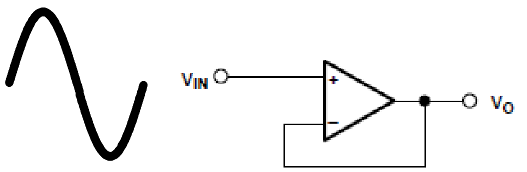
Operational amplifier classroom problems

Problem 1

An operational amplifier is available whose frequency response is shown in Figure P1.1. The slew rate of this operational amplifier is 16V/µs. It is intended to use such an amplifier in a voltage follower configuration.



Estimate which can be the maximum frequency of a sine wave that applied to the VIN input will be amplified without presenting any type of distortion or phase shift with respect to the input signal for the following cases:

a) Input signal amplitude V_{in} = 50 mV
b) Input signal amplitude V_{in} = 3 V.

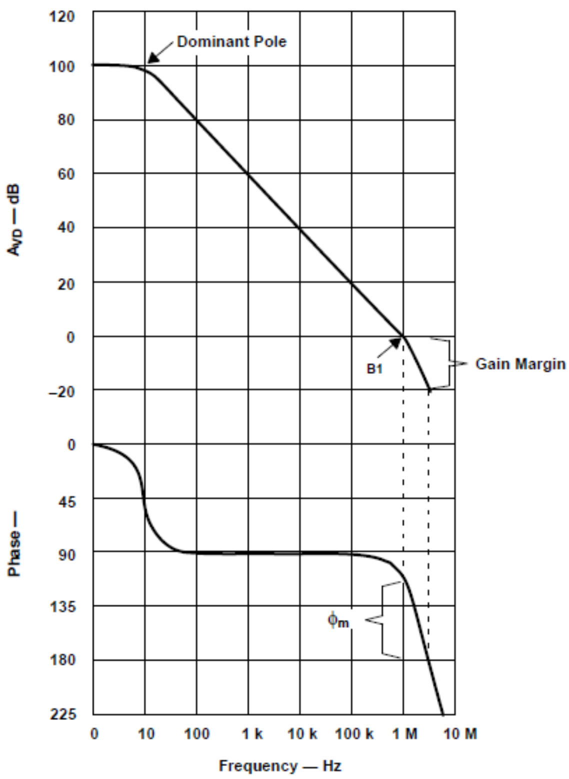


Figure P1.1

Solution:

La definición del "slew rate" es:

$$SR = \left(\frac{dv}{dt}\right)_{max}$$

Para averiguar la frecuencia máxima para la que este parámetro no significará una limitación, se supone una tensión de entrada sinusoidal en la entrada del amplificador:

$$v_g = A \cdot \operatorname{Sen} \omega t$$

Por tanto:

$$\frac{dv}{dt} = \frac{d}{dt}\left[A \cdot \operatorname{Sen} \omega t\right] = A \cdot \omega \cdot \operatorname{Cos} \omega t$$

$$\left(\frac{dv}{dt}\right)_{max} = \left(A \cdot \omega\right)_{max} = A \cdot \omega_{max} = SR$$

y de esta expresión se tiene:

$$A \cdot 2\pi f_{max} = SR \longrightarrow \boxed{f_{max} = \frac{SR}{2\pi A}}$$

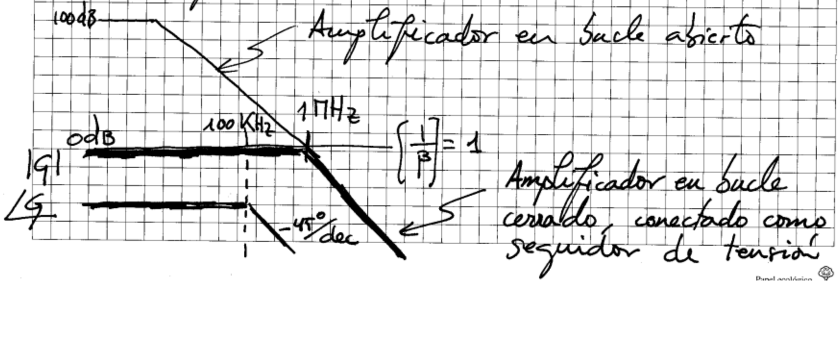
TENSIÓN DE GRAN SEÑAL : $\hat{V}_{in} = 3V$

$$f_{max} = \frac{16\ V/\mu s}{2\pi \cdot 3V} = 848\ KHz$$

TENSIÓN DE PEQUEÑA SEÑAL

$$f_{max} = \frac{16\ V/\mu s}{2\pi \cdot 50 \cdot 10^{-3}} = 51\ MHz$$

Por otro lado, el ancho de banda del amplificador operacional, también supone una limitación a la frecuencia máxima:



Por tanto la frecuencia máxima que puede amplificar el seguidor de tensión sin que aparezca desfase en la salida son 100 KHz

	SR	f_{max}	ΔB
$\hat{V}_{in} = 3\,V$	848 KHz		$\boxed{100\ KHz}$
$\hat{V}_{in} = 50\,mV$	51 MHz		$\boxed{100\ KHz}$

CASO MAS DESFAVORABLE

$$f_{max} = 100\ KHz$$

Problem 2

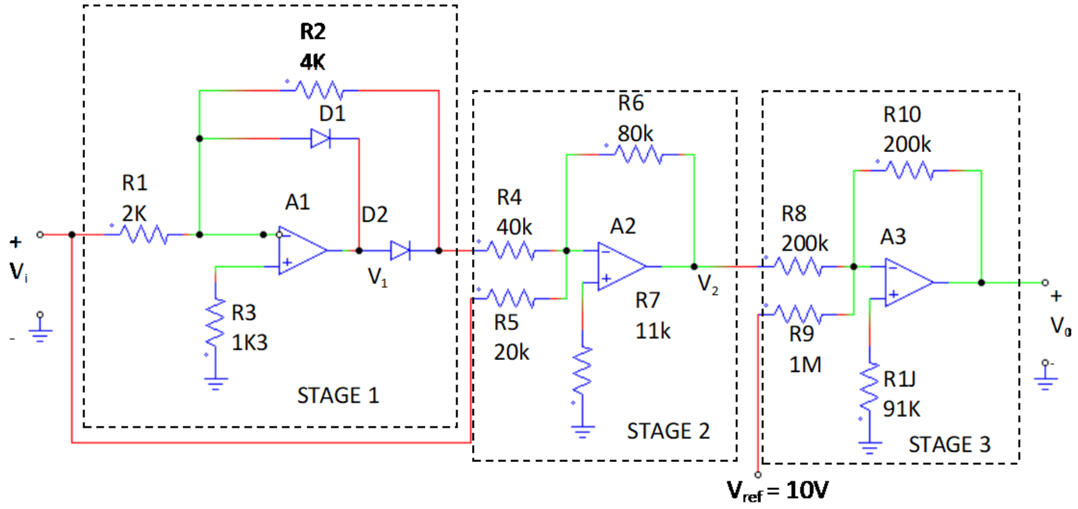


Figure P2.1

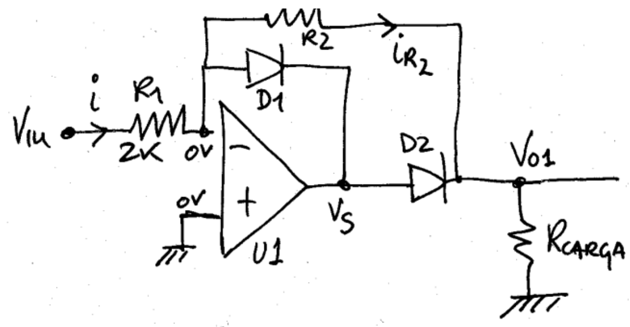
Given the circuit of the figure P2.1,

a) Analyze the function that each stage would perform in isolation.
b) Indicate the function performed by each of the stages in this circuit.
c) Obtain and represent the transfer function, Vo as a function of Vi.
d) The input signal of the circuit is a sinusoidal signal with voltage amplitude of 0.5 V and frequency of 1 kHz. Represent the voltage waveforms of Vi, V1, V2, and Vo with appropriate dimensioning.

Solution:

BLOQUE 1

Rectificador de semiciclos negativos.



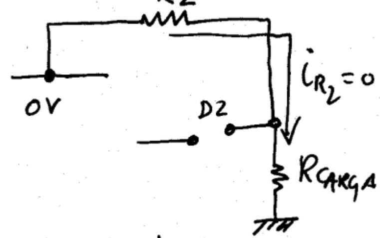
→ Representa la R_i de la siguiente etapa

$$\boxed{V_{in} > 0} \Rightarrow \quad i = \frac{V_{in}}{R_1} > 0$$

HIPÓTESIS: Suponemos D_1 conduce y D_2 cortado, entonces:

$V_S = -0'7\,V$ (caída de tensión directa del diodo)

$V_{o_1} = 0V$ ya que al no conducir D2 se forma la siguiente malla:

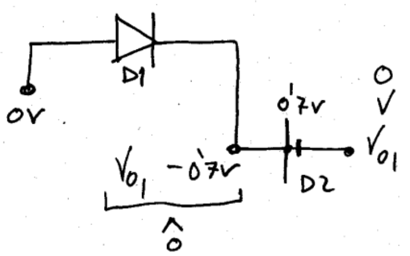


COMPROBACIÓN:

Con $V_{o_1} = 0V$ efectivamente $V_{AK_{D1}} = -0'7V \longrightarrow$ CORTADO

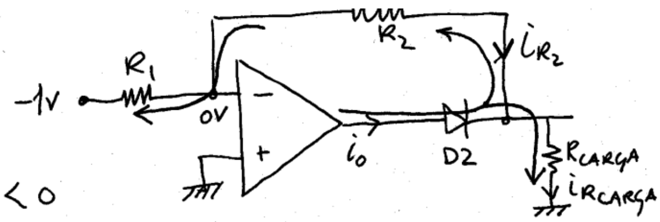
OTRA HIPÓTESIS: Suponemos D_1 NO conduce y D2 CONDUCE

Si D_2 conduce $\Rightarrow V_S = -0'7V$ y por tanto D_1 también conduciría, lo cual es contrario a la hipótesis de partida por lo que este 2° escenario se descarta.



Además, si D2 conduce sería negativa, ya que el operacional U_1 estaría realimentado y $V_{o_1} = -\frac{R_2}{R_1} V_{in}$ $\hat{0}$ por ser V_{in} positivo

$\boxed{V_{in} < 0}$

Ej: 

$V_{in} < 0 \Rightarrow i = \frac{V_{in}}{R_1} < 0$

Esta corriente negativa puede proceder de i_{D1} o de i_{R_2}.

i_{D1} no puede ser negativa, por lo que D_1 NO CONDUCE.

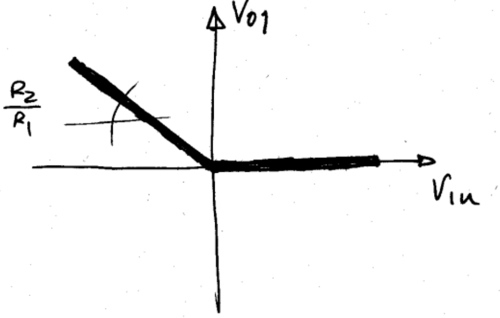
Sin embargo el operacional puede ceder corriente ($i_o > 0$) que polarice directamente a D_2, ceda $i_{RCARGA} > 0$ (como corresponde a $V_{o1} > 0$) y ceda una corriente ~~que~~ $-i_{R_2}$ que circulara en sentido contrario a i a través de R_1.

RESUMEN :

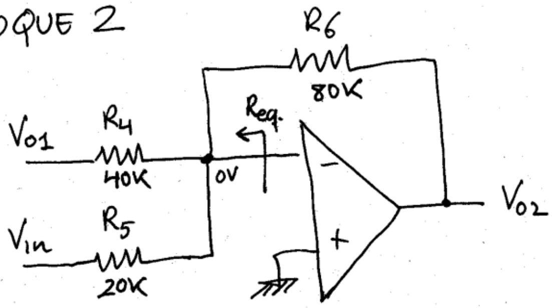
$V_{in} > 0 \Rightarrow V_{o1} = 0$

$V_{in} < 0 \Rightarrow V_{o1} = -\frac{R_2}{R_1} \cdot V_{in}$ ($V_{o1} > 0$ ya que $V_{in} < 0$)

GANANCIA



BLOQUE 2



R_7 reduce los errores en D
pero funcionalmente
NO HACE NADA

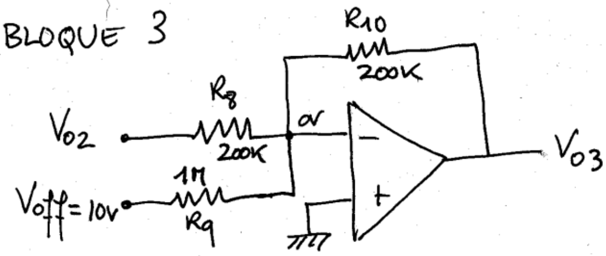
$$R_7 = R_4 \| R_5 \| R_6 = R_{eq}.$$

$$\frac{V_{o1}}{R_4} + \frac{V_{in}}{R_5} = \frac{0 - V_{o2}}{R_6} \quad ; \quad V_{o2} = -\frac{R_6}{R_4} \cdot V_{o1} - \frac{R_6}{R_5} V_{in}$$

$$V_{o2} = -\frac{80}{40} \cdot V_{o1} - \frac{80}{20} V_{in} = -2V_{o1} - 4V_{in}$$

$$\boxed{V_{o2} = -2V_{o1} - 4V_{in}} \quad (1)$$

BLOQUE 3



$$V_{o3} = -\frac{R_{10}}{R_8} \cdot V_{o2} - \frac{R_{10}}{R_9} \cdot V_{OFF}$$

$$\boxed{V_{o3} = -V_{o2} - 2} \quad (2)$$

COMBINANDO (1) y (2)

$$V_{o3} = -\left[-2V_{o1} - 4V_{in}\right] - 2 = 2V_{o1} + 4V_{in} - 2$$

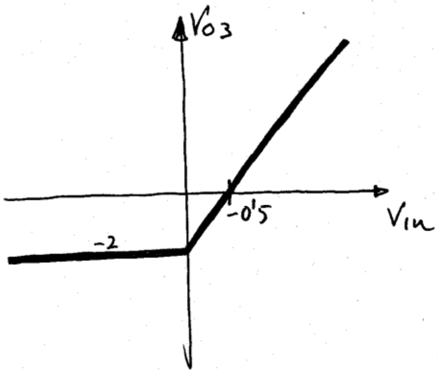
$V_{o3} =$
- $V_{in} > 0 \Rightarrow V_{o1} = 0 \Rightarrow V_{o3} = 4V_{in} - 2$
- $V_{in} < 0 \Rightarrow V_{o1} = -\frac{R_2}{R_1} V_{in} = -2V_{in} \Rightarrow V_{o3} = 2(-2V_{in}) + 4V_{in} - 2$

$$V_{o3} = -2\,V$$

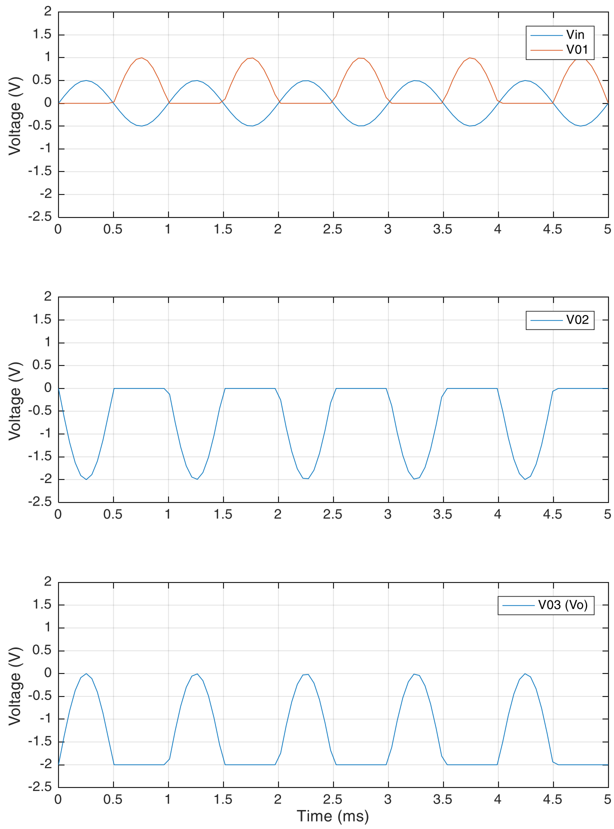
FINALMENTE

$$V_{in} > 0 \Rightarrow V_{o3} = 4\,V_{in} - 2$$

$$V_{in} < 0 \Rightarrow V_{o3} = -2\,V$$



d)



Problem 3

A sinusoidal interference of 1 MHz has been detected in a data acquisition circuit for instrumentation. We want to actively attenuate this interference using the Sallen-Key type second-order low-pass filter in Figure P3.1.

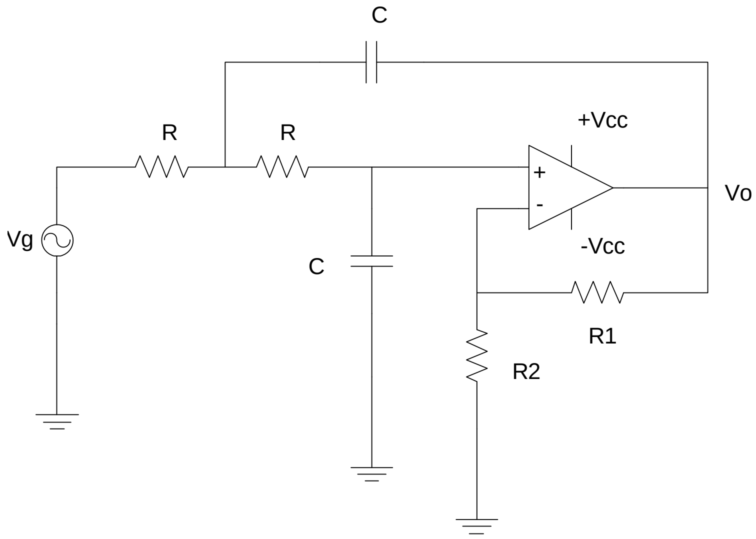


Figure P3.1

$$H(s)=\frac{V_o}{V_g}=\frac{K\omega_0^2}{s^2+\frac{\omega_0}{Q}s+\omega_0^2}=\frac{K}{s^2(RC)^2+sRC(3-K)+1}$$

1. Calculate the cutoff frequency of the filter wo, its DC gain K and its quality factor Q as a function of the passive components of the circuit R, C, R1 and R2.
2. Determine the quality factor of the filter and its DC gain to amplify 3dB the input at the cutoff frequency.
3. Determine the cutoff frequency of the filter so that the input power is attenuated by 40 dB at 1MHz.
4. Calculate C and R1 assuming that R2 = R = 1kΩ.
5. Calculate the minimum Slew-Rate in V / µs that the amplifier needs to obtain a sine wave without distortion at the filter output when a sine wave of 1V amplitude and 100KHz frequency is applied to the input filter. To do this, take into account the gain factor at that frequency and use the following approximation: $\lim_{x\to 0} A\sin(x) \approx Ax$
6. Using the DC gain of the filter, determine the minimum GBW product required by your op amp. If you have not calculated the previous sections, assume a DC gain of 1.76 and a filter cutoff frequency of 75 kHz.

Solution

1) Multiplying the numerator and denominator by $1/(RC)^2$

$$H(s)=\frac{V_o}{V_g}=\frac{K\omega_0{}^2}{s^2+\frac{\omega_0}{Q}s+\omega_0{}^2}=\frac{\frac{K}{(RC)^2}}{s^2+\frac{3-K}{RC}s+\frac{1}{(RC)^2}}$$

and identifying terms:

$$\omega_0=\frac{1}{RC};\ Q=\frac{1}{3-K}$$

Analyzing the circuit in DC, all capacitors will act like an open circuit (Zc=1/(jw)->∞), obtaining a non-inverting amplifier.

$$V_g=V_o\frac{R_2}{R_1+R_2}\rightarrow K=\frac{V_o}{V_g}=\frac{R_1+R_2}{R_2}=1+\frac{R_1}{R_2}$$

Finally, Q in terms of R1 and R2:

$$Q=\frac{R_2}{2R_2-R_1}$$

2) The gain at the cutoff frequency will be

$$20\log_{10}\left(\left|H(j\omega_0)\right|\right)=3dB\rightarrow\left|H(j\omega_0)\right|=10^{\frac{3}{20}}\left[\frac{V}{V}\right]$$

The modulus of the transfer junction is

$$H(j\omega)=\left|-\frac{K\omega_0{}^2}{-\omega^2+j\frac{\omega_0}{Q}\omega+\omega_0{}^2}\right|=\left|\frac{K\omega_0{}^2}{\left(\omega_0{}^2-\omega^2\right)+j\frac{\omega_0}{Q}\omega}\right|$$

At the cut-off frequency

$$H(j\omega_0)=\left|\frac{K\omega_0{}^2}{j\frac{\omega_0{}^2}{Q}}\right|=KQ=10^{\frac{3}{20}}=a$$

$$Q=\frac{1}{3-K}=\frac{a}{K}\Rightarrow K=\frac{3a}{1+a}=\frac{3.10^{\frac{3}{20}}}{1+10^{\frac{3}{20}}}=1.76\Rightarrow Q=0.8$$

3) As the filter is a second order filter, and the gain at the cutoff frequency is 3dBm, if we have -40 dB of attenuation at 1 MHz the cutoff frequency must be more that one decade before 1MHz. Therefore, we safely can assume that $f_1=1MHz>>f_0$. For frequencies much greater than the cutoff frequency the modulus of the filters transfer function became:

$$H(j\omega)\Big|_{\omega>>\omega_0} = \left| \frac{K\omega_0^{\ 2}}{\left(\omega_0^{\ 2}-\omega^2\right)+j\dfrac{\omega_0}{Q}\omega} \right|_{\omega>>\omega_0} \approx \frac{K\omega_0^{\ 2}}{\omega^2} = \frac{Kf_0^{\ 2}}{f^{\ 2}}$$

At f_1=1 MHz we have an attenuation of -40 dB then

$$20\log_{10}\left(\left|H(j\omega_1)\right|\right) = -40\,dB \rightarrow \left|H(j\omega_1)\right| = 10^{-\frac{40}{20}} = 0.01\left[\frac{V}{V}\right]$$

Finally,

$$20\log_{10}\left(\left|H(j\omega_1)\right|\right) = 0.01 \approx \frac{Kf_0^{\ 2}}{(1MHz)^2} \rightarrow f_0 \approx 1MHz\sqrt{\frac{0.01}{1.76}} = 75.4kHz$$

The cutoff frequency can be calculated using the modulus without any approximation or using the asymptotic Bode, as well. For instance, using the asymptotic Bode first we have to calculate the DC gain in dBs:

$$20\log_{10}\left(1.76\right) = 4.9dB$$

Then, using the asymptotic approximation of the second order transfer function (~1/w^2):

$$20\log_{10}\left(\left|H(j\omega_1)\right|\right) - 20\log_{10}\left(\left|H(j\omega_0)\right|\right) = -40-(+4.9) = -44.9dB \rightarrow$$

$$20\log_{10}\left(\frac{\left|H(j\omega_1)\right|}{\left|H(j\omega_0)\right|}\right) = 20\log_{10}\left(\frac{1/\omega_1^{\ 2}}{1/\omega_0^{\ 2}}\right) = 20\log_{10}\left(\frac{\omega_0^{\ 2}}{\omega_1^{\ 2}}\right) = 40\log_{10}\left(\frac{\omega_0}{\omega_1}\right) = -44.9dB \rightarrow$$

$$\frac{\omega_0}{\omega_1} = \frac{f_0}{f_1} = 10^{-\frac{44.9}{40}} \rightarrow f_0 = f_1 10^{-\frac{44.9}{40}} = 1MHz.10^{-\frac{44.9}{40}} = 75.4kHz$$

4) From the cutoff frequency obtained in 1) and using R_2=R=1 kΩ

$$f_o = \frac{1}{2\pi RC} \rightarrow C = \frac{1}{2\pi Rf} = 2.1nF$$

$$K = 1+\frac{R_1}{R_2} \rightarrow R_1 = R_2(K-1) = 760\,\Omega$$

5) At the frequency of the input signal, 100 kHz, the gain will be:

$$\left|H(j2\pi 100kHz)\right| = 0.96\frac{V}{V} \approx 1\frac{V}{V}$$

The output signal will be

$$V_o = \left|H(j\omega)\right| A\sin(\omega t+\varphi)$$

The slew rate must be higher than the maximum slope of the output signal at 100kHz to obtain a sine wave without distortion:

$$\left(\frac{dV_o}{dt}\right)_{\max} = \left|H(j\omega)\right| A\omega \Rightarrow SR > \left(\frac{dV_o}{dt}\right)_{\max} = \left|H(j\omega)\right| A\omega = 1*1V*2\pi 100kHz = 0.63\frac{V}{\mu s}$$

6) Considering the negative feedback of the op amp, a GBW = K*f_0 provides a constant gain along all the bandwidth of the filter. However, if the phase characteristic of the filter is required to be constant to avoid phase shifts or distortion at the output then the GBW should be at least one decade above: GBW=10*K* f_0 = 1.3MHz.