

c) 2. Problem 1.

c)  $x[n]$  periodic with  $N=6$  and  $x[n] = \left(\frac{1}{2}\right)^{|n|}$ ,  $n \in [-2..3]$

$$a_k = \frac{1}{6} \sum_{n=-2}^3 \left(\frac{1}{2}\right)^{|n|} e^{j\frac{2\pi}{6}kn} =$$

$$= \frac{1}{6} \left( \sum_{n=-2}^0 \left(\frac{1}{2}\right)^{-n} e^{j\frac{2\pi}{6}kn} + \sum_{n=1}^3 \left(\frac{1}{2}\right)^n e^{j\frac{2\pi}{6}kn} \right) =$$

$$= \frac{1}{6} \left( \sum_{n=0}^2 \left(\frac{1}{2}\right)^n e^{j\frac{2\pi}{6}kn} + \sum_{n=0}^2 \left(\frac{1}{2}\right)^{n+1} e^{j\frac{2\pi}{6}k(n+1)} \right) =$$

$$= \frac{1}{6} \left( \sum_{n=0}^2 \left(\frac{1}{2}\right)^n e^{j\frac{2\pi}{6}kn} + \frac{1}{2} \sum_{n=0}^2 \left(\frac{1}{2}\right)^n e^{j\frac{2\pi}{6}k(n+1)} \right) =$$

$$= \frac{1}{6} \left( \sum_{n=0}^2 \left(\frac{1}{2} e^{j\frac{2\pi}{6}k}\right)^n + \sum_{n=0}^2 \left(\frac{1}{2} e^{j\frac{2\pi}{6}k}\right)^{n+1} \right) = \frac{1}{6} \left( \frac{1 - \left(\frac{1}{2} e^{j\frac{2\pi}{6}k}\right)^3}{1 - \frac{1}{2} e^{j\frac{2\pi}{6}k}} + \left(\frac{1}{2} e^{j\frac{2\pi}{6}k}\right) \frac{1 - \left(\frac{1}{2} e^{j\frac{2\pi}{6}k}\right)^3}{1 - \frac{1}{2} e^{j\frac{2\pi}{6}k}} \right) =$$

$$= \frac{1}{6} \left( \frac{1 - \frac{1}{8} e^{j\pi k}}{1 - \frac{1}{2} e^{j\frac{2\pi}{6}k}} + \frac{\left(\frac{1}{2} e^{j\frac{2\pi}{6}k}\right) \left(1 - \frac{1}{8} e^{j\pi k}\right)}{1 - \frac{1}{2} e^{j\frac{2\pi}{6}k}} \right) = \frac{1}{6} \left( \frac{\left(1 - \frac{1}{8}(-1)^k\right) \left(1 - \frac{1}{2} e^{j\frac{2\pi}{6}k}\right) + \left(\frac{1}{2} e^{j\frac{2\pi}{6}k}\right) \left(1 - \frac{1}{8}(-1)^k\right)}{\left(1 - \frac{1}{2} e^{j\frac{2\pi}{6}k}\right) \left(1 - \frac{1}{2} e^{j\frac{2\pi}{6}k}\right)} \right) =$$

$$= \frac{1}{6} \frac{\left(1 - \frac{1}{8}(-1)^k\right) \left(1 - \frac{1}{2} e^{j\frac{2\pi}{6}k}\right) + \left(\frac{1}{2} e^{j\frac{2\pi}{6}k} - \frac{1}{4}\right)}{1 - \frac{1}{2} e^{j\frac{2\pi}{6}k} - \frac{1}{2} e^{j\frac{2\pi}{6}k} + \frac{1}{4}} = \frac{1}{6} \frac{\left(1 - \frac{1}{8}(-1)^k\right) \left(1 - 0 - \frac{1}{4}\right)}{\frac{5}{4} - \cos\left(\frac{\pi}{3}k\right)} = \frac{3}{6} \frac{1 - \frac{1}{8}(-1)^k}{5 - 4\cos\left(\frac{\pi}{3}k\right)} = \boxed{\frac{1 - \frac{1}{8}(-1)^k}{10 - 8\cos\left(\frac{\pi}{3}k\right)}}$$