Linear Systems
(2023-2024)

EXERCISE 3

$\checkmark 1\ \checkmark 2\ \ 3\ \checkmark 4\ \checkmark 5$
$\checkmark 6\ \checkmark 7\ \ 8\ \checkmark 9$

E3. Problem 1. Sampling a Sinc-Square

Consider $x(t) = \text{sinc}^2(\frac{t}{2})$, $t \in \mathbb{R}$ with samples $x[n] = x(nT_s)$, $n \in \mathbb{Z}$

a) How small must T_s be s.t. we can perfectly ~~red~~ recover $x(\cdot)$ from $x[\cdot]$?

b) For $T_s = 1$, plot the F.T of $x[\cdot]$. Is there aliasing?

c) For $T_s = 2$, plot the F.T of $x[\cdot]$. Is there aliasing?

a) $x(t) = \text{sinc}^2(\frac{t}{2}) = \text{sinc}(\frac{t}{2}) \cdot \text{sinc}(\frac{t}{2}) = y(t) \cdot y(t)$

$y(t) = \text{sinc}(\frac{t}{2}) \Rightarrow \hat{y}(f) = 2\text{rect}(2f) = 2\mathbb{I}\{\underbrace{|2f| \leq \tfrac{1}{2}}_{|f| \leq \frac{1}{4}}\}$

$\hat{x}(f) = \hat{y}(f) * \hat{y}(f) = 2\text{rect}(2f) * 2\text{rect}(2f) = \cancel{4\text{rect}}\ 4\,\text{tri}(2f) = \begin{cases} 4 - 4|2f| & \text{if } \underbrace{|2f| \leq 1}_{|f| \leq \frac{1}{2}} \\ 0 & \text{otherwise} \end{cases}$

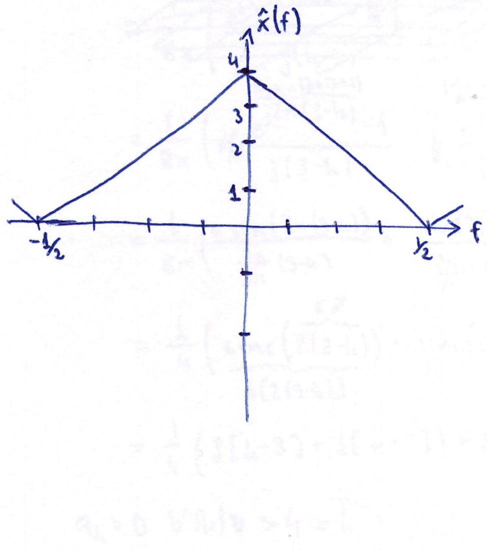
$\hat{x}(f) = 0\ \forall\ |f| \leq \frac{1}{2} \Rightarrow f_0 = \frac{1}{2}$

$f_s \geq 2 \cdot f_0 = 1 \Rightarrow T_s = \frac{1}{f_s} \leq \frac{1}{1} = \boxed{1}$

b) $x[n] = x(n \cdot 1) = \cancel{\ }\ \text{sinc}^2(\frac{n}{2}) = y[n] \cdot y[n]$

~~$\hat{x}(f) =$~~ $y[n] = \text{sinc}(\frac{n}{2}) \Rightarrow \hat{y}(f) = 2\text{rect}(2f) = 2\begin{cases} 1 & \text{if } \underbrace{|2f| \leq \tfrac{1}{2}}_{|f| \leq \frac{1}{4}} \\ 0 & \text{otherwise} \end{cases}, \quad -\frac{1}{2} \leq f \leq \frac{1}{2}$

$\hat{x}(f) = \hat{y}(f) * \hat{y}(f) = 4\text{tri}(2f) = \begin{cases} 4 - 4|2f| & \text{if } \underbrace{|2f| \leq 1}_{|f| \leq \frac{1}{2}} \\ 0 & \text{otherwise} \end{cases}, \quad -\frac{1}{2} \leq f \leq \frac{1}{2}$



$\boxed{\text{No aliasing}}$