# TRANSMISSION LINES: SMITH CHART

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This chapter presents the Smith chart, which is one of the basic tools in the analysis and design of any microwave circuit.

Smith chart is an abacus that represents in a POLAR plane the reflection coefficient at a specific position of a loaded line. The fact that there is a bijective relation between the reflection coefficient at that point and the impedance enables Smith chart to represent in a unique and constrained manner for passive loads, the impedance at that point on the line.

# Learning goals

- Knowledge: concept of reflection coefficient and standing wave ratio; relationship with impedance.
- Tools: Smith chart
- Use of Smith chart:
  - Direct representation of reflection coefficients
  - Direct reading of impedance
  - Movement along a transmission line.
  - Impedance matching.

# Outline

- Telegrapher’s equations
- Loaded line parameters
- Power considerations in a loaded line: mismatching coefficient
- Smith chart
- Smith chart applications:
  - Relationship between impedances and reflection coefficient
  - Movement along a line
  - Impedance matching
- Bode-Fano criterion

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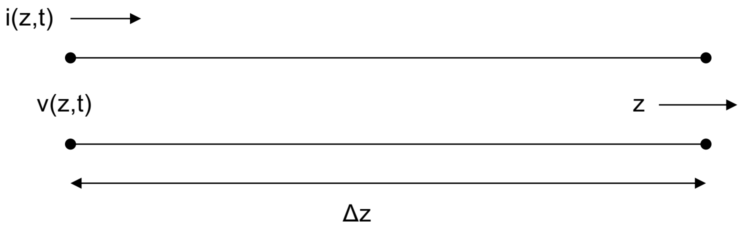
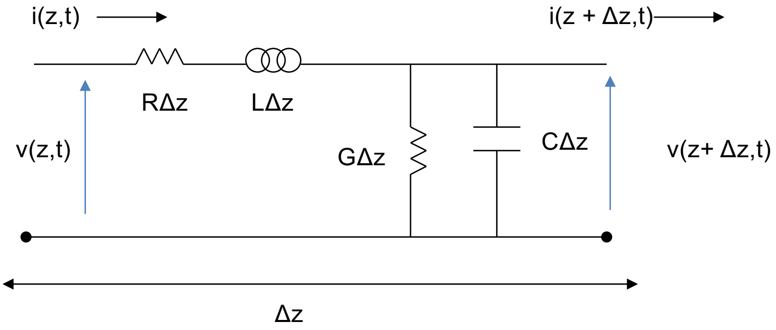
# Propagation equations in a line

Given a transmission line:




An equivalent circuit model for it can be obtained…

# Propagation equations in a line




R = series resistance per unit length, Ω/m

L = series inductance per unit length, H/m

G = shunt conductance per unit length, S/m

C = capacitance per unit length, F/m

# Telegrapher's equation

By Kirchoff's law:

$$v(z,t)-R\Delta z\cdot i(z,t)-L\cdot\Delta z\cdot\frac{\partial i(z,t)}{\partial t}-v(z+\Delta z,t)=0$$

$$i(z,t)-G\Delta z\cdot V(z+\Delta z,t)-C\Delta z\frac{\partial v(z+\Delta z,t)}{\partial t}-i(z+\Delta z,t)=0$$

$\Delta z \longrightarrow 0$

$$\frac{\partial v(z,t)}{\partial z}=-R\cdot i(z,t)-L\cdot\frac{\partial i(z,t)}{\partial t}$$

$$\frac{\partial i(z,t)}{\partial z}=-G\cdot v(z,t)-C\cdot\frac{\partial v(z,t)}{\partial t}$$

Application of Fourier transformation in t

# Propagation equation in a transmission line

$$\left.\begin{aligned}\frac{dV(z)}{dz} &= -(R + jwL)\cdot I(z)\\ \frac{dI(z)}{dz} &= -(G + jwC)\cdot V(z)\end{aligned}\right\}$$ Similarity with Maxwell's equations

$$\boxed{\frac{d^2V(z)}{dz} - \gamma^2 V(z) = 0}$$

$$\boxed{\frac{d^2I(z)}{dz} - \gamma^2 I(z) = 0}$$

$$\gamma = \alpha + j\beta = \sqrt{(R + jwL)\cdot(G + jwC)}$$

PROPAGATION CONSTANT

# Propagation equation in a transmission line

$$V(z) = V_o^+ \cdot e^{-\gamma z} + V_o^- \cdot e^{\gamma z}$$

$$I(z) = I_o^+ \cdot e^{-\gamma z} + I_o^- \cdot e^{\gamma z}$$

$$\Downarrow$$

$$I(z) = \frac{1}{Z_o}\left[V_o^+ \cdot e^{-\gamma z} - V_o^- \cdot e^{\gamma z}\right] = \frac{\gamma}{R + jwL}\left[V_o^+ \cdot e^{-\gamma z} - V_o^- \cdot e^{\gamma z}\right]$$

$$\frac{V_o^+}{I_o^+} = Z_o = -\frac{V_o^-}{I_o^-}$$

$$Z_o = \frac{R + jwL}{\gamma} = \sqrt{\frac{R + jwL}{G + jwC}} \qquad I(z) = \frac{V_o^+ \cdot e^{-\gamma z} - V_o^- \cdot e^{\gamma z}}{Z_o}$$

# Propagation equation in a transmission line

(time domain)

$$v(z,t) = \left|V_o^+\right| \cdot \cos\left(wt - \beta z + \phi^+\right) \cdot e^{-\alpha z} + \left|V_o^-\right| \cdot \cos\left(wt + \beta z + \phi^-\right) \cdot e^{\alpha z}$$

$$\lambda = \frac{2\pi}{\beta} \qquad v_p = \frac{w}{\beta} = \lambda \cdot f$$

# Equation in lossless transmission lines

$$\left.\gamma = \alpha + j\beta = jw\sqrt{LC}\right\} \quad \begin{matrix} \beta = w\sqrt{LC} \\ \alpha = 0 \end{matrix} \quad Z_o = \sqrt{\frac{L}{C}}$$

$$V(z) = V_o^+ \cdot e^{-j\beta z} + V_o^- \cdot e^{j\beta z}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{w\sqrt{LC}}$$

$$I(z) = \frac{V_o^+}{Z_o} \cdot e^{-j\beta z} - \frac{V_o^-}{Z_o} \cdot e^{j\beta z}$$

$$v_p = \frac{w}{\beta} = \frac{1}{\sqrt{LC}}$$

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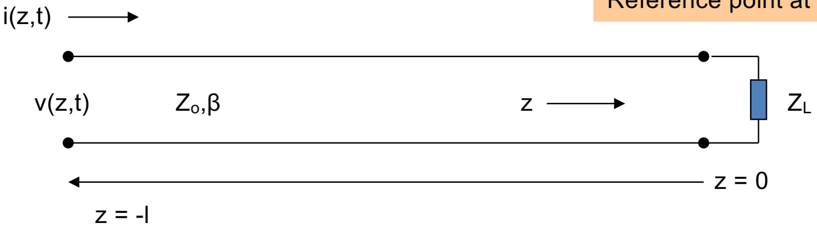
# Loaded lossless transmission lines

A regressive wave appears when the line has a termination condition

$$V(z) = V_o^+ \cdot e^{-j\beta z} + V_o^- \cdot e^{j\beta z}$$

$$I(z) = \frac{V_o^+}{Z_o} \cdot e^{-j\beta z} - \frac{V_o^-}{Z_o} \cdot e^{j\beta z}$$

Reference point at load




With the reference point at load, we force z'=-z

$$\boxed{V(z) = V_o^+ e^{j\beta z} + V_o^- \cdot e^{-j\beta z}}$$

$$\boxed{I(z) = \frac{1}{Z_o}\left[V_o^+ e^{j\beta z} - V_o^- \cdot e^{-j\beta z}\right]}$$

# Loaded lossles transmission lines: definition of reflection coefficient

$$Z_L = \frac{V(0)}{I(0)} = \frac{V_o^+ + V_o^-}{V_o^+ - V_o^-} Z_o \Rightarrow V_o^- = \frac{Z_L - Z_o}{Z_L + Z_o} V_o^+$$

$$\boxed{\Gamma_L = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_o}{Z_L + Z_o}}$$

Bilinear relationship between reflection coefficient and impedance: BIJECTIVE, each reflection coefficient corresponds to one and ONLY ONE impedance

$$V(z) = V_o^+ \left[ e^{j\beta l} + \Gamma \cdot e^{-j\beta l} \right]$$

$$I(z) = \frac{V_o^+}{Z_o} \left[ e^{j\beta l} - \Gamma \cdot e^{-j\beta l} \right]$$

Current reflection coefficient

$$\Gamma_I = \frac{-V_o^-}{V_o^+} = -\Gamma_V = -\frac{Z_L - Z_o}{Z_L + Z_o} = \frac{Z_o - Z_L}{Z_o + Z_L} = \frac{Y_L - Y_o}{Y_L + Y_o}$$

We will use the voltage reflection coefficient to avoid ambiguities

# Loaded lossless transmission lines: standing wave ratio

- The wave on the line is formed by an incident and a reflected wave, resulting in a standing wave.
- The magnitude of its envelope is fixed at specific positions:

$$\left|V(z)\right| = \left|V_o^+\right| \cdot \left|1 + \Gamma \cdot e^{2j\beta z}\right| = \left|V_o^+\right| \cdot \left|1 + \Gamma \cdot e^{-2j\beta l}\right| = \left|V_o^+\right| \cdot \left|1 + \left|\Gamma\right| \cdot e^{j(\theta - 2\beta l)}\right|$$

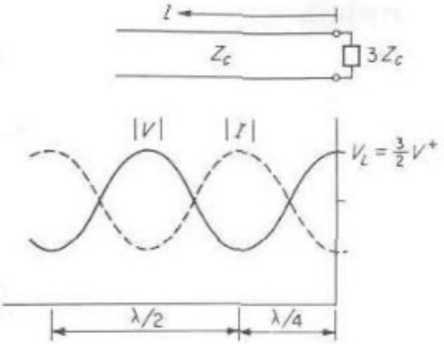



Image taken from Collin.

- Parameters:
  - Return losses:

$$P_{av} = \frac{1}{2}\frac{\left|V_o^+\right|^2}{Z_o}\left(1 - \left|\Gamma\right|^2\right) \qquad \boxed{RL = -20 \cdot \log\left(\left|\Gamma\right|\right)} \text{ dB}$$

  - Standing wave ratio:

$$V_{max} = \left|V_o^+\right| \cdot \left(1 + \left|\Gamma\right|\right) \qquad V_{min} = \left|V_o^+\right| \cdot \left(1 - \left|\Gamma\right|\right) \qquad \boxed{ROE = SWR = \frac{V_{max}}{V_{min}} = \frac{1 + \left|\Gamma\right|}{1 - \left|\Gamma\right|}} \qquad 1 < SWR < \infty$$

# Reflection coefficient at a point on the line

$$\Gamma(l) = \frac{V^{-}{}_{o} \cdot e^{-j\beta l}}{V^{+}{}_{o} \cdot e^{j\beta l}} = \Gamma(0) e^{-2j\beta l}$$

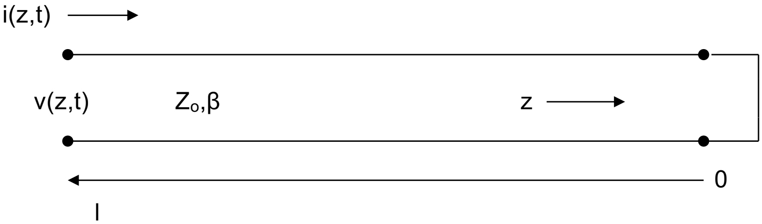
REFLECTION COEFFICIENT IN THE REST OF THE LINE

The reflection coefficient in a lossless line has a constant amplitude as the wave propagates along the line: it is represented by a circle

$$Z_{in}(z) = \frac{V(-z)}{I(-z)} = \frac{1+\Gamma e^{-2j\beta Z}}{1-\Gamma e^{-2j\beta Z}} Z_o = Z_o \frac{Z_L \cos(\beta z) + jZ_o \sin(\beta z)}{Z_o \cos(\beta z) + jZ_L \sin(\beta z)} = Z_o \frac{Z_L + jZ_o \tan(\beta z)}{Z_o + jZ_L \tan(\beta z)}$$

Examples of particular cases…

# Particular cases: short-circuited line

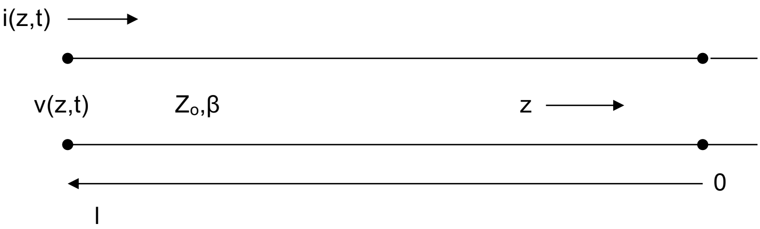



$$V(z)=V_o^+\left[e^{-j\beta z}-e^{j\beta z}\right]=-2j\cdot V_o^+\cdot sen(\beta z)$$

$$I(z)=\frac{V_o^+}{Z_0}\left[e^{-j\beta z}+e^{j\beta z}\right]=\frac{2V_o^+}{Z_0}\cdot\cos(\beta z)$$

$$\boxed{Z_{in}=jZ_o\tan(\beta l)}$$

# Particular cases: open-circuited line

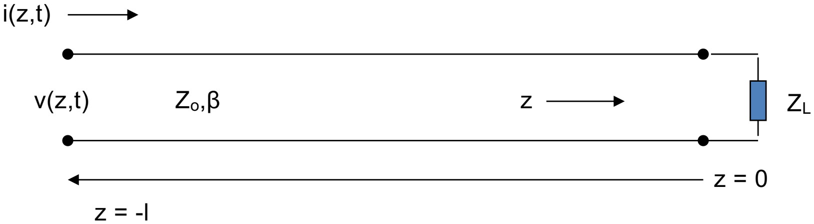



$$V(z) = V_o^+\left[e^{-j\beta z} + e^{j\beta z}\right] = 2V_o^+ \cdot \cos(\beta z)$$

$$I(z) = \frac{V_o^+}{Z_0}\left[e^{-j\beta z} - e^{j\beta z}\right] = \frac{2j \cdot V_o^+}{Z_0} \cdot sen(\beta z)$$

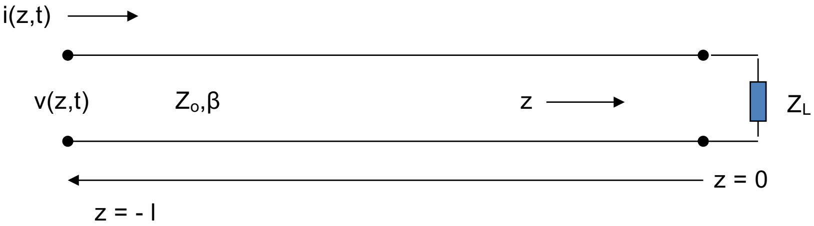
$$\boxed{Z_{in} = -j \cdot Z_o \cot(\beta l)}$$

# Particular cases: λ/2 line



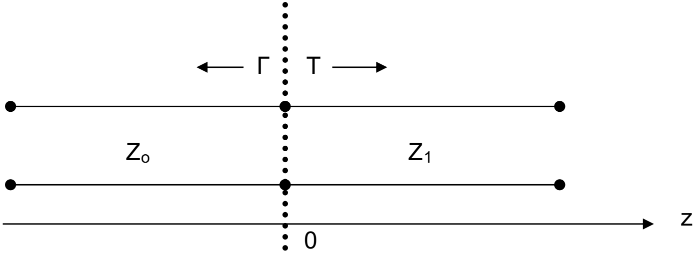

$$\boxed{Z_{in} = Z_L}$$

# Particular cases: λ/4 line




$$\boxed{Z_{in} = \frac{Z^2{}_o}{Z_L}}$$

# Particular cases: line coupled to another line



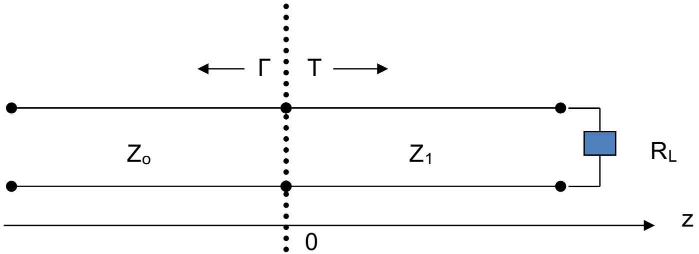

$$\Gamma = \frac{Z_1 - Z_o}{Z_1 + Z_o}$$

$$V(z) = V_o^+ \left[ e^{-j\beta z} + \Gamma \cdot e^{j\beta z} \right] \qquad z<0$$

$$V(z) = V_o^+ \cdot T \cdot e^{-j\beta z} \qquad z>0$$

$$T = 1 + \Gamma = \frac{2Z_1}{Z_1 + Z_o}$$

# Particular cases: multiple reflections theory




$\Gamma_1$ ⟵  $T_1$ ⟶  ⟵ $\Gamma_3$

$\Gamma_2$ ⟶

$$\Gamma_1 = \frac{Z_1 - Z_o}{Z_1 + Z_o}$$

$$\Gamma_2 = \frac{Z_0 - Z_1}{Z_0 + Z_1} = -\Gamma_1$$

$$\Gamma_3 = \frac{R_L - Z_1}{R_L + Z_1}$$

$$T_1 = 1 + \Gamma_1 = \frac{2Z_1}{Z_1 + Z_o}$$

$$T_2 = \frac{2Z_o}{Z_1 + Z_o}$$

# Particular cases: multiple reflections theory

$$\Gamma = \Gamma_1 - T_1 T_2 \Gamma_3 + T_1 T_2 \Gamma_2 {\Gamma^2}_3 - T_1 T_2 {\Gamma^2}_2 {\Gamma^3}_3 + \ldots =$$

$$= \Gamma_1 - T_1 T_2 \Gamma_3 \sum_{n=0}^{\infty} \left(-\Gamma_2 \Gamma_3\right)^n$$

$$\Downarrow$$

Serie geométrica

$$\Gamma = \frac{\Gamma_1 + \Gamma_1 \Gamma_2 \Gamma_3 - T_1 T_2 \Gamma_3}{1 + \Gamma_2 \Gamma_3} = \frac{2\left({Z^2}_1 - Z_o R_L\right)}{\left(Z_1 + Z_o\right)\left(R_L + Z_1\right)}$$

# Lossy transmission line

$$\gamma = \alpha + j\beta = \sqrt{(R + jwL)\cdot(G + jwC)}$$

$$R << wL$$
$$G << wC$$

$$\Rightarrow \begin{cases} \alpha \cong \dfrac{1}{2}\left(\dfrac{R}{Z_o} + GZ_o\right) \\ \beta \cong w\sqrt{LC} \end{cases} \qquad Z_o \cong \sqrt{\dfrac{L}{C}}$$

Heaviside's line $\Rightarrow$ $$\frac{R}{L} = \frac{G}{C} \Rightarrow \begin{cases} \alpha = R\sqrt{\dfrac{C}{L}} \\ \beta = w\sqrt{LC} \end{cases}$$

# Lossy transmission line

$$V(z) = V_o^+\left[e^{-\gamma z} + \Gamma \cdot e^{\gamma z}\right] \qquad I(z) = \frac{V_o^+}{Z_o}\left[e^{-\gamma z} - \Gamma \cdot e^{\gamma z}\right]$$

$$\Gamma(l) = \frac{V_o^- \cdot e^{-\gamma l}}{V_o^+ \cdot e^{\gamma l}} = \Gamma(0)e^{-2\alpha l}e^{-2j\beta l}$$

The reflection coefficient in a lossy line has a decreasing amplitude with the attenuation factor as it propagates along the line: it is represented by a spiral

$$Z_{in} = \frac{V(-l)}{I(-l)} = Z_o\frac{Z_L + Z_o\tanh(\gamma l)}{Z_o + Z_L\tanh(\gamma l)}$$

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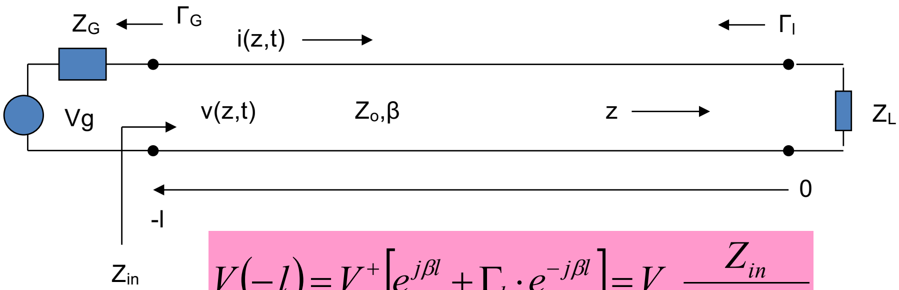
# Power in a transmission line (lossless)

**Load and generator mismatching**




$$V(-l)=V_o^+\left[e^{j\beta l}+\Gamma_l\cdot e^{-j\beta l}\right]=V_g\frac{Z_{in}}{Z_{in}+Z_g}$$

$$V_o^+=V_g\frac{Z_o}{Z_o+Z_g}\frac{e^{-j\beta\cdot l}}{1-\Gamma_l\cdot\Gamma_g e^{-2j\beta\cdot l}}$$

$$\Gamma_g=\frac{Z_g-Z_o}{Z_g+Z_o}$$

# Power in a transmission line (lossless)

$$P = \frac{1}{2}\mathrm{Re}\left\{V_{in} I^{*}_{in}\right\} = \frac{1}{2}\left|V_g\right|^2 \mathrm{Re}\left\{\frac{1}{Z_{in}}\right\} =$$

$$\frac{1}{2}\left|V_g\right|^2 \frac{R_{in}}{\left(R_{in}+R_g\right)^2+\left(X_{in}+X_g\right)^2}$$

Power delivered to load

# Power in a transmission line (lossless)

- There are three scenarios: load matched to the line, generator matched to the line and conjugate matching

**Load matched to the line**

$$P = \frac{1}{2}|V_g|^2 \frac{Z_o}{(Z_o + R_g)^2 + (X_g)^2}$$

**Generator matched to loaded line**

$$P = \frac{1}{2}|V_g|^2 \frac{R_g}{4(R_g + X_g)^2}$$

**Conjugate matching**

$$P = \frac{1}{2}|V_g|^2 \frac{1}{4R_g}$$

$$Z_{in} = Z_g^*$$

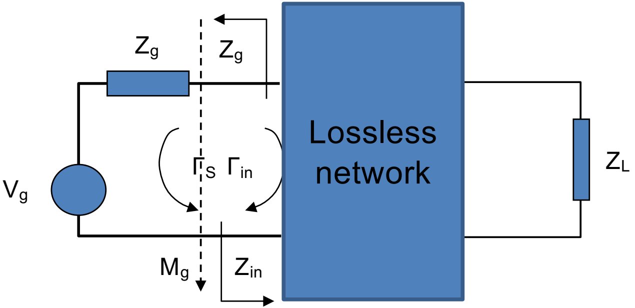
# Mismatch coefficient concept

- Available power of a generator

$$\boxed{P_{dg} = \frac{1}{8} \cdot \frac{|V_g|^2}{R_g}}$$

- Input power to the lossless network

$$\boxed{P_{in} = \frac{1}{8} \frac{|V_g|^2}{R_g} \left[ \frac{4 \cdot R_g \cdot R_{in}}{|Z_g + Z_{in}|^2} \right] = P_{dg} \cdot M_g}$$




- Conjugate matching for maximum power transmission

$$Z_{in} = Z_g^*$$

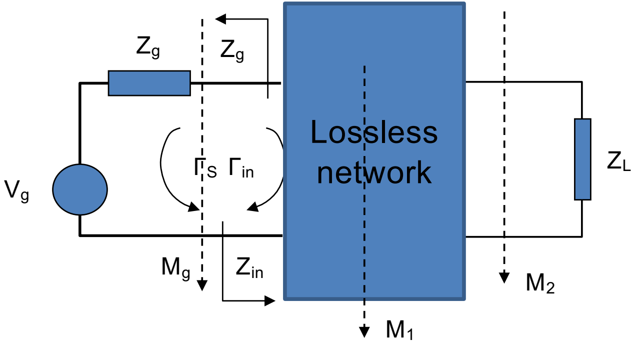
- Conjugate reflection coefficient:

$$\rho_{in} = \frac{Z_{in} - Z_g^*}{Z_{in} + Z_g}$$

- Relationship between conjugate reflection coefficient and mismatch coefficient:

$$\boxed{M_g = 1 - \rho_{in}^2}$$

# Mismatch coefficient concept




- Theorem: the mismatch coefficient across a lossless matching network remains constant throughout the entire structure.

$$\boxed{M_g = M_1 = M_2}$$

# Power in a transmission line (lossy)

$$V(z) = V_o^+\left[e^{-\gamma z} + \Gamma \cdot e^{\gamma z}\right] \qquad I(z) = \frac{V_o^+}{Z_o}\left[e^{-\gamma z} - \Gamma \cdot e^{\gamma z}\right]$$

The reflection coefficient in a lossy line has a decreasing amplitude with the attenuation factor as it propagates along the line: it is represented by a spiral

$$\boxed{Z_{in} = \frac{V(-l)}{I(-l)} = Z_o \frac{Z_L + Z_o \tanh(\gamma l)}{Z_o + Z_L \tanh(\gamma l)}}$$

$$P_{in} = \frac{\left|V_o^+\right|^2}{2Z_o}\left[1 - \left|\Gamma(l)\right|^2\right] e^{2\alpha l} \qquad P_L = \frac{\left|V_o^+\right|^2}{2Z_o}\left[1 - \left|\Gamma\right|^2\right]$$

Being $P_L$ the power at the load

$$\underbrace{\qquad\qquad}$$

$$P_{loss} = P_{in} - P_L = \frac{\left|V_o^+\right|^2}{2Z_o}\left[\left(e^{2\alpha l} - 1\right) + \left|\Gamma\right|^2\left(1 - e^{2\alpha l}\right)\right]$$

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# Reflection coefficient definition

$$Z_L = \frac{V(0)}{I(0)} = \frac{V_o^+ + V_o^-}{V_o^+ - V_o^-} Z_o$$

$$V_o^- = \frac{Z_L - Z_o}{Z_L + Z_o} V_o^+$$

$$\boxed{\Gamma = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_o}{Z_L + Z_o}}$$

$$V(z) = V_o^+ \left[ e^{j\beta l} + \Gamma \cdot e^{-j\beta l} \right]$$

**Bilinear relationship between impedance and reflection coefficient**

$$I(z) = \frac{V_o^+}{Z_o} \left[ e^{j\beta l} - \Gamma \cdot e^{-j\beta l} \right]$$

$$\boxed{ROE = SWR = \frac{V_{max}}{V_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}}$$

# Reflection coefficient at any point of the line

$$\Gamma(l)=\frac{V^{-}{}_{o}\cdot e^{-j\beta l}}{V^{+}{}_{o}\cdot e^{j\beta l}}=\Gamma(0)e^{-2j\beta l}$$

REFLECTION COEFFICIENT IN THE REST OF THE LINE

$$\boxed{Z_{in}=\frac{V(-l)}{I(-l)}=\frac{1+\Gamma(0)e^{-2j\beta l}}{1-\Gamma(0)e^{-2j\beta l}}Z_o=Z_o\frac{Z_L+jZ_o\tan(\beta l)}{Z_o+jZ_L\tan(\beta l)}}$$

Examples of particular cases…

# Properties of the reflection coefficient and the standing wave

- As a consequence of reflection at the load, both voltage and current amplitude remains stationary along each line abscissa.
- The maxima occur when $(\theta-2\beta l) = 2n\pi$
- The minima occur when $(\theta-2\beta l) = (2n-1)\pi$
- Voltage maxima coincide with current minima, and viceversa.
- In a lossless line the magnitude of the reflection coefficient remains constant. $\Gamma(l)=\Gamma(0)e^{-2j\beta l}$
- This locus is a circle in the complex plane of $\Gamma(l)$

# Properties of the reflection coefficient and the standing wave

- Exists a bilinear relationship between impedances and coefficients:

$$\boxed{Z(l)=\frac{1+\Gamma(0)e^{-2j\beta l}}{1-\Gamma(0)e^{-2j\beta l}}\cdot Z_o=\frac{1+\Gamma(l)}{1-\Gamma(l)}\cdot Z_o\Rightarrow\Gamma(l)=\frac{Z(l)-Z_0}{Z(l)+Z_0}}$$

- Each reflection coefficient corresponds to one and only one value of impedance.
- Concept of impedance normalization.

# Smith chart

$$Z_L = \frac{1+\Gamma_L}{1-\Gamma_L}$$

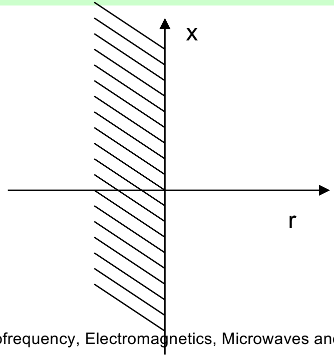
**One-to-one match**

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{\overline{Z_L} - 1}{\overline{Z_L} + 1}$$

Complex plane of impedances.
Cartesian representation.
Half-plane.

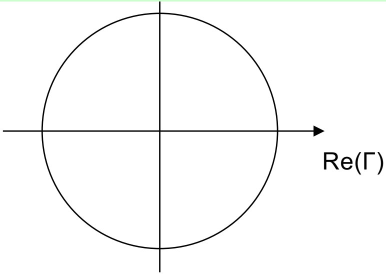
Complex plane of coefficients $\Gamma_L$.
Polar representation.
Plane limited by the circle $|\Gamma_L|=1$.

2 families of perpendicular lines




**Bijection**

2 families of perpendicular circles

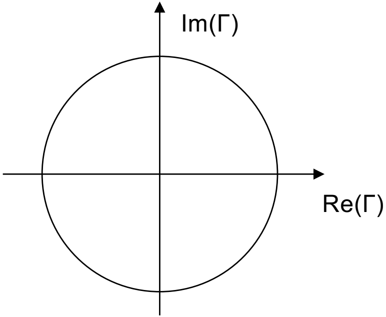


# Smith chart

Complex plane of coefficients $\Gamma_L$.

Polar representation: variables the radius and the angle

Plane limited by the circle $|\Gamma_L|=1$.



# Smith chart

Normalization

$$Z(l)=\frac{1+\Gamma(l)}{1-\Gamma(l)}Z_o \quad\longrightarrow\quad \overline{Z(l)}=\frac{Z(l)}{Z_o}=r+jx$$

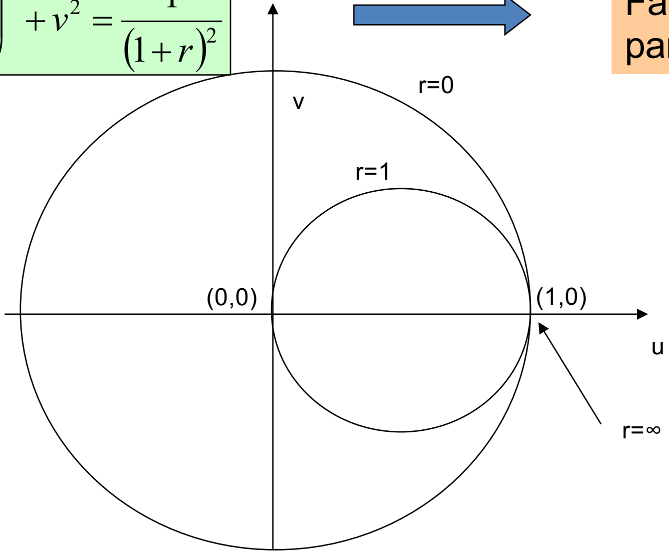
$$\left.\begin{array}{l} w=u+jv=\Gamma_L e^{-2j\beta l} \\ \Gamma_L=\dfrac{Z_L-Z_o}{Z_L+Z_o}=\dfrac{\overline{Z_L}-1}{\overline{Z_L}+1} \end{array}\right\} \quad r+jx=\frac{1+(u+jv)}{1-(u+jv)}$$

$$\left.\begin{array}{l} r=\dfrac{1-\left(u^2+v^2\right)}{(1-u)^2+v^2} \\ x=\dfrac{2v}{(1-u)^2+v^2} \end{array}\right\} \longrightarrow \left\{\begin{array}{l} \boxed{\left(u-\dfrac{r}{1+r}\right)^2+v^2=\dfrac{1}{(1+r)^2}} \\ \boxed{(u-1)^2+\left(v-\dfrac{1}{x}\right)^2=\dfrac{1}{x^2}} \end{array}\right.$$

# Smith chart: resistance curves

$$\left(u-\frac{r}{1+r}\right)^2+v^2=\frac{1}{(1+r)^2}$$

Family of circles with r as parameter

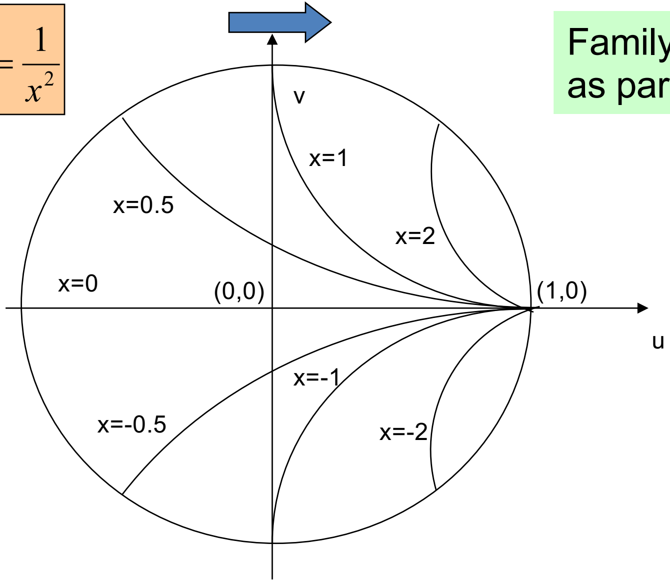



*Center* $\left(\frac{\mathrm{r}}{1+\mathrm{r}},0\right)$

*Radius* $\frac{1}{1+r}$

# Smith chart: reactance curves

$$(u-1)^2+\left(v-\frac{1}{x}\right)^2=\frac{1}{x^2}$$

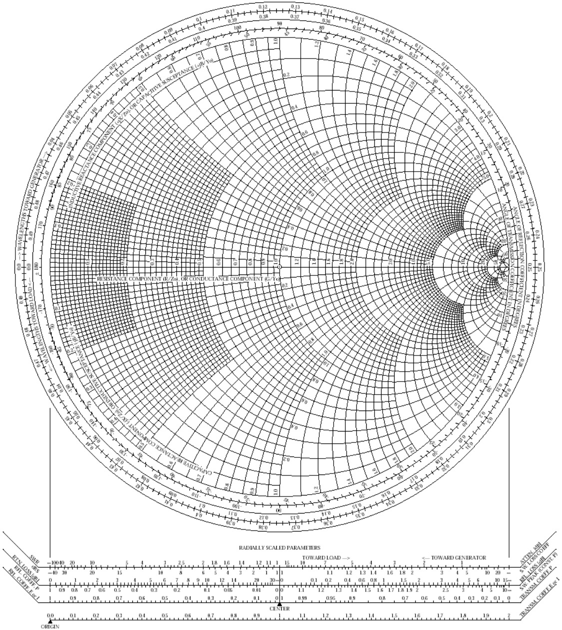



Family of circles with x as parameter

*Center* $\left(1,\frac{1}{x}\right)$

*Radius* $\frac{1}{x}$

# Smith chart

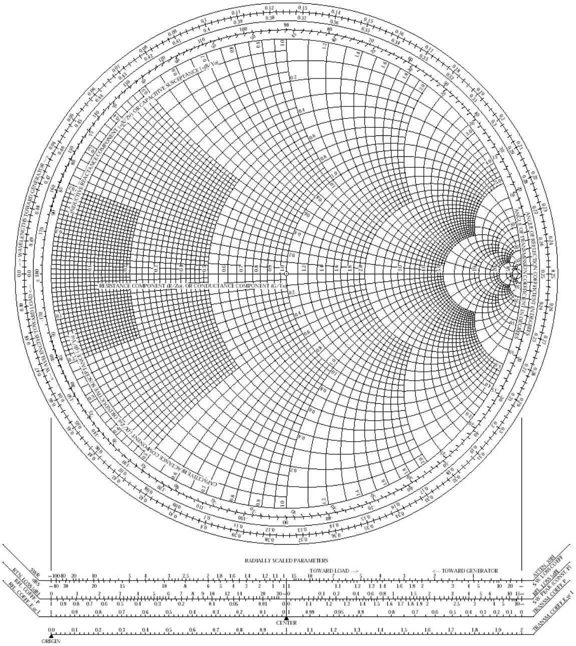

RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
RADIALLY SCALED PARAMETERS
TOWARD LOAD —>
<— TOWARD GENERATOR
CENTER
ORIGIN

# Outline

- Telegrapher's equations
- Loaded line parameters
- Power considerations in a loaded line: mismatching coefficient
- Smith chart
- Smith chart applications:
  - Relationship between impedances and reflection coefficient
  - Movement along a line
  - Impedance matching
- Bode-Fano criterion

# Smith chart: movement along a line



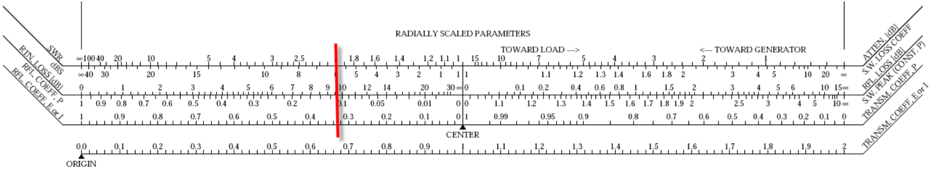


Meaning of the direction of movement on the chart?

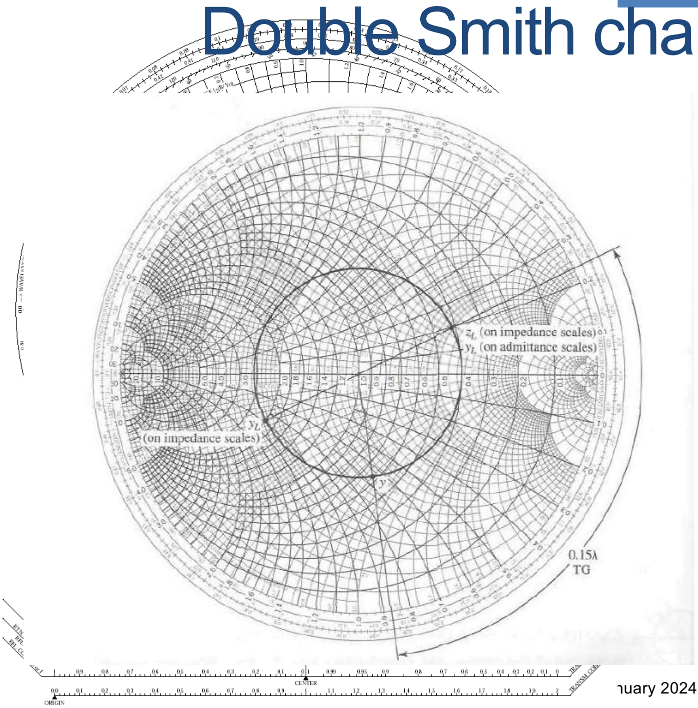
Clockwise: toward generator
Anti-clockwise: toward load

# Calculator (abacus) in Smith chart

For a VSWR of 2, drawing a vertical line, we can see that the voltage reflection coefficient is 0.33, the power reflection coefficient is 0.11, which, in dB, is 9.54 dB.



# Double Smith chart ZY




The coefficient $\Gamma_v$ = - $\Gamma_I$

Switching from impedance to admitances involves rotating 180º the previous chart or make a dual reading on the double ZY chart

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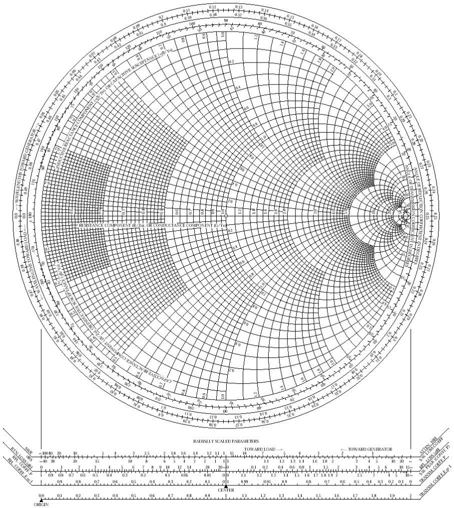
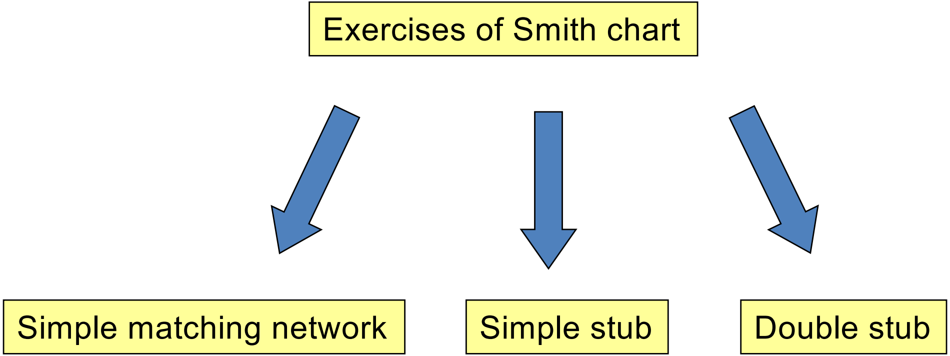
# Impedance matching

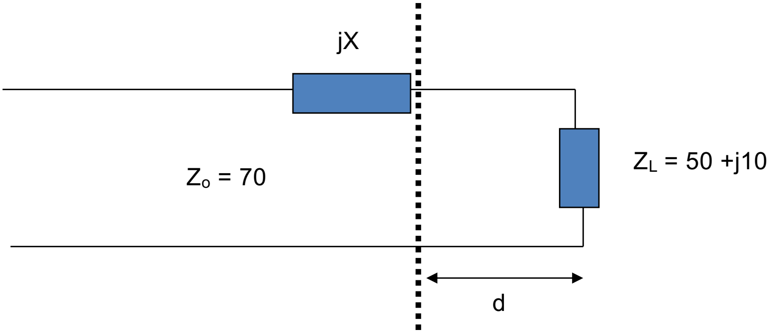
- Involves moving from an initial point of reflection coefficient (impedance) to a final one.
- Tipically, but no always, the final point is the origin: reflection coefficient 0 or normalized impedance 1.
- To make that movement, we can only move along circles of constant parameter:
  - Movement along a lossless line: circle with constant amplitude of reflection coefficient.
  - Inclusion of a lossless matching cell: movement along a cirle of constant r or g.
  - Inclusion of a matching cell with losses (only): movement along a circle of constant reactance (not common).



# Impedance matching



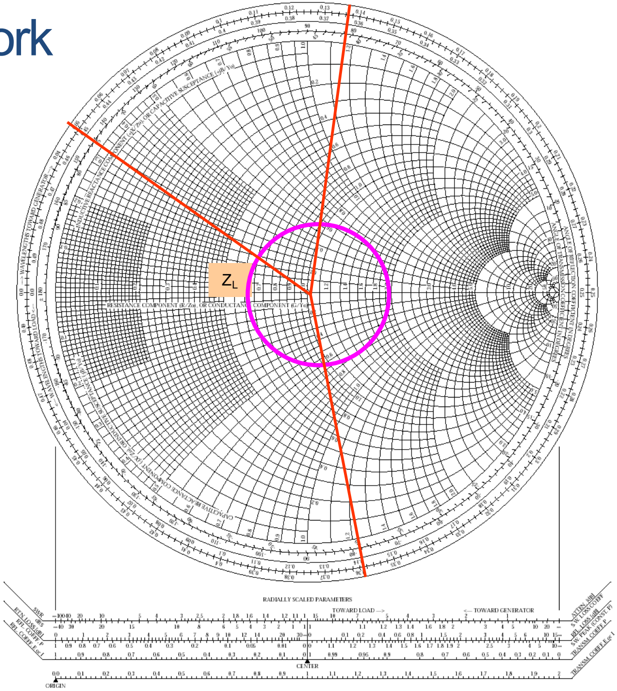
# Simple matching network




Find the position and reactance value to achieve matching in the line…

# Simple matching network

$$\overline{Z_L} = \frac{Z_L}{Z_o} = 0.714 + j0.142$$



# Simple matching network

$$\overline{Z_L} = \frac{Z_L}{Z_o} = 0.714 + j0.142$$

Solution A:

Azimuth = 0.141 λ

Impedance seen = 1 + j0.38

d = (0.141-0.043)λ = 0.098 λ

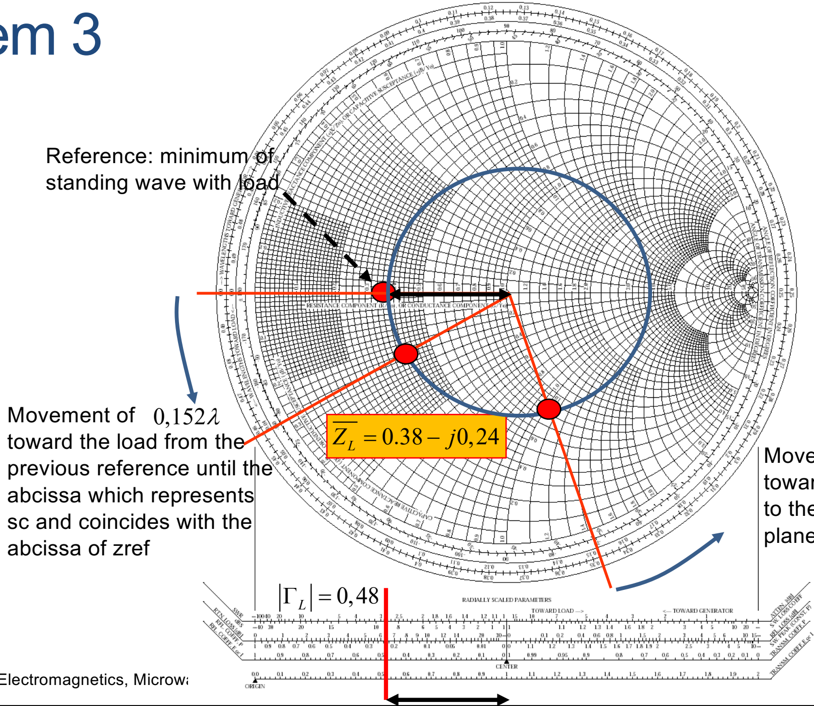
Solución B:

Azimuth = 0.359 λ

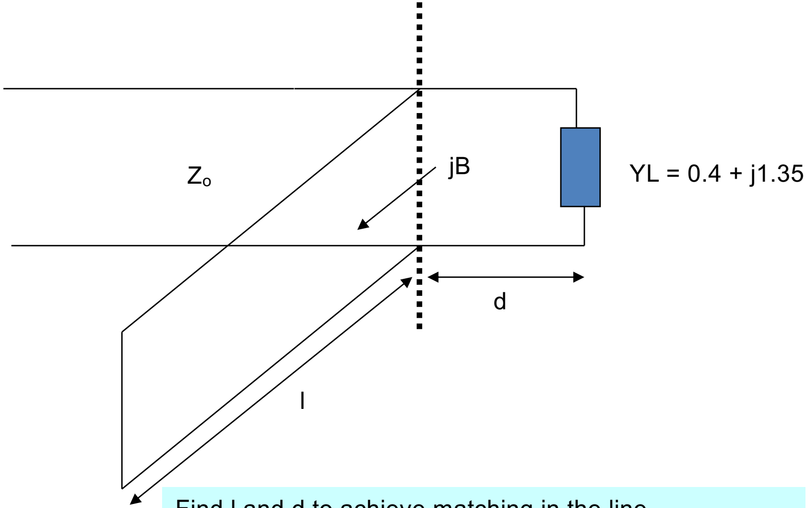
Impedance seen = 1 - j0.38

d = (0.359-0.043)λ = 0.316 λ

# Problem 3

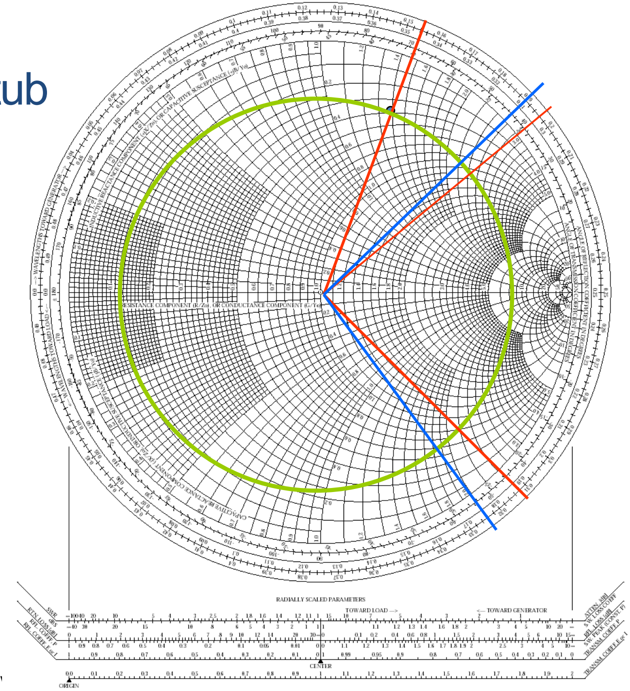

Reference: minimum of standing wave with load
Movement of $0,152\lambda$ toward the load from the previous reference until the abcissa which represents sc and coincides with the abcissa of zref
$\overline{Z_L} = 0.38 - j0,24$
Movement of 1,39λ (=0,39λ) toward the load to come back to the position of the reference plane where the load is
$|\Gamma_L| = 0,48$

# Simple matching networks: simple stub




Find l and d to achieve matching in the line…

# Simple matching network: simple stub



# Simple matching network: simple stub

Solution A:

Azimuth = 0.193 λ

d = (0.193-0.153)λ = 0.04 λ

Admitance seen = 1 + j2.3

Azimuth of -j2.3 = 0.315 λ

l = (0.315 – 0.25) λ = 0.065 λ

Solution B:

Azimuht = 0.307 λ

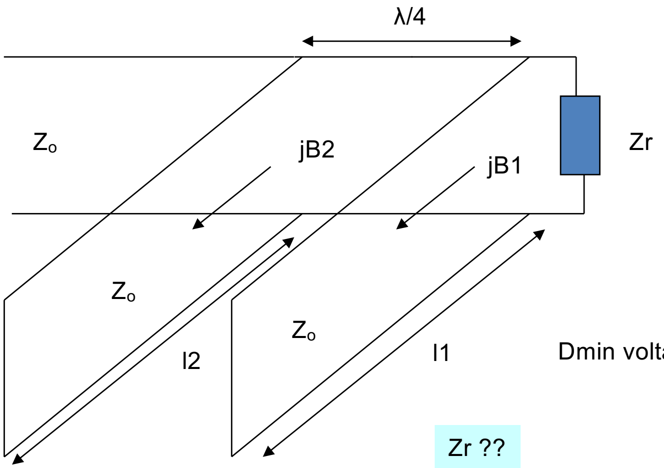
d = (0.307-0.153)λ = 0.154 λ

Admitance seen = 1 - j2.3

Azimuth of j2.3 = 0.185 λ

l = (0.25 + 0.185) λ = 0.435 λ

# Double matching network: double stub




Zo = 200 Ω

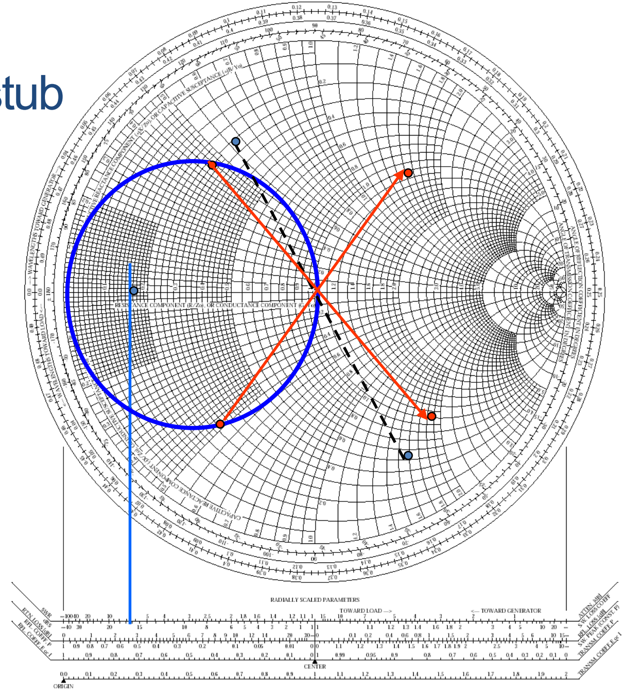
SWR = 6.5

Dmin voltage to the load = 0.168 λ

Zr ??

l1 and l2 for line matching ??

# Double matching network: double stub



# Double matching network: double stub

Moving 0.168 λ toward the load:

Zr = Zo(0.6 – j1.6) = 120 – j320 Ω

Yr = 0.21 + j0.55

Solution A:

Yr = 0.21 + j 0.41

Stub admitance = j0.41 – j0.55 = -j 0.14

Azimuth of –j0.14 = 0.478 λ

l1 = (0.478-0.25) λ = 0.228 λ

Yin = 1 – j1.95

Azimuth of j1.95 = 0.174 λ

l = (0.25 + 0.174) λ = 0.424 λ

Solution B:

Yr = 0.21 - j 0.41

Stub admitance = -j0.41 – j0.55 = -j 0.96

Azimuth of –j0.96 = 0.379 λ

l1 = (0.379-0.25) λ = 0.129 λ

Yin = 1 + j1.95

Azimuth of -j1.95 = 0.326 λ

l = (0.326 – 0.25) λ = 0.076 λ

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# Bode-Fano criterion

Proving the criterion is very difficult

H. W. Bode, *Network Analysis and Feedback Amplifier Design, NY, 1945.*

R. M. Fano, *Theeoritical limitations on the broad band matching of arbitrary impedances,* Journal of the Franklin Institute, vol. 249, pp. 57-83, January 1950, and pp. 139-154 February 1950.

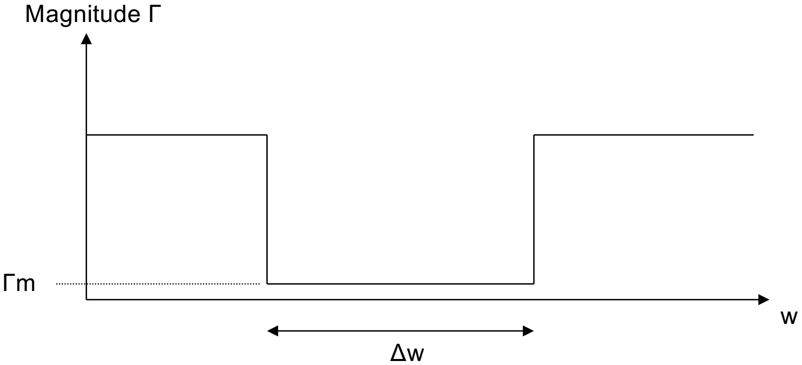
Can perfect matching be achieved for a specified bandwidth?

If it is not possible, what is the relationship between the maximum reflection coefficient that we can afford in the line and the bandwidth?

Can the complexity of the matching network be evaluated?

# Bode-Fano criterion

$$\int_0^\infty \ln \frac{1}{|\Gamma(w)|}\, dw \le \frac{\pi}{RC} \quad \Rightarrow \quad \Delta w \ln \frac{1}{\Gamma_m} \le \frac{\pi}{RC}$$



# Bode-Fano criterion

Main conclusions of Bode-Fano criterion

For a given load, a large bandwidth can be achieved with the cost of increasing the reflection coefficient….

The reflection coefficient can only be zero at discrete frequencies….

Circuits with higher Q are harder to match than those with lower Q: (high Q implies high values of R and/or C)

# Conclusions (I)

- The terminated transmission line has been introduced, producing a standing wave.
- This standing wave is characterized by the reflection coefficient at each point of the line.
  - In a lossless line, it is constant in amplitud. This is represented by a circle.
  - In a lossy line, the amplitude decreases with the phase variation. This is represented by a spiral.
  - Having a bijective mapping between each reflection coefficient and each impedance means that each reflection coefficient corresponds to one and only one impedance.

# Conclusions (II)

- The Smith chart is the basic tool to analyse any microwave circuit.
- Consists on a representation of the reflection coefficients in the POLAR PLANE.
- Due to the bijective mapping between reflection coefficients and impedances, each reflection coefficient in the polar plane corresponds to a specific value of impedance or admittance.

# Conclusions (III)

- Functionalities of the Smith chart:
  - Direct reading of the reflection coefficient in magnitude and phase (by overlapping curves of resistance – conductance – and reactance – susceptance - the value of the impedance can also be read).
  - Obtaining the value of the reflection coefficient at any point of a line is as simple as rotating through a circle of constant reflection coefficient (centered at the origin with a radius R).
  - Representation of admitances/impedances symply by making a 180º rotation (in the conventional Smith chart).
  - Impedance matching by moving, mainly, two families of circles: constant reflection coefficient and constant resistance (conductance).

# References


1. David M.Pozar: "Microwave Engeneering" Second Edition 1998, John Wiley&Sons. (capítulo 5)
2. Robert E. Collin: "Foundations for microwave engineering" New York McGraw-Hill, 1992. (capítulo 5)
3. Bahl y Bhartia: "Microwave Solid State Circuit Design", Wiley Interscience, 1988, segunda edición. (capítulo 4).
4. Roberto Sorrentino and G. Bianchi: Microwave and RF engineering, Wiley 2010
5. M. Steer: Fundamentals of Microwave and RF design; North Caroline State University. url: https://repository.lib.ncsu.edu/handle/1840.20/36776