Problem 1 *Fourier Series Coefficients*

(a) $N = 8$.

$$a_k = \begin{cases} \frac{1}{2j} e^{-j\frac{\pi}{4}k}, & k = \dots, -7, 1, 9, \dots \\ -\frac{1}{2j} e^{-j\frac{\pi}{4}k}, & k = \dots, -9, -1, 7, \dots \\ 0, & \text{otherwise.} \end{cases}$$

(b) $N = 8$.

$$a_k = \begin{cases} \frac{1}{2} e^{-j\frac{\pi}{3}}, & k = \dots, -5, 3, 11, \dots \\ \frac{1}{2} e^{j\frac{\pi}{3}}, & k = \dots, -3, 5, 13, \dots \\ 0, & \text{otherwise.} \end{cases}$$

(c) $N = 6$.

$a_k = \frac{3-\frac{5}{8}(-1)^k}{5-4\cos\left(\frac{\pi}{3}k\right)}$, $k \in \mathbb{Z}$.

(d) $a_k = \displaystyle\sum_{\ell=-\infty}^{\infty} \delta[k+4\ell] + \frac{j}{4}\frac{1}{1-e^{-j\frac{\pi}{2}(k-1/2)}} - \frac{j}{4}\frac{1}{1-e^{-j\frac{\pi}{2}(k+1/2)}}$, $k \in \mathbb{Z}$.

(e) $a_k = \begin{cases} \frac{4}{7}, & k = \dots, -7, 0, 7, \dots \\ \frac{1}{7} e^{-j\frac{4\pi}{7}k} \frac{\sin\left(\frac{5\pi}{7}k\right)}{\sin\left(\frac{\pi}{7}k\right)}, & \text{otherwise.} \end{cases}$

(f) $a_k = \begin{cases} \frac{5}{6}, & k = \dots, -6, 0, 6, \dots \\ \frac{1}{6} \frac{e^{-j\frac{\pi}{3}k}\sin\left(\frac{\pi}{2}\right) + e^{-j\frac{\pi}{2}k}\sin\left(\frac{\pi}{3}k\right)}{\sin\left(\frac{\pi}{6}k\right)}, & \text{otherwise.} \end{cases}$

Problem 2 *Properties of the Fourier Series*

(a) $b_k = a_k e^{-j\frac{2\pi}{N}kn_0}$, $k \in \mathbb{Z}$.

(b) $b_k = a_k \left(1 - e^{-j\frac{2\pi}{N}k}\right)$, $k \in \mathbb{Z}$.

(c) $b_k = a_k + a_k^*$, $k \in \mathbb{Z}$.

(d) $b_k = a_{k-N/2}$, $k \in \mathbb{Z}$.

(e) $b_k = \frac{a_k}{m}$, $k \in \mathbb{Z}$ and period changes to mN.

Problem 3 *The Fourier Series of Real Periodic Signals*

(a) ...

(b) ...

(c) ...

Problem 4 *Modulation Property of Fourier Series*

(a) ...

(b) $c_k = \begin{cases} \frac{7}{24} + \frac{1}{24}\frac{\sin\left(\frac{14\pi}{3}\right)}{\sin\left(\frac{2\pi}{3}\right)}, & k = \pm 2 \\ \frac{1}{24}\frac{\sin\left(\frac{7\pi}{12}(k-2)\right)}{\frac{\pi}{12}(k-2)} + \frac{1}{24}\frac{\sin\left(\frac{7\pi}{12}(k+2)\right)}{\frac{\pi}{12}(k+2)}, & \text{otherwise.} \end{cases}$

Problem 5 *Fourier Transforms I*

(a) $\hat{x}(f) = \frac{3}{5-4\cos(2\pi f)}$, $-1/2 \leq f \leq 1/2$.

(b) $\hat{x}(f) = \frac{\sin^2(4\pi f)}{\sin^2(\pi f)}$, $-1/2 \leq f \leq 1/2$.

(c) $\hat{x}(f) = e^{-j3\pi f}\frac{\sin(4\pi f)}{\sin(\pi f)}$, $-1/2 \leq f \leq 1/2$.

(d) $\hat{x}(f) = \frac{1-\frac{1}{2}e^{j2\pi f}}{\frac{5}{4}-\cos(2\pi f)}$, $-1/2 \leq f \leq 1/2$.

(e) $\hat{x}(f) = j\left[\left(N+\frac{1}{2}\right)\frac{\cos(\pi f(2N+1))}{\sin(\pi f)} - \frac{1}{2}\frac{\sin(\pi f(2N+1))}{\sin(\pi f)}\frac{\cos(\pi f)}{\sin(\pi f)}\right]$, $-1/2 \leq f \leq 1/2$.

(f) $\hat{x}(f) = 2 - 4|f|$, $-1/2 \leq f \leq 1/2$.

Problem 6 *Fourier Transforms II*

(a) $\hat{y}(f) = \frac{1}{2}\hat{x}(f) + \frac{1}{2}\hat{x}^*(-f)$, $-1/2 \leq f \leq 1/2$.

(b) $\hat{y}(f) = \hat{x}^*(f)$, $-1/2 \leq f \leq 1/2$.

(c) $\hat{y}(f) = \hat{x}(f) - \hat{x}(-f)$, $-1/2 \leq f \leq 1/2$.

(d) $\hat{y}(f) = 2\int_{-\infty}^{\infty}\hat{x}(\nu)\hat{x}(f-\nu)e^{-j10\pi(f-\nu)}\,\mathrm{d}\nu$, $-1/2 \leq f \leq 1/2$.

Problem 7 *A Discrete-Time System*

(a) Yes.

(b) No.

(c) $y[n] = \begin{cases} 2, & n = 0 \\ 1, & n = 1 \\ 0, & \text{otherwise.} \end{cases}$

Problem 8 *LTI System 1*

- $\hat{y}(f) = \frac{1}{1-\frac{1}{2}e^{-j2\pi f}}\frac{1}{1-\frac{3}{4}e^{-j2\pi f}}$, $-1/2 \leq f \leq 1/2$.

  $$y[n] = \begin{cases} 3\left(\frac{3}{4}\right)^n - 2\left(\frac{1}{2}\right)^n, & n = 0, 1, \ldots \\ 0, & n = -1, -2, \ldots \end{cases}$$

- $\hat{y}(f) = \frac{1}{1-\frac{1}{2}e^{-j2\pi f}}\frac{1}{\left(1-\frac{1}{4}e^{-j2\pi f}\right)^2}$, $-1/2 \leq f \leq 1/2$.

  $$y[n] = \begin{cases} 4\left(\frac{1}{2}\right)^n - 2\left(\frac{1}{4}\right)^n - (n+1)\left(\frac{1}{4}\right)^n, & n = 0, 1, \ldots \\ 0, & n = -1, -2, \ldots \end{cases}$$

Problem 9 *LTI System 2*

(a) $\hat{h}(f) = \frac{1}{1+\frac{1}{2}e^{-j2\pi f}}$, $-1/2 \leq f \leq 1/2$.

(b) (i) $\hat{y}(f) = \frac{1}{1-\frac{1}{4}e^{-j4\pi f}}$, $-1/2 \leq f \leq 1/2$.

$$y[n] = \begin{cases} \left(\frac{1}{2}\right)^n, & n \text{ is even} \\ 0, & n \text{ is odd.} \end{cases}$$

(ii) $\hat{y}(f) = \frac{1}{\left(1+\frac{1}{2}e^{-j2\pi f}\right)^2}$, $-1/2 \leq f \leq 1/2$.

$$y[n] = \begin{cases} (n+1)\left(-\frac{1}{2}\right)^n, & n = 0, 1, \ldots \\ 0, & n = -1, -2, \ldots \end{cases}$$

(iii) $\hat{y}(f) = 1$, $-1/2 \leq f \leq 1/2$.

$$y[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0. \end{cases}$$

(iv) $\hat{y}(f) = \frac{1-\frac{1}{2}e^{-j2\pi f}}{1+\frac{1}{2}e^{-j2\pi f}}$, $-1/2 \leq f \leq 1/2$.

$$y[n] = \begin{cases} 1, & n = 0 \\ 2\left(-\frac{1}{2}\right)^n, & n \geq 1. \end{cases}$$

Problem 10 *Minimum Phase System*

(a) ...

(b) ...