✓ E4. Problem 5. Power-Series Expansion

By using ~~the~~ $\log(1-w) = -\sum_{i=1}^{\infty} \frac{w^i}{i}$, $|w|<1$, determine the inverse z-transform.

a) $\hat{x}(z) = \log(1-2z)$, $|z|<\frac{1}{2}$

$$|z|<\frac{1}{2} \Rightarrow |2z| < 2\cdot\frac{1}{2} = 1 \Rightarrow \hat{x}(z) = -\sum_{i=1}^{\infty} \frac{(2z)^i}{i} = -\sum_{i=1}^{\infty} \frac{2^i}{i} z^i = \sum_{j=-\infty}^{-1} \frac{2^{-j}}{j} z^{-j}$$

$$\hat{x}(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n} = \underbrace{\sum_{n=-\infty}^{-1} x[n]z^{-n}}_{\sum_{j=-\infty}^{-1}\frac{2^{-j}}{j}z^{-j}} + \underbrace{\sum_{n=0}^{\infty} x[n]z^{-n}}_{0} \Rightarrow \boxed{x[n] = \begin{cases}\frac{1}{n2^n}, & n<0\\ 0, & n\geq 0\end{cases}}$$

b) $\hat{x}(z) = \log(1-\frac{1}{2}z^{-1})$, $|z|>\frac{1}{2}$

$\cancel{\text{Let } \hat{y}(z) = \hat{x}(z^{-1}) = \log(1-\tfrac{1}{2}z),\ |z^{-1}|>\tfrac{1}{2}}$

$\cancel{|z|<2 \Rightarrow |\tfrac{1}{2}z| < \tfrac{1}{2}\cdot 2 = 1 \Rightarrow}$

$\cancel{\Rightarrow \hat{y}(z) = -\sum_{i=1}^{\infty} \frac{(\frac{1}{2}z)^i}{i}}$

$$|z|>\frac{1}{2} \Rightarrow |z^{-1}|<2 \Rightarrow |\tfrac{1}{2}z^{-1}| < \tfrac{1}{2}\cdot 2 = 1 \Rightarrow$$

$$\Rightarrow \hat{x}(z) = -\sum_{i=1}^{\infty} \frac{(\frac{1}{2z})^i}{i} = \sum_{i=1}^{\infty} \frac{1}{-i2^i} z^{-i}$$

$\cancel{\hat{x}(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n} = \sum_{n=-\infty}^{-1} x[n]z^{-n} + \sum_{n=0}^{\infty} x[n]z^{-n} \Rightarrow x[n] = \begin{cases}\frac{1}{n2^n}, & n<0\\ 0, & n\geq 0\end{cases}}$

$$\hat{x}(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n} = \underbrace{\sum_{n=-\infty}^{0} x[n]z^{-n}}_{0} + \underbrace{\sum_{n=1}^{\infty} x[n]z^{-n}}_{\sum_{i=1}^{\infty}\frac{-1}{i2^n}z^{-i}} \Rightarrow \boxed{x[n] = \begin{cases}\frac{-1}{n2^n}, & n>0\\ 0, & n\leq 0\end{cases}}$$