

Solutions to Exercise 3

Linear Systems 2023/2024

Problem 1 Sampling a Sinc-Square

- (a) $T_s = 1 \text{ s.}$
- (b) There is no aliasing.
- (c) There is aliasing.

Problem 2 Sampling a Periodic Signal

- (a) $T_0 = 2\pi$ N = 9.
- (b) There is no aliasing.
- (c) There is aliasing. The reconstructed continuous-time signal is $\tilde{x}(t) = \frac{1}{2}\cos(2t) 2\sin(t), t \in \mathbb{R}$.

Problem 3 Product of Two Bandlimited Signals

$$T_s = \frac{1}{2W_1 + 2W_2}.$$

Problem 4 Energy of Sampled Signal

$$E_s = \frac{1}{T_s} E_c.$$

Problem 5 Upsampling

- (a) $x_1(\cdot)$ needs to be upsampled by a factor of $N_1 = 5$ and $x_2(\cdot)$ needs to be upsampled by a factor of $N_2 = 3$.
- (b) ...

Problem 6 Echo echo echo

- (a) $h[n] = \delta[n] + \frac{1}{8}\delta[n-1]$.
- (b) $h[n] = \delta[n] \frac{(-1)^n}{8\pi(n-1/2)}$.

Problem 7 Fourier Series Coefficients of Sampled Periodic Signal

- (a) N = 21.
- (b) The Fourier series coefficients of $x[\cdot]$ are periodic with period 21 and $a_{k,\mathrm{DT}} = \left(\frac{1}{2}\right)^k, \ k = -10,\ldots,10.$

Problem 8 Digital System That Processes a Continuous-Time Signal

$$\hat{h}(f) = \begin{cases} \frac{1}{1 - \frac{1}{2}e^{-j2\pi f T_s}}, & |f| \leq \frac{1}{2T_s} \\ 0, & |f| > \frac{1}{2T_s}. \end{cases}$$

Problem 9 Analog System That Processes a Discrete-Time Signal

$$\hat{h}(f) = \frac{1}{3 + 4j2\pi f/T_s + (j2\pi f/T_s)^2}, |f| \le \frac{1}{2}.$$