E4. Problem 4. Inverse z-Transform II

A right-sided sequence $x[\cdot]$ has $\hat{x}(z) = \dfrac{3z^{-10}+z^{-7}-5z^{-2}+4z^{-1}+1}{z^{-10}-5z^{-7}+z^{-3}}$. Determine $x[n]$, $n<0$

$$
\begin{array}{r|l}
 & z^{3}+4z^{2}-5z \\
\hline
z^{-3}-5z^{-7}+z^{-10} & 1+4z^{-1}-5z^{-2}+0+0+0+0+z^{-7}+0+0+3z^{-10} \\
 & 1 \qquad -5z^{-4} \qquad +z^{-7} \\
\hline
 & 4z^{-1}-5z^{-2}+0+5z^{-4} \qquad +0 \qquad +3z^{-10} \\
 & 4z^{-1} \qquad -20z^{-5} \qquad +4z^{-8} \\
\hline
 & -5z^{-2} \qquad +5z^{-4}+20z^{-5} \qquad -4z^{-8} \qquad +3z^{-10} \\
 & -5z^{-2} \qquad +25z^{-6} \qquad -5z^{-9} \\
\hline
 & 5z^{-4}+20z^{-5}-25z^{-6} \quad -4z^{-8}+5z^{-9}+3z^{-10}
\end{array}
$$

$$\hat{x}(z) = \underbrace{z^{3}+4z^{2}-5z}_{n<0} + \overbrace{\frac{5z^{-4}+20z^{-5}-25z^{-6}-4z^{-8}+5z^{-9}+3z^{-10}}{z^{-10}-5z^{-7}+z^{-3}}}^{n>0}$$

$$\hat{x}(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n} = \underbrace{\cdots}_{0} + \underbrace{x[-3]z^{-(-3)}}_{z^{3}} + \underbrace{x[-2]z^{-(-2)}}_{4z^{2}} + \underbrace{x[-1]z^{-(-1)}}_{-5z} + \cdots \Rightarrow \boxed{\begin{cases} x[n]=0 & \forall\ n<-3 \\ x[-3]=1 \\ x[-2]=4 \\ x[-1]=-5 \end{cases}}$$