E2. Problem 6. Fourier transform II

Let $\hat{x}(\cdot)$ be the F.T. of $x(\cdot)$. Derive expressions in terms of $x(\cdot)$ for ~~those~~ the FTs of these signals.

a) $y[n] = \mathrm{Re}\{x[n]\}$, $n \in \mathbb{Z}$

$$y[n] = \frac{x[n] + x^*[n]}{2} \Rightarrow \hat{y}(f) = \frac{1}{2}\hat{x}(f) + \frac{1}{2}\hat{x}^*(f), \quad -\tfrac{1}{2} \le f \le \tfrac{1}{2}$$

b) $y[n] = x^*[-n]$, $n \in \mathbb{Z}$

$$\hat{y}(f) = \hat{x}^*(f), \quad -\tfrac{1}{2} < f \le \tfrac{1}{2}$$

c) $y[n] = x[n] - x[-n]$, $n \in \mathbb{Z}$

$$\hat{y}(f) = \hat{x}(f) - \hat{x}(-f), \quad -\tfrac{1}{2} \le f \le \tfrac{1}{2}$$

d) $y[n] = 2x[n]x[n-5]$, $n \in \mathbb{Z}$

Let $z[n] = x[n-5] \Rightarrow \hat{z}(f) = \hat{x}(f)\, e^{j2\pi f \cdot 5}$

$$\hat{y}(f) = 2(\hat{x} * \hat{z})(f) = 2\int_{-\infty}^{\infty} \hat{x}(\nu)\hat{x}(f-\nu)e^{-j2\pi\cdot 5(f-\nu)}\, d\nu \quad -\tfrac{1}{2} \le f \le \tfrac{1}{2}$$