(4) Ex. Problem 1.

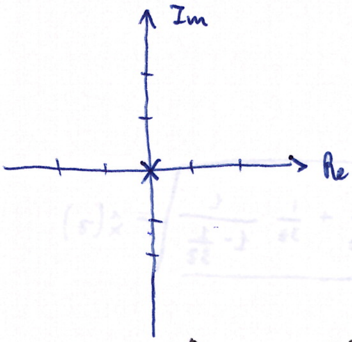
g) $x[n] = \begin{cases} 0 , & n<0 \\ 1 , & 0 \le n \le \cancel{10}\ 9 \\ 0 , & n > \cancel{10}\ 9 \end{cases}$

$$\hat{x}(z) = \sum_{n=0}^{9} z^{-n} = \frac{1-\frac{1}{z^{10}}}{1-\frac{1}{z}} = \frac{\frac{z^{10}-1}{z^{10}}}{\frac{z-1}{z}} = \boxed{\frac{z^{10}-1}{z^{9}(z-1)} = \hat{x}(z)}$$

$$z^9(z-1) = 0 \iff \begin{cases} z=0 \to \lim\limits_{z\to 0} \hat{x}(z) = \lim\limits_{z\to 0} \frac{z^{10}-1}{z^9(z-1)} = \frac{-1}{0\cdot(-1)} = \pm\infty \\ z=1 \to \lim\limits_{z\to 1} \hat{x}(z) = \lim\limits_{z\to 1} \frac{z^{10}-1}{z^9(z-1)} = \lim\limits_{z\to 1} \frac{z^{10}-1}{z-1} = \lim\limits_{z\to 1} \frac{10 z^9}{1} = 10 \Rightarrow 1 \in ROC \end{cases}$$

$\boxed{ROC = \mathbb{C} \setminus \{0\}}$

$\{|z|=1\} \in ROC \Rightarrow \boxed{\text{F.T. exists}}$



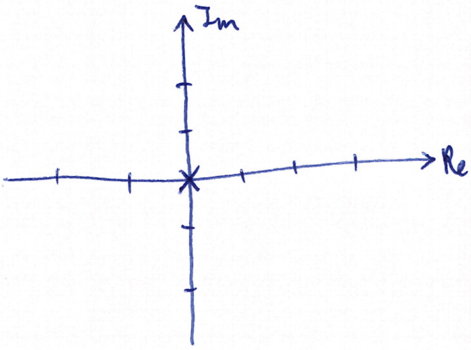
h) $x[n] = \begin{cases} 0 , & n<0 \\ (\frac{1}{4})^n , & 0 \le n \le 10 \\ 0 , & n > 10 \end{cases}$

$$\hat{x}(z) = \sum_{n=0}^{10} (\tfrac{1}{4})^n z^{-n} = \frac{1-(\frac{1}{4z})^{11}}{1-\frac{1}{4z}} = \boxed{\frac{1-\frac{1}{4^{11}z^{11}}}{1-\frac{1}{4z}} = \hat{x}(z)} \quad ; \cancel{\left|\tfrac{1}{4z}\right| < 1 \iff |z| > \tfrac{1}{4} = ROC}$$

$$\frac{1}{4z} \to \infty \iff z \to 0 : \lim_{z\to 0} \hat{x}(z) = \lim_{z\to 0} \frac{1-\frac{1}{4^{11}z^{11}}}{1-\frac{1}{4z}} = \frac{1-\frac{1}{4^{11}}\infty^{11}}{1-\frac{1}{4}\infty} = \infty \Rightarrow z=0 \notin ROC$$

$$1-\frac{1}{4z} = 0 \iff \frac{1}{4z} = 1 \iff z = \frac{1}{4} : \lim_{z\to \frac14} \hat{x}(z) = \lim_{z\to \frac14} \frac{1-\frac{1}{4^{11}z^{11}}}{1-\frac{1}{4z}} = \lim_{z\to \frac14} \frac{4^{11}z^{11}-1}{4^{10}z^{10}(4z-1)} =$$

$$= \lim_{z\to \frac14} \frac{4^{11}z^{11}-1}{4z-1} = \lim_{z\to \frac14} \frac{11\cdot 4^{11} z^{10}}{4} = 11 \cdot 4^{10} \cdot (\tfrac{1}{4})^{10} = 11 \Rightarrow z=\tfrac{1}{4} \in ROC$$



$\boxed{ROC = \mathbb{C} \setminus \{0\}}$

$\{|z|=1\} \in ROC \Rightarrow \boxed{\text{F.T. exists}}$