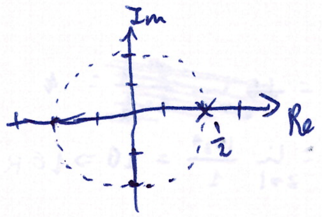
(3) E4. Problem 1.

(✓) d) $x[n] = \begin{cases} \left(\frac{1}{2}\right)^n, & n \leq 0 \\ 0, & n > 0 \end{cases}$

$$\hat{x}(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = \sum_{n=-\infty}^{0} \left(\frac{1}{2}\right)^n z^{-n} = \sum_{n=0}^{\infty} 2^n z^n = \boxed{\frac{1}{1-2z} = \hat{x}(z)}$$

$|2z| < 1 \Leftrightarrow \boxed{|z| < \frac{1}{2} : ROC}$; $\{|z|=1\} \notin ROC \Rightarrow$ $\boxed{\text{F.T. doesn't exist}}$

$1-2z=0 \Leftrightarrow 1=2z \Leftrightarrow z=\frac{1}{2}$



(e)

(✓) f) $x[n] = \left(\frac{1}{3}\right)^{|n|}, \quad n \in \mathbb{Z}$

$$\hat{x}(z) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{3}\right)^{|n|} z^{-n} = \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n z^n + \overbrace{\sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^{(n+1)} z^{-(n+1)}}^{-\infty \to 0} = \boxed{\frac{1}{1-\frac{1}{3}z} + \frac{1}{3z} \frac{1}{1-\frac{1}{3z}}} = \hat{x}(z)$$

$\left|\frac{1}{3}z\right| < 1 \Leftrightarrow |z| < 3$

$\left|\frac{1}{3z}\right| < 1 \Leftrightarrow |z| > \frac{1}{3}$

$\Rightarrow ROC = \{|z|<3\} \cap \{|z|>\frac{1}{3}\} = \boxed{\{\frac{1}{3} < |z| < 3\} = ROC}$

$\{|z|=1\} \in ROC \Rightarrow \boxed{\text{F.T. exists}}$

$1-\frac{1}{3}z = 0 \Leftrightarrow \frac{1}{3}z = 1 \Leftrightarrow z=3$

$1-\frac{1}{3z} = 0 \Leftrightarrow \frac{1}{3z} = 1 \Leftrightarrow z=\frac{1}{3}$