Solutions to the exercises of stability study using Bode plots

Exercise 1

The transfer function of a voltage amplifier with differential input, in open loop is:

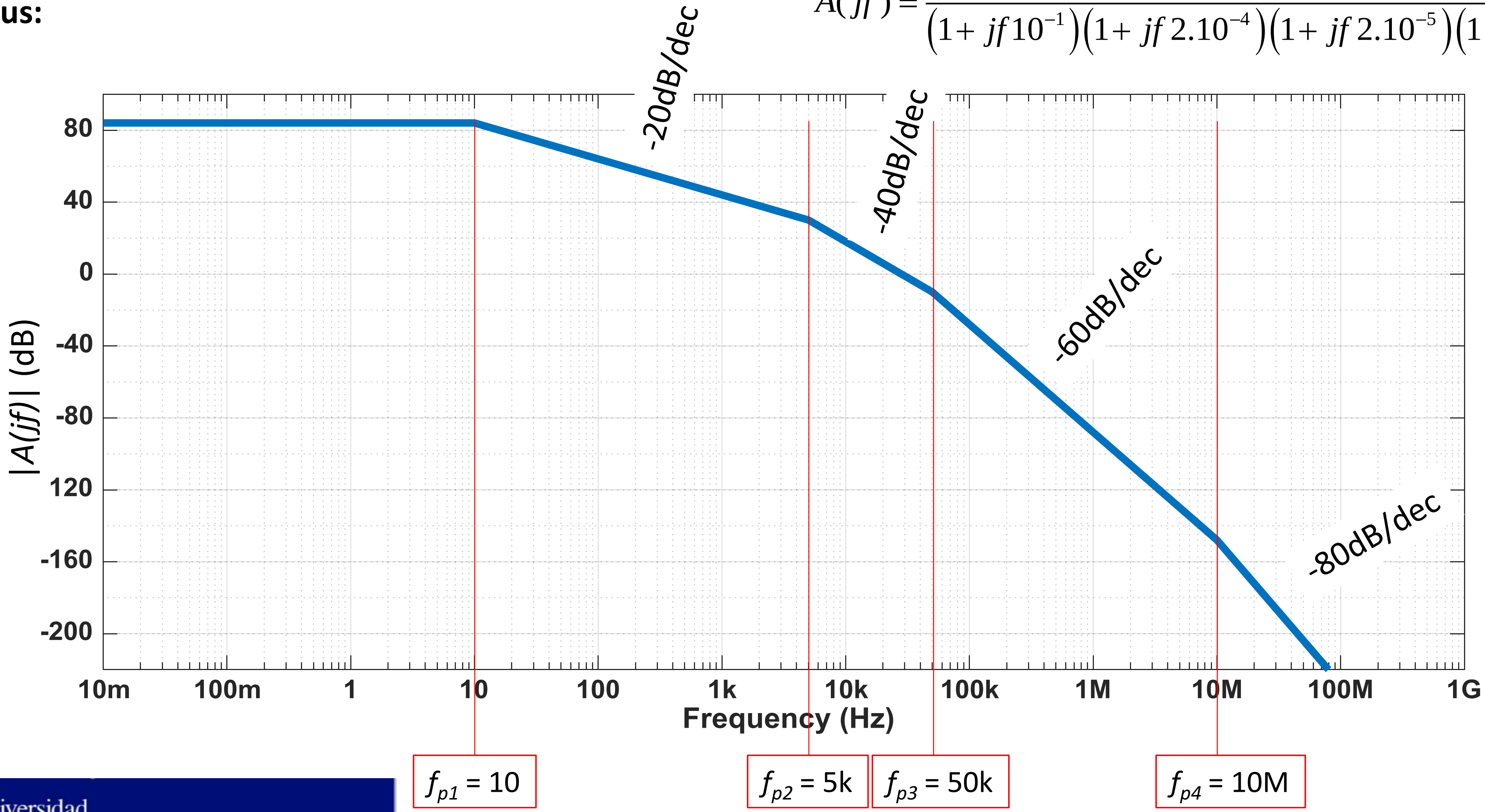
$$A(jf) = \frac{16.10^{3}}{\left(1+jf10^{-1}\right)\left(1+jf\,2.10^{-4}\right)\left(1+jf\,2.10^{-5}\right)\left(1+jf10^{-7}\right)}$$ with f expressed in Hz

1. Represent the asymptotic Bode diagram (modulus and phase) of the amplifier.
2. Determine if this amplifier is stable when it is used as a voltage follower. Give reasons for your answer.
3. If the amplifier is feedback with a non-inverting configuration, what is the minimum value of the closed-loop gain for which the amplifier is stable with a phase margin of 45°?
4. Represent the schematic of the feedback amplifier and calculate the values of the resistors of the β network for the minimum gain value calculated in the previous section.

1. Represent the asymptotic Bode diagram (modulus and phase) of the amplifier.

Modulus:

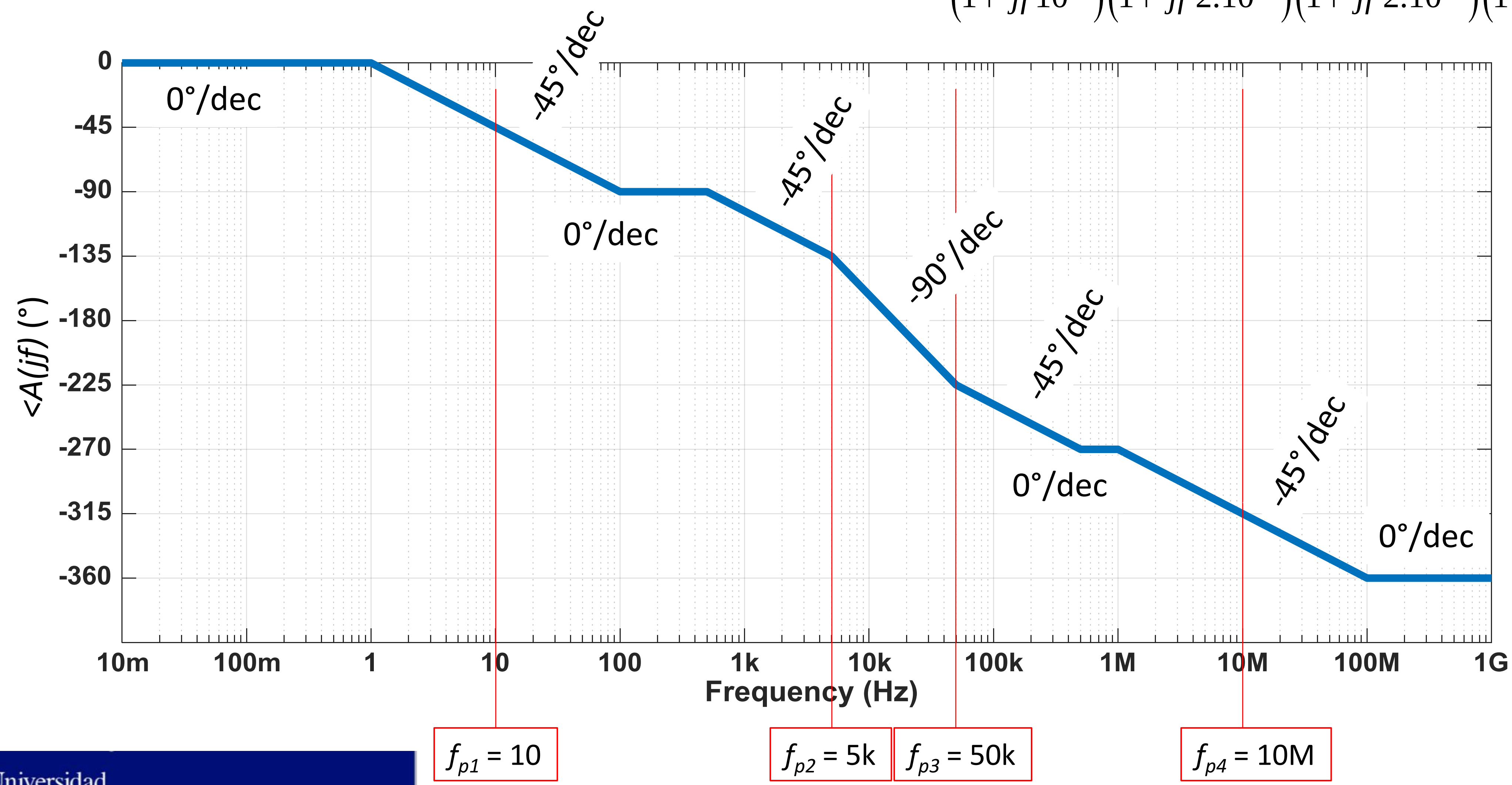
$$A(jf) = \frac{16.10^3}{\left(1+jf10^{-1}\right)\left(1+jf\,2.10^{-4}\right)\left(1+jf\,2.10^{-5}\right)\left(1+jf10^{-7}\right)}$$



1. Represent the asymptotic Bode diagram (modulus and phase) of the amplifier.

Phase:

$$A(jf) = \frac{16.10^3}{\left(1+jf10^{-1}\right)\left(1+jf\,2.10^{-4}\right)\left(1+jf\,2.10^{-5}\right)\left(1+jf10^{-7}\right)}$$

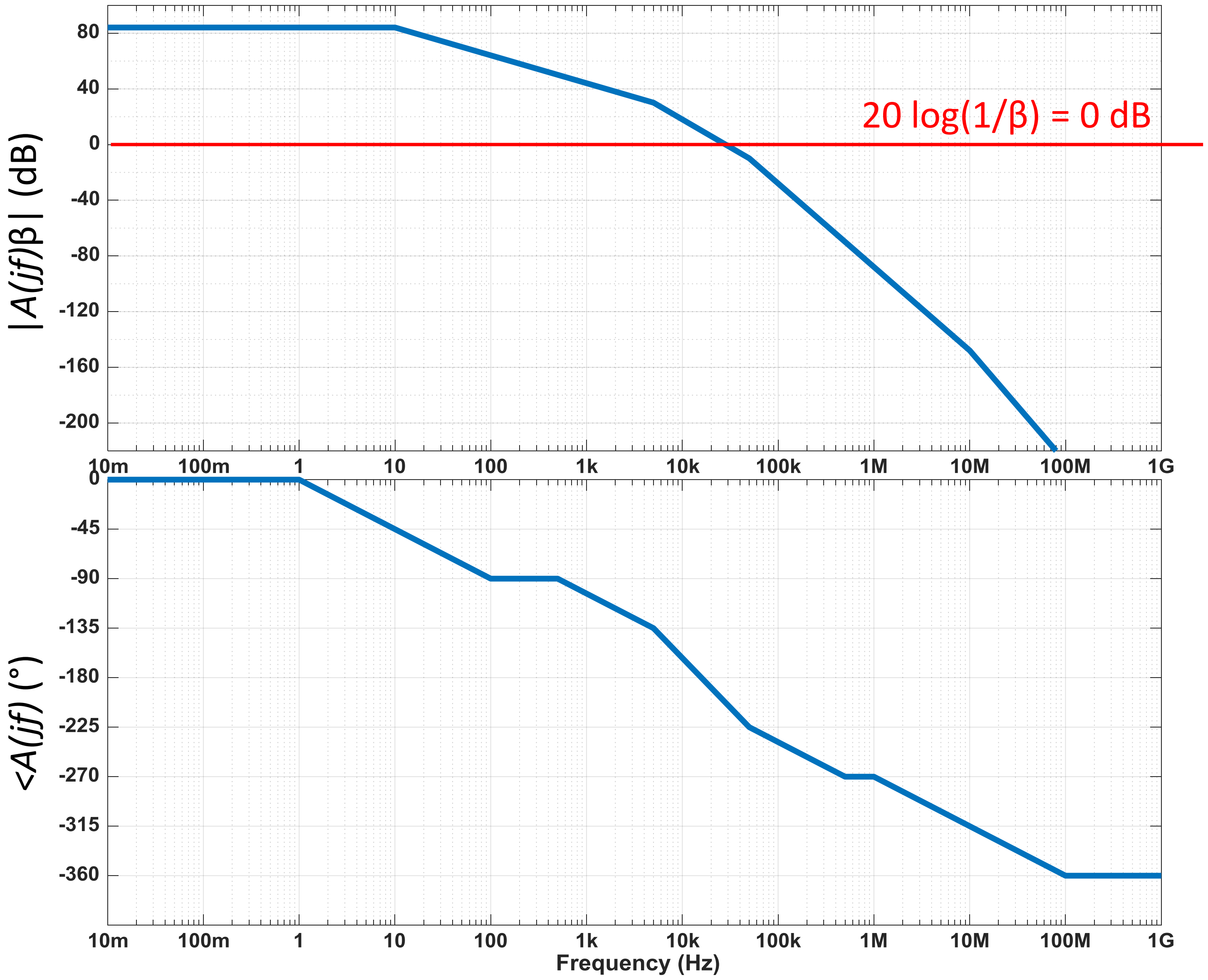


2. Determine if this amplifier is stable when it is used as a voltage follower. Give reasons for your answer.

The gain of the feedback amplifier as a voltage follower is 1, so

$$G_f = \frac{A_v}{1 + A_v\beta} = 1 \Rightarrow \beta = \frac{A_v - 1}{A_v} = \frac{16000 - 1}{16000} \cong 1$$

Voltage follower => β = 1



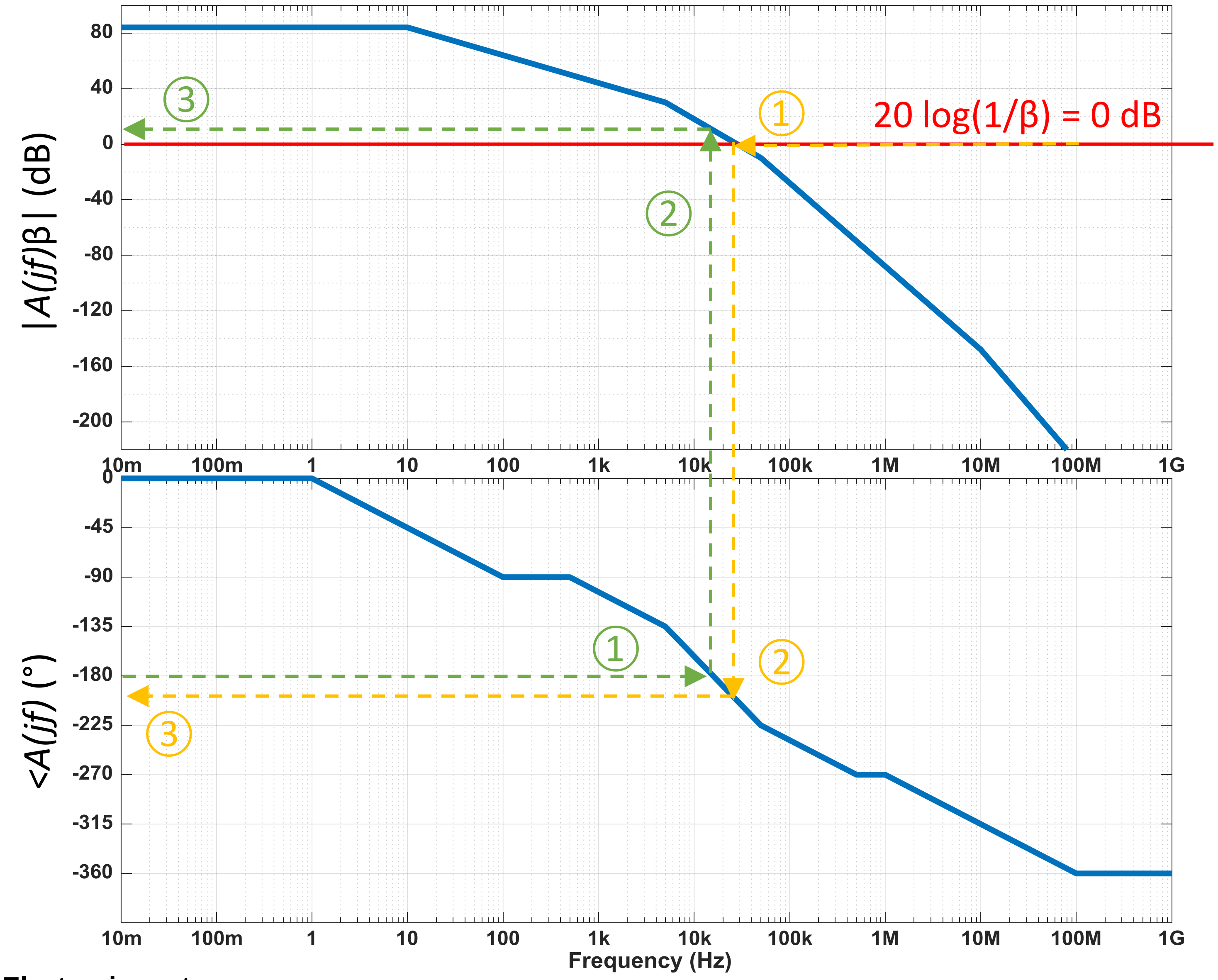
2. Determine if this amplifier is stable when it is used as a voltage follower. Give reasons for your answer.

Voltage follower => β = 1

$|A.\ \beta_{(f\text{-}180°)}| > 0\text{dB}$ => unstable

or, equivalently,

$|\angle(A.\ \beta)_{(f\text{-}0dB)}| > 180°$ => **unstable**



3. If the amplifier is feedback with a non-inverting configuration, what is the minimum value of the closed-loop gain for which the amplifier is stable with a phase margin of 45°?

If the gain loop is Aβ >> 1, then the total gain of the feedback amplifier will be:

$$G_f = \frac{1}{\beta}$$

For a phase margin equal to 45° we have:

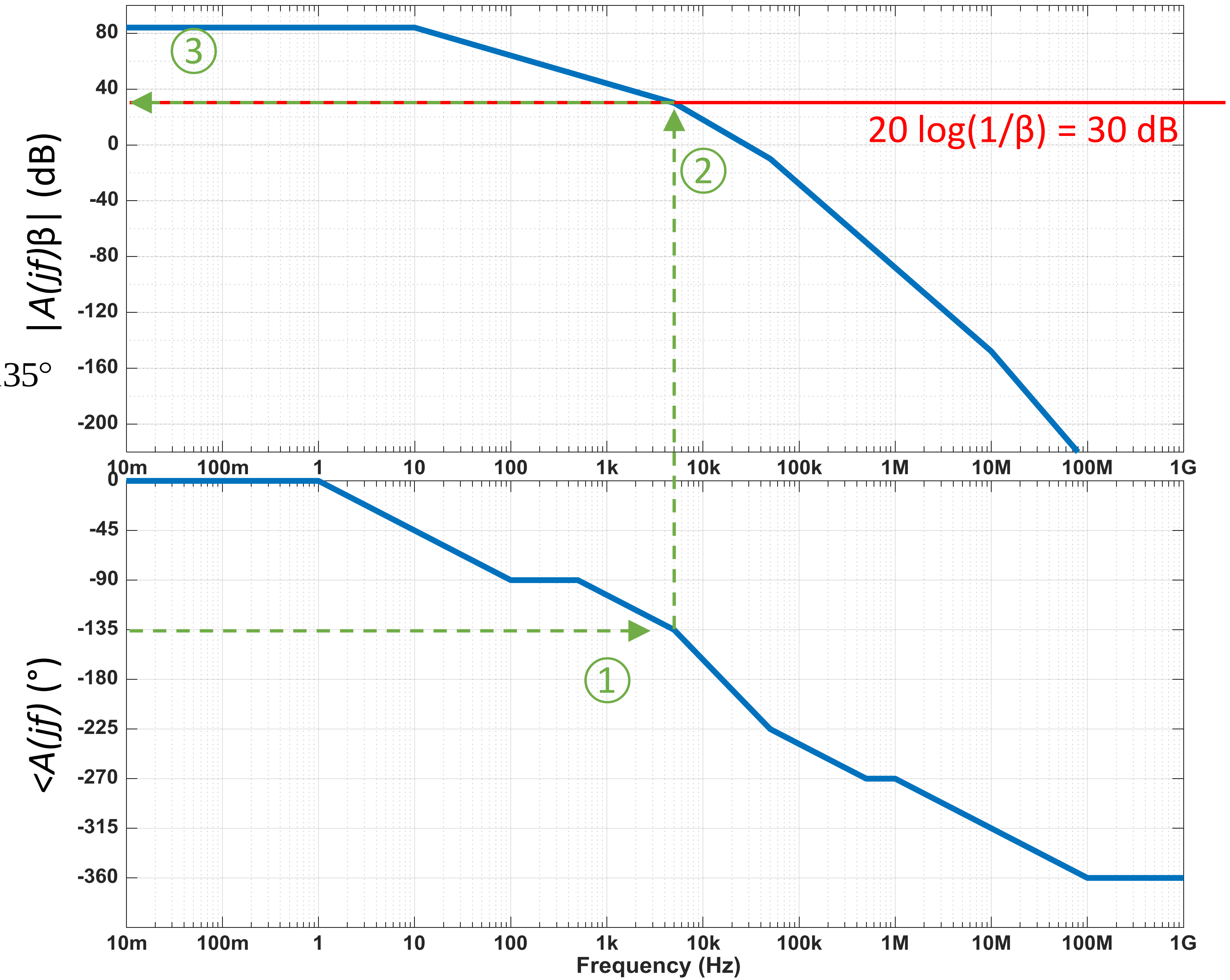
$$PM = 180^\circ + \angle A.\beta\Big|_{f_{0dB}} = 45^\circ \Rightarrow \angle A.\beta\Big|_{f_{0dB}} = -135^\circ$$

From the bode diagram we found that,

$$20\log\frac{1}{\beta} = 30dB \Rightarrow \frac{1}{\beta} \cong 32$$

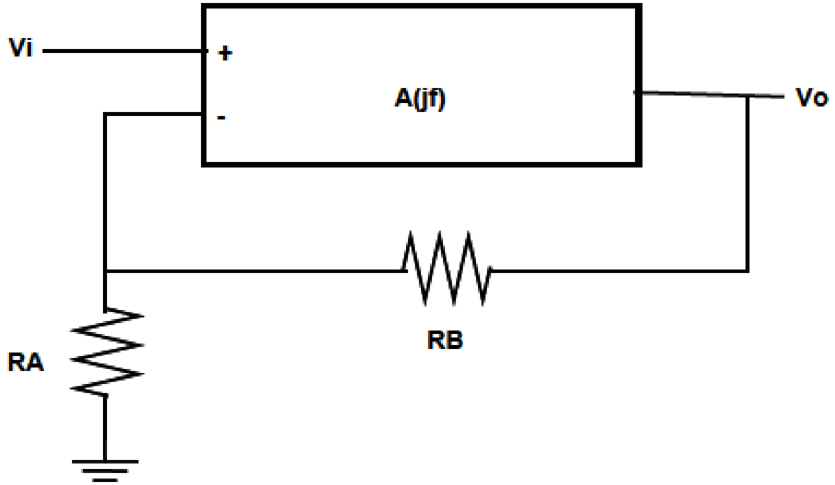
and it is satisfied that Aβ >> 1, as it was supposed at the beginning.

The minimum value of the closed-loop gain for which the amplifier is stable with a phase margin of 45° is 32.



4. Represent the schematic of the feedback amplifier and calculate the values of the resistors of the β network for the minimum gain value calculated in the previous section.

The schematic of the feedback amplifier in an non-inverting configuration is:



As the gain of the feedback amplifier loop is Aβ >>1, the gain of the feedback amplifier will be approximately

$$G_f = \frac{1}{\beta} = 1 + \frac{R_B}{R_A} = 32 \Rightarrow R_B = 31 R_A$$

For instance, using E6 series resistor values (20% tolerance): R_A = 1 kΩ, R_A = 33 kΩ.

Exercise 2

The transfer function of a voltage amplifier with differential input, in open loop is:

$$A(jf) = \frac{10^3}{\left(1 + j\dfrac{f}{100}\right)\left(1 + j\dfrac{f}{10^3}\right)\left(1 + j\dfrac{f}{2.10^4}\right)}$$

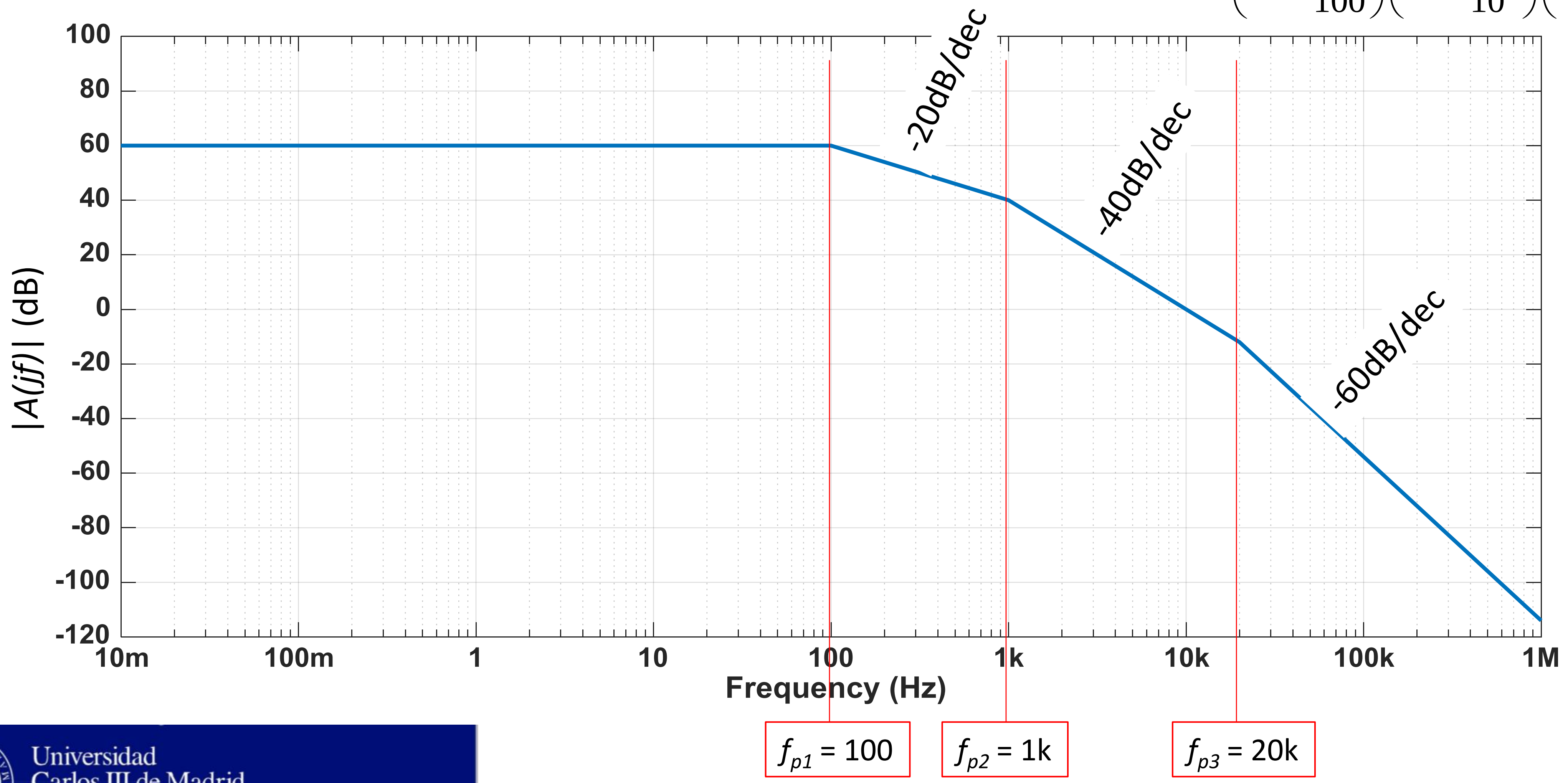
with f expressed in Hz

1. Draw the asymptotic Bode diagram, modulus and phase, of the amplifier, for frequencies between 10 mHz and 1 MHz. Clearly indicate the most significant points and slopes of each section.
2. Determine, from the Bode diagram above, if the amplifier is stable if it is feedback as a voltage follower. Justify your answer.
3. At what frequency would the first pole of the amplifier have to be moved so that the feedback amplifier as a voltage follower is stable with a phase margin of 45°? Justify your answer.

1. **Draw the asymptotic Bode diagram, modulus and phase, of the amplifier, for frequencies between 10 mHz and 1 MHz. Clearly indicate the most significant points and slopes of each section.**

$$A(jf) = \frac{10^3}{\left(1+j\frac{f}{100}\right)\left(1+j\frac{f}{10^3}\right)\left(1+j\frac{f}{2.10^4}\right)}$$

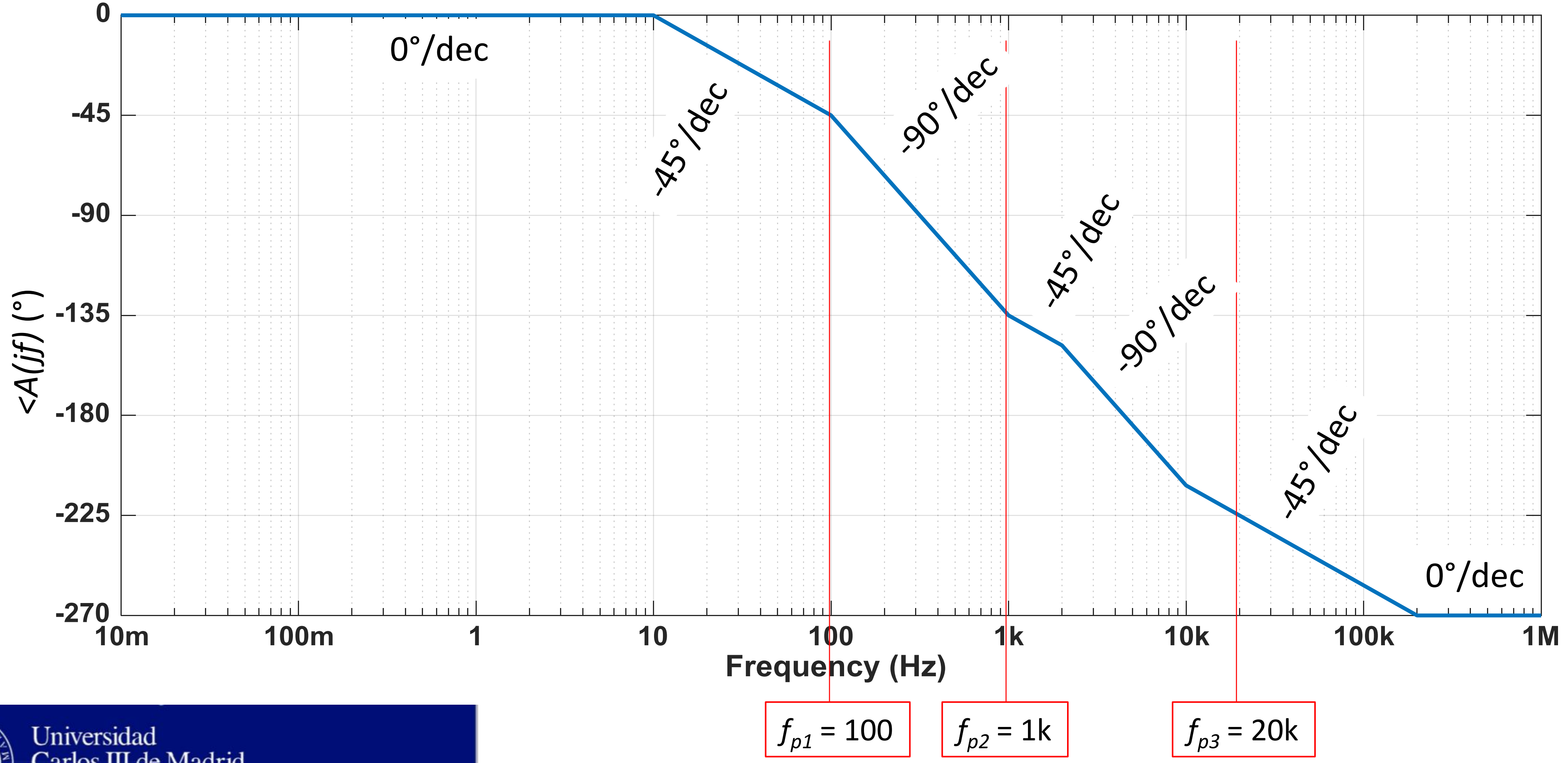
Modulus:



1. **Draw the asymptotic Bode diagram, modulus and phase, of the amplifier, for frequencies between 10 mHz and 1 MHz. Clearly indicate the most significant points and slopes of each section.**

Phase:

$$A(jf)=\frac{10^3}{\left(1+j\frac{f}{100}\right)\left(1+j\frac{f}{10^3}\right)\left(1+j\frac{f}{2.10^4}\right)}$$



2. Determine, from the Bode diagram above, if the amplifier is stable if it is feedback as a voltage follower. Justify your answer.

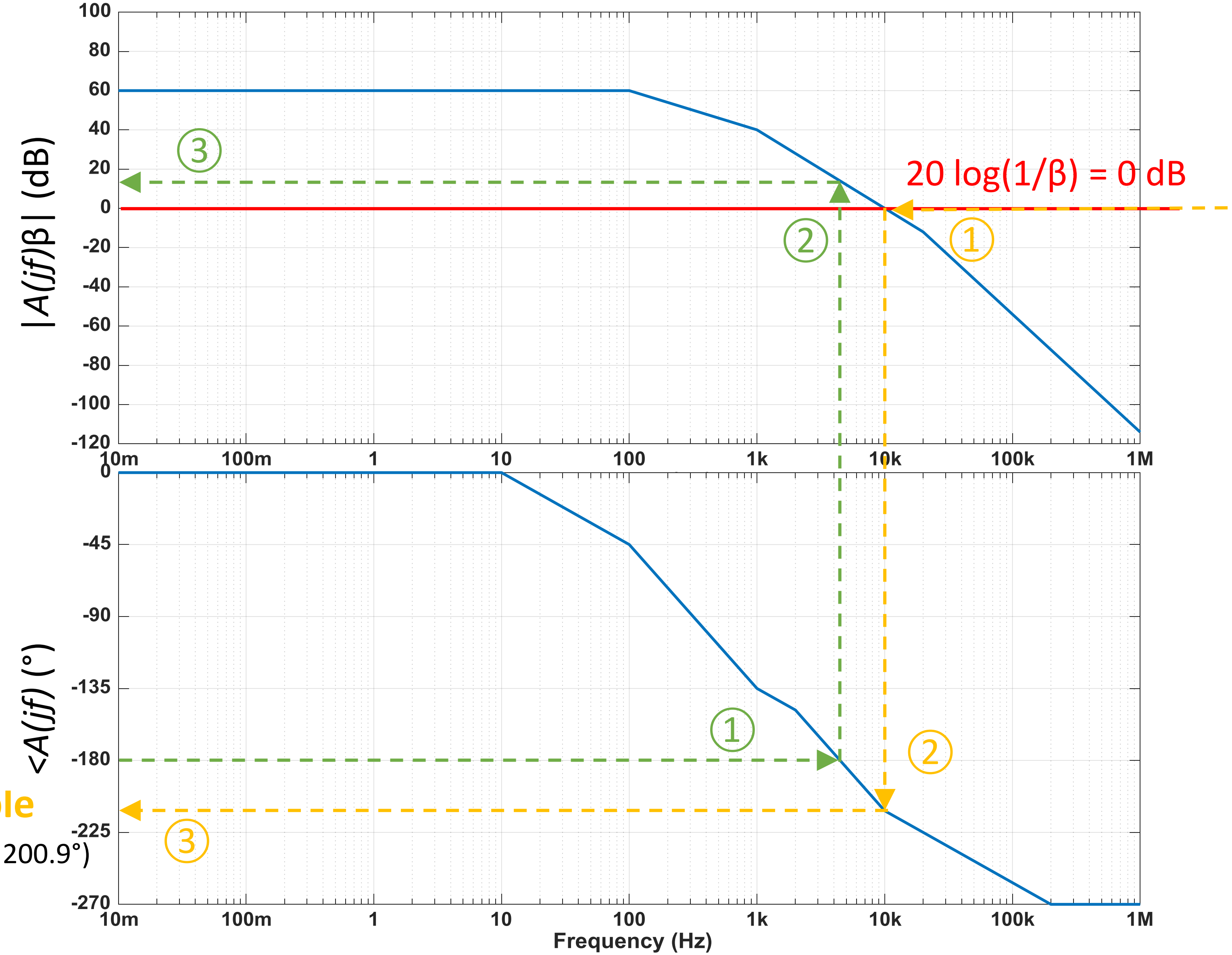
Voltage follower => β = 1

|A. β $_{(f\text{-}180°)}$ | ≈ 15dB > 0dB => unstable

or, equivalently,

<(A. β) $_{(f\text{-}0dB)}$≈-211.5° < -180° => **unstable**

(closer value to the actual phase -180°+5.7°-atan(0,5) ≈ 200.9°)



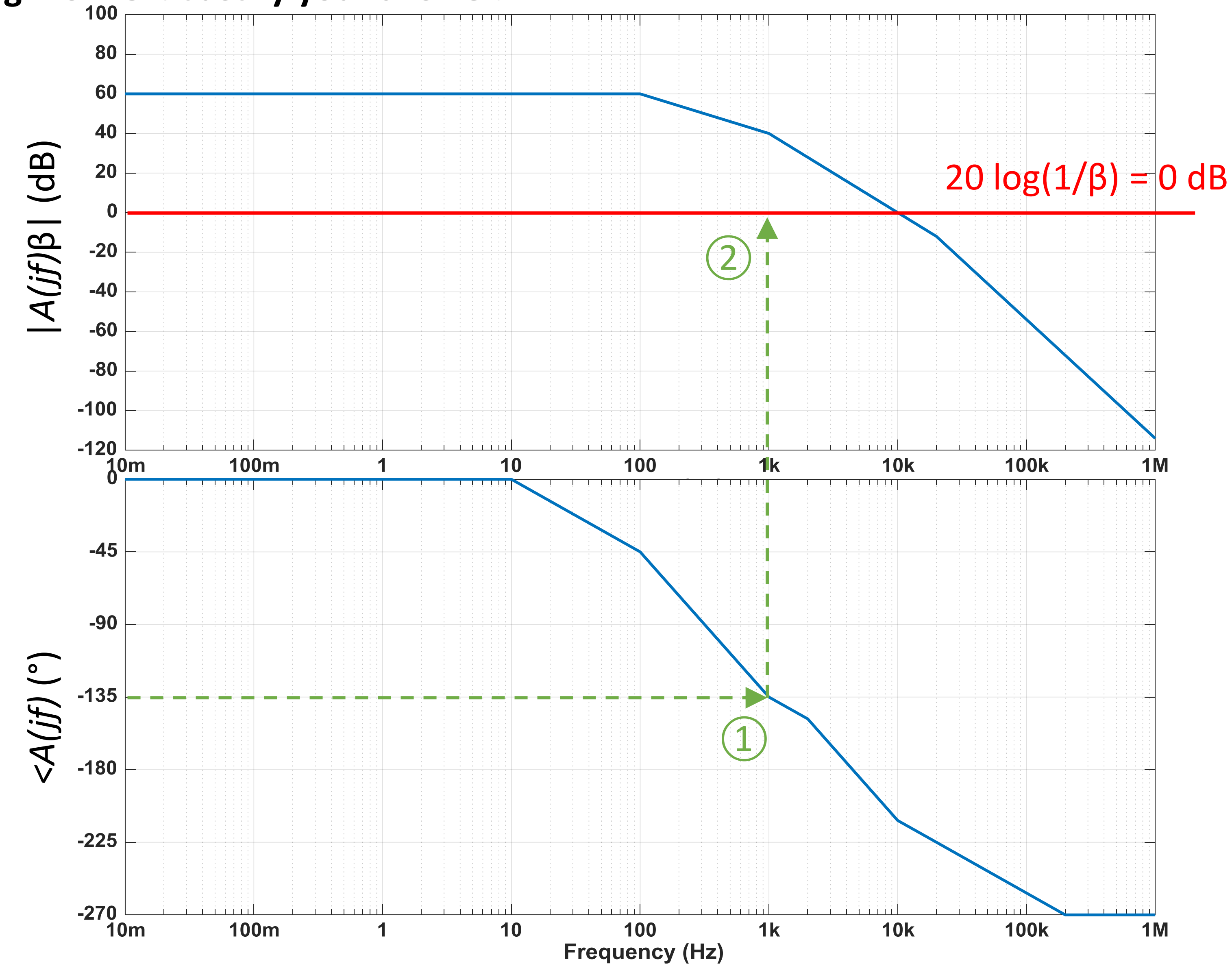
3. At what frequency would the first pole of the amplifier have to be moved so that the feedback amplifier as a voltage follower is stable with a phase margin of 45°? Justify your answer.

Voltage follower => β = 1

For a phase margin equal to 45° we have:

$$PM = 180° + \angle A.\beta\big|_{f_{0dB}} = 45° \Rightarrow \angle A.\beta\big|_{f_{0dB}} = -135°$$

As one pole introduces a phase shift of -90° in two decades, and the distance between the poles is at least one decade the phase at the corner frequency of the second pole will be always -135°.



3. At what frequency would the first pole of the amplifier have to be moved so that the feedback amplifier as a voltage follower is stable with a phase margin of 45°? Justify your answer.

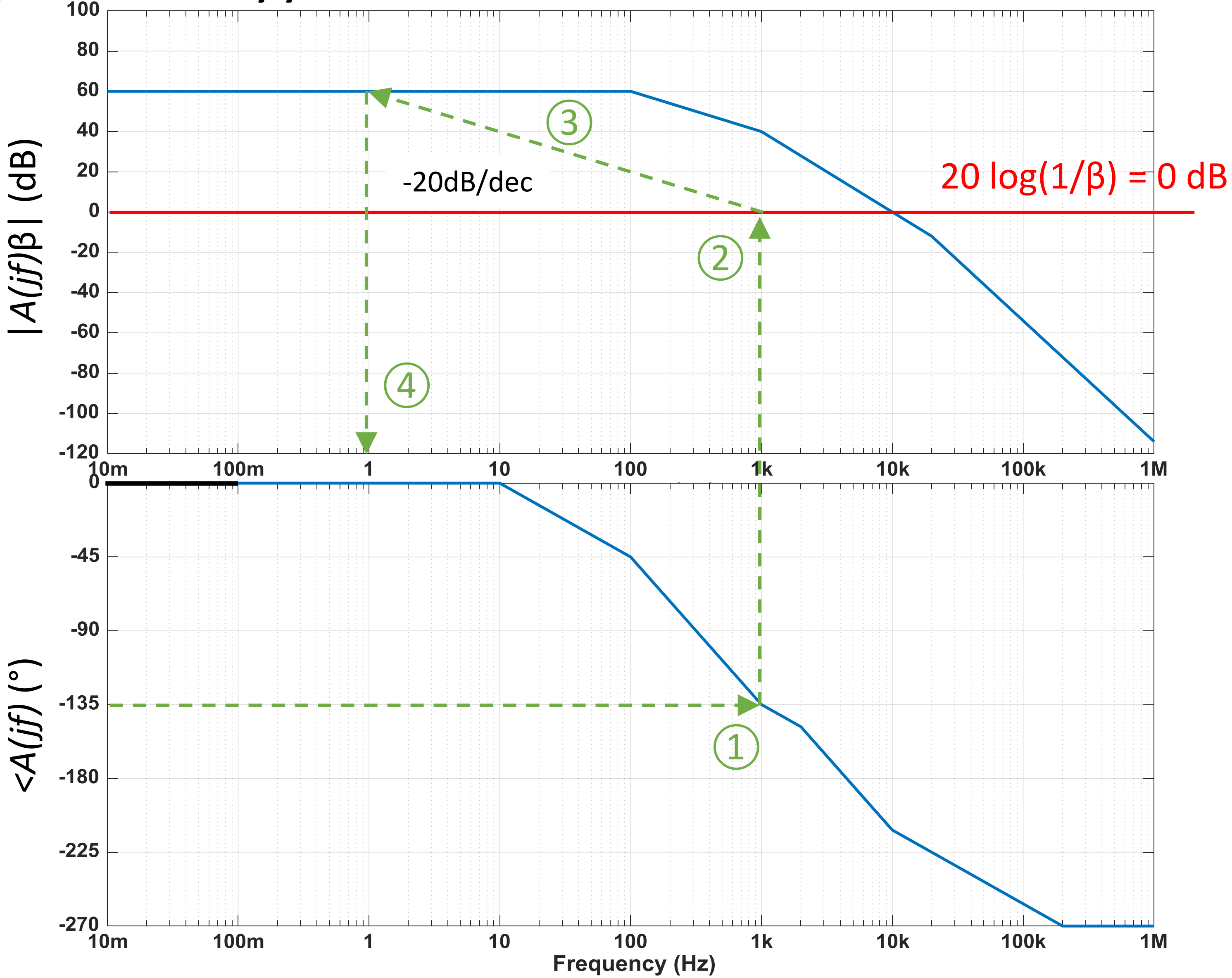
Voltage follower => β = 1

For a phase margin equal to 45° we have:

$$PM = 180° + \angle A.\beta\big|_{f_{0dB}} = 45° \Rightarrow \angle A.\beta\big|_{f_{0dB}} = -135°$$

As one pole introduces a phase shift of -90° in two decades, and the distance between the poles is at least one decade the phase at the corner frequency of the second pole will be always -135°.

From the bode diagram we found that we have to move the dominant pole from 100 Hz to 1 Hz



3. At what frequency would the first pole of the amplifier have to be moved so that the feedback amplifier as a voltage follower is stable with a phase margin of 45°? Justify your answer.

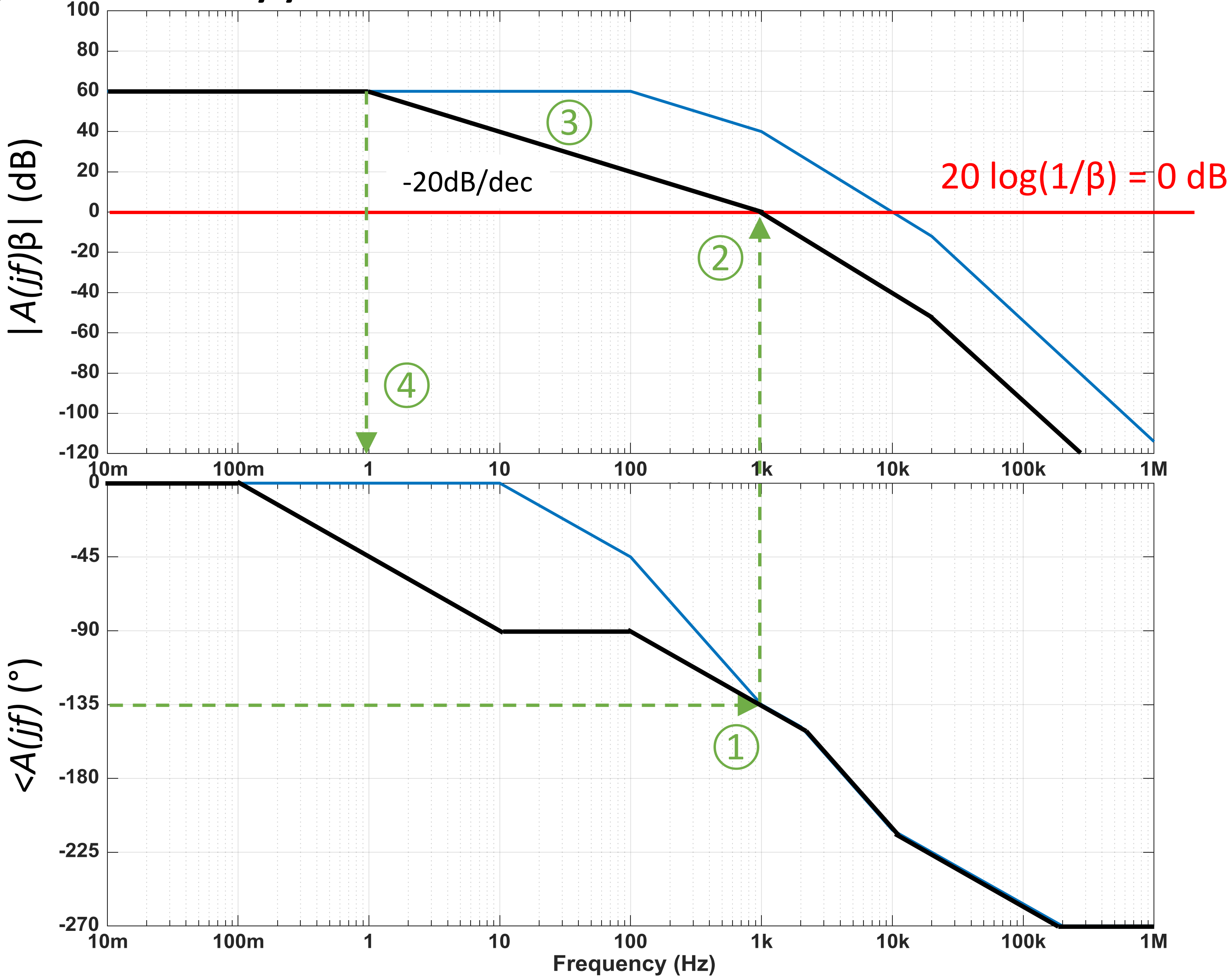
Voltage follower => β = 1

For a phase margin equal to 45° we have:

$$PM = 180° + \angle A\beta\big|_{f_{0dB}} = 45° \Rightarrow \angle A\beta\big|_{f_{0dB}} = -135°$$

As one pole introduces a phase shift of -90° in two decades, and the distance between the poles is at least one decade the phase at the corner frequency of the second pole will be always -135°.

From the bode diagram we found that we have to move the dominant pole from 100 Hz to 1 Hz



Exercise 3

The transfer function of a multistage amplifier with differential input is as follows:

$$A(jf) = \frac{10^3}{\left(1 + j\frac{f}{10^5}\right)\left(1 + j\frac{f}{10^3}\right)\left(1 + j\frac{f}{10^4}\right)}$$

with f expressed in Hz

1. Represent the asymptotic Bode diagram of the frequency response (modulus and phase) of the amplifier. Is this amplifier stable? Give reasons for the answer.

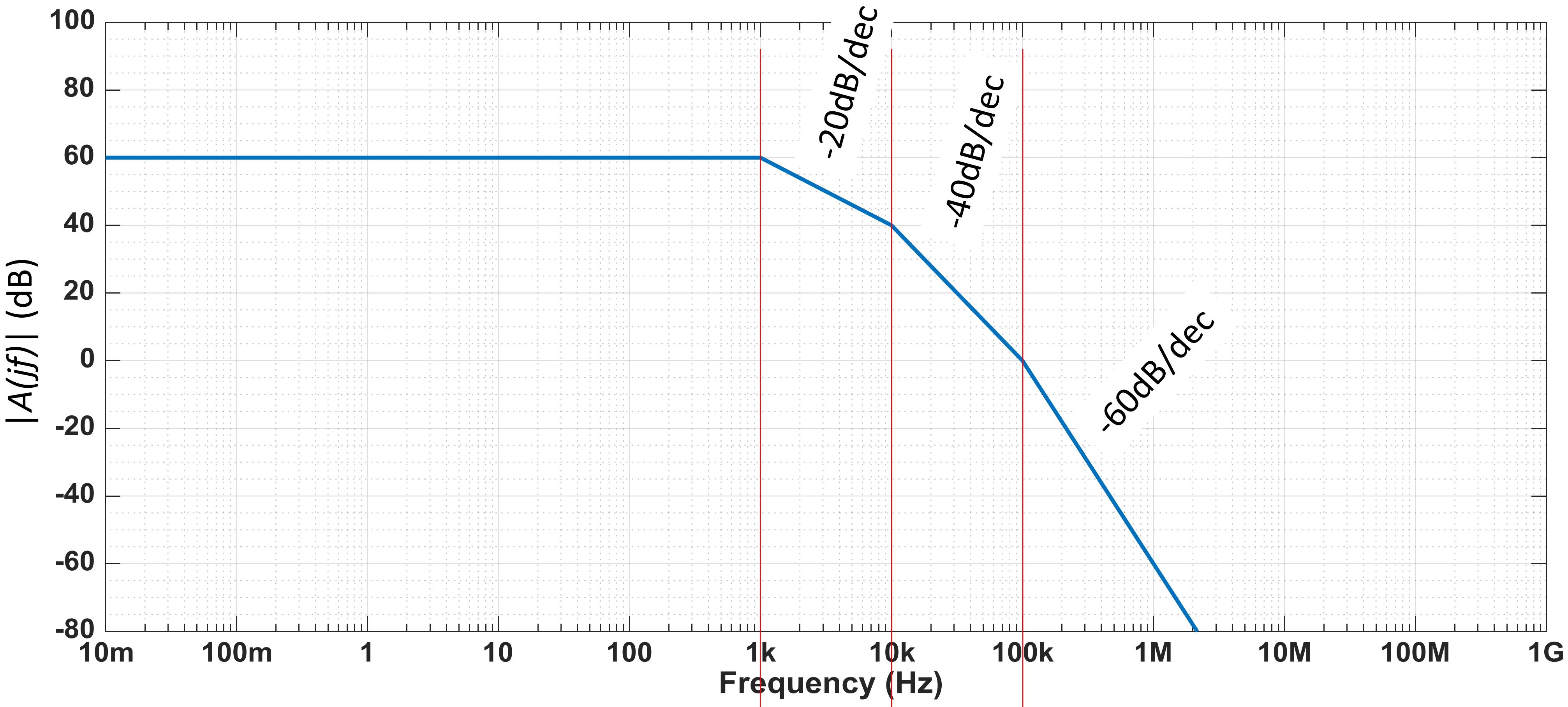
You want to use this amplifier, in a feedback configuration, as a voltage follower. In this case:

2. Value of β to be used in the feedback network. Is this feedback amplifier stable as a follower? Give reasons for the answer.
3. In the event that the feedback amplifier as a voltage follower is unstable, proceed to compensate it by means of a dominant pole so that the compensated system has a phase margin of 45° (PM = 45°).

1. **Represent the asymptotic Bode diagram of the frequency response (modulus and phase) of the amplifier.**

$$A(jf)=\frac{10^3}{\left(1+j\frac{f}{10^5}\right)\left(1+j\frac{f}{10^3}\right)\left(1+j\frac{f}{10^4}\right)}$$

Modulus:

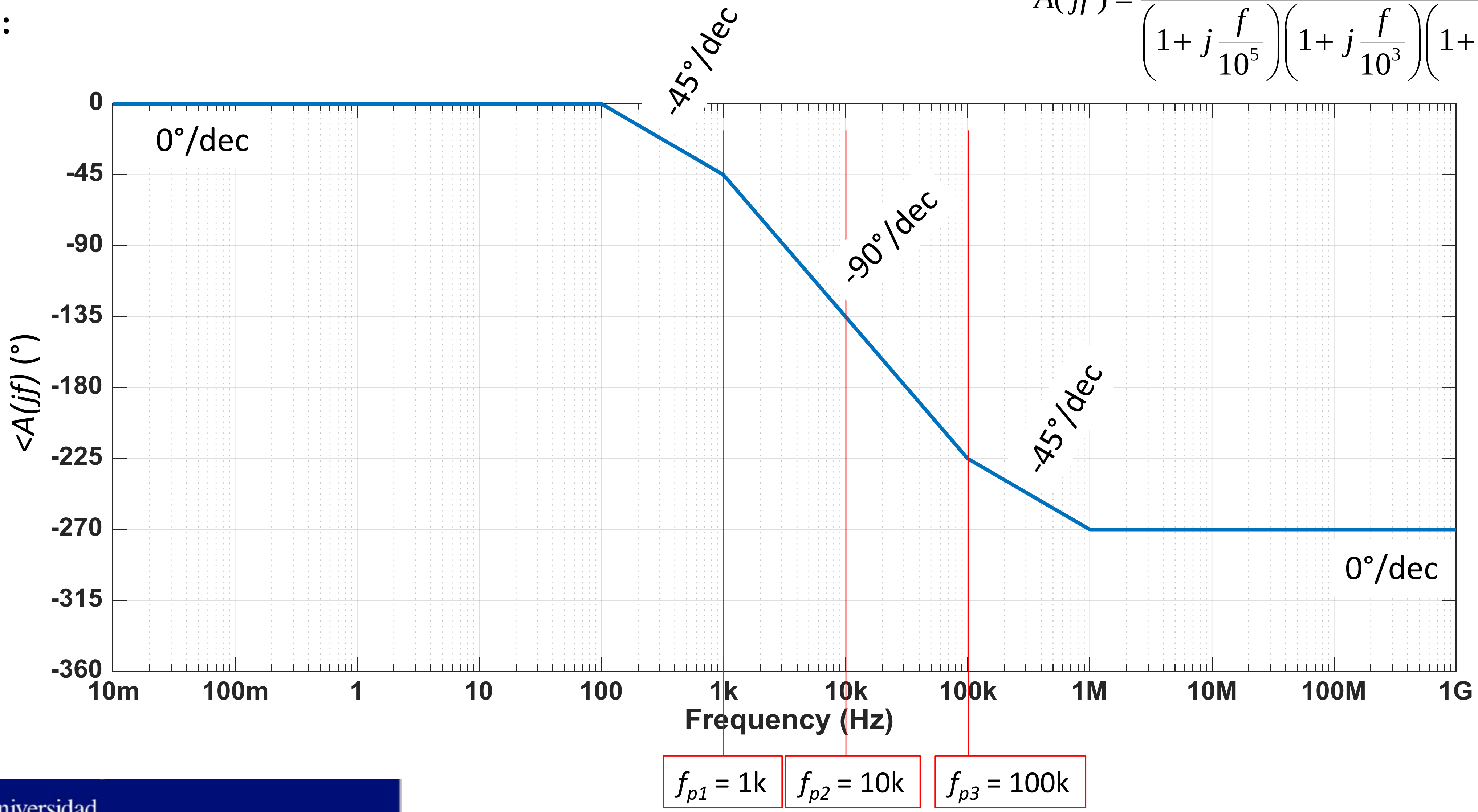


f_{p1} = 1k f_{p2} = 10k f_{p3} = 100k

1. **Represent the asymptotic Bode diagram of the frequency response (modulus and phase) of the amplifier.**

$$A(jf)=\frac{10^3}{\left(1+j\frac{f}{10^5}\right)\left(1+j\frac{f}{10^3}\right)\left(1+j\frac{f}{10^4}\right)}$$

Phase:



1.b Is this amplifier stable? . Give reasons for your answer.

$$A(jf) = \frac{10^3}{\left(1 + j\frac{f}{10^5}\right)\left(1 + j\frac{f}{10^3}\right)\left(1 + j\frac{f}{10^4}\right)}$$

Yes, it is. The amplifier is stable since all his poles are in the left half of the s-plane.

Remember: *A causal LTI system with a rational transfer function H*(*s*) *is stable if and only if all poles of H*(*s*) *are in the left half of the s-plane, i.e., the real parts of all poles are negative.*

You want to use this amplifier, in a feedback configuration, as a voltage follower. In this case:

2. **Value of β to be used in the feedback network. Is this feedback amplifier stable as a follower?**

The gain of the feedback amplifier as a voltage follower is 1, so

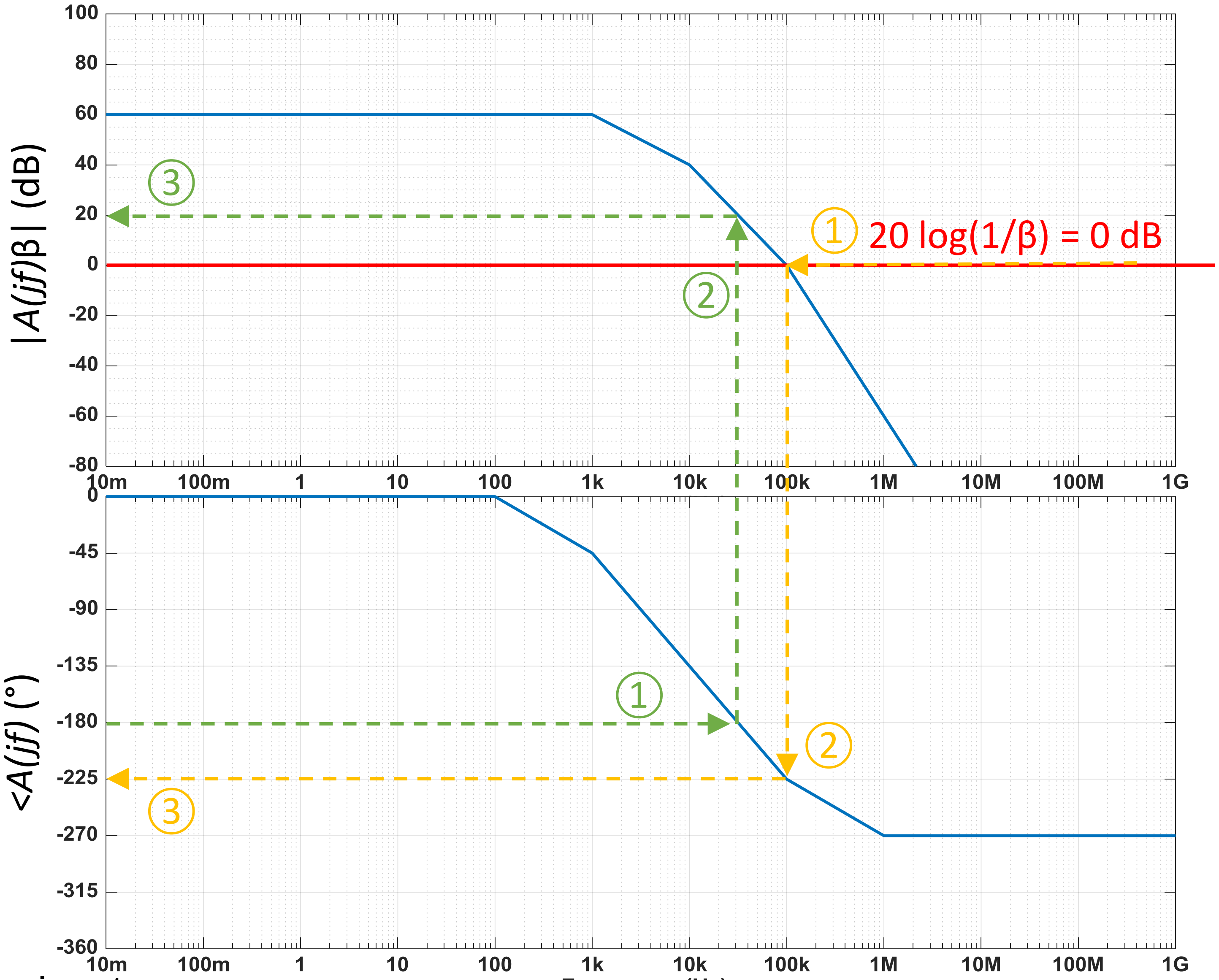
$$G_f = \frac{A_v}{1+A_v\beta} = 1 \Rightarrow \beta = \frac{A_v - 1}{A_v} = \frac{1000-1}{1000} \cong 1$$

Voltage follower => β = 1

|A. β $_{(f\text{-}180°)}$ | > 0dB => unstable

or, equivalently,

| <(A. β) $_{(f\text{-}0dB)}$| > 180° => **unstable**



You want to use this amplifier, in a feedback configuration, as a voltage follower. In this case:

2. Value of β to be used in the feedback network. Is this feedback amplifier stable as a follower?

The gain of the feedback amplifier as a voltage follower is 1, so

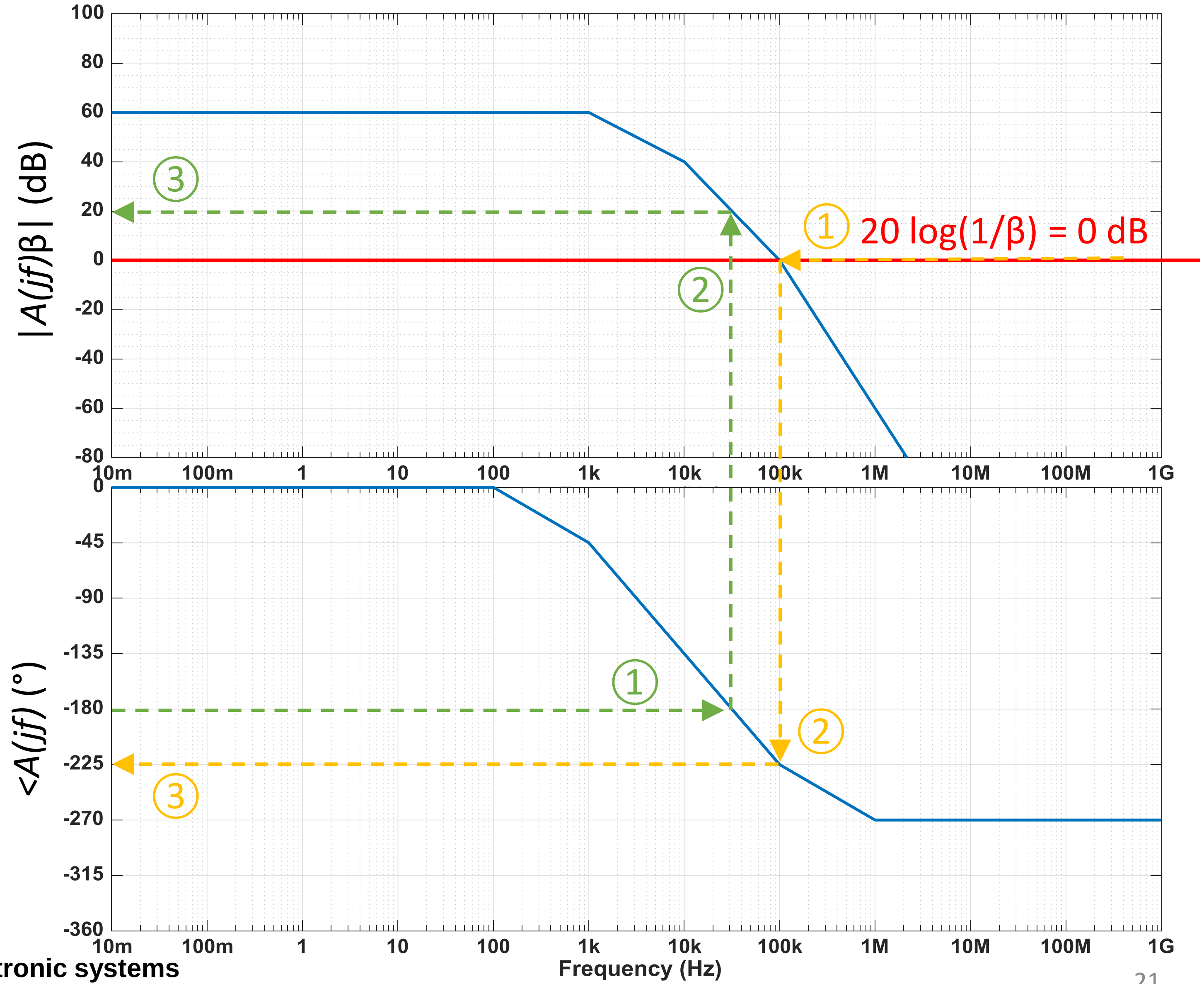
$$G_f = \frac{A_v}{1+A_v\beta} = 1 \Rightarrow \beta = \frac{A_v - 1}{A_v} = \frac{1000-1}{1000} \cong 1$$

Voltage follower => β = 1

$|A.\ \beta_{(f\text{-}180°)}| > 0\text{dB} \Rightarrow$ unstable

or, equivalently,

$|\angle(A.\ \beta)_{(f\text{-}0\text{dB})}| > 180° \Rightarrow$ **unstable**



3. In the event that the feedback amplifier as a voltage follower is unstable, proceed to compensate it by means of a dominant pole so that the compensated system has a phase margin of 45° (PM = 45°).

Voltage follower => β = 1

For a phase margin equal to 45° we have:

$$PM = 180° + \angle Dp.A.\beta\big|_{f_{0dB}} = 45° \Rightarrow$$

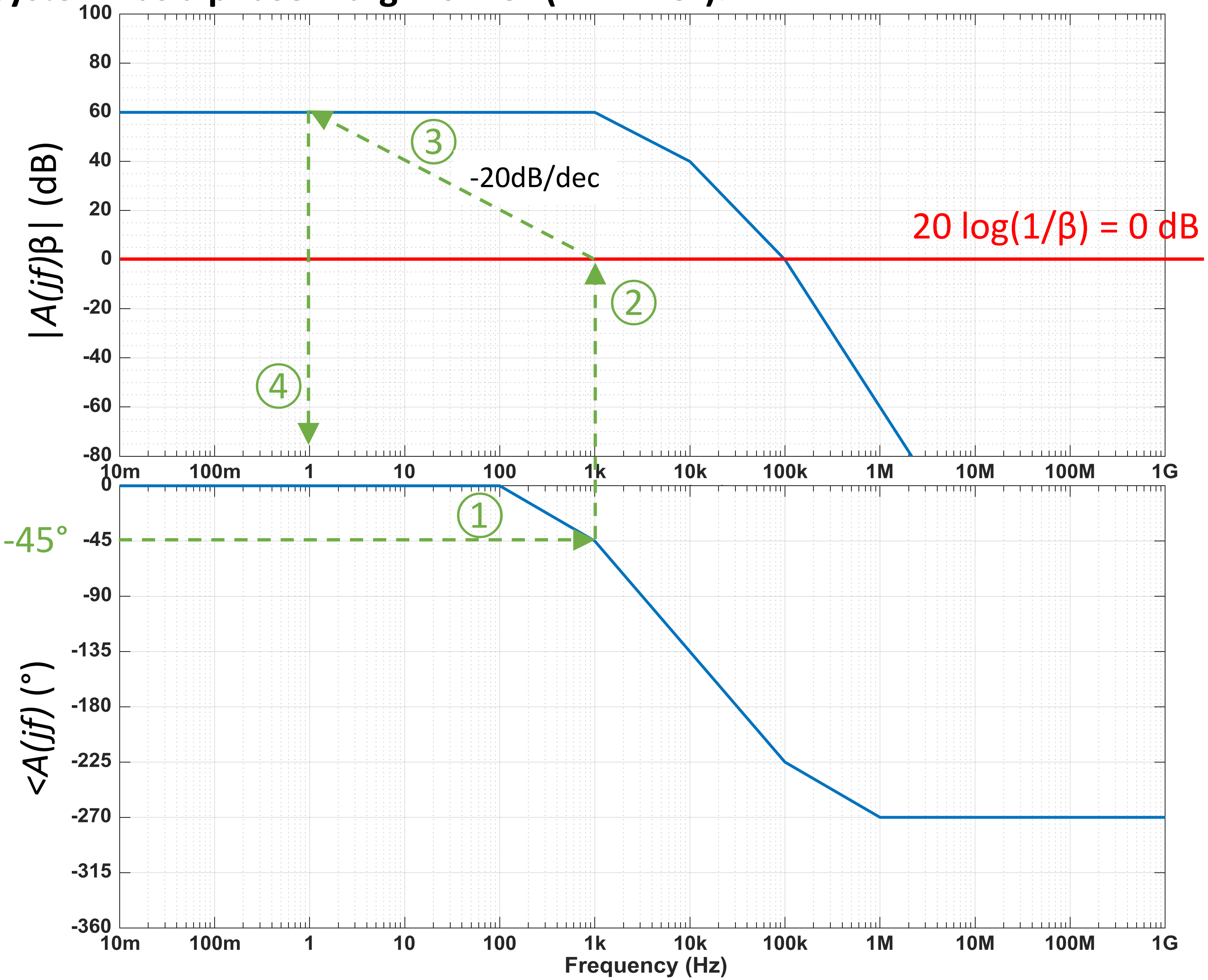
$$\angle Dp.A.\beta\big|_{f_{0dB}} = -135° \Rightarrow$$

$$\angle Dp + \angle A.\beta\big|_{f_{0dB}} = -135° \Rightarrow$$

$$\angle A.\beta\big|_{f_{0dB}} = -135° - \angle Dp = -45°$$

From the bode diagram we found that we have to introduce a dominant pole at 1 Hz

-135° + 90° = -45°



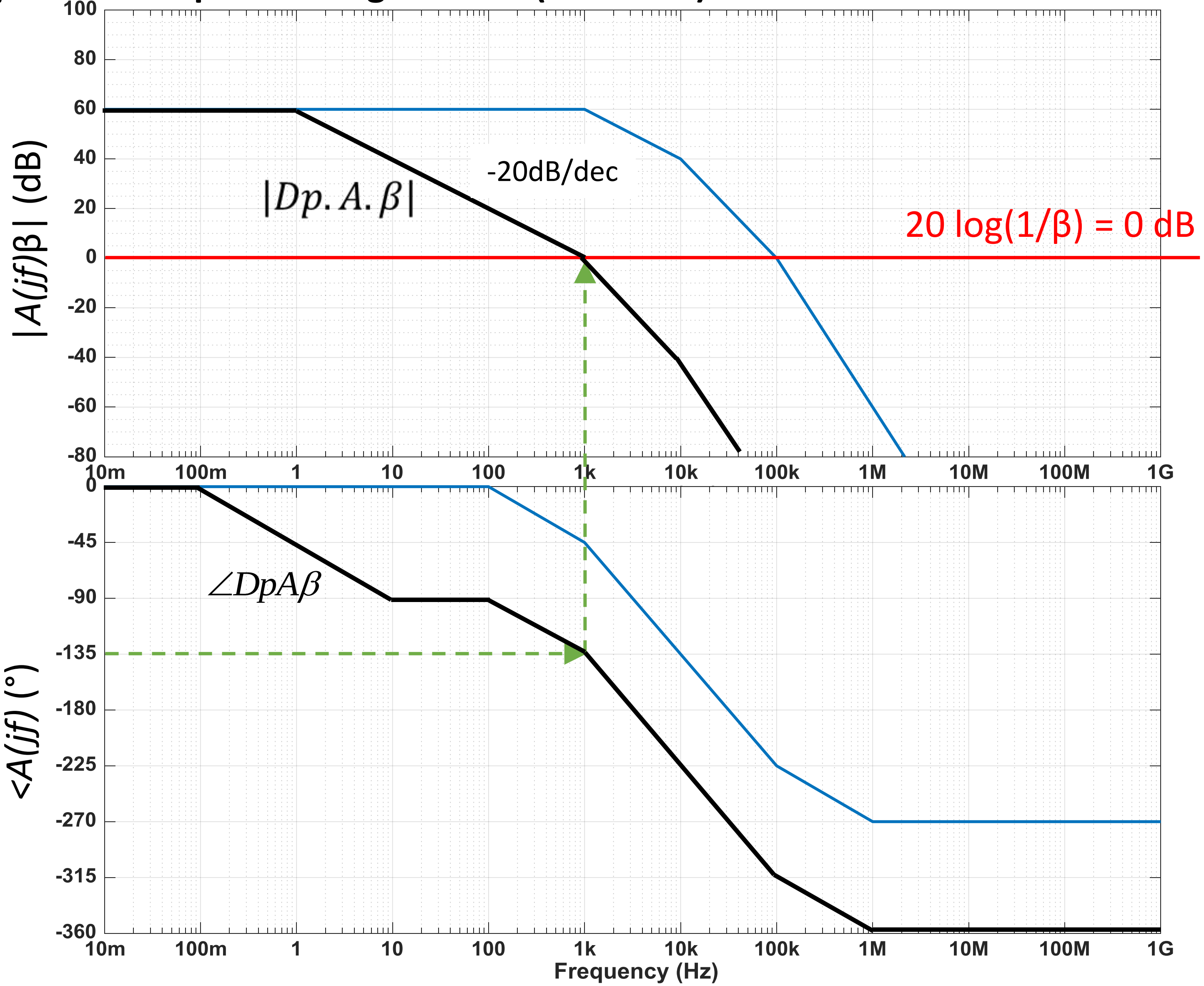
3. In the event that the feedback amplifier as a voltage follower is unstable, proceed to compensate it by means of a dominant pole so that the compensated system has a phase margin of 45° (PM = 45°).

Voltage follower => β = 1

For a phase margin equal to 45° we have:

$$PM = 180° + \angle A.\beta\big|_{f_{0dB}} = 45° \Rightarrow \angle A.\beta\big|_{f_{0dB}} = -135°$$

From the bode diagram we found that we have to introduce a dominant pole at 1 Hz



Exercise 4

Figure 1 represents the asymptotic Bode diagram of a voltage amplifier with differential input in open loop,

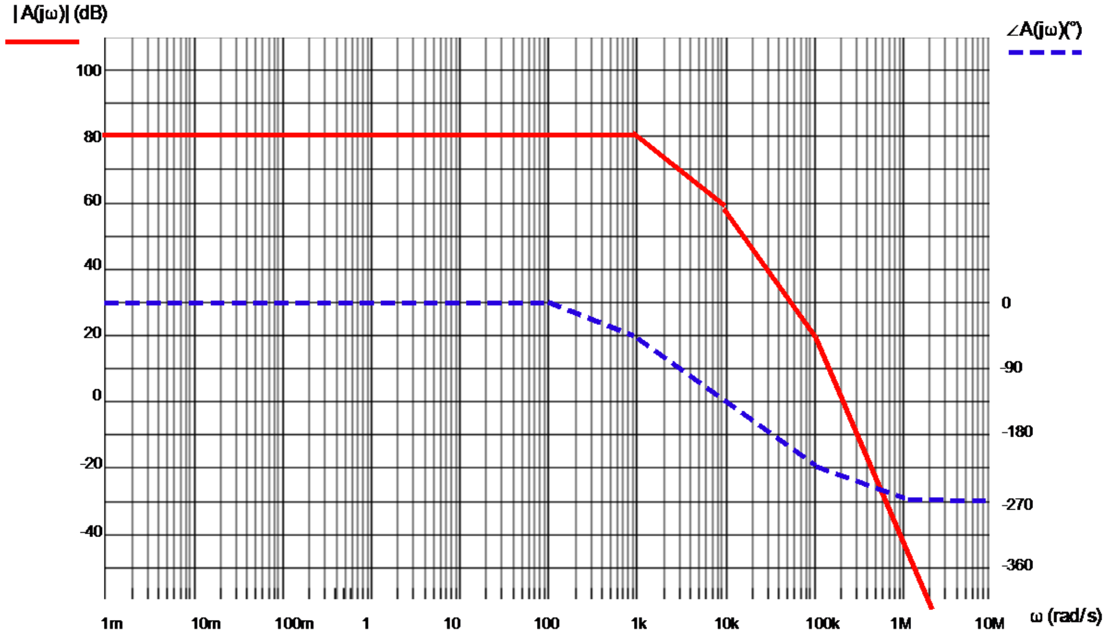


Figure 1

1. Determine the frequencies of the poles and zeros of the amplifier, and its gain at mid frequencies. Give reasons for the answer.

You want to use this amplifier in a feedback configuration as shown in figure 2

1. Is the feedback amplifier in Figure 2 stable? Give reasons for the answer.
2. In the assumption that the feedback amplifier in figure 2 is unstable, proceed to compensate it using the dominant pole so that the compensated system has a gain margin equal to 20dB, GM = 20dB.

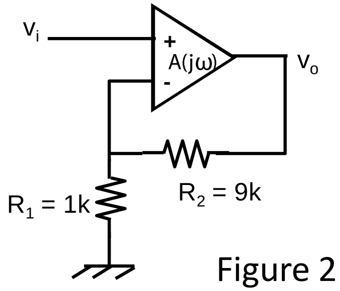


Figure 2

1. Determine the frequencies of the poles and zeros of the amplifier, and its gain at mid frequencies. Give reasons for the answer

Poles:

$\square_{p1}$= 1 krad/s

$\square_{p2}$= 10 krad/s

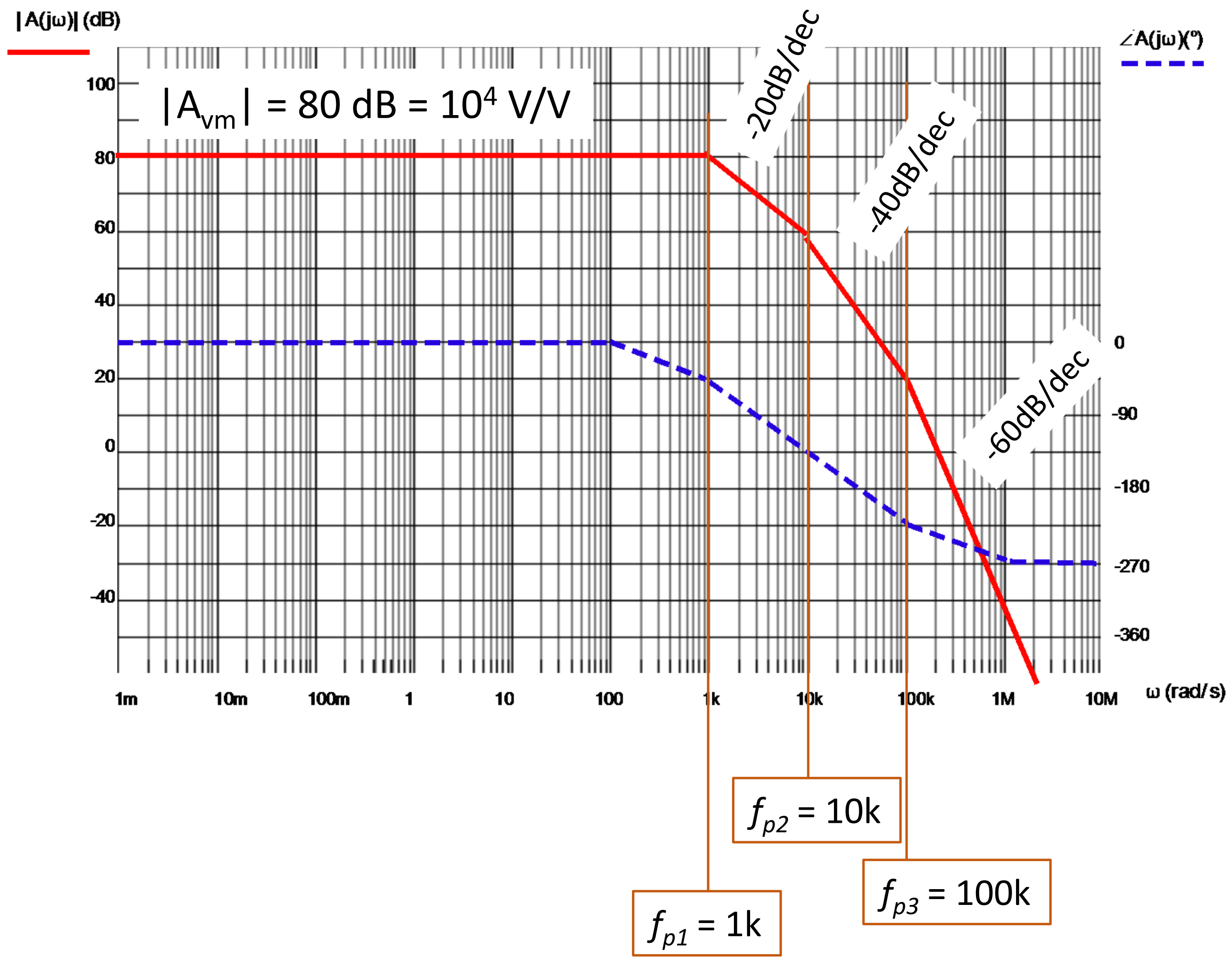
$\square_{p3}$= 100 krad/s

Zeros:

$\square_{z1}$, $\square_{z2}$, $\square_{z3} \rightarrow \infty$

Gain at mid frequencies:

$|A_{vm}|$ = 80 dB = 10^4 V/V, $<A_{vm}$ = 0° => A_{vm} = 10^4 V/V



You want to use this amplifier in a feedback configuration as shown in figure 2

2. Is the feedback amplifier in Figure 2 stable? Give reasons for the answer.

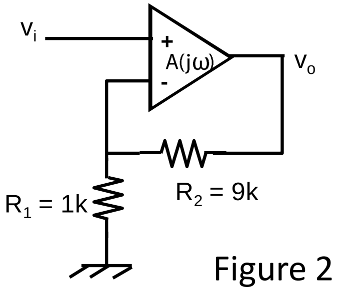


Figure 2

The non-inverting feedback amplifier is a series-parallel configuration, so

$$\beta = \left.\frac{V_1}{V_2}\right|_{i_1=0} = \frac{R_1}{R_1+R_2} = 0.1 \rightarrow \frac{1}{\beta} = 1 + \frac{R_2}{R_1} = 10$$

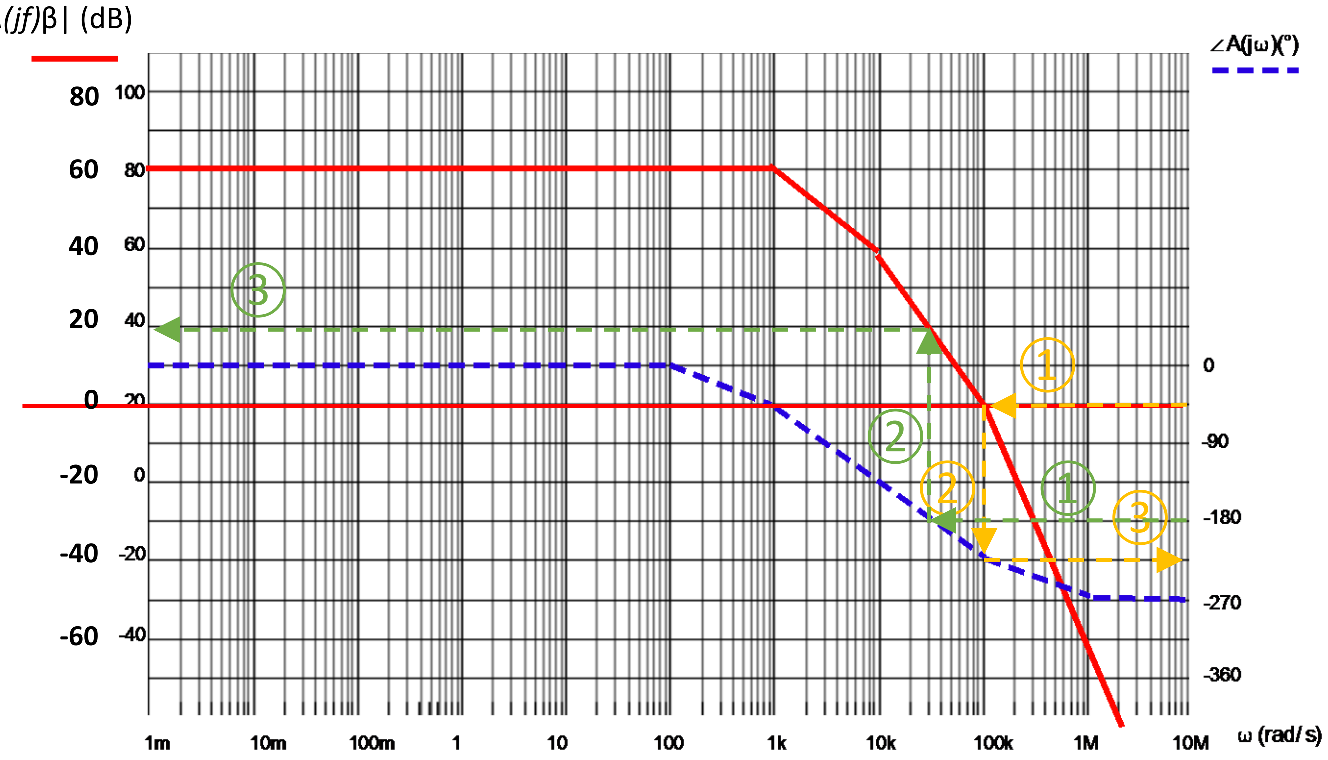
Now we check the stability of feedback amplifier with the Bode diagram of the close loop gain : A(j□)β

|A. β $_{(f\text{-}180°)}$ | ≈ 20 dB > 0dB => unstable

or, equivalently,

<(A. β) $_{(f\text{-}0dB)}$ ≈ -225 < -180° => **unstable**

Then , the feedback amplifier is unstable.



2. In the assumption that the feedback amplifier in figure 2 is unstable, proceed to compensate it using the dominant pole so that the compensated system has a gain margin equal to 20dB, GM = 20dB.

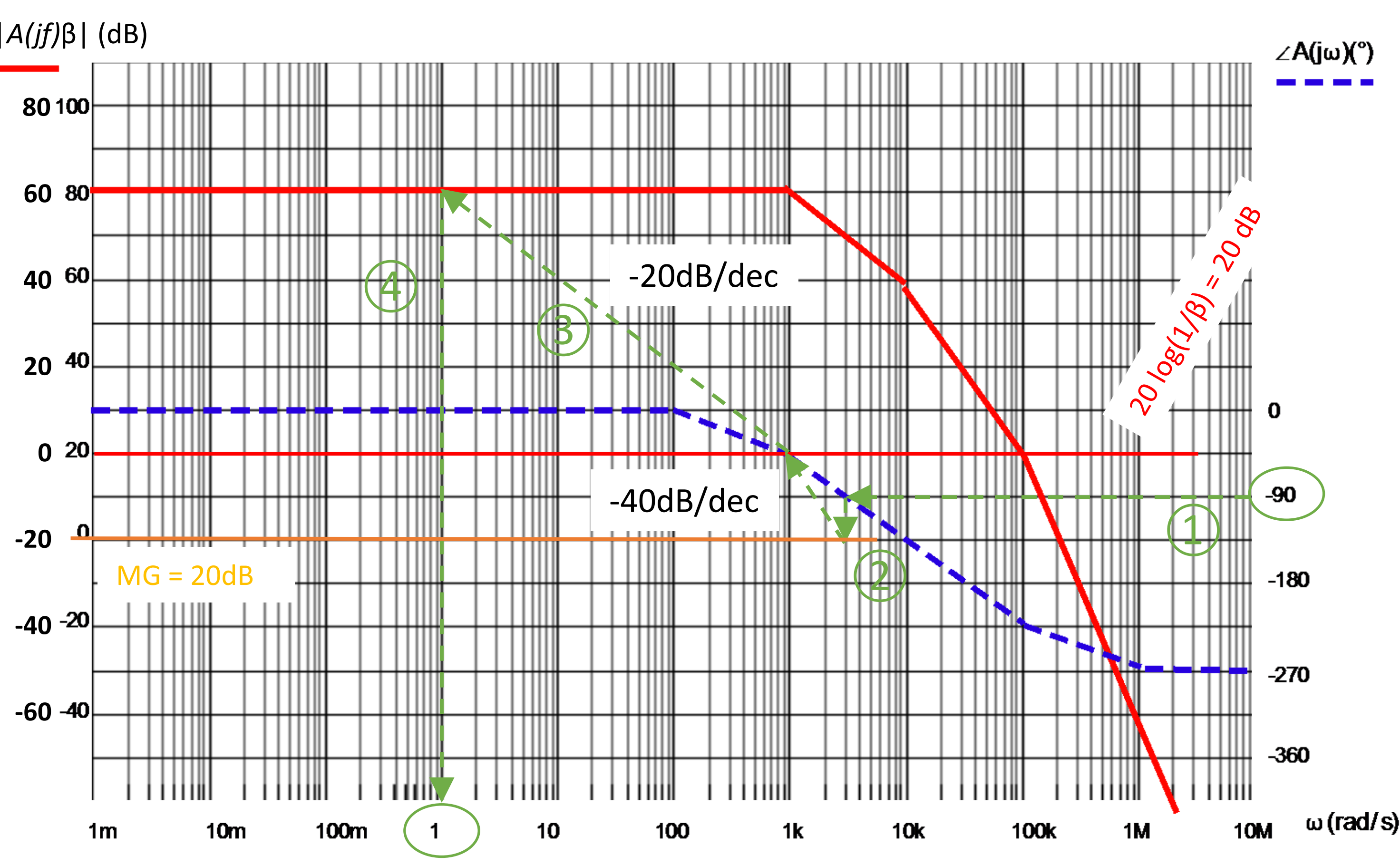
For a gain margin equal to 20dB we have:

$$MG = 0dB - \left|Dp.A.\beta\right|_{f_{180^\circ}} = 20dB \Rightarrow \left|Dp.A.\beta\right|_{f_{180^\circ}} = -20dB$$

After introducing the dominant pole will be a phase shift of -90°. So the frequency at Pd*A*β has a phase shift equal to -180° after introducing the dominant pole, is the one that is equal to -90° in A*β.

$$\angle Dp = -90^\circ$$
$$\angle DpA\beta = \angle Dp + \angle A\beta = \angle Dp + \angle A = -180 \Rightarrow$$
$$\angle A = -90^\circ$$

From the bode diagram we found that we have to introduce a dominant pole at 1 rad/s.



2. In the assumption that the feedback amplifier in figure 2 is unstable, proceed to compensate it using the dominant pole so that the compensated system has a gain margin equal to 20dB, GM = 20dB.

For a gain margin equal to 20dB we have:

$$MG = 0dB - \left|Dp.A.\beta\right|_{f_{180°}} = 20dB \Rightarrow \left|Dp.A.\beta\right|_{f_{180°}} = -20dB$$

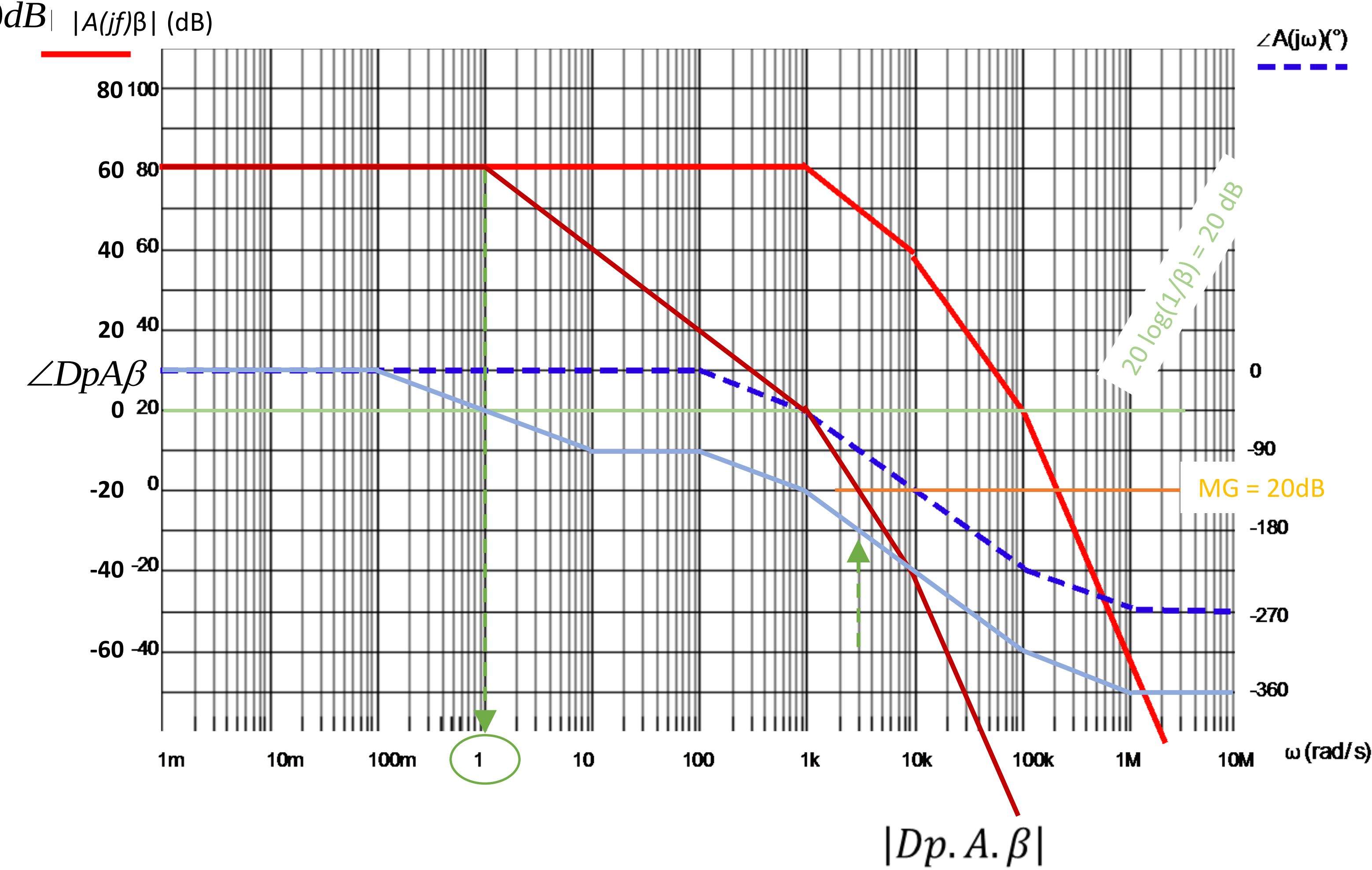
After introducing the dominant pole will be a phase shift of -90°. So the frequency at Pd*A*β has a phase shift equal to -180° after introducing the dominant pole, is the one that is equal to -90° in A*β.

$$\angle Dp = -90°$$

$$\angle DpA\beta = \angle Dp + \angle A\beta = \angle Dp + \angle A = -180 \Rightarrow$$

$$\angle A = -90°$$

From the bode diagram we found that we have to introduce a dominant pole at 1 rad/s.



Exercise 5

The transfer function of a voltage amplifier with differential input is:

$$A_V(jf) = \frac{500}{\left(1 + j\frac{f}{1kHz}\right)\left(1 + j\frac{f}{1kHz}\right)\left(1 + j\frac{f}{10kHz}\right)}$$

with f expressed in Hz

1. Represent the Asymptotic Bode Diagram (modulus and phase) of the amplifier, $A_V(jf)$. Is this amplifier stable? Justify your answer.

This amplifier is used in a feedback configuration as a voltage follower, as shown in Figure 3.

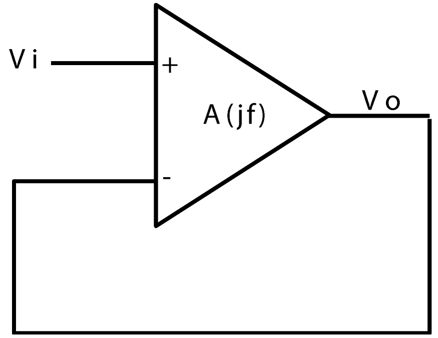


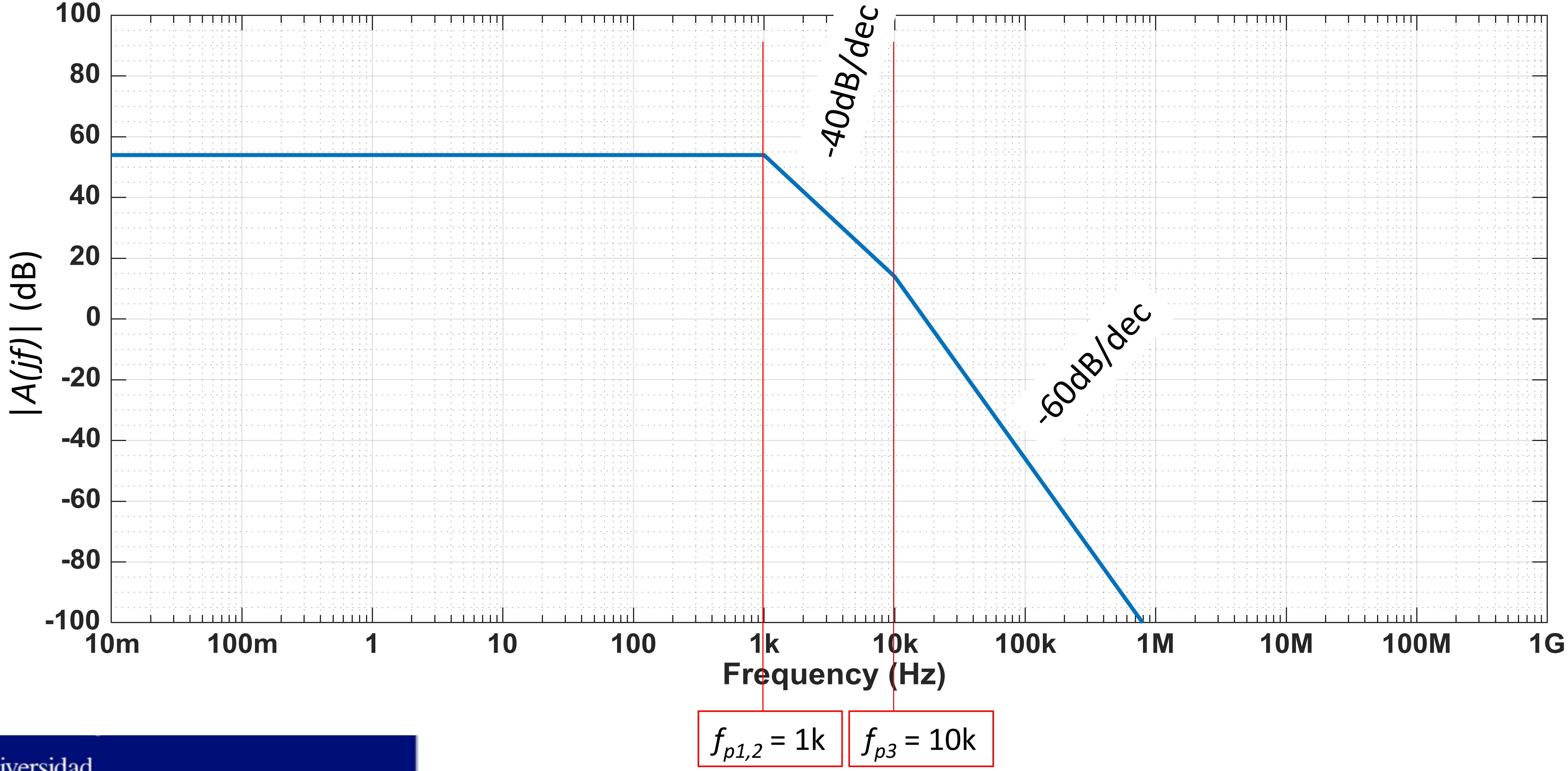
Figure 3

2. Is this feedback amplifier stable as a voltage follower? Justify your answer.
3. At what frequency would a dominant pole have to be inserted in the feedback amplifier so that it is stable as a voltage follower with a gain margin of 20dB? Justify your answer.

1. **Represent the asymptotic Bode diagram of the frequency response (modulus and phase) of the amplifier.**

Modulus:

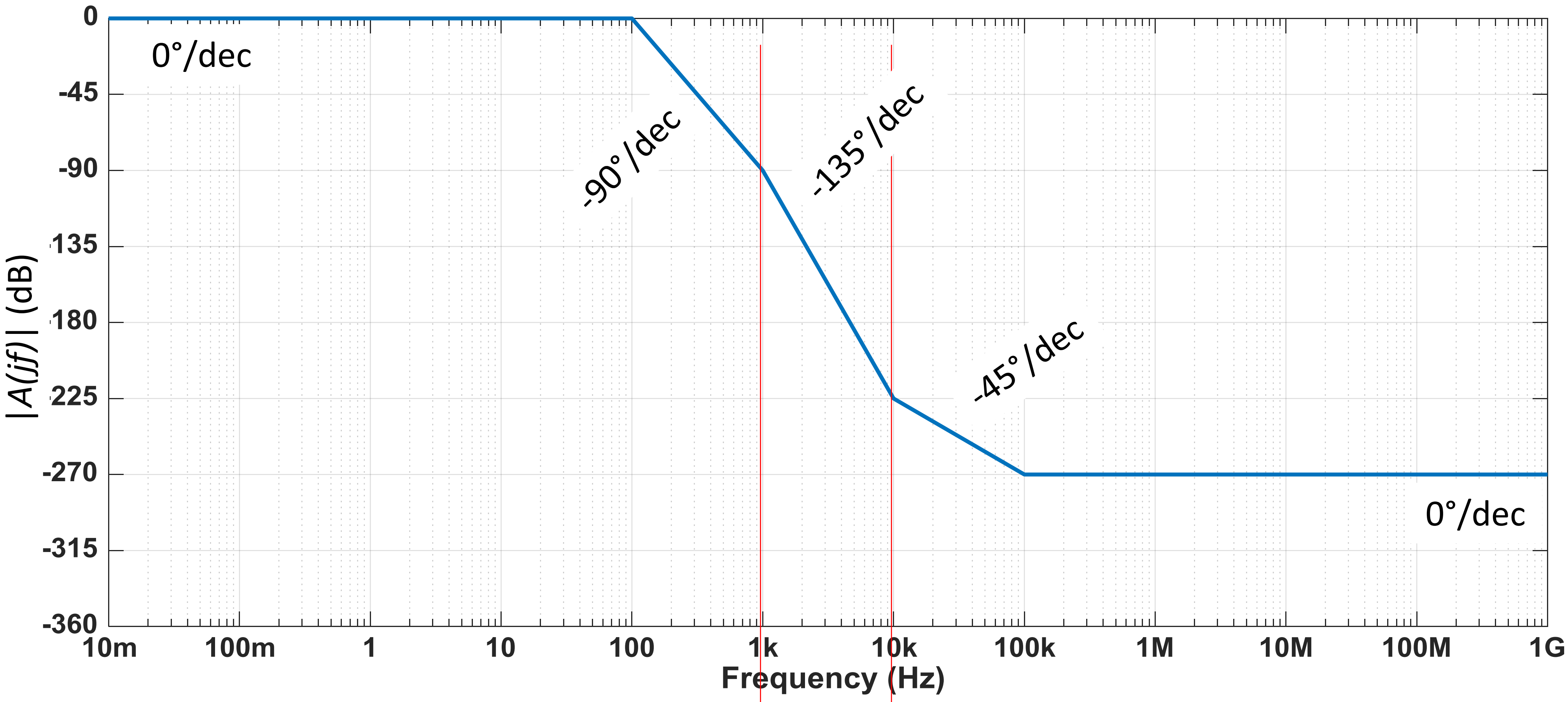
$$A_V(jf) = \frac{500}{\left(1 + j\frac{f}{1kHz}\right)\left(1 + j\frac{f}{1kHz}\right)\left(1 + j\frac{f}{10kHz}\right)}$$



1. Represent the asymptotic Bode diagram of the frequency response (modulus and phase) of the amplifier.

$$A_V(jf) = \frac{500}{\left(1 + j\frac{f}{1kHz}\right)\left(1 + j\frac{f}{1kHz}\right)\left(1 + j\frac{f}{10kHz}\right)}$$

Modulus:



$f_{p1,2}$ = 1k f_{p3} = 10k

1.b Is this amplifier stable? . Give reasons for your answer.

$$A_V(jf) = \frac{500}{\left(1+j\frac{f}{1kHz}\right)\left(1+j\frac{f}{1kHz}\right)\left(1+j\frac{f}{10kHz}\right)}$$

Yes, it is. The amplifier is stable since all his poles are in the left half of the s-plane.

Remember: *A causal LTI system with a rational transfer function H*(*s*) *is stable if and only if all poles of H*(*s*) *are in the left half of the s-plane, i.e., the real parts of all poles are negative.*

This amplifier is used in a feedback configuration as a voltage follower, as shown in Figure 3.
Is this feedback amplifier stable as a voltage follower? Justify your answer.

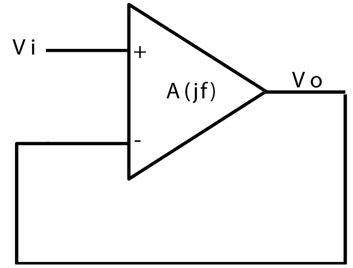


Figure 3

The gain of the feedback amplifier as a voltage follower is 1, so

$$G_f = \frac{A_{vm}}{1 + A_{vm}\beta} = 1 \Rightarrow \beta = \frac{A_{vm} - 1}{A_{vm}} = \frac{500 - 1}{500} \cong 1$$

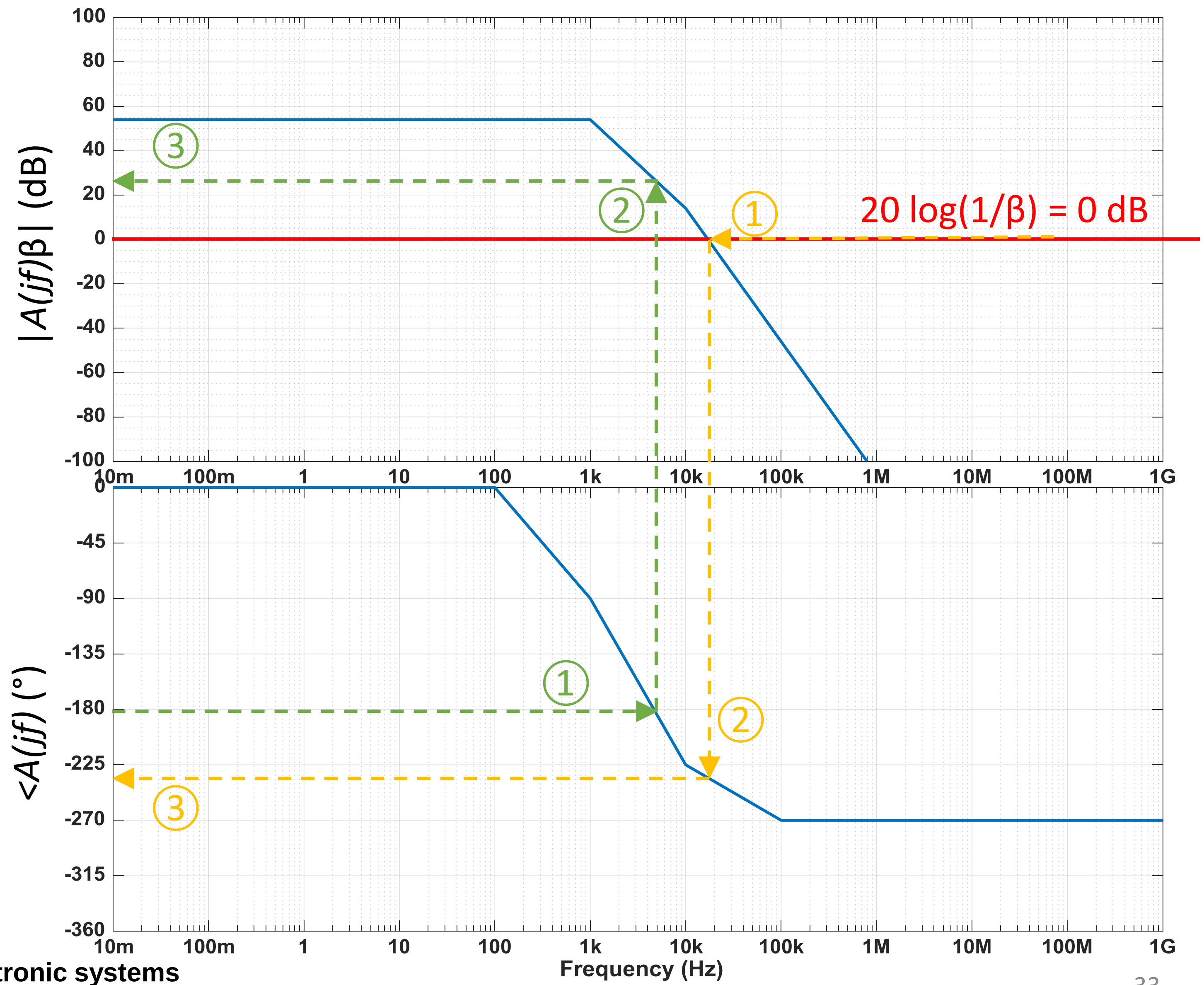
Voltage follower => β = 1

|A. β $_{(f\text{-}180°)}$ | ≈25dB > 0dB => unstable

or, equivalently,

| <(A. β) $_{(f\text{-}0dB)}$ | > 180° => **unstable**

In conclusion, the feedback amplifier is unstable.



3. At what frequency would a dominant pole have to be inserted in the feedback amplifier so that it is stable as a voltage follower with a gain margin of 20dB? Justify your answer.

Voltage follower => β = 1

For a gain margin equal to 20dB we have:

$$MG = 0dB - \left|Dp.A.\beta\right|_{f_{180^\circ}} = 20dB \Rightarrow \left|Dp.A.\beta\right|_{f_{180^\circ}} = -20dB$$

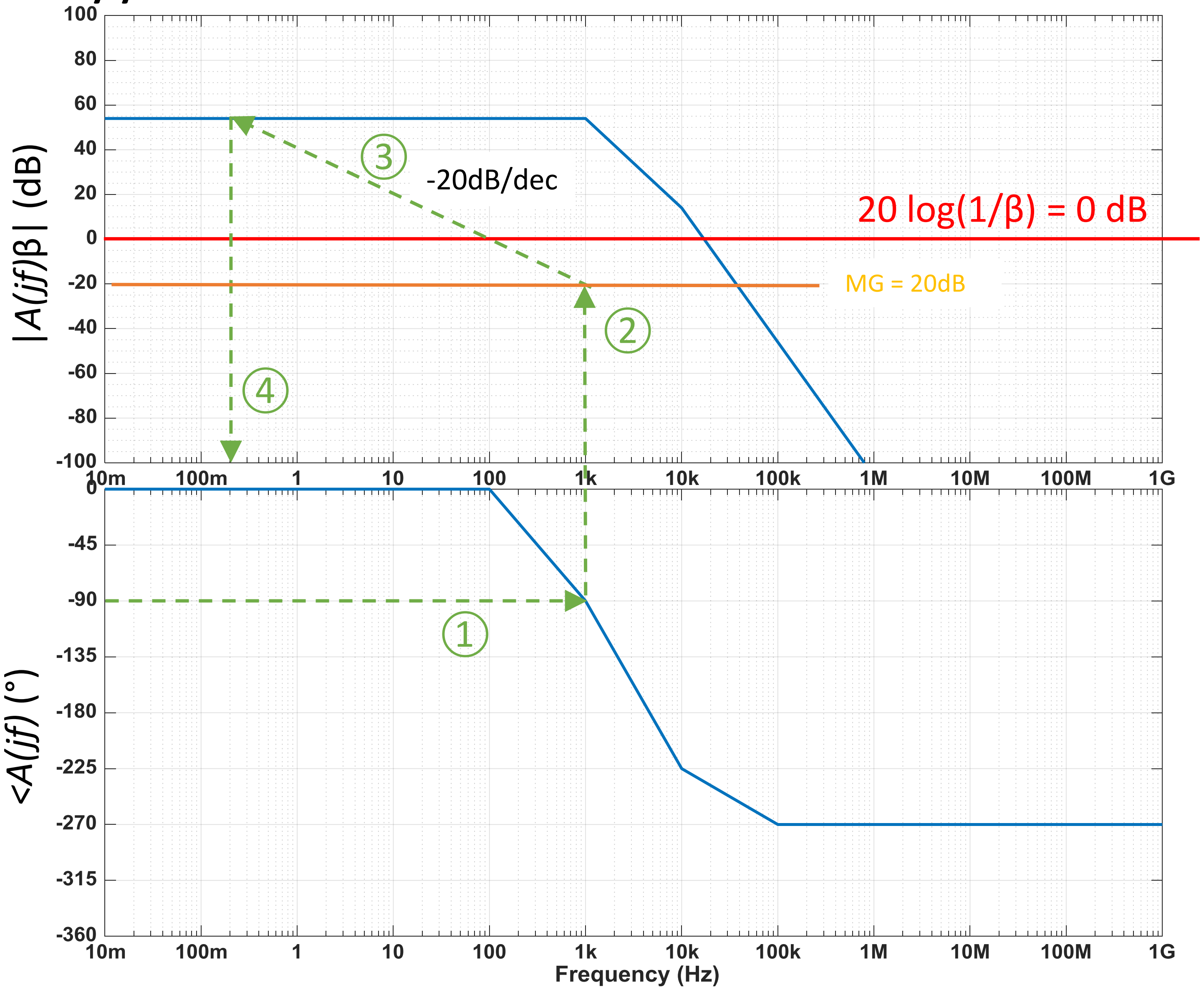
After introducing the dominant pole will be a phase shift of -90°. So the frequency at Pd*A*β has a phase shift equal to -180° after introducing the dominant pole, is the one that is equal to -90° in A*β.

$$\angle Dp = -90^\circ$$

$$\angle DpA\beta = \angle Dp + \angle A\beta = \angle Dp + \angle A = -180 \Rightarrow$$

$$\angle A = -90^\circ$$

From the bode diagram we found that we have to introduce a dominant pole at 200 mHz.



3. At what frequency would a dominant pole have to be inserted in the feedback amplifier so that it is stable as a voltage follower with a gain margin of 20dB? Justify your answer.

Voltage follower => β = 1

For a gain margin equal to 20dB we have:

$$MG = 0dB - \left|Dp.A.\beta\right|_{f_{180^\circ}} = 20dB \Rightarrow \left|Dp.A.\beta\right|_{f_{180^\circ}} = -20dB$$

After introducing the dominant pole will be a phase shift of -90°. So the frequency at Pd*A*β has a phase shift equal to -180° after introducing the dominant pole, is the one that is equal to -90° in A*β.

$$\angle Dp = -90^\circ$$
$$\angle DpA\beta = \angle Dp + \angle A\beta = \angle Dp + \angle A = -180 \Rightarrow$$
$$\angle A = -90^\circ$$

From the bode diagram we found that we have to introduce a dominant pole at 200 mHz.