(4) E2-Problem 1.

d) $x[\cdot]$ is periodic with period 4 and $x[n] = 1-\sin\left(\frac{\pi n}{4}\right),\ n \in [0..3]$

$$a_k = \frac{1}{4}\sum_{n=0}^{3}\left(1-\sin\left(\frac{\pi n}{4}\right)\right)e^{-j\frac{2\pi}{4}kn} = \frac{1}{4}\left(\sum_{n=0}^{3} e^{-j\frac{\pi}{2}kn} - \frac{1}{2j}\sum_{n=0}^{3} e^{j\left(-\frac{2\pi}{4}kn+\frac{\pi}{4}n\right)} + \frac{1}{2j}\sum_{n=0}^{3} e^{-j\left(\frac{2\pi}{4}kn-\frac{\pi}{4}n\right)}\right) =$$

$$= \frac{1}{4}\left(\frac{1-e^{-j2\pi k}}{1-e^{-j\frac{\pi}{2}k}} + j\frac{1}{2}\,\frac{1-e^{j(-2\pi k+\pi)}}{1-e^{j\left(-\frac{\pi}{2}k+\frac{\pi}{4}\right)}} - j\frac{1}{2}\,\frac{1-e^{-j(2\pi k+\pi)}}{1-e^{-j\left(\frac{\pi}{2}k+\frac{\pi}{4}\right)}}\right) =$$

$$= \frac{1}{4}\left(\frac{\overbrace{+j\sin 2\pi k}^{0}}{\underbrace{1-e^{-j\frac{\pi}{2}k}}_{=0 \Leftrightarrow \frac{\pi}{2}k = 2\pi m \Rightarrow k=4m}} + j\frac{1}{2}\,\frac{1-\cos(-2\pi k+\pi) - j\sin(2\pi k+\pi)}{1-e^{j\left(-\frac{\pi}{2}k+\frac{\pi}{4}\right)}} - j\frac{1}{2}\,\frac{1-\cos(2\pi k+\pi)+j\sin(2\pi k+\pi)}{1-e^{j\left(\frac{\pi}{2}k+\frac{\pi}{4}\right)}}\right) =$$

$$= \frac{1}{4}\left(\begin{cases}\lim_{k\to 0}\frac{j\sin 2\pi k}{1-\cos\left(\frac{\pi}{2}k\right)+j\sin\left(\frac{\pi}{2}k\right)} & \text{if } k=4m \\ 0 & \text{otherwise}\end{cases} + j\frac{1}{2}\,\frac{1+1-j\sin(\pi)}{1-e^{j\left(-\frac{\pi}{2}k+\frac{\pi}{4}\right)}} - j\frac{1}{2}\,\frac{1+1-j\sin\pi}{1-e^{-j\left(\frac{\pi}{2}k+\frac{\pi}{4}\right)}}\right) =$$

$$= \frac{1}{4}\left(\begin{cases}\lim_{k\to 0}\frac{2\pi k+o(k)}{\frac{\pi}{2}k+o(k)} & \text{if } k=4m \\ 0 & \text{otherwise}\end{cases} + j\frac{1}{1-e^{-j\left(\frac{\pi}{2}k-\frac{\pi}{4}\right)}} - j\frac{1}{1-e^{-j\left(\frac{\pi}{2}k+\frac{\pi}{4}\right)}}\right) =$$

$$= \boxed{\sum_{m=-\infty}^{\infty}\delta[k-4m] + j\frac{1}{4}\,\frac{1}{1-e^{-j\left(\frac{\pi}{2}k-\frac{\pi}{4}\right)}} - j\frac{1}{4}\,\frac{1}{1-e^{-j\left(\frac{\pi}{2}k+\frac{\pi}{4}\right)}}}$$