High Frequency Technologies

Problems on waveguides

Problem 1

Consider a waveguide with square cross-section of side a.

a) Compute the cutoff frequencies of its modes.

b) Check that there are two dominant modes.

c) Compute the power transmitted by each of the two modes, in terms of the maximum amplitude of the electric field.

d) Compute the power transmitted by the superposition of both modes.

Answer

a) $f_{c,mn} = \dfrac{c_0}{2a\sqrt{\varepsilon_r}}\sqrt{m^2+n^2}$

b) TE_{10} and TE_{01}.

c) The power is the same for each mode, if both amplitudes are equal $P_{\mathrm{TE}_{10}} = \dfrac{|E_0|^2\, a^2}{4Z_{\mathrm{TE}_{10}}}$.

d) $P = P_{\mathrm{TE}_{10}} + P_{\mathrm{TE}_{01}}$.

Problem 2

A rectangular waveguide has a cross-section with dimensions $a = 86\,\mathrm{mm}$ and $b = 43\,\mathrm{mm}$. Compute:

a) The single-mode bandwidth.

b) The maximum power that the fundamental mode can transmit in the single-mode band, if the maximum allowable electric field is limited to $30\,\mathrm{V/cm}$.

Answer

a) $1.743\,\mathrm{GHz}$ b) $19.13\,\mathrm{W}$

Problem 3

Consider an empty cylindrical waveguide, with a fundamental mode that has the same cutoff frequency than the rectangular waveguide from the previous problem.

a) Compute the radius of the cylindrical waveguide.

b) Compare the single-mode bandwidth of both waveguides.

Answer

a) 50.4 mm. b) 0.534 GHz, 30.6 % of the rectangular waveguide.

Problem 4

Consider a rectangular waveguide with cross-section of dimensions $a \times b$.

a) Find the locations in the x axis so that the amplitudes of the components H_x and H_z of the fundamental mode magnetic field are equal (this means that the field has circular polarization).

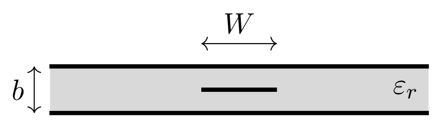
b) Solve the frequency so that the condition is met at $x = a/4$.

Answer

a) $x = \dfrac{a}{\pi} \arctan\left(\dfrac{\pi}{\beta_{10}\, a}\right)$ b) $\sqrt{2} f_c$

Problem 5

Consider a symmetric stripline transmission line (see figure), where W is the width of the strip, and b is the separation between the ground plane plates. The relative permittivity of the dielectric that fills this space is ε_r:



a) Sort the particular cases in the table in terms of their characteristic impedances, using the relationships "less than" ($<$) and "approximately equal to" ($\approx$) (Example: $Z_0^A < Z_0^C \approx Z_0^D < Z_0^B < Z_0^E$). Reason your answer.

	A	B	C	D	E
Width	W	$2W$	$2W$	W	$3W$
Separation	b	$2b$	b	$2b$	$2b$

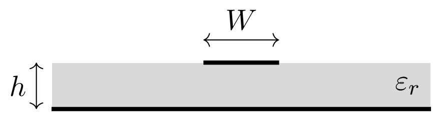
b) If $Z_{0A} = 50\,\Omega$ and $\varepsilon_r = 4.5$, what is the characteristic impedance if the dielectric is replaced with another one of $\varepsilon_r = 9$?

Answer

a) $Z_0^C < Z_0^E < Z_0^A \approx Z_0^B < Z_0^D$. b) $35.36\,\Omega$.

Problem 6

Consider a microstrip transmission line (see figure), where W is the width of the strip and h is the thickness of the dielectric substrate, with relative permittivity ε_r.



a) Sort the particular cases in the table in terms of their phase velocities, using the relationships "less than" ($<$) and "approximately equal to" ($\approx$) (Example: $v_p^A < v_p^C \approx v_p^D < v_p^B < v_p^E$). Reason your answer.

	A	B	C	D	E
Width	W	$2W$	$2W$	W	$3W$
Thickness	h	$2h$	h	$2h$	$2h$

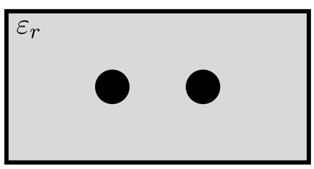
b) If the dielectric substrate is replaced by another one with higher ε_r, will the phase velocities increase or decrease?

Answer

a) $v_f^C < v_f^E < v_f^A \approx v_f^B < v_f^D$. b) They decrease.

Problem 7

The diagram below shows the cross-section of a waveguide, where the black lines represent perfect conductor, and the grey region is certain dielectric with ε_r.



a) What types of modes can propagate?

b) How many TEM modes (if any) are supported?

c) For each TEM mode that is supported, what is the phase velocity?

d) If the dielectric is replaced with another one whose permittivity is double ($\varepsilon_r' = 2\varepsilon_r$), what are the new phase velocities, in terms of the original ones?

Answer

a) TE, TM and TEM.

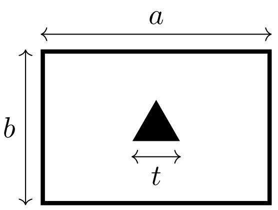
b) Two.

c) For both, $v_f = c_0/\sqrt{\varepsilon_r}$.

d) $v_f' = v_f/\sqrt{2}$.

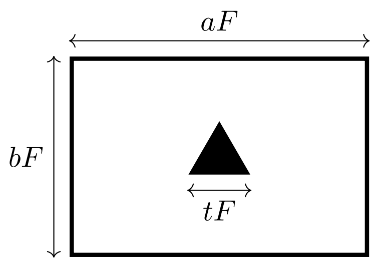
Problem 8

The diagram below shows the cross-section of a waveguide, where the black lines represent perfect conductor, and the white region is empty:

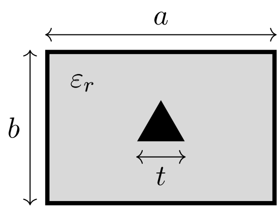


Original waveguide

Also consider two new waveguides with the cross-sections shown below, that are versions of the original waveguide: one of them is scaled with scale factor F, the other one is filled with certain dielectric of ε_r (represented in grey).



Scaled waveguide



Filled waveguide

a) What types of modes can propagate?

b) In the case of the fundamental modes of the original and scaled waveguides, determine the relationship between their characteristic impedances, and also their phase velocities.

c) Repeat the question, but comparing the original and the filled waveguides.

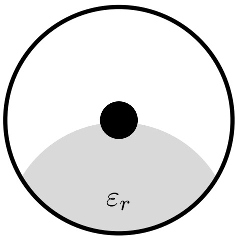
Answer

a) Both the characteristic impedances and phase velocities are the same.

b) Both decrease by a factor $\sqrt{\varepsilon_r}$.

Problem 9

The diagram below shows the cross-section of a waveguide, where the black lines represent perfect conductor, the white region is empty, and the grey region is filled with certain dielectric of permittivity ε_r. The waveguide is a coaxial cable that is mainly empty, with a dielectric support for the inner conductor.



Answer and reason the following questions:

a) What types of modes can propagate?

b) Indicate two characteristics of the fundamental mode.

c) For the fundamental mode, estimate a lower and a higher bound of the phase velocity.

d) From the previous result, what happens if $\varepsilon_r = 1.01$?

Answer

a) Hybrid modes. The fundamental mode is quasi-TEM.

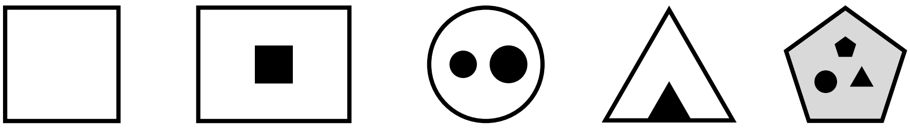
b) No cutoff (or zero) frequency, slightly dispersive.

c) $c_0 > v_p > c_0/\sqrt{\varepsilon_r}$.

d) The phase velocity is $v_p \approx c_0$, and therefore the mode is almost non-dispersive. In other words, the medium is almost homogeneous with dielectric constant $\varepsilon \approx \varepsilon_0$, and the fundamental mode can be considered TEM.

Problem 10

The diagrams shown below represent the cross-sections of five different waveguides. Black parts are perfect conductor, grey regions lossless dielectric of permittivity $\varepsilon > \varepsilon_0$, while white regions are empty.

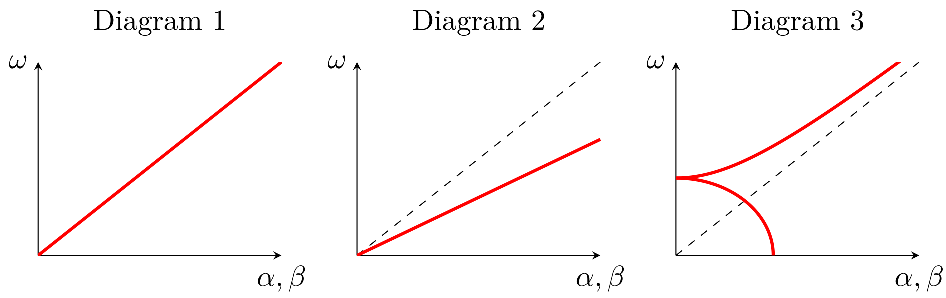


(a) (b) (c) (d) (e)

Answer the following questions:

a) For each case, indicate and reason which types of modes are supported. If TEM modes are supported, how many of them?

b) The plots below are three dispersion diagrams. The slope of the dashed line is the speed of light in vacuum. Indicate which dispersion diagram corresponds to each cross-section from the previous question, providing justification for your answer.



Answer

a) All the cases support TE and TM modes. The number of supported TEM modes is 0, 1, 2, 0, and 3, respectively.

b) The first dispersion diagram corresponds to a TEM mode with dielectric of $\varepsilon_r > 1$ (e). The second one is a TEM mode with vacuum (b, c). Finally, the third one is a TE or TM mode with vacuum (a, d).

Problem 11

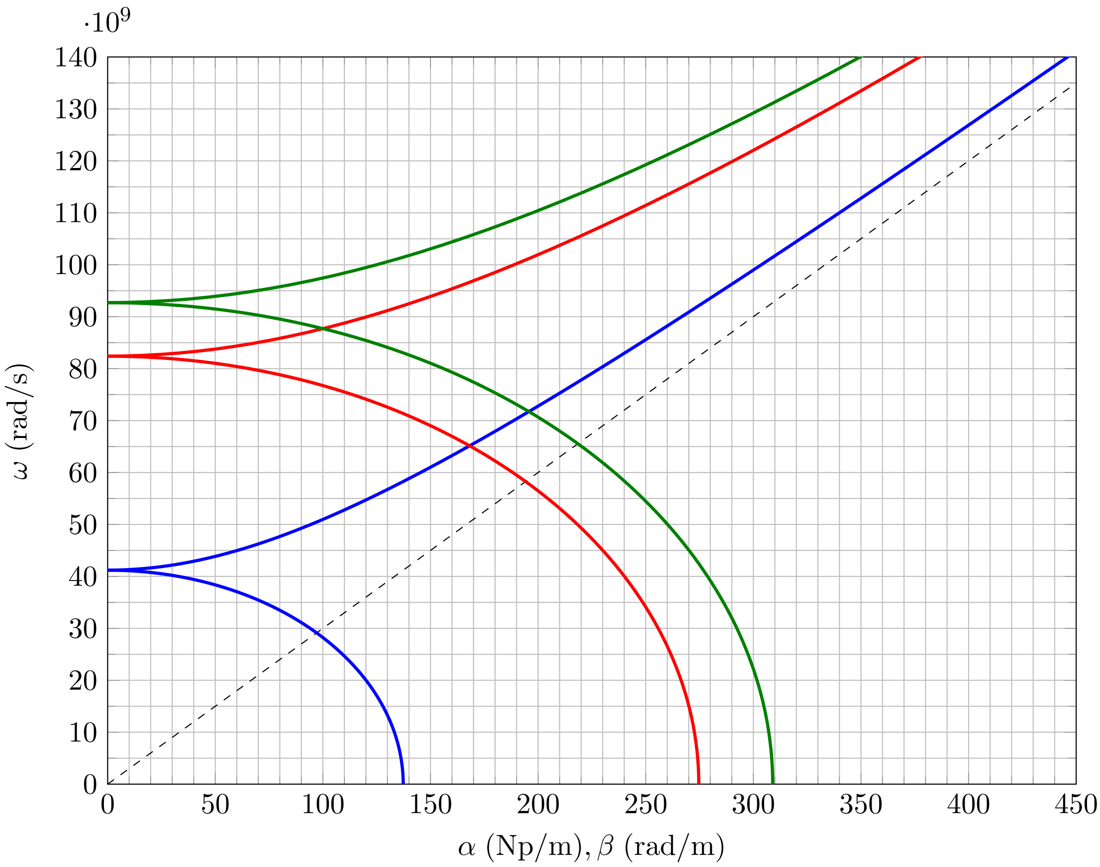
The plot below shows the dispersion diagram of the first three modes supported by a WR-90 rectangular waveguide. Using the diagram estimate the following values:

a) Single-mode bandwidth, in GHz.

b) Attenuation and phase constant of the fundamental mode, at 3.18 GHz and 11.9 GHz.

c) Phase velocity and group velocity of the fundamental mode, at 11.9 GHz.

d) Internal dimensions of the WR-90 waveguide cross-section.



Answer

a) 6.550 GHz.

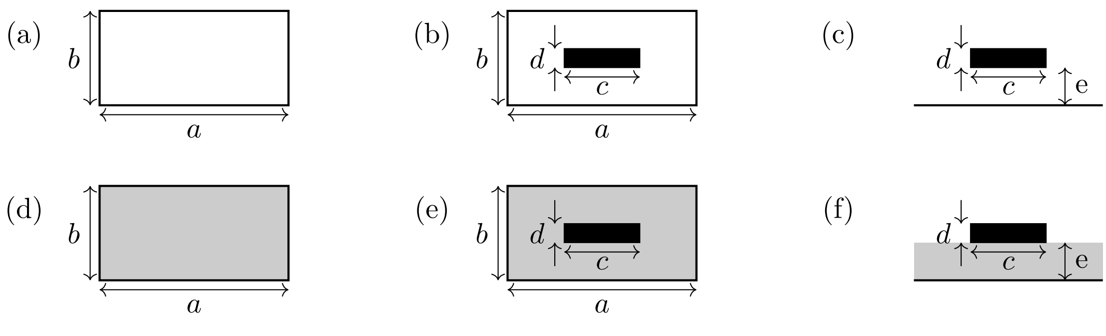
b) $\alpha = 120.18\,\text{Np/m}$ and $\beta = 208.13\,\text{rad/m}$.

c) $v_p = 3.592 \times 10^8\,\text{m/s}$ and $v_g = 2.502 \times 10^8\,\text{m/s}$.

d) $22.85\,\text{mm} \times 10.16\,\text{mm}$.

Problem 12

The following figures represent the cross-sections of several waveguides. Black parts are perfect conductor, grey areas are certain dielectric with relative permittivity $\varepsilon_r = 4$, while white areas are just vacuum. The dimensions are $a = 25\,\text{mm}$, $b = 12\,\text{mm}$, $c = 2\,\text{mm}$, $d = 0.1\,\text{mm}$, and $e = 1\,\text{mm}$.



Answer the following questions:

a) For each case, indicate the types of modes that are supported, identifying the fundamental mode, and computing its cutoff frequency.

b) For each case, compute the wavelength of the fundamental mode at $f = 2\,\text{GHz}$, if possible.

c) For each waveguide, sketch the dispersion diagram of the fundamental mode. Make use of the axis provided, where the dashed line represent the speed of light in vacuum. Justify your answer.

Answer

a) (a) TE+TM, TE_{10}, $f_{c,10} = 6\,\text{GHz}$
   (b) TE+TM+TEM, TEM, $f_c = 0$
   (c) TE+TM+TEM, TEM, $f_c = 0$
   (d) TE+TM, TE_{10}, $f_{c,10} = 3\,\text{GHz}$
   (e) TE+TM+TEM, TEM, $f_c = 0$
   (f) Hybrid modes, Quasi-TEM, $f_c = 0$

b) (a) Below cutoff
   (b) 15 cm
   (c) 15 cm
   (d) Below cutoff
   (e) 7.5 cm
   (f) 8.565 cm ($\varepsilon_{r,\text{ef}} \approx 3.067$)