

16) ... E2. Problem 1.

✓ f) $x[n]$ periodic with $N=6$ and $x[n] = \begin{cases} 1, & n=0 \\ 2, & n=1,2 \\ 0, & n=3,4,5 \end{cases}$

$$a_k = \frac{1}{6} \sum_{n=0}^{5} x[n] e^{j \frac{2\pi}{6} kn} = \frac{1}{6} \left(e^{j \frac{\pi}{3} \cdot 0} + \sum_{n=1}^2 2 e^{j \frac{\pi}{3} kn} \right) = \frac{1}{6} \left(1 + 2 e^{j \frac{\pi}{3} k} + 2 e^{j \frac{2\pi}{3} k} \right) =$$

$$= -\frac{1}{6} + \frac{1}{3} \frac{1 - e^{j \pi k}}{1 - e^{j \frac{\pi}{3} k}} = -\frac{1}{6} + \frac{1}{3} e^{j(\frac{\pi}{2} - \frac{\pi}{6})k} \frac{\sin(\frac{\pi}{2}k)}{\sin(\frac{\pi}{6}k)} = -\frac{1}{6} + \frac{1}{3} \begin{cases} e^{j \frac{\pi}{3} k} \frac{\sin(\frac{\pi}{2}k)}{\sin(\frac{\pi}{6}k)}, & k \in \{ \dots, -6, 0, 6, \dots \} \\ e^{j \frac{\pi}{3} k} \frac{\sin(\frac{\pi}{2}k)}{\sin(\frac{\pi}{6}k)}, & \text{otherwise} \end{cases}$$

$$= \begin{cases} \frac{5}{6}, & k \in \{ \dots, -6, 0, 6, \dots \} \\ \frac{1}{3} e^{j \frac{\pi}{3} k} \frac{\sin(\frac{\pi}{2}k)}{\sin(\frac{\pi}{6}k)} - \frac{1}{6}, & \text{otherwise} \end{cases} = a_k$$