E1. Problem 2. Periodic Signal.

Consider the signal $x(t)=\cos(2\pi t)$, $t\in\mathbb{R}$. Since $x(\cdot)$ is periodic with $T_0=1$, it is also periodic with period N for any ~~$N\in\mathbb{Z}, N\neq 0$~~ positive integer N. Calculate its Fourier series coefficients if we regard it as a periodic signal with period 3.

$$a_k = \frac{1}{3}\int_0^3 \cos(2\pi t)\, e^{-j\frac{2\pi}{3}kt}\,dt = \frac{1}{3}\int_0^3 \frac{e^{j2\pi t}+e^{-j2\pi t}}{2}\, e^{-j\frac{2\pi}{3}kt}\,dt = \frac{1}{6}\left(\int_0^3 e^{jt\left(2\pi-\frac{2\pi}{3}k\right)}\,dt + \int_0^3 e^{-jt\left(2\pi+\frac{2\pi}{3}k\right)}\,dt\right) =$$

$$= \frac{1}{6}\left(\frac{e^{jt\left(2\pi-\frac{2\pi}{3}k\right)}}{j\left(2\pi-\frac{2\pi}{3}k\right)} + \frac{e^{-jt\left(2\pi+\frac{2\pi}{3}k\right)}}{-j\left(2\pi+\frac{2\pi}{3}k\right)}\right)\Bigg|_{t=0}^{3} = \frac{1}{6}\left(-j\,\frac{e^{j\left(6\pi-2\pi k\right)}-e^{0}}{2\pi-\frac{2\pi}{3}k} + j\,\frac{e^{-j\left(6\pi+2\pi k\right)}-e^{0}}{2\pi+\frac{2\pi}{3}k}\right) =$$

$$= \frac{1}{2}\left(-j\,\frac{\overbrace{\cos(6\pi-2\pi k)}^{1}+j\sin(6\pi-2\pi k)-1}{6\pi-2\pi k} + j\,\frac{\overbrace{\cos(6\pi+2\pi k)}^{1}-j\sin(6\pi+2\pi k)-1}{6\pi+2\pi k}\right) =$$

$$= \frac{1}{2}\left(\operatorname{sinc}(6-2k)+\operatorname{sinc}(6+2k)\right) = \frac{1}{2}\left(\delta[3-k]+\delta[3+k]\right) = \boxed{\begin{cases}\frac{1}{2} & \text{if } k\in\{3,-3\}\\ 0 & \text{otherwise}\end{cases}}$$