

Problem 1 *z-Transforms*

- (a) $\hat{x}(z) = 1$, ROC: \mathbb{C} . The Fourier transform exists.
- (b) $\hat{x}(z) = z^{-1}$, ROC: $\mathbb{C} \setminus \{0\}$. The Fourier transform exists.
- (c) $\hat{x}(z) = z$, ROC: $\mathbb{C} \setminus \{\infty\}$. The Fourier transform exists.
- (d) $\hat{x}(z) = \frac{1}{1-2z}$, ROC: $|z| < \frac{1}{2}$. The Fourier transform does not exist.
- (e) $\hat{x}(z) = \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{1}{1-\frac{1}{4}z^{-1}}$, ROC: $|z| > \frac{1}{2}$. The Fourier transform exists.
- (f) $\hat{x}(z) = \frac{1}{1-\frac{1}{3}z^{-1}} + \frac{\frac{1}{3}z}{1-\frac{1}{3}z}$, ROC: $\frac{1}{3} < |z| < 3$. The Fourier transform exists.
- (g) $\hat{x}(z) = \frac{1-z^{-10}}{1-z^{-1}}$, ROC: $\mathbb{C} \setminus \{0\}$. The Fourier transform exists.
- (h) $\hat{x}(z) = \frac{1-\frac{1}{4^{11}}z^{-11}}{1-\frac{1}{4}z^{-1}}$, ROC: $\mathbb{C} \setminus \{0\}$. The Fourier transform exists.

Problem 2 *Inverse z-Transform I*

- (a) $x[n] = \left(-\frac{1}{2}\right)^n \mathbf{I}\{n \geq 0\}$.
- (b) $x[n] = -\left(-\frac{1}{2}\right)^n \mathbf{I}\{n < 0\}$.
- (c) $x[n] = \left[-3\left(-\frac{1}{4}\right)^n + 4\left(-\frac{1}{2}\right)^n\right] \mathbf{I}\{n \geq 0\}$.
- (d) $x[n] = \delta[n] - \left(\frac{1}{4}\right)^n \mathbf{I}\{n \geq 1\}$.
- (e) $x[n] = a^{-n+1} \mathbf{I}\{n \geq 1\} - a^{-n-1} \mathbf{I}\{n \geq 0\}$.

Problem 3 *Partial Fraction Decomposition*

- (a) $\hat{x}(z) = 1 - \frac{1-3z}{1-3z+2z^2}$.
- (b) $\hat{x}(z) = 1 + \frac{1}{1-2z} - \frac{2}{1-z}$.
- (c) $x[n] = \left[\left(\frac{1}{2}\right)^n - 2\right] \mathbf{I}\{n < 0\}$

Problem 4 *Inverse z-Transform II*

$$\begin{aligned}
 x[n] &= 0, \quad n < -3. \\
 x[-3] &= 1. \\
 x[-2] &= 4. \\
 x[-1] &= -5.
 \end{aligned}$$

Problem 5 *Power-Series Expansion*

(a) $x[n] = \left(\frac{1}{2}\right)^n \frac{1}{n} \mathbf{I}\{n < 0\}.$

(b) $x[n] = -\left(\frac{1}{2}\right)^n \frac{1}{n} \mathbf{I}\{n > 0\}.$

Problem 6 *Real-Valued Sequences*

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Problem 7 *Time Reversal*

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Problem 8 *Even Sequences*

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Problem 9 *LTI System I*

(a) $\hat{h}_1(z) = 1 - e^{-8\alpha} z^{-8}$, ROC: $\mathbb{C} \setminus \{0\}.$

(b) $\hat{h}_2(z) = \frac{1}{1 - e^{-8\alpha} z^{-8}}$, ROC: $|z| < e^{-\alpha}$ or $|z| > e^{-\alpha}.$

(c) $h_2[n] = \begin{cases} e^{-\alpha n}, & \text{if } n = 0, 8, \dots \\ 0, & \text{otherwise} \end{cases}$

or

$$h_2[n] = \begin{cases} -e^{-\alpha n}, & \text{if } n = -8, -16, \dots \\ 0, & \text{otherwise} \end{cases}$$

Problem 10 *LTI System II*

$$y[n] = \frac{1 - a^{n+1}}{1 - a} \mathbf{I}\{0 \leq n < N\} + \frac{a^{n+1-N} - a^{n+1}}{1 - a} \mathbf{I}\{n \geq N\}$$