E3. Problem 5. Upsampling

Consider two continuous-time signals $x_1(\cdot)$ and $x_2(\cdot)$ of $W_1 = 3\text{kHz}$ and $W_2 = 5\text{kHz}$. Let $x_i[\cdot]$, $i \in \{1,2\}$ be obtained by sampling $x_i(\cdot)$ at $T_{si} = \frac{1}{2W_i}$. $x_1[\cdot]$ and $x_2[\cdot]$ are processed by a digital signal processor, and both signals need to operate at the same sampling rate.

a) By what factors do the signals need to be upsampled by?

b) Sketch the fourier transforms of $x_i(\cdot)$, the sampled signals, and the upsampled signals.

a) $T_{s1} = \frac{1}{2W_1} = \frac{1}{2 \cdot 3\text{kHz}} \Rightarrow f_{s1} = \frac{1}{T_{s1}} = 6\text{kHz}$

$T_{s2} = \frac{1}{2W_2} = \frac{1}{2 \cdot 5\text{kHz}} \Rightarrow f_{s2} = \frac{1}{T_{s2}} = 10\text{kHz}$

$f_{sc} = \text{mcm}(f_{s1}, f_{s2}) = 30\text{kHz} = 5 f_{s1} = 3 f_{s2}$

Signals $x_1[\cdot]$ and $x_2[\cdot]$ need to be upsampled by factors of 5 and 3 respectively.

b)