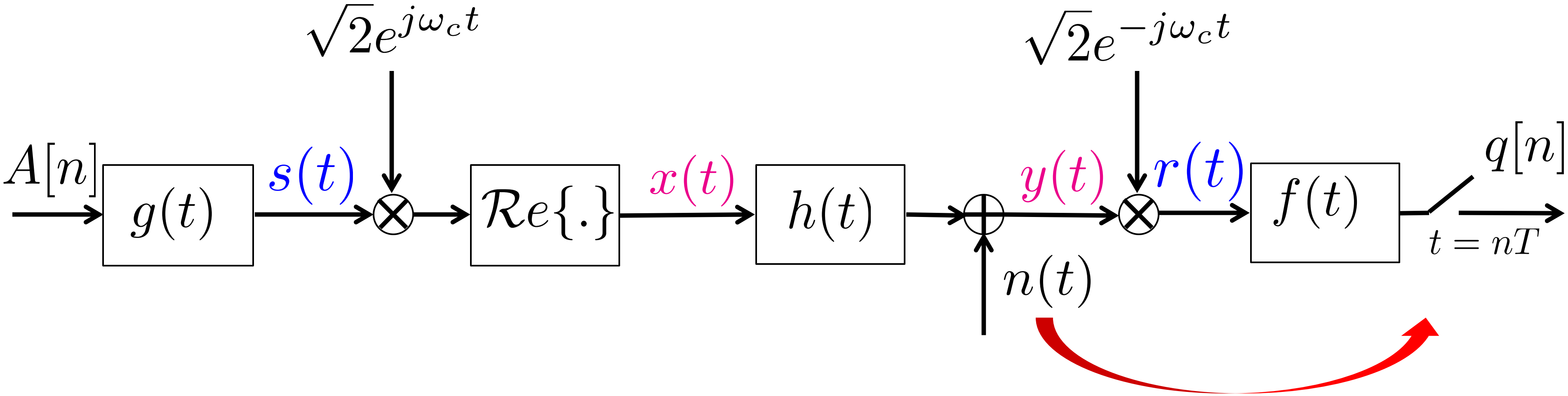
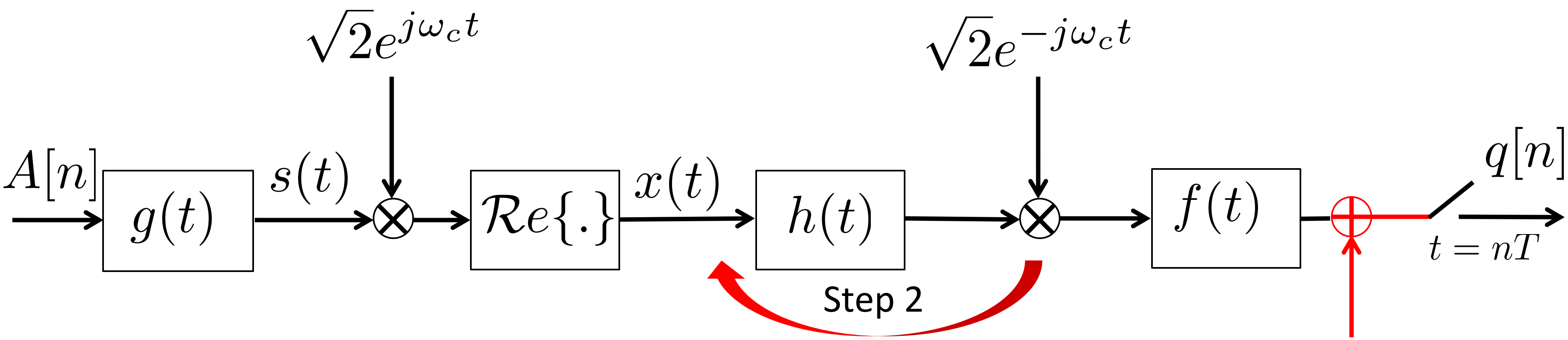
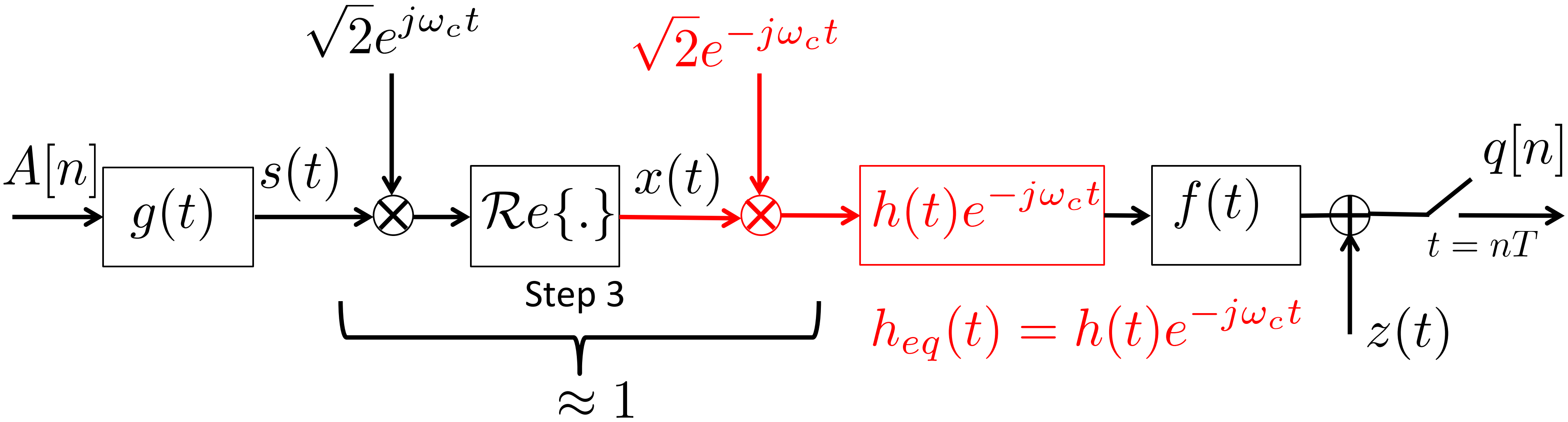


$$S_X(j\omega) = \frac{1}{2}\left(S_S(j\omega - j\omega_c) + S_S^*(-j\omega - j\omega_c)\right)$$

$$S_S(j\omega) = \frac{1}{T} S_A(e^{j\omega T})|G(j\omega)|^2$$



$$z(t) = \left(\sqrt{2}n(t)e^{-j\omega_c t}\right) * f(t)$$



Proof of step 2:

$$
\begin{aligned}
\sqrt{2}\left(x(t) * h(t)\right) e^{-j\omega_c t} &= \sqrt{2}\int_{-\infty}^{\infty} x(\tau)h(t-\tau)e^{-j\omega_c t} d\tau = \\
&= \sqrt{2}\int_{-\infty}^{\infty} x(\tau)e^{-j\omega_c \tau} h(t-\tau)e^{-j\omega_c (t-\tau)} d\tau = \\
&= \sqrt{2}\int_{-\infty}^{\infty} x(\tau)e^{-j\omega_c \tau} h_{eq}(t-\tau) d\tau = \\
&= \left(x(t)\sqrt{2}e^{-j\omega_c t}\right) * h_{eq}(t)
\end{aligned}
$$

Proof Step 3:

$$2\mathcal{R}e\left\{s(t)e^{j\omega_c t}\right\}e^{-j\omega_c t} =$$

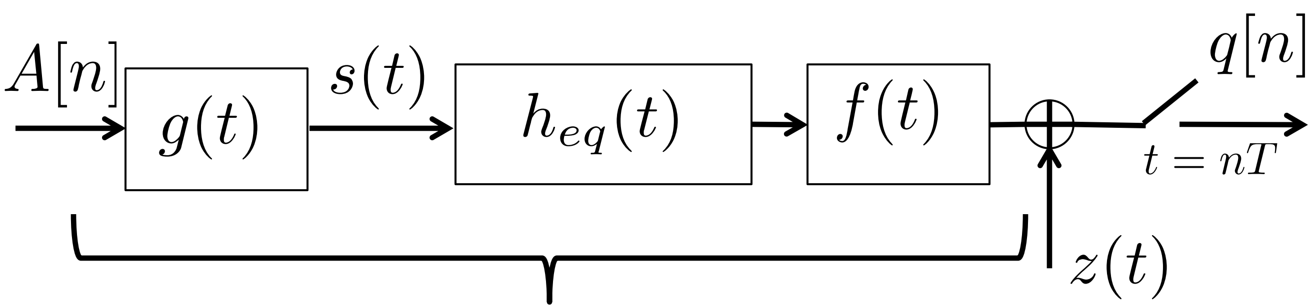
$$2\mathcal{R}e\left\{\left(s_I(t) + js_Q(t)\right)e^{j\omega_c t}\right\}e^{-j\omega_c t} =$$

$$= 2\left(s_I(t)\cos\omega_c t - s_Q(t)\sin\omega_c t\right)e^{-j\omega_c t} =$$

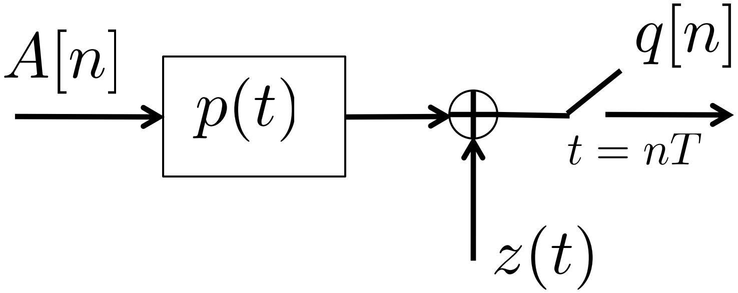
$$= 2\left(s_I(t)\cos^2\omega_c t - s_Q(t)\sin\omega_c t\cos\omega_c t - js_I(t)\cos\omega_c t\sin\omega_c t + js_Q(t)\sin^2\omega_c t\right)$$

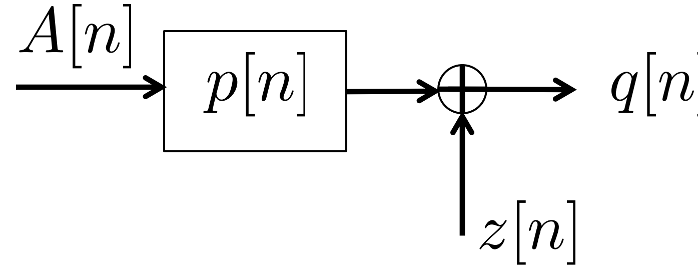
$$= 2\left(s_I(t)\left(\tfrac{1+\cos 2\omega_c t}{2}\right) - s_Q(t)\tfrac{\sin 2\omega_c t}{2} - js_I(t)\tfrac{\sin 2\omega_c t}{2} + js_Q(t)\left(\tfrac{1-\cos 2\omega_c t}{2}\right)\right) =$$

$= s(t) +$ several terms modulated @ $2\omega_c \approx s(t)$



$$p(t) = g(t) * h_{eq}(t) * f(t)$$





Equivalent discrete channel

Summary:

$$p[n] = p(nT) = \left(g(t) * h_{eq}(t) * f(t)\right)_{t=nT}$$

$$z[n] = z(nT) = \left(\left(\sqrt{2}n(t)e^{-j\omega_c t}\right) * f(t)\right)|_{t=nT}$$

PSD of the noise in the bandpass system:

$$\mathcal{DEP}\left\{z(t)\right\} = \mathcal{S}_z\left(j\omega\right) = 2\mathcal{S}_n\left(j\omega + j\omega_c\right)\left|F(j\omega)\right|^2$$

$$\mathcal{DEP}\left\{z[n]\right\} = \mathcal{S}_z\left(e^{j\omega}\right) = \frac{2}{T}\sum_k \mathcal{S}_n\left(j\frac{\omega}{T} + j\frac{\omega_c}{T} - j\frac{2\pi k}{T}\right)\left|F\left(j\frac{\omega}{T} - j\frac{2\pi k}{T}\right)\right|^2$$

$$= \frac{2}{T}\sum_k \frac{N_0}{2}\left|F\left(j\frac{\omega}{T} - j\frac{2\pi k}{T}\right)\right|^2$$