E2. Problem 7. A DT system.

A ~~AT~~ DT system transforms $x[\cdot]$ to $y[\cdot]$, as described by $\hat{y}(f) = 2\hat{x}(f) + e^{-j2\pi f}\hat{x}(f) + \frac{d}{df}\hat{x}(f)$, $-\frac{1}{2} \le f \le \frac{1}{2}$.

a) Is it linear?

$$y[n] = 2x[n] + x[n-1] - j2\pi n\, x[n]$$

$$\alpha y_1[n] + \beta y_2[n] = \alpha 2x_1[n] + \alpha x_1[n-1] - \alpha j2\pi n x_1[n] + \beta 2x_2[n] + \beta x_2[n-1] - \alpha j2\pi n x_2[n] =$$

$$= 2(\alpha x_1[n] + \beta x_2[n]) + (\alpha x_1[n-1] + \beta x_2[n-1]) - j2\pi n(\alpha x_1[n] + \beta x_2[n]) \Rightarrow \text{Linear}$$

$\boxed{\text{Yes}}$

b) Is it time-invariant?

$$y[n-k] = 2x[n-k] + x[n-k-1] - j2\pi(n-k)x[n-k]$$

given $x'[n] = x[n-k]$: $y'[n] = 2x[n-k] + x[n-k-1] - j2\pi n\, x[n-k] \neq y[n-k] \Rightarrow$ not TI.

$\boxed{\text{No}}$

c) What is $y[\cdot]$ if $x[n] = \delta[n]$, $n \in \mathbb{Z}$

$$y[n] = 2\delta[n] + \delta[n-1] - j2\pi n\,\delta[n] = \boxed{2\delta[n] + \delta[n-1]}$$