ELECTROMAGNETIC FIELDS Bachelor in Telecommunications Engineering
GITT+GICME+GIT. Academic Year 23/24

Lab 3: GUIDED WAVES

Part I: Brillouin Diagram and Mode Impedance

An ideal rectangular air-filled waveguide has dimensions $a = 2$cm and $b = 1.5$cm so:

m	n	f_c(GHz)
1	0	7.5
0	1	10
1	1	12.5
2	0	15

By means of the expressions:

$$\gamma = \alpha + j\beta = \sqrt{-\omega^2\mu\epsilon_c + \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}; \qquad Z_{TE_{mn}} = \frac{j\omega\mu}{\gamma_{mn}}$$

and assuming a frequency range from 0.1 to 20 GHz, in MATLAB™:

✎ **1)** Represent the Brillouin diagram for the fundamental mode.

✎ **2)** Represent the variation of the mode impedance with frequency ($\mu_0 = 4\pi 10^{-7} H/m$). Limit the range of the impedance representation by using the following comand: `axis([0 max(f) 0 2000])`

✎ **3)** Repeat the previous two cases for a dielectric with $\tan\delta = 0.01$. How are the losses affecting the propagation constant and the mode impedance?

Part II: Analysis of TE_{mn} modes (ideal case)

Suppose an operating frequency of $f = 10$ GHz and the domain discretization:

```
x=0:0.02*a:a;
y=0:0.02*b:b;
```

Calculate all the EM field componentes according to (set the incident long. magnetic field amplitude to $H^i_{mn} = j$ to avoid complex numbers in the representations of E_y in propagating modes):

$$\begin{aligned}
H^i_{mn} &= j \\
H_z &= H^i_{mn}\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right) \\
H_x &= \frac{\gamma_{mn} m\pi}{k_c^2 a} H^i_{mn}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right) \\
H_y &= \frac{\gamma_{mn} n\pi}{k_c^2 b} H^i_{mn}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) \\
E_x &= \frac{j\omega\mu n\pi}{k_c^2 b} H^i_{mn}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) \\
E_y &= \frac{-j\omega\mu m\pi}{k_c^2 a} H^i_{mn}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)
\end{aligned}$$

where:

$$k_c = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}; \qquad \gamma_{mn} = \sqrt{-\omega^2\mu\epsilon + k_c^2}$$

```
Hmn=j;            % Hz Amplitude (excitation)
for ii=1:length(x);
for jj=1:length(y);
    Hz(ii,jj)=Hmn*cos(m*pi*x(ii)/a)*cos(n*pi*y(jj)/b);
    Hx(ii,jj)=(gamma*m*pi)/(kc^2*a)*Hmn*sin(m*pi*x(ii)/a)*cos(n*pi*y(jj)/b);
    Hy(ii,jj)=(gamma*n*pi)/(kc^2*b)*Hmn*cos(m*pi*x(ii)/a)*sin(n*pi*y(jj)/b);
    Ex(ii,jj)=(j*w*mu*n*pi)/(kc^2*b)*Hmn*cos(m*pi*x(ii)/a)*sin(n*pi*y(jj)/b);
    Ey(ii,jj)=(-j*w*mu*m*pi)/(kc^2*a)*Hmn*sin(m*pi*x(ii)/a)*cos(n*pi*y(jj)/b);
end;
end;
```

For the TE_{10} mode, represent:

4) The magnitude of E_y vs. x for $y = b/2$.

```
figure
plot(x,abs(Ey(:,25)))
xlabel('x(m)');ylabel('|Ey|');title('Ey')
```

5) The transverse E field:

```
figure
quiver(x,y,real(Ex)',real(Ey)')
xlabel('x(m)'); ylabel('y(m)'); axis('equal'); title('E_t (V/m)')
```

6) The transverse H field:

```
figure
quiver(x,y,real(Hx)',real(Hy)')
xlabel('x(m)'); ylabel('y(m)'); axis('equal'); title('H_t (A/m)')
```

7) Poynting vector:

```
figure
contourf(x,y,real((abs(Ey).^2+abs(Ex).^2)/(2*zmode))',10)
xlabel('x(m)'); ylabel('y(m)'); axis('equal'); title('S (W/m^2)')
colorbar
```

For the TE_{11} mode:

8) H_y vs. x in the center of the waveguide.

9) Transverse H field in the cross section. How is the magnetic field over the contour?

10) Poynting vector in the cross section. Calculate again the Poynting vector if the frequency changes to f=20 GHz.

Part III: Time domain full wave simulations with CST-Microwave Studio

a) Create a waveguide in *CST Microwave Studio*™with the aforementioned dimensions and a length $d = 5$cm, using a brick made of air and PEC lateral boundary conditions for the six sides of the simulation domain (Aula Global File: lab3.cst).

b) Excite an electromagnetic field at $f_0 = 9$ GHz using CST *Waveguide Ports* and analyse the following cases, visualizing the indicated instantaneous field components:

✎ **11)** For a TE_{10} mode:

- The variation of E_t in the cross section (Folder '2D/3D Results: Port Modes') and along the z axis.

- The variation of H_t in the cross section (Folder 'Port Modes') and along the z axis.

- The Average Poynting vector in the cross section.

✎ **12)** For a TE_{11} mode:

- The variation of E_t in the cross section (Folder 'Port Modes') and along the z axis.

- The Average Poynting vector in the cross section.

✎ **13)** For a TE_{20} mode:

- The variation of E_t in the cross section (Folder 'Port Modes') and along the z axis.

- The variation of E_t along the z axis for a new operating frequency of $f_0 = 20$ GHz.