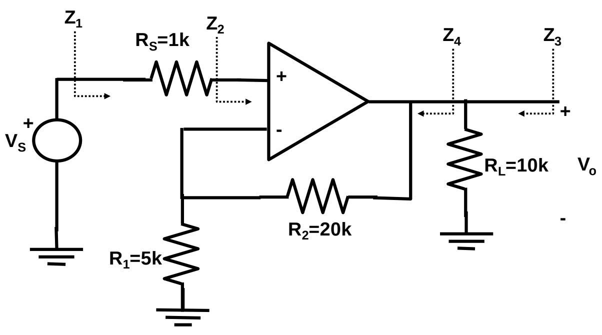
EXAMPLE 1 OF PRACTICAL ANALYSIS OF NEGATIVE FEEDBACK AMPLIFIERS

Given the following feedback amplifier:



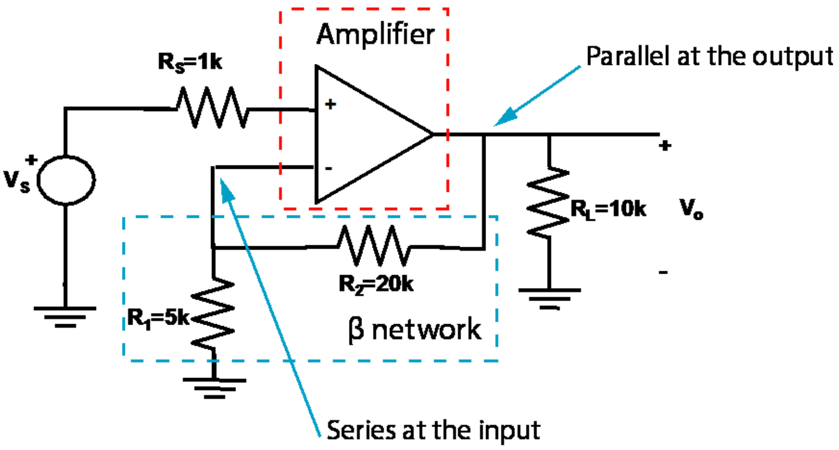
DATA:

Parameters of the small signal equivalent circuit of the amplifier at mid frequencies:

$$R_i = 100k\Omega$$
$$A_v = 3000\frac{V}{V}$$
$$R_o = 10k\Omega$$

Calculate: V_o/V_S, Z_1, Z_2, Z_3 y Z_4

The feedback topology is **series-parallel,** the feedback amplifier mixes voltage at the input and sense voltage at the output (**Transvoltage amplifier**)



Is the feedback negative?

From the output, if $\Delta V_o \uparrow \Rightarrow V_- \downarrow \Rightarrow V_i = V_s - V_- \downarrow (V_s \text{ is const.}) \Rightarrow V_o = AV_i \downarrow$, so, indeed, **there is negative feedback.**

Other way to study if there is negative feedback is to check that the product $A\beta$ is positive. The value of $A\beta$ can be determine as follows:

1. Set the input signal to zero, $x_s = 0$.
2. Break the feedback loop at a convenient location, ensuring that the values of *A* and β does not change. Since we assumed that the feedback does not load the amplifier output, we can break the loop at the amplifier output with out causing *A* to change.
3. Apply a test signal x_t to the input look (where the break has made) and determine the returned signal at the loop output (at the other side of the break): $x_r = -A\beta x_t$, so the loop gain will be:

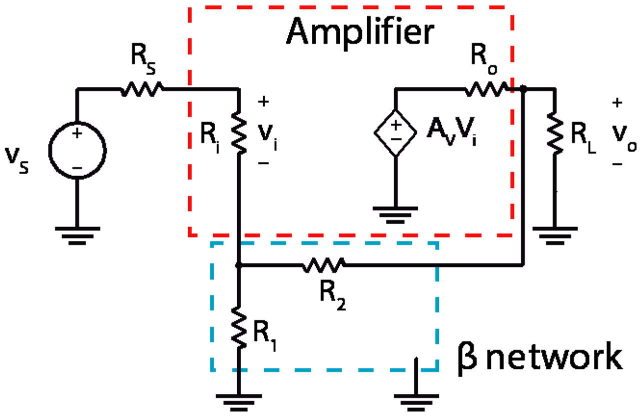
$$A\beta = -\frac{x_r}{x_t}$$

If we apply this procedure in this case,

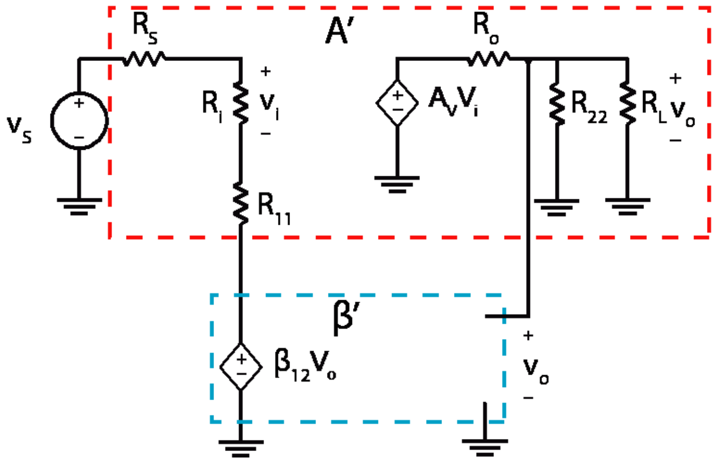
$$V_- = \frac{R_1}{R_1+R_2}V_t \rightarrow V_r = -AV_- = -A\frac{R_1}{R_1+R_2}V_t \rightarrow A\beta = -\frac{X_r}{X_t} = A\frac{R_1}{R_1+R_2} > 0$$

So, as the loop gain $A\beta > 0$ the feedback is negative.

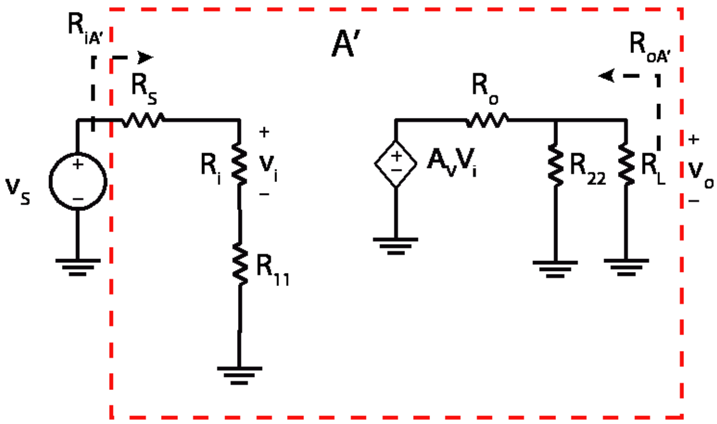
Let's use the practical method to analyze the gain, and the input and output impedances of the feedback amplifier



We can redraw an equivalent circuit considering the load effects of the β network obtaining the new A' and β' networks.



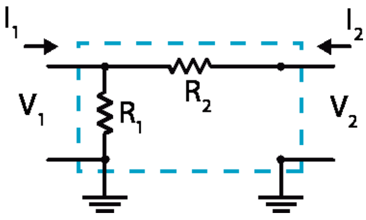
The A' network is obtained by augmenting the basic amplifier at the input with the source resistance and R_{11}, and at the output with the load resistance and R_{22}.



Since it is a transvoltage amplifier we have to calculate the voltage gain of the A' network:

$$A_V' = \frac{v_o'}{v_S'} = \frac{v_i}{v_S'}\frac{v_o'}{v_i} = \frac{R_i}{R_s + R_i + R_{11}} \cdot A_V \frac{R_L \parallel R_{22}}{R_o + R_L \parallel R_{22}} = A_V \cdot \frac{R_L \parallel R_{22}}{R_o + R_L \parallel R_{22}} \cdot \frac{R_i}{R_S + R_i + R_{11}}$$

Now, we can calculate the parameters of the β network quadrupole.



The feedback network quadrupole can be represented by

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

that corresponds to h parameters. Remember that the independent variables are the common magnitudes at the input and at the output, in this case, they are I_1 and V_2. The loading effect of the feedback network at the input is obtained by short circuiting the port 2 because it is connected in parallel and looking into port 1.

$$R_{11} = \beta_{11} = \left.\frac{v_1}{i_1}\right|_{v_2=0} = R_1 \parallel R_2$$

The loading effect of the feedback network at the output side is obtained by open-circuiting the port 1, because it is connected in series, and looking into the port 2:

$$R_{22} = 1/\beta_{22} = \left.\frac{v_2}{i_2}\right|_{i_1=0} = R_1 + R_2$$

Substituting the gain of the A' network will be:

$$A_V' = \frac{v_o'}{v_S'} = A_V \cdot \frac{R_L \parallel (R_1+R_2)}{R_o + R_L \parallel (R_1+R_2)} \cdot \frac{R_i}{R_S + R_i + \dfrac{R_1 R_2}{R_1+R_2}} = A_v \frac{\dfrac{R_L(R_1+R_2)}{R_L+R_1+R_2}}{R_o + \dfrac{R_L(R_1+R_2)}{R_L+R_1+R_2}} \cdot \frac{R_i}{R_S + R_i + \dfrac{R_1 R_2}{R_1+R_2}} =$$

$$= A_v \frac{R_L(R_1+R_2)}{R_o(R_L+R_1+R_2)+R_L(R_1+R_2)} \cdot \frac{R_i}{R_S+R_i+\dfrac{R_1R_2}{R_1+R_2}} \approx 1200\left[\frac{V}{V}\right]$$

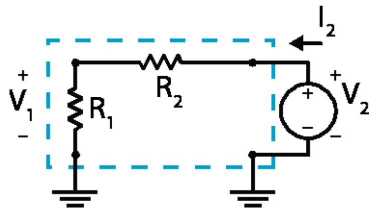
The input and output resistance of the A' network will be required to obtain the input and output resistant of the feedback amplifier. The input resistance can be easily found by inspection of the A' network:

$$R_{iA'} = R_S + R_i + R_{11} = R_S + R_i + R_1 \parallel R_2 \cong 104k\Omega,$$

as well as the output resistance of the A' network:

$$R_{oA'} = R_o \parallel R_L \parallel R_{2\beta} \cong 4.15k\Omega.$$

The β network gain is the term β_{12} and can be determined taking into account the figure:



$$\beta_V = \beta_{12} = \left.\frac{v_1}{v_2}\right|_{i_1=0} = \frac{R_1}{R_1+R_2} = 0.2\left[\frac{V}{V}\right]$$

Feedback amplifier parameters (Gain, A_{VCR}, input resistance, R_{iCR}, and output resistance, R_{oCR})

The voltage gain with feedback can now be obtained as

$$A_{Vf} = \frac{v_o}{v_S} = \frac{A_V'}{1 + A_V'\beta_V} \cong 4.98 \left(\cong \frac{1}{\beta} \right)$$

The input resistance R_{if} determined by the feedback equations is the resistance seen **by the external source (Z_1)**

$$R_{if} = R_{iA'}\left(1 + A_V'\beta_V\right) \cong 25M\Omega$$

The resistance R_{of} given by the feedback equations is the output resistance of the feedback amplifier, including the load resistance R_L, i.e., Z_3

$$R_{of} = \frac{R_{oA'}}{\left(1 + A_V'\beta_V\right)} \cong 17.2\Omega$$

To obtain Z_2 we have to subtract R_S from R_{if}:

$$Z_2 = R_{if} - R_S \cong R_{if} \cong 25M\Omega$$

The resistance asked for, Z_3 , is the output resistance of the feedback amplifier excluding R_L. From the circuit

$$R_{of} = Z_4 \| R_L \Rightarrow Z_4 = \frac{R_L R_{of}}{R_L - R_{of}} \cong R_{of} \cong 17.2\Omega$$

To sum up, the requested parameters of the circuit V_o/V_S, Z_1, Z_2, Z_3 and Z_4 are:

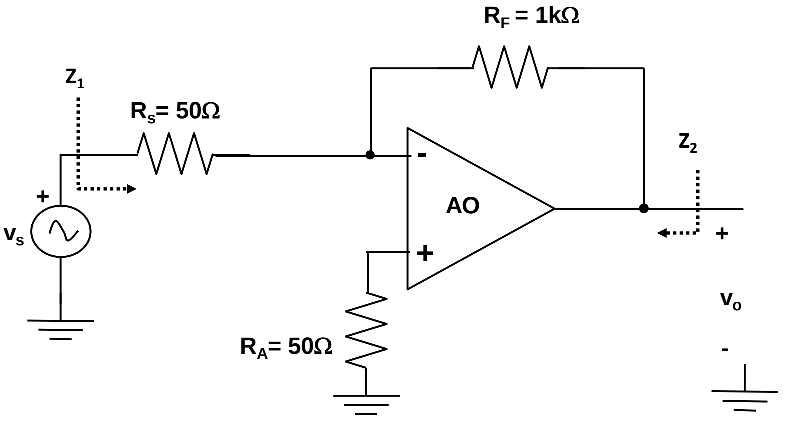
$$\frac{v_o}{v_S} = A_{Vf} \cong 4.98$$

$$Z_1 = R_{if} \cong 25M\Omega \qquad Z_2 \cong 25M\Omega$$

$$Z_3 = R_{of} \cong 17.2\Omega \qquad Z_4 \cong 17.2\Omega$$

EXAMPLE 2 OF PRACTICAL ANALYSIS OF NEGATIVE FEEDBACK AMPLIFIERS

Given the following feedback amplifier:



DATOS:

Parameters of the small signal equivalent circuit of the amplifier at mid frequencies:

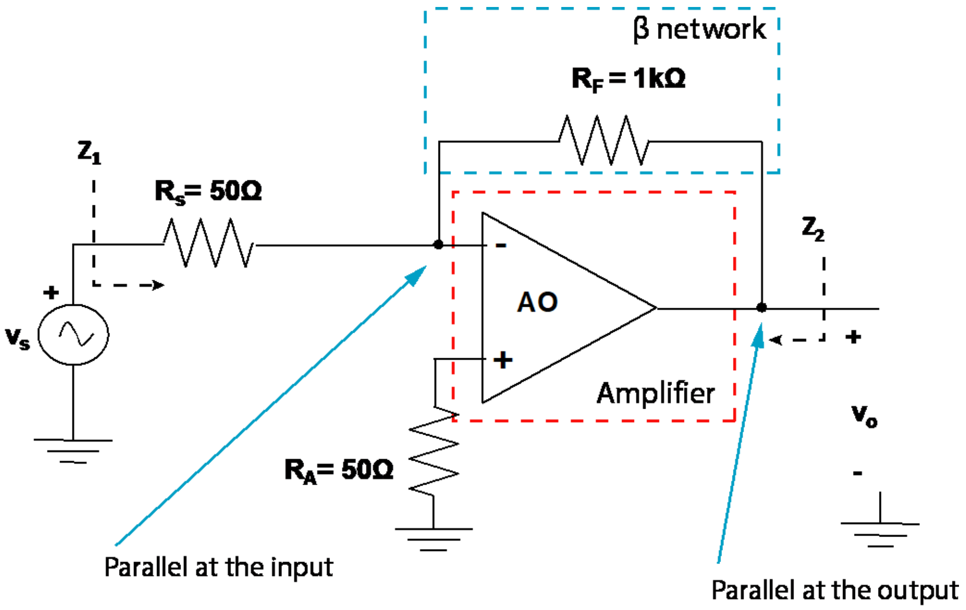
$$R_i = 2M\Omega$$

$$A_v = 2000\,\frac{V}{V}$$

$$R_o = 150\Omega$$

Calculate: V_o/V_S, Z_1, Z_2

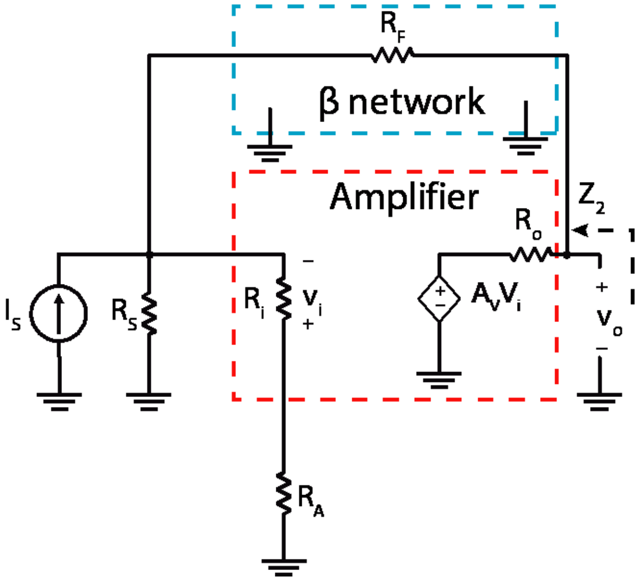
The feedback topology is **parallel-parallel,** the feedback amplifier mixes current at the input and sense voltage at the output (**Transimpedance amplifier).**



Is the feedback negative?

From the output, if $\Delta V_o \uparrow \Rightarrow -I_{R_F} \uparrow \rightarrow I_{R_S} \uparrow (I_S = I_{R_S} - I_{R_F}\, is\, const.) \Rightarrow V_- \uparrow \Rightarrow V_o = -AV_- \downarrow$, so, indeed, **there is negative feedback**.

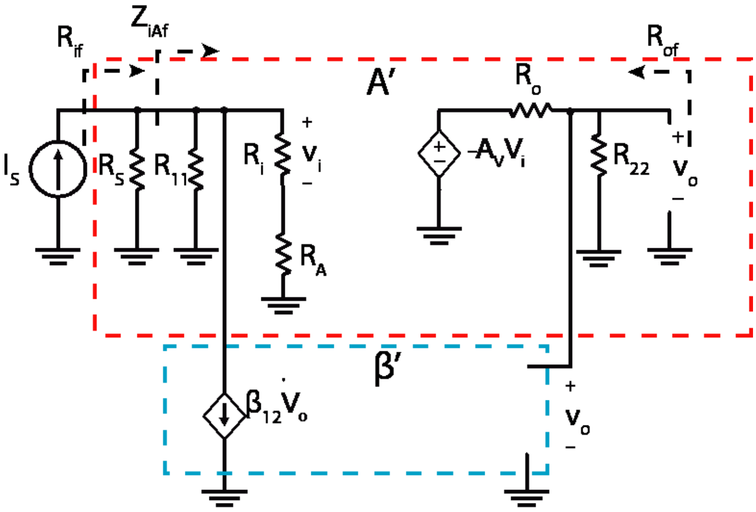
This amplifier is a transimpedance amplifier, i.e, a current to voltage amplifier. The gain is given by the ratio V_o/I_S. It is convenient to show explicitly the input current by substituting the voltage source by its Norton equivalent current source:



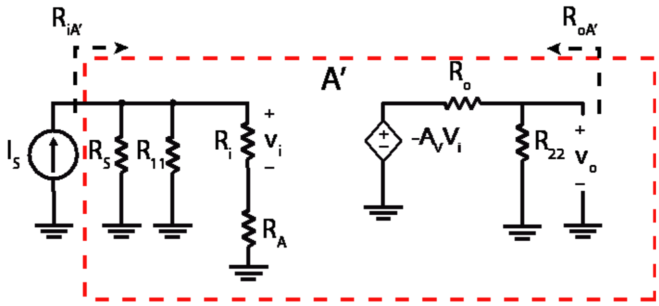
where the input current is given by

$$I_s = \frac{V_s}{R_s}$$

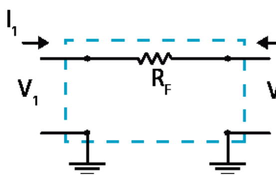
We can redraw an equivalent circuit considering the load effects of the β network obtaining the new A' and β' networks.



The A' network is obtained by augmenting the basic amplifier at the input with the source resistance and R_{11}, and at the output with the load resistance and R_{22}.



We obtain the values of R_{11} and R_{22} analyzing the β network quadrupole



The feedback network quadrupole can be represented by

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

that corresponds to y parameters. Remember that the independent variables are the common magnitudes at the input and at the output, in this case, they are V_1 and V_2. The loading effect of the feedback network at the input is obtained by short circuiting the port 2 because it is connected in parallel and looking into port 1.

$$R_{11} = \frac{1}{\beta_{11}} = \left.\frac{v_1}{i_1}\right|_{V_2=0} = R_F$$

The loading effect of the feedback network at the output side is obtained by short -circuiting the port 1, because it is connected in parallel as well, and looking into the port 2:

$$R_{22} = \frac{1}{\beta_{22}} = \left.\frac{v_2}{i_2}\right|_{V_1=0} = R_F$$

Since it is a transimpedance amplifier we have to calculate the current to voltage gain of the A' network:

$$A_Z' = \frac{v_o'}{i_S'} = \frac{v_i}{i_S'}\frac{v_o'}{v_i} \rightarrow \begin{cases} v_i = R_s \parallel R_F \parallel (R_i + R_A)\dfrac{R_i}{R_i + R_A} i_S' \approx R_s i_S' \\ v_o' = -A_V v_i \dfrac{R_F}{R_o + R_F} \approx -A_V v_i \end{cases} \Rightarrow$$

$$A_Z' = \frac{v_o'}{i_S'} = \frac{v_i}{i_S'}\frac{v_o'}{v_i} \approx -A_V R_s = 100\,k\Omega$$

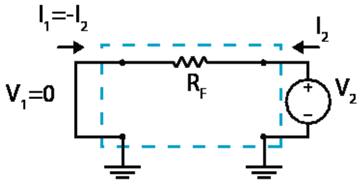
The input and output resistance of the A' network will be required to obtain the input and output resistant of the feedback amplifier. The input resistance can be easily found by inspection of the A' network:

$$R_{iA'} = R_S \parallel R_F \parallel (R_i + R_A) \approx R_S = 50\,\Omega,$$

as well as the output resistance of the A' network:

$$R_{oA'} = R_o \parallel R_F \cong 130\,\Omega.$$

Finally, we can calculate the β network gain, β_{12}. From the following figure we have:



$$\beta_Y = \beta_{12} = \left.\frac{i_1}{v_2}\right|_{v1=0} = -\frac{1}{R_F} = -1m\Omega^{-1}$$

Feedback amplifier parameters (gain, A_{Zf}, input resistance, R_{if}, and output resistance, R_{of})

The transimpedance gain with feedback can be obtained as

$$A_{Zf} = \frac{v_o}{i_S} = \frac{A_Z'}{1 + A_Z'\beta_Y} \cong \frac{1}{\beta_Y} = -1k\Omega$$

where $A_Z'\beta_Y = 100 >> 1$. The input resistance R_{if} determined by the feedback equations is the resistance seen **by the external source, note that in this case is the current equivalent source**

$$R_{if} = \frac{R_{iA'}}{1 + A_Z'\beta_Y} \cong 0.5\Omega$$

The resistance R_{of} given by the feedback equations is the output resistance of the feedback amplifier, i.e., Z_2

$$R_{of} = \frac{R_{oA'}}{(1 + A_Z'\beta_Y)} \cong 1.3\Omega$$

Requested parameters of the circuit V_o/V_S, Z_1 and Z_2

To obtain the **voltage gain** we have to undo the Norton equivalent source in order to recover the initial voltage source:

$$\frac{v_o}{v_s} = \frac{v_o}{i_s \cdot R_S} = \frac{A_{Zf}}{R_S} = \frac{-1k\Omega}{50\Omega} = -20 \left[\frac{V}{V}\right]$$

The input impedance, Z_1, is the one seen by the **voltage source** and is going to be equal to

$$Z_1 = R_s + Z_{iAf}$$

where Z_{iAf} is the input impedance of the amplifier. We can calculate Z_{iAf} from R_{if} as (note the impedances marked in the 3rd figure of this solution)

$$R_{if} = R_s \parallel Z_{iAf} \Rightarrow Z_{iAf} = \frac{R_S R_{if}}{R_S - R_{if}} \cong R_{if} = 0.5\Omega$$

Finally, the input impedance is

$$Z_1 = R_s + Z_{iAf} \approx R_s = 50\Omega$$

The output impendace, Z_2, is just the already calculated R_{of}

$$Z_2 = R_{of} = 1.3\ \Omega$$

To sum up, the requested parameters of the circuit V_o/V_S, Z_1, Z_2 are:

$$\frac{v_o}{v_s} = \frac{A_{Zf}}{R_S} = -20\left[\frac{V}{V}\right]$$
$$Z_1 \cong R_S = 50\Omega$$
$$Z_2 = R_{of} = 1.3\Omega$$

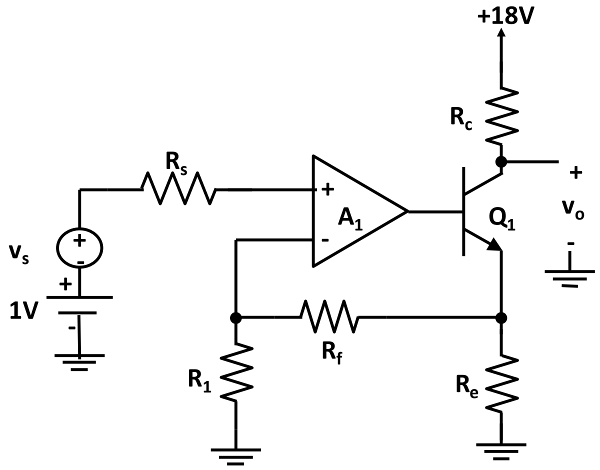
EXAMPLE 3 OF PRACTICAL ANALYSIS OF NEGATIVE FEEDBACK AMPLIFIERS

Considering the feedback amplifier depicted in the figure:

a. Draw the small signal equivalent circuit at mid frequencies.

b. Demonstrate that there is negative feedback. Identify the topology and obtain the equivalent circuits of A' and β.

c. Calculate the numerical value of A', β and v_o/v_s.



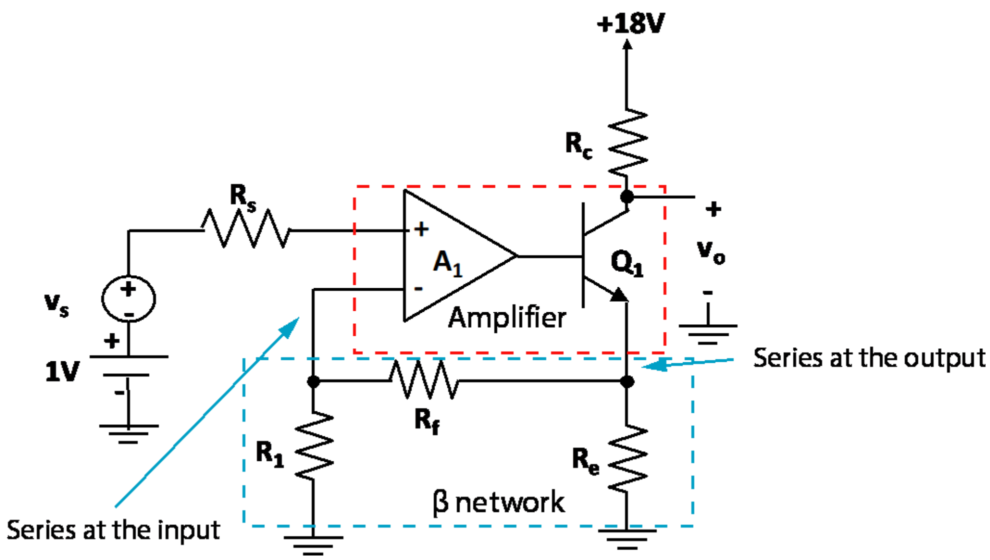
Data:

Q₁: $r_\pi \cong 2.8k\Omega$ $g_m \cong 0.072\Omega^{-1}$

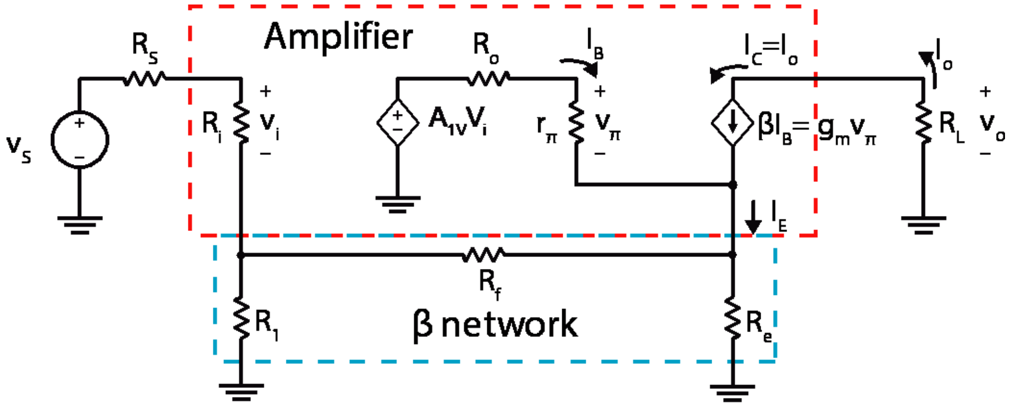
A₁ is a voltaje amplifier: $A_{1v} = 2\cdot 10^5$ (V/V), $R_i = 1M\Omega$, $R_o = 150\Omega$

Other components: $R_s = 50\Omega$, $R_1 = 1.1k\Omega$, $R_f = 2.2k\Omega$, $R_c = 6.8k\Omega$ y $R_e = 3.3k\Omega$

The feedback topology is **series-series** (see the following figure)**,** the feedback amplifier mixes voltage at the input and sense current at the output (**Transadmitance amplifier).** This amplifier is a Transadmitance amplifier, i.e, a voltage to current amplifier.



a) The small signal equivalent circuit a mid frequencies is:



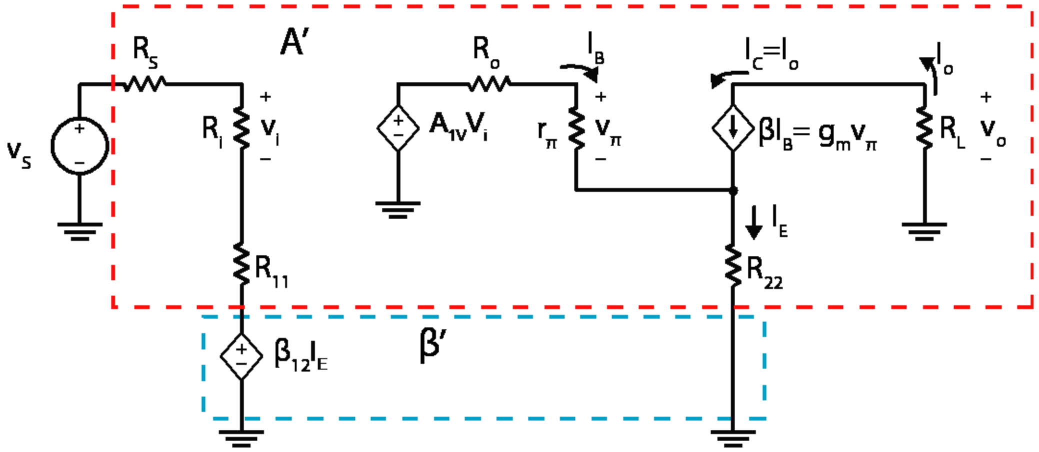
Note that in this case the amplifier cannot be represented by a quadrupole because in the output port, the current that enters into the port, $I_o = I_c$, is not equal to the current that leaves the port, I_E, as a consequence of having the BJT transistor. Special care has to be taken when we calculate the output impedance in these cases. More on this in the next exercise. If instead of a BTJ we have a MOSFET at the output of the amplifier, then we would could represent the amplifier by a quadrupole.

b) Is the feedback negative?

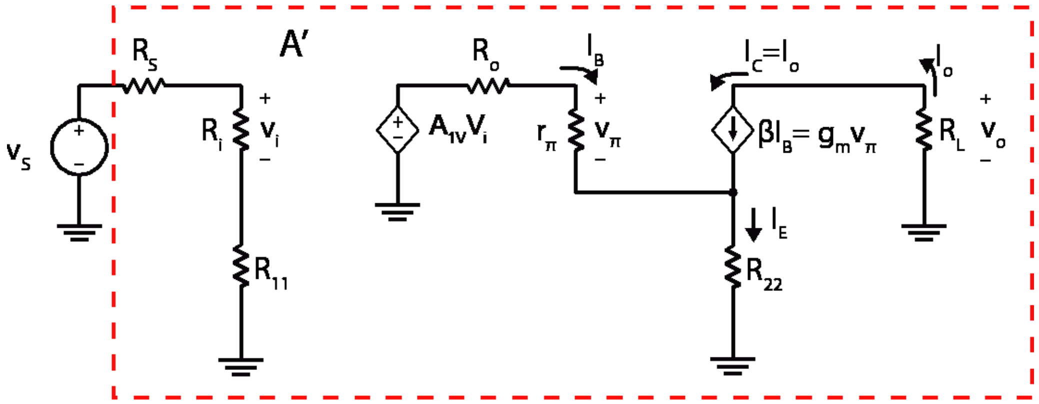
From the output, if $\Delta i_o \uparrow \Rightarrow i_e \uparrow \Rightarrow V_{R_e} = R_e I_e \uparrow \Rightarrow V_{R_1} \uparrow \Rightarrow \left(V_s = V_{R_s} + V_i + V_{R_1} = const.\right) V_i \downarrow \Rightarrow V_o = AV_i \downarrow \Rightarrow I_B \downarrow \Rightarrow$

$\Rightarrow I_C = I_o \downarrow$, so, indeed, **there is negative feedback**.

We can redraw an equivalent circuit considering the load effects of the β network obtaining the new A' and β' networks.



The A' network is obtained by augmenting the basic amplifier at the input with the source resistance and R_{11}, and at the output with the load resistance and R_{22}.



Since it is a transadmitance amplifier we have to calculate the voltage to current gain of the A' network:

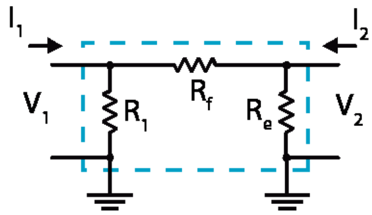
$$A_Y' = \frac{i_o'}{v_S'} = \frac{v_i}{v_S'}\frac{i_B}{v_i}\frac{i_o'}{i_B} = \begin{cases} v_i = \dfrac{R_i}{R_s + R_i + R_{11}} v_s' \\ i_B = \dfrac{A_{1v} v_i}{R_o + r_\pi + (\beta_{Q_1} + 1) R_{22}} \\ i_o' = \beta_{Q_1} i_B \end{cases} \Rightarrow$$

$$A_Y' = \frac{i_o'}{v_S'} = \frac{v_i}{v_S'}\frac{i_B}{v_i}\frac{i_o'}{i_B} = \frac{R_i}{R_s + R_i + R_{11}} \frac{A_{1v}\beta_{Q_1}}{R_o + r_\pi + (\beta_{Q_1} + 1) R_{22}}$$

where β_{Q_1} is the β parameter of the BJT Q1 and it is equal to:

$$\beta_{Q_1} = g_m r_\pi \approx 202$$

Now, we can calculate the parameters of the β network quadrupole.



The feedback network quadrupole can be represented by

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

that corresponds to z parameters. Remember that the independent variables are the common magnitudes at the input and at the output, in this case, they are I_1 and I_2. The loading effect of the feedback network at the input is obtained by open circuiting the port 2 because it is connected in series, and looking into port 1.

$$R_{11} = \beta_{11} = \left.\frac{v_1}{i_1}\right|_{i_2=0} = R_1 \,||\, (R_f + R_e) = \frac{R_1(R_f + R_e)}{R_1 + R_f + R_e} = 0.92\,k\Omega$$

The loading effect of the feedback network at the output side is obtained by open-circuiting the port 1, because it is connected in series, and looking into the port 2:

$$R_{22} = \beta_{22} = \left.\frac{v_2}{i_2}\right|_{i_1=0} = R_e \,||\, (R_1 + R_f) = \frac{R_e(R_1 + R_f)}{R_e + R_1 + R_f} = 1.65\,k\Omega$$

Substituting the gain of the A' network will be:

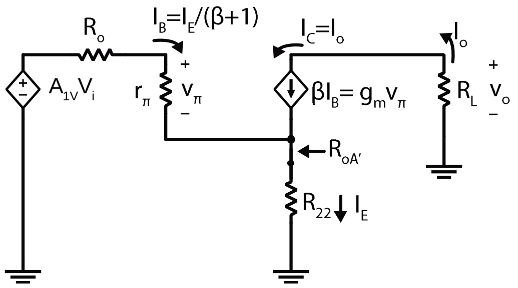
$$A_Y' = \frac{i_o'}{v_S'} = \frac{R_i}{R_s + R_i + R_{11}} \frac{A_v\beta_{Q_1}}{R_o + r_\pi + (\beta_{Q_1} + 1) R_{22}} \approx \frac{R_i}{R_i}\frac{A_v\beta_{Q_1}}{\beta_{Q_1} R_{22}} \approx \frac{A_v}{R_{22}} \approx 121.2\,\Omega^{-1}$$

In this exercise it is not asked to obtain the input and output resistance of the amplifier. However, lets indicate that the input resistance of the A' network will be by inspection:

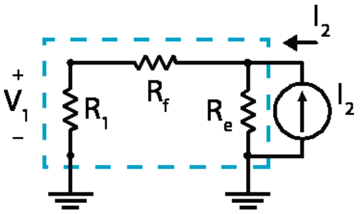
$$R_{iA'} = R_S + R_i + R_{11} \cong R_i = 1M\Omega,$$

The output resistance of the A' network is not so straightforward since we have to realize that the **sensed current is I_e (current in the emitter) and not the actual output of the amplifier I_o (current in the collector)**. The output impedance of the A' is obtained by breaking the circuit between the emitter and R_{22} (see next figure). The resistance found between these two points is

$$R_{oA'} = R_{22} + \frac{r_\pi + R_o}{\beta_{Q_1} + 1} \cong 1.66\,k\Omega.$$



The β **network gain** is the term β_{12} and can be determined taking into account the figure:



$$\beta_Z = \beta_{12} = \left.\frac{v_1}{i_2}\right|_{i_1=0} = \frac{(R_1+R_f)\,||\,R_e}{R_1+R_f}R_1 = \frac{R_1R_e}{R_1+R_f+R_e} \approx 0.55\,k\Omega$$

Note that $A_y\,'\beta_z \approx 67k >> 1$ is positive as should be if we have negative feedback.

The **transadmitance gain** of the feedback amplifier will be:

$$A_{Yf} = \frac{i_o}{v_s} = \frac{A_Y\,'}{1+A_Y\,'\beta_Z} \cong \frac{1}{\beta_Z} \cong 1.8\,m\Omega^{-1}$$

It is asked for the voltage gain, then

$$v_o = -i_o \cdot R_C \Rightarrow \frac{v_o}{v_s} = -\frac{i_o}{v_s} \cdot R_C = -A_{Yf} \cdot R_C \cong -12.2\frac{V}{V}$$

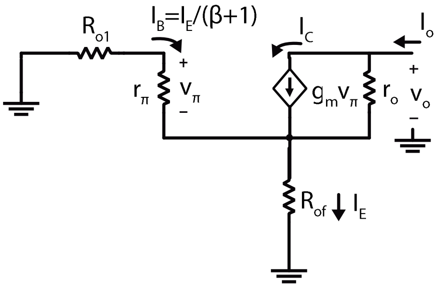
The input resistance R_{if} determined by the feedback equations is the resistance seen **by the external source,**

$$R_{if} = R_{iA'}\left(1+A_Y\,'\beta_Z\right) \cong 67\,G\,\Omega$$

The resistance R_{of} given by the feedback equations is the output resistance of the feedback amplifier seen **from the emitter**:

$$R_{of} = R_{oA'}\left(1+A_Y\,'\beta_Z\right) \cong 111M\,\Omega$$

The output resistance, the one saw from the collector is calculated from the following equivalent circuit where we have introduced the equivalent resistance seen from the emitter with feedback and in general, we have to consider the r_o to account for the Early effect.

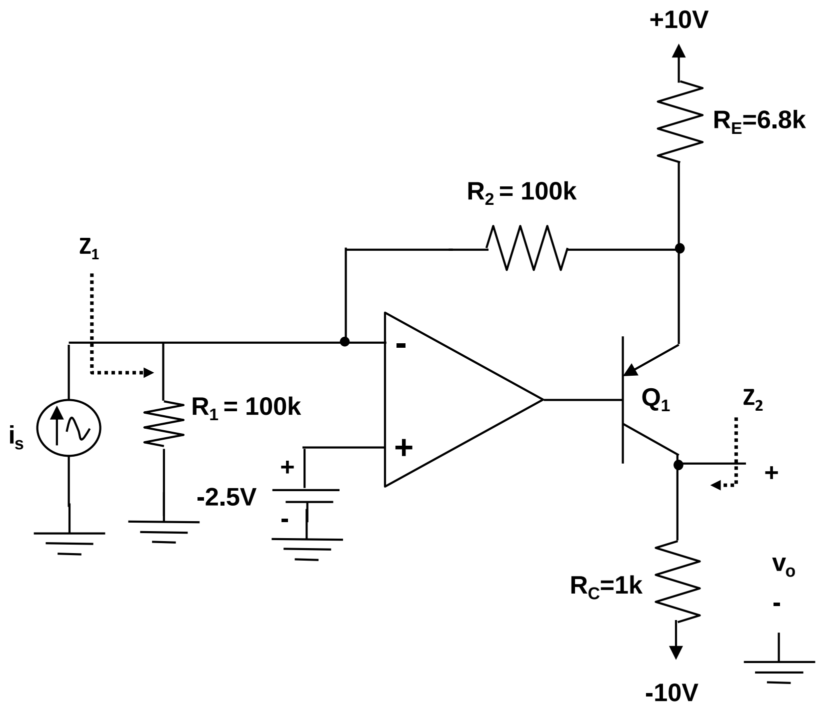


$$Z_o = \frac{v_o}{i_o} = r_o + \left[R_{of}\,||\left(R_{o1}+r_\pi\right)\right]\left(1+r_o g_m \frac{r_\pi}{r_\pi+R_{o1}}\right)$$

Note that, in this case, the actual output resistance of the amplifier is not multiplied by the loop gain factor. The loop gain factor appears in R_{of} that is the resistance seen from the emitter because is the emitter current what is sensed by the feedback network. More about this in the next exercise.

EXAMPLE 4 OF PRACTICAL ANALYSIS OF NEGATIVE FEEDBACK AMPLIFIERS

Given the following feedback amplifier:



DATA:

Parameters of the small signal equivalent circuit of the amplifier at mid frequencies:

$$R_i = 2M\Omega$$

$$A_v = 2 \cdot 10^5 \frac{V}{V}$$

$$R_o = 100\Omega$$

Data:

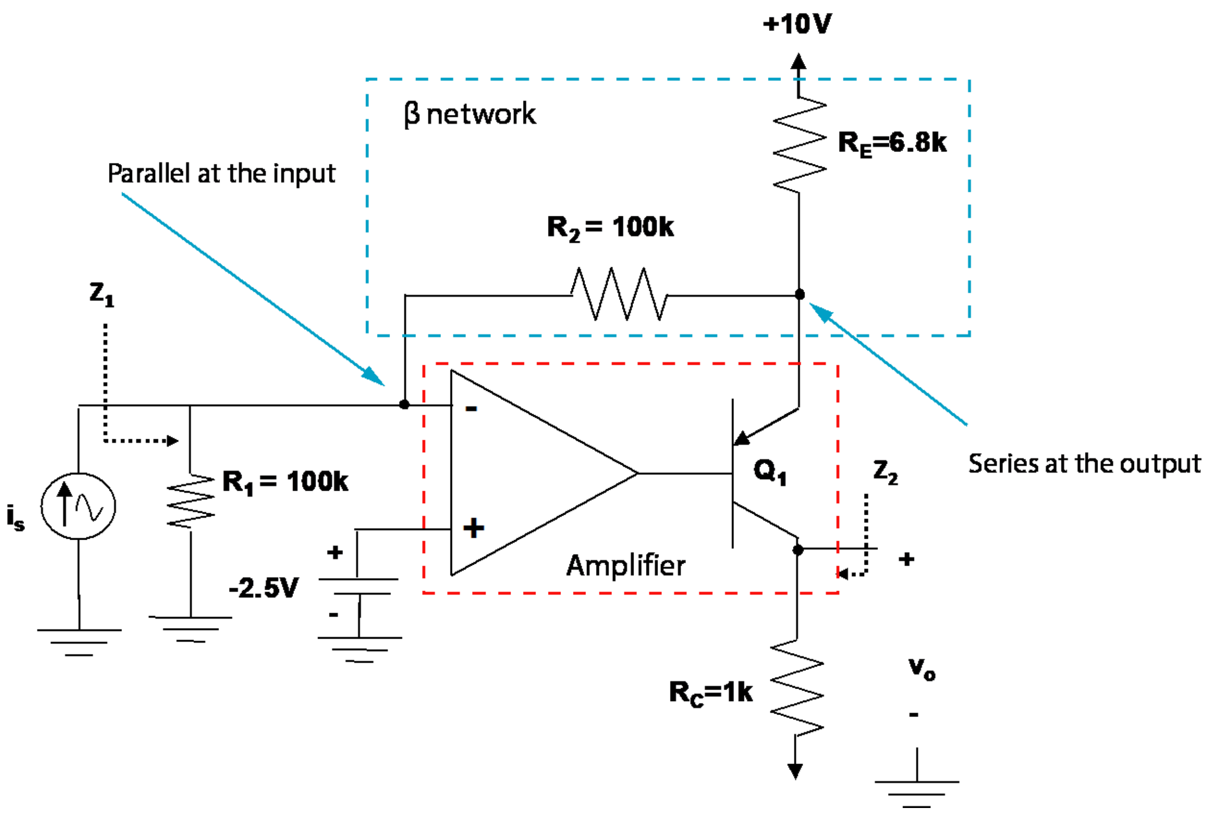
Q_1:

$r_o \rightarrow \infty$ $\quad r_\pi \cong 3.3k\Omega$ $\quad g_m \cong 0.09\Omega^{-1}$

Se pide:

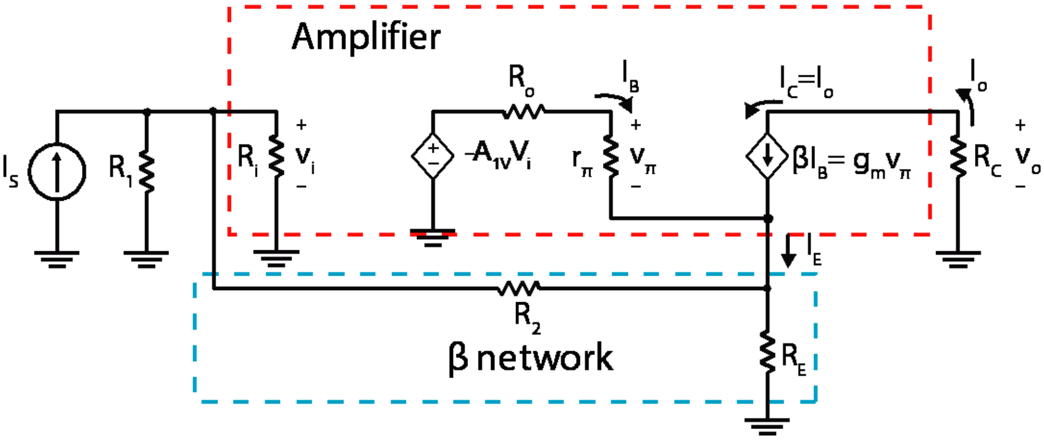
1. Demonstrate that there is negative feedback, indicating its topology and the transfer function that stabilizes.
2. Obtain the equivalent circuits of A’ and β.
3. Calculate A’, β and V_o/i_s
4. Calculate Z_1 and Z_2

The feedback topology is **parallel-series** (see the following figure)**,** the feedback amplifier mixes current at the input and sense current at the output (**Transcurrent amplifier).** This amplifier is a transcurrent amplifier, i.e, a current to current amplifier.



Note that in this case the amplifier cannot be represented by a quadrupole because in the output port, the current that exits the port, $I_o= I_c$, is not equal to the current that enters the port, I_E, as a consequence of having the PNP transistor. Special care has to be taken when we calculate the output impedance in these cases. If instead of a BTJ we have a MOSFET at the output of the amplifier, then we would could represent the amplifier by a quadrupole.

The equivalent small-signal circuit is:

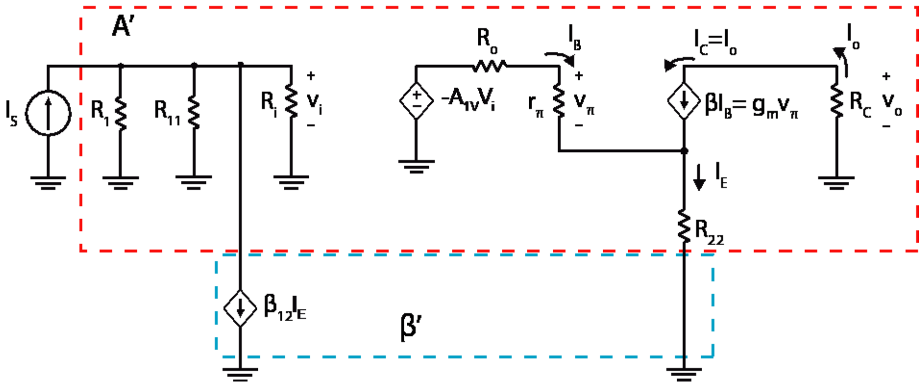


Is the feedback negative?

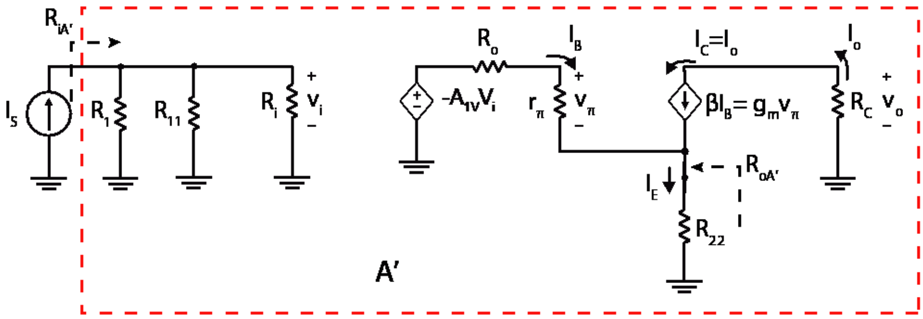
From the output,

if $|\Delta i_o|\uparrow \Rightarrow |\Delta i_c|\uparrow \Rightarrow |i_e| = |i_{R_2}+i_{R_e}|\uparrow \Rightarrow |i_{R_2}|\uparrow \Rightarrow (i_s = i_{R_1}+i_{R_2}-i_{R_i} = const., and\ i_{R_i}\to 0)|i_{R_1}|\downarrow \Rightarrow |V_i|\downarrow \Rightarrow |V_o| = |A_v V|_i \downarrow \Rightarrow$
$\Rightarrow |I_B|\downarrow \Rightarrow |I_C|\downarrow$, so, indeed, **there is negative feedback**.

We can redraw an equivalent circuit considering the load effects of the β network obtaining the new A' and β' networks.



Remember that the small signal model of the PNP transistor is exactly the same as the NPN transistor. Negative currents will flow in opposite directions as the marked ones. The A' network is obtained by augmenting the basic amplifier at the input with the source resistance and R_{11}, and at the output with the load resistance and R_{22}.



Since it is a transcurrent amplifier we have to calculate the current to current gain of the A' network:

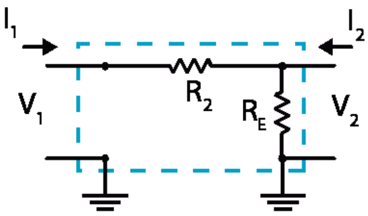
$$A_I' = \frac{i_o'}{i_s'} = \frac{v_i}{i_S'}\frac{i_B}{v_i}\frac{i_o'}{i_B} = \begin{cases} v_i = (R_1 \| R_{11} \| R_i)\, i_s' \\ i_B = -\dfrac{A_{1v} v_i}{R_o + r_\pi + (\beta_{Q_1}+1)R_{22}} \\ i_o' = \beta_{Q_1} i_B \end{cases} \Rightarrow$$

$$A_I' = \frac{i_o'}{i_s'} = \frac{v_i}{i_S'}\frac{i_B}{v_i}\frac{i_o'}{i_B} = -R_1 \| R_{11} \| R_i \frac{A_{1v}\beta_{Q_1}}{R_o + r_\pi + (\beta_{Q_1}+1)R_{22}}$$

where β_{Q_1} is the β parameter of the BJT Q1 and it is equal to:

$$\beta_{Q_1} = g_m r_\pi = 297$$

Now, we can calculate the parameters of the β network quadrupole.



The feedback network quadrupole can be represented by

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

that corresponds to g parameters. Remember that the independent variables are the common magnitudes at the input and at the output, in this case, they are V_1 and I_2. The loading effect of the feedback network at the input is obtained by open circuiting the port 2 because it is connected in series, and looking into port 1.

$$R_{11} = \frac{1}{\beta_{11}} = \left.\frac{v_1}{i_1}\right|_{i_2=0} = R_2 + R_E = 106.8k\Omega$$

The loading effect of the feedback network at the output side is obtained by short-circuiting the port 1, because it is connected in parallel, and looking into the port 2:

$$R_{22} = \beta_{22} = \left.\frac{v_2}{i_2}\right|_{v_1=0} = R_E \,||\, R_2 = 6.4k\Omega$$

Substituting the gain of the A' network will be:

$$A_I' = \frac{i_o'}{i_s'} = -R_1 \,||\, R_{11} \,||\, R_i \frac{A_{1v}\beta_{Q_1}}{R_o + r_\pi + (\beta_{Q_1} + 1)R_{22}} \approx -1.57 \cdot 10^6 \left[\frac{A}{A}\right]$$

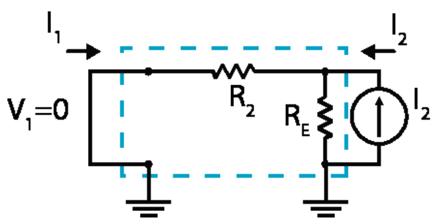
The input resistance of the A' network will be by inspection:

$$R_{iA'} = -R_1 \,||\, R_{11} \,||\, R_i \cong 50k\Omega$$

The output resistance of the A' network is not so straightforward since we have to realize that the **sensed current is I_e (current in the emitter) and not the actual output of the amplifier I_o (current in the collector)**. The output impedance of the A' is obtained by breaking the circuit between the emitter and R_{22} (see A' network figure at the emitter). The resistance found between these two points is

$$R_{oA'} = R_{22} + \frac{r_\pi + R_o}{\beta_{Q_1} + 1} \cong 6.4k\Omega.$$

The ***β* network gain** is the term β_{12} and can be determined taking into account the figure:



$$\beta_I = \beta_{12} = \left.\frac{i_1}{i_2}\right|_{v_1=0} = -\frac{R_2 \,||\, R_E}{R_2} = -\frac{R_E}{R_2 + R_E} \approx -0.064 \left[\frac{A}{A}\right]$$

Note that $A_I'\beta_I \approx 100k >> 1$ is positive as should be if we have negative feedback.

The **transcurrent gain** of the feedback amplifier will be:

$$A_{If} = \frac{i_o}{i_s} = \frac{A_I'}{1 + A_I'\beta_I} \cong \frac{1}{\beta_I} \cong -15.7$$

It is asked for the transimpedance gain. We can calculate it if we take into account that the output voltage is given by

$$v_o = -i_o \cdot R_C \Rightarrow \frac{v_o}{i_s} = -\frac{i_o}{i_s} \cdot R_C = -A_{If} \cdot R_C \cong 15.7\,k\Omega$$

The input resistance R_{if} determined by the feedback equations is the resistance seen **by the external source,**

$$R_{if} = \frac{R_{iA'}}{1 + A_I'\beta_I} \cong 0.5\ \Omega$$

And is just the asked input impedance Z_1:

$$Z_1 = R_{if} \cong 0.5\ \Omega$$

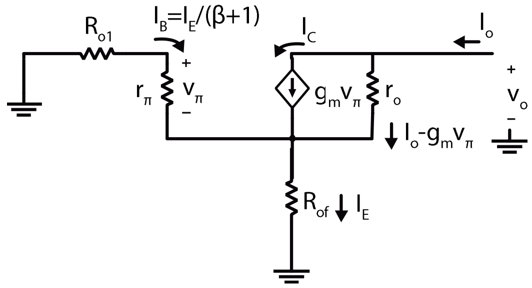
The resistance R_{of} given by the feedback equations is the output resistance of the feedback amplifier seen **from the emitter**:

$$R_{of} = R_{oA'}\left(1 + A_I'\beta_I\right) \cong 640M\ \Omega$$

However, it is asked for the resistance, Z_2, that is the parallel between the output impedance of the amplifier with feedback and R_C:

$$Z_2 = Z_0 \parallel R_C$$

Z_o is the output impedance of the amplifier, the one saw from the collector. It can be calculated from the following equivalent circuit where we have introduced the equivalent resistance seen from the emitter with feedback and in general, we have to consider the r_o of the BJT to account for the Early effect.



Taking into account that the base current is $I_B = \frac{V_\pi}{r_\pi}$ we can write the output voltage as:

$$v_o = r_o\left(i_o - g_m V_\pi\right) + \left(i_o + \frac{V_\pi}{r_\pi}\right) R_{of} \tag{1.1}$$

On the other hand, analyzing the base-emitter loop:

$$V_\pi + \frac{V_\pi}{r_\pi} R_{o1} + \left(i_o + \frac{V_\pi}{r_\pi}\right) R_{of} = 0 \Rightarrow -i_o R_{of} = V_\pi \left(1 + \frac{1}{r_\pi} + \frac{R_{o1}}{r_\pi}\right) \Rightarrow$$

$$V_\pi = -\frac{r_\pi R_{of}}{R_o + r_\pi + R_{of}} i_o \tag{1.2}$$

Substituting (1.2) in (1.1):

$$v_o = i_o r_o \left(1 + g_m \frac{r_\pi R_{of}}{R_{o1} + r_\pi + R_{of}}\right) + i_o R_{of}\left(1 - \frac{R_{of}}{R_{o1} + r_\pi + R_{of}}\right) \Rightarrow$$

$$Z_o = \frac{v_o}{i_o} = r_o + r_o g_m \frac{r_\pi R_{of}}{R_{o1} + r_\pi + R_{of}} + \frac{R_{of}\left(r_\pi + R_{o1}\right)}{R_{o1} + r_\pi + R_{of}} =$$

$$= r_o + r_o g_m \frac{r_\pi R_{of}}{R_o + r_\pi + R_{of}} \frac{r_\pi + R_{o1}}{r_\pi + R_{o1}} + R_{of} \parallel \left(r_\pi + R_{o1}\right) =$$

$$= r_o + R_{of} \parallel \left(R_{o1} + r_\pi\right) + r_o g_m \left(R_{of} \parallel \left(R_{o1} + r_\pi\right)\right) \frac{r_\pi}{r_\pi + R_{o1}} =$$

Factorizing we finally obtain

$$Z_o = \frac{v_o}{i_o} = r_o + \left[R_{of} \parallel \left(R_{o1} + r_\pi\right)\right]\left(1 + r_o g_m \frac{r_\pi}{r_\pi + R_{o1}}\right)$$

Note that, in this case, the actual output resistance of the amplifier is not multiplied by the loop gain factor. The loop gain factor appears in R_{of}, that is the resistance seen from the emitter because is the emitter current what is sensed by the feedback network.

In our case, as $r_o \to \infty$, we have that the output impedance is

$$Z_o = r_o + \left[R_{of} \parallel \left(R_{o1} + r_\pi\right)\right]\left(1 + r_o g_m \frac{r_\pi}{r_\pi + R_{o1}}\right) \approx r_o \to \infty$$

As expected for a common emitter BJT amplifier when $r_o \to \infty$. Finally, the asked impedance is:

$$Z_2 = R_C \parallel \infty = R_C = 1k\Omega$$