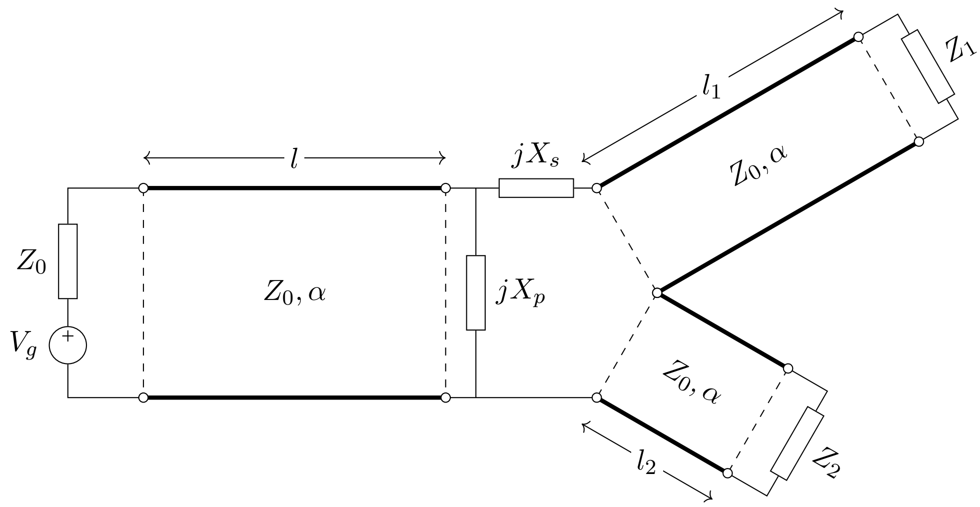
High Frequency Technologies

Problems on transmission lines

Problem 1

We want to feed two antennas with input impedances $Z_1 = 40\,\Omega$ and $Z_2 = 60\,\Omega$, with a generator of peak voltage $V_g = 10\,\mathrm{V}$, frequency $f_0 = 2\,\mathrm{GHz}$, and internal impedance $Z_0 = 50\,\Omega$. To do so, a lossless three-port network will be used, with an equivalent circuit shown in the figure (the part composed of the reactances $\mathrm{j}X_p$ and $\mathrm{j}X_s$). The three-port network is connected to the other elements through coaxial cables with characteristic impedance Z_0. The length of the cable that connects the common port and the generator is $\beta l = \frac{3\pi}{4}$ rad, while the antenna cables that connect the other ports with the loads Z_1 and Z_2 have lengths $\beta l_1 = \frac{\pi}{2}$ rad and $\beta l_2 = \pi$ rad, respectively. Find the power delivered to each antenna.



Note The coaxial cable attenuation is $3.5 \times 10^{-3}\,\mathrm{Np/cm}$. The matching network reactances are $X_s = 10\,\Omega$ and $X_p = 100\,\Omega$.

Answer

$P_1 = 86.129\,\mathrm{mW}$, $P_2 = 80.239\,\mathrm{mW}$.

Problem 2

A shunt resistance $R = 100\,\Omega$ is connected at an arbitrary point between the two conductors of a lossless transmission line with characteristic impedance $Z_0 = 50\,\Omega$. One of the transmission line ends is connected to a generator with internal impedance Z_0 and available power $P_{\mathrm{av}} = 250\,\mathrm{mW}$, while the other end is loaded with Z_0. Determine:

a) The incident and reflected voltage and current waves between the generator and the $100\,\Omega$ resistor, and also between the resistor and the load.

b) The power dissipated in the resistor, the power delivered to the load, and also the power reflected to the generator.

Answer

- Power reflected to the generator: $10\,\mathrm{mW}$
- Power delivered to the load: $160\,\mathrm{mW}$
- Power dissipated in R: $80\,\mathrm{mW}$

Problem 3

Find the ratio between the maximum power that can be transmitted in a lossless transmission line when its load is matched (P_1), and the transmitted power when there is an arbitrary standing wave ratio S (P_2). Suppose that the only limitation is the electric breakdown, so that the maximum electric field strength supported by the dielectric is E_{rup}.

Answer

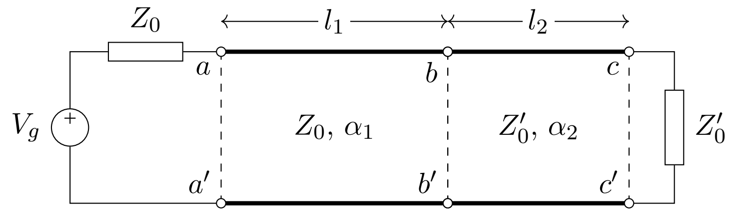
$\frac{P_1}{P_2} = \text{SWR}$

Problem 4

Using the computer, characterize the electrical parameters along a transmission line terminated with loads corresponding to different reflection coefficient values Γ_L. Try this five loads: short circuit, open circuit, pure inductance, pure capacitance, and a complex load with inductive character. Draw both the voltage and current waves, as well as the reflection coefficient. Do it for different time instants.

Problem 5

Consider the following network, composed of two sections of transmission line, with a source and a load.



Find:

a) The power P_a, delivered at the input of the first transmission line section (a/a').

b) The power P_b, delivered at the input of the second transmission line section (b/b').

c) The power P_c, delivered to the load (c/c').

d) If $\alpha_1 = \alpha_2 = 0$ (lossless transmission lines), sketch the envelope of the voltage along the transmission line sections.

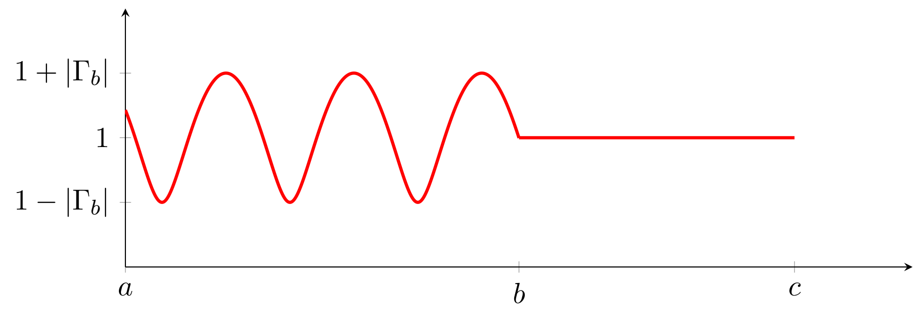
Answer

a) $P_a = \frac{|V_g|^2}{8Z_0}\left(1 - \left|\frac{Z_0'-Z_0}{Z_0'+Z_0}\right|^2 e^{-4\alpha_1 l_1}\right)$

b) $P_b = \frac{|V_g|^2}{8Z_0}\left(1 - \left|\frac{Z_0'-Z_0}{Z_0'+Z_0}\right|^2\right) e^{-2\alpha_1 l_1}$

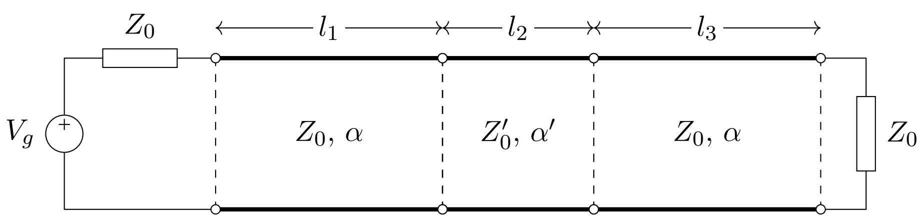
c) $P_c = \frac{|V_g|^2}{8Z_0}\left(1 - \left|\frac{Z_0'-Z_0}{Z_0'+Z_0}\right|^2\right) e^{-2(\alpha_1 l_1 + \alpha_2 l_2)}$

d) Voltage envelope (with $\Gamma_b = \frac{Z_0'-Z_0}{Z_0'+Z_0}$):



Problem 6

Compute the power delivered to the load $Z_L = Z_0$ from the source with frequency 1 GHz, peak voltage $V_g = 10\,\mathrm{V}$, and internal impedance Z_0, when they are connected by three cascaded sections of transmission lines, as shown in the schematic,



where the dielectric of the transmission lines has $\varepsilon_r = 1$, their lengths are $l_1 = 2\,\mathrm{m}$, $l_2 = 10\,\mathrm{cm}$, $l_3 = 1\,\mathrm{m}$, and their respective characteristic impedances and attenuations are:

- First and third sections: $Z_0 = 50\,\Omega$, $\alpha = 3.5 \times 10^{-3}\,\mathrm{Np/cm}$.
- Second section: $Z_0' = 75\,\Omega$, $\alpha' = 3 \times 10^{-3}\,\mathrm{Np/cm}$.

Answer

The power delivered to the load is

$$P_L = P_{\text{av}} \frac{(1-|\Gamma_{1o}|^2)(1-|\Gamma_{2o}|^2)(1-|\Gamma_{3o}|^2)}{(1-|\Gamma_{2i}|^2)(1-|\Gamma_{3i}|^2)}\, e^{-2(\alpha l_1 + \alpha' l_2 + \alpha l_3)} = 25.57\,\text{mW}$$

where Γ_{ki} and Γ_{ko} are, respectively, the reflection coefficients seen at the input and output of the k^{th} transmission line section,

$$\begin{aligned} \Gamma_{1i} &= 0.0814\, e^{-2.5687j} & \Gamma_{1o} &= 0.3300\, e^{-0.474j} \\ \Gamma_{2i} &= 0.1884\, e^{-1.047j} & \Gamma_{2o} &= -0.2 \\ \Gamma_{3i} &= 0 & \Gamma_{3o} &= 0 \end{aligned}$$

and $P_{\text{av}} = 250\,\text{mW}$ is the available power of the source.