E4. Problem 9. LTI System I

Consider an LTI system $s[\cdot] \longmapsto x[\cdot]$ described by $x[n] = s[n] - e^{-8\alpha} s[n-8]$, $0<\alpha<1$

a) Find $\hat{h}_1(z) = \frac{\hat{x}(z)}{\hat{s}(z)}$ and plot the poles and zeros. Indicate the ROC

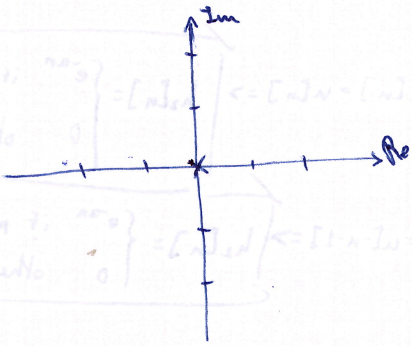
b) We wish to recover $s[\cdot]$ from $x[\cdot]$ with another LTI system. Find $\hat{h}_2(z) = \frac{\hat{y}(z)}{\hat{x}(z)}$ ~~such~~ such that $y[n] = s[n]$ $\forall$ $n \in \mathbb{Z}$. Find all possible ROCs for $\hat{h}_2(z)$ and indicate the causality and stability.

c) Find all possible $h_2[\cdot]$ choices s.t. $y[n] = (h_2 * x)[n] = s[n]$, $n \in \mathbb{Z}$

a) $\hat{x}(z) = \hat{s}(z) - e^{-8\alpha} z^{-8} \hat{s}(z)$

$$\hat{h}_1(z) = \frac{\hat{x}(z)}{\hat{s}(z)} = \frac{\hat{s}(z) - e^{-8\alpha} z^{-8} \hat{s}(z)}{\hat{s}(z)} = \boxed{1 - e^{-8\alpha} z^{-8} = \hat{h}_1(z)}$$

$\boxed{\text{ROC}: \mathbb{C} \setminus \{0\}}$



b) $y[n] = s[n] \Rightarrow \hat{y}(z) = \hat{s}(z) \Rightarrow \hat{h}_2(z) = \frac{\hat{s}(z)}{\hat{s}(z) - e^{-8\alpha} z^{-8} \hat{s}(z)} = \boxed{\frac{1}{1 - e^{-8\alpha} z^{-8}} = \hat{h}_2(z)}$

$1 - e^{-8\alpha} z^{-8} = 0 \Leftrightarrow e^{-8\alpha} = z^{8} \Leftrightarrow e^{-\alpha} = z \notin \text{ROC}$

when $z = 0$: $\lim_{z\to 0} \hat{h}_2(z) = \lim_{z \to 0} \frac{1}{1 - e^{-8\alpha} z^{-8}} = \frac{1}{1 - e^{-8\alpha} \frac{1}{0}} = \frac{1}{1-\infty} = 0 \Rightarrow \{z = 0\} \in \text{ROC}$ (possibly)

$\text{ROC}_1: \lvert z\rvert < e^{-\alpha}$	anticausal, unstable	$\Leftarrow e^{-x} < 1\ \forall x > 0$
$\text{ROC}_2: \lvert z\rvert > e^{-\alpha}$	causal, stable	$\Leftarrow e^{-x} < 1\ \forall x > 0$