Planar waveguides

Stripline and microstrip

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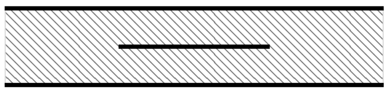
1 Stripline



Characteristics:

- Two ground planes that touch at ∞, one signal strip.
- Therefore, two disjoint conductors and one TEM mode.

$$\gamma = \gamma_0 = j\omega\sqrt{\mu\varepsilon} \qquad Z_{\mathrm{TEM}} = \eta = \sqrt{\frac{\mu}{\varepsilon}}$$

Undesired modes:

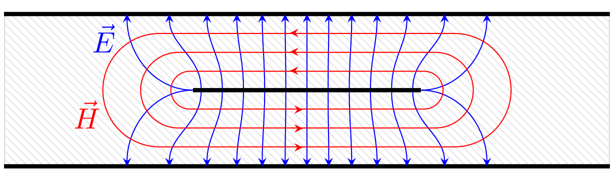
- TEM mode between ground planes (parallel plate waveguide): short circuit them with metallic screws or posts.
- Higer order TE and TM modes: keep separation between ground planes $< \lambda/4$.

The structure can be constructed by stacking two dielectric boards fully metallized on one face. The strip can be engraved (photo-etching, milling machine, laser cutting) on the opposite face of one board (or both). Both layer can be pressed and hold together by screws, that also ensure the electric continuity between ground planes.

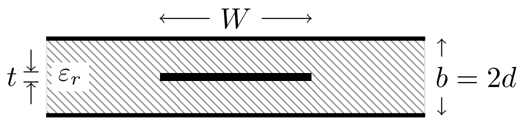
Analysis and synthesis:

1. Semi-empiric equations.
2. Graphs.
3. Quasi-static approximation (i.e., compute C' or L' as a static problem, suppose that it is a good approximation when $f > 0$, and use it to obtain Z_0).

1.1 TEM mode fields



1.2 Equations



Characteristic impedance:

$$Z_0 = \frac{30\pi}{\sqrt{\varepsilon_r}} \frac{b}{W_e + 0.441b} \quad \text{where} \quad \frac{W_e}{b} = \begin{cases} \dfrac{W}{b} & \text{for } W/b \geq 0.35 \\ \dfrac{W}{b} - \left(0.35 - \dfrac{W}{b}\right)^2 & \text{for } W/b < 0.35 \end{cases}$$

Design equation:

$$\frac{W}{b} = \begin{cases} x & \text{for } \sqrt{\varepsilon_r} Z_0 \leq 120\,\Omega \\ 0.85 - \sqrt{0.6 - x} & \text{for } \sqrt{\varepsilon_r} Z_0 > 120\,\Omega \end{cases} \quad \text{where} \quad x = \frac{30\pi}{\sqrt{\varepsilon_r} Z_0} - 0.441$$

Conductor attenuation constant (Np/m):

$$\alpha_c = \begin{cases} \dfrac{2.7 \times 10^{-3} R_s \varepsilon_r Z_0}{30\pi(b-t)} A & \text{for } \sqrt{\varepsilon_r} Z_0 \leq 120\,\Omega \\ \dfrac{0.16 R_s}{Z_0 b} B & \text{for } \sqrt{\varepsilon_r} Z_0 > 120\,\Omega \end{cases}$$

where t is the conductor (strip) thickness and

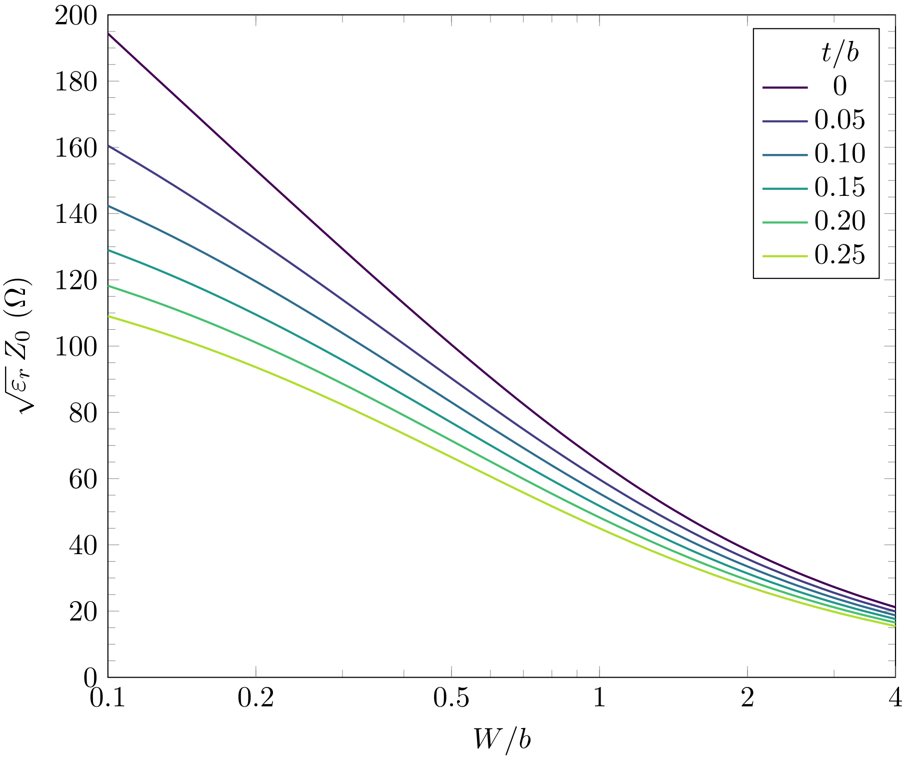
$$A = 1 + \frac{2W}{b-t} + \frac{1}{\pi} \frac{b+t}{b-t} \ln \frac{2b-t}{t}, \qquad B = 1 + \frac{b}{0.5W + 0.7t} \left(0.5 + \frac{0.414t}{W} + \frac{1}{2\pi} \ln \frac{4\pi W}{t}\right)$$

Dielectric attenuation constant: just like **any TEM mode**. The complex relative dielectric constant is $\varepsilon_r = \varepsilon_r' - j\varepsilon_r'' = \varepsilon_r'(1 - \tan\delta)$

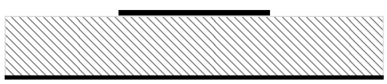
$$\tan\delta = \frac{2\alpha_d}{\beta} = \frac{\varepsilon_r''}{\varepsilon_r'} \quad \Rightarrow \quad \alpha_d = \frac{\beta \tan\delta}{2} = \frac{\omega}{2c} \frac{\varepsilon_r''}{\varepsilon_r'} = \frac{\omega \varepsilon_r'' \eta}{2}$$

Stripline online calculator: https://bit.ly/34v0Xvq

1.3 Analysis/synthesis graph



2 Microstrip



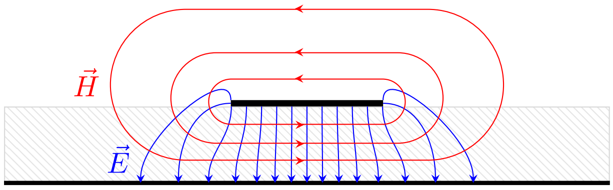
Characteristics:

- One ground plane, one signal strip.
- Open structure: it may radiate.
- Two different dielectrics: air above, substrate below: **non homogeneous**

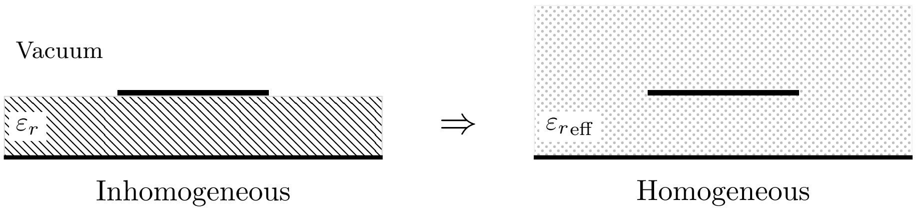
Consequences of non-homogeneity:

- No TE, TM or TEM modes: hybrid modes (with both $\vec{E}_t \neq 0$ and $\vec{H}_t \neq 0$).
- But one mode with *almost* same properties than a TEM mode (e.g. $f_c = 0$, well defined $V(z)$ and $I(z)$): **quasi-TEM mode**.
- The properties of the quasi-TEM mode are not constant with frequency: **dispersion**.

2.1 Quasi-TEM mode fields



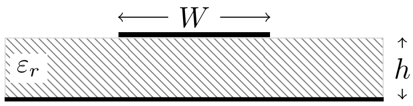
Concept of effective permittivity:



$$1 < \varepsilon_{r\text{eff}} < \varepsilon_r$$

$$v_p = \frac{c_0}{\sqrt{\varepsilon_{r\text{eff}}}} \qquad \beta = \frac{\omega}{v_p} = \frac{\omega}{c_0}\sqrt{\varepsilon_{r\text{eff}}} = k_0\sqrt{\varepsilon_{r\text{eff}}}$$

2.2 Equations



Effective permittivity:

$$\varepsilon_{r\text{eff}} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\frac{1}{\sqrt{1 + 12h/W}} \quad \text{where } \varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

Characteristic impedance:

$$Z_0 = \begin{cases} \dfrac{60}{\sqrt{\varepsilon_{r\text{eff}}}} \ln\left(\dfrac{8h}{W} + \dfrac{W}{4h}\right) & \text{for } W/h \le 1 \\[2ex] \dfrac{120\pi}{\sqrt{\varepsilon_{r\text{eff}}}\left[W/h + 1.393 + 0.667\ln(W/h + 1.444)\right]} & \text{for } W/h \ge 1 \end{cases}$$

Design equation:

$$\frac{W}{h} = \begin{cases} \dfrac{8\,e^A}{e^{2A} - 2} & \text{for } W/h < 2 \\[2ex] \dfrac{2}{\pi}\left[B - 1 - \ln(2B - 1) + \dfrac{\varepsilon_r - 1}{2\varepsilon_r}\left(\ln(B - 1) + 0.39 - \dfrac{0.61}{\varepsilon_r}\right)\right] & \text{for } W/h > 2 \end{cases}$$

where

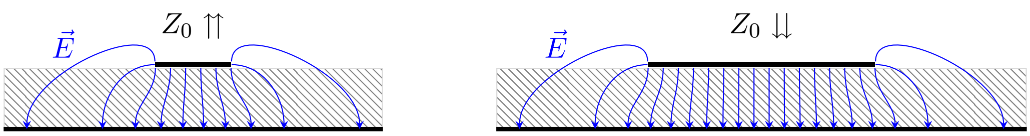
$$A = \frac{Z_0}{60}\sqrt{\frac{\varepsilon_r + 1}{2}} + \frac{\varepsilon_r - 1}{\varepsilon_r + 1}\left(0.23 + \frac{0.11}{\varepsilon_r}\right) \qquad B = \frac{377\pi}{2Z_0\sqrt{\varepsilon_r}}$$

Conductor and dielectric attenuation constants (Np/m):

$$\alpha_c = \frac{R_s}{Z_0 W} \qquad \alpha_d = \frac{8.686\pi}{\lambda}\frac{\varepsilon_{reff} - 1}{\varepsilon_r - 1}\frac{\varepsilon_r}{\varepsilon_{reff}}\tan\delta$$

where $\lambda = \frac{v_p}{f} = \frac{c}{f\sqrt{\varepsilon_{reff}}}$ is the wavelength.

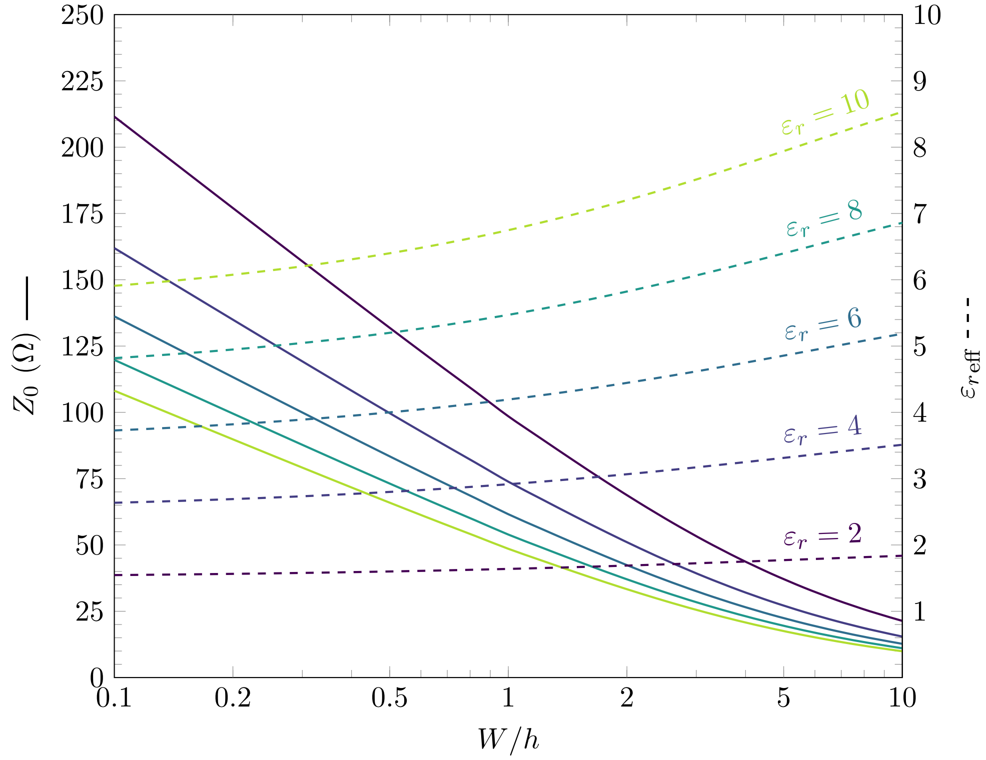
Characteristic impedance vs. effective permittivity.



$$1 < \varepsilon_{reff\ Z_0\upuparrows} < \varepsilon_{reff\ Z_0\downdownarrows} \lesssim \varepsilon_r$$

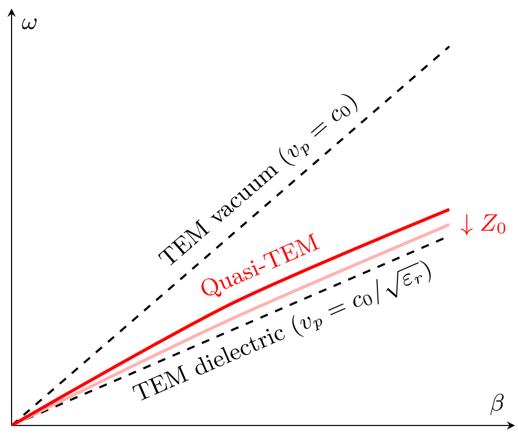
> Microstrip online calculator: https://bit.ly/3GIzsf8

2.3 Analysis/synthesis graph



2.4 Dispersion

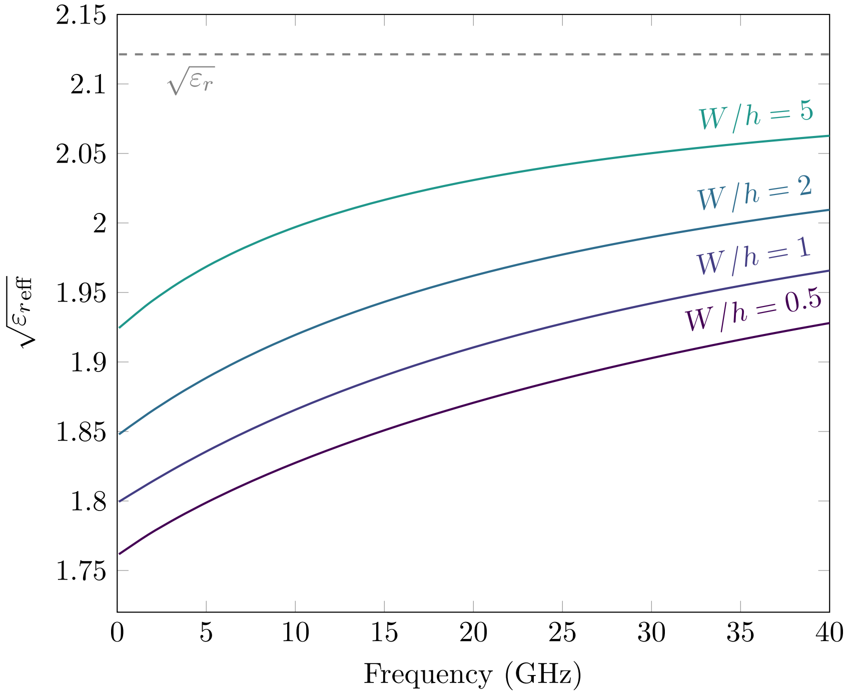
Microstrip dispersion diagram:



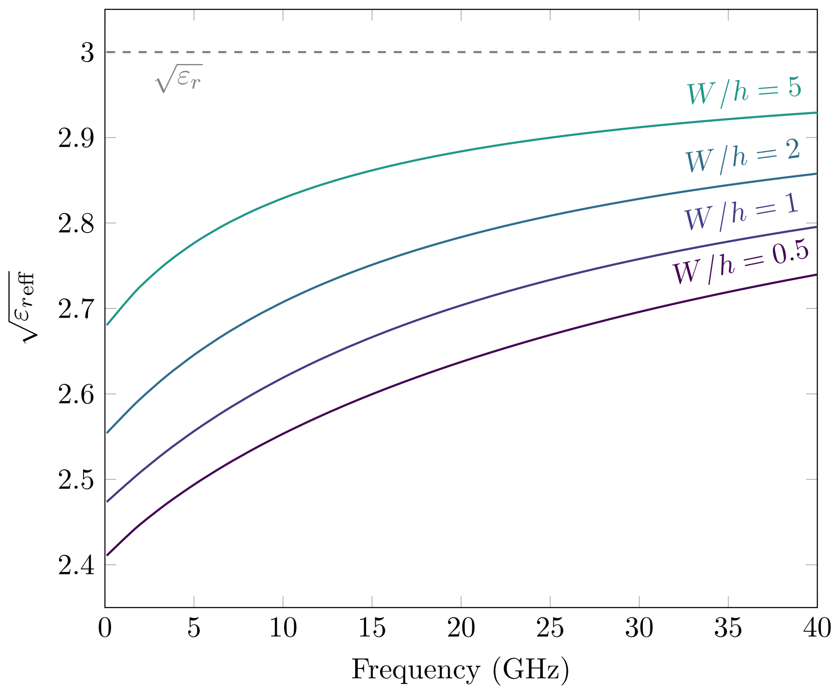
$$v_p = \frac{\omega}{\beta} = \frac{c_0}{\sqrt{\varepsilon_{r\text{eff}}}}$$

Variation of the effective permittivity with frequency (for $\varepsilon_r = 9$):

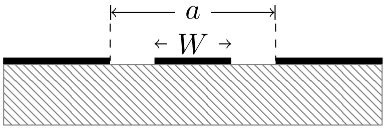
- For $\varepsilon_r = 4.5$



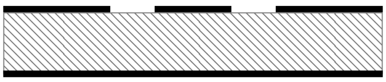
- For $\varepsilon_r = 9$



3 Coplanar waveguide



Coplanar waveguide (CPW)



Conductor-backed coplanar waveguide (CBCPW)

Characteristics:

- Two ground planes, one signal strip.
- Additional back plane (optional): CBCPW.
- Two different dielectrics: air above, substrate below: **non homogeneous**
- One low dispersion quasi-TEM mode (at least at low frequencies).
- Other undesired quasi-TEM modes that should be mitigated (short-circuited ground planes with air bridges and/or screws).

$$\varepsilon_{r\text{eff}} \approx \frac{\varepsilon_r + 1}{2}$$

$$Z_0 = \begin{cases} \dfrac{\eta}{\pi\sqrt{\varepsilon_{r\text{eff}}}} \ln\left(2\sqrt{\dfrac{a}{W}}\right) & \text{for } 0 < W/a \leq 0.173 \\ \dfrac{\pi\eta}{4\sqrt{\varepsilon_{r\text{eff}}}} \left[\ln\left(2\frac{1+\sqrt{W/a}}{1-\sqrt{W/a}}\sqrt{\dfrac{a}{W}}\right)\right]^{-1} & \text{for } 0.173 < W/a < 1 \end{cases}$$