... E1. Problem 6.

b) If $x(\cdot)$ is even, (i.e. $x(-t) = x(t)$) show that $\hat{x}(\cdot)$ is real and even.

$$\hat{x}(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft}\, dt = \int_{\infty}^{-\infty} x(-t)\, e^{-j2\pi f(-t)}\, d(-t) = -\int_{-\infty}^{\infty} x(t)\, e^{j2\pi ft}\, dt = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi(-f)t}\, dt = \hat{x}(-f) \Rightarrow$$

$\Rightarrow \hat{x}(\cdot)$ is even

$$\hat{x}(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft}\, dt = \int_{-\infty}^{0} x(t)\, e^{-j2\pi ft}\, dt + \int_{0}^{\infty} x(t)\, e^{-j2\pi ft}\, dt = \int_{\infty}^{0} x(-t)\, e^{-j2\pi f(-t)}\, d(-t) + \int_{0}^{\infty} x(t)\, e^{-j2\pi ft}\, dt =$$

$$= -\int_{0}^{\infty} x(t)\, e^{j2\pi ft}\, dt + \int_{0}^{\infty} x(t)\, e^{-j2\pi ft}\, dt = \int_{0}^{\infty} x(t)\left(e^{+j2\pi ft} + e^{-j2\pi ft}\right) dt = \int_{0}^{\infty} x(t) \cdot 2\cos(2\pi ft)\, dt \quad \in \mathbb{R}$$

integral of real functions $\in \mathbb{R}$