

... Problem 1.

c)  $x(t)$  periodic with  $T_0=2$  and:  $x(t)=e^{-t}$ ,  $-1 < t \leq 1$

$$a_k = \frac{1}{T_0} \int_{-1}^1 x(t) e^{-j \frac{2\pi}{T_0} kt} dt = \frac{1}{2} \int_{-1}^1 e^{-t} e^{-j \frac{2\pi}{2} kt} dt = \frac{1}{2} \int_{-1}^1 e^{-(1+j\pi k)t} dt$$
~~$$= \frac{1}{2} \left[ \frac{e^{-(1+j\pi k)t}}{-(1+j\pi k)} \right]_{-1}^1 = \frac{1}{2} \frac{e^{-(1+j\pi k)} - e^{-(1-j\pi k)}}{-(1+j\pi k)}$$

$$= \frac{1}{2} \frac{e^{-1} (e^{-j\pi k} - e^{j\pi k})}{-(1+j\pi k)} = \frac{1}{2} \frac{e^{-1} (-2j \sin \pi k)}{-(1+j\pi k)} = \frac{j e^{-1} \sin \pi k}{1+j\pi k}$$~~

$$= \frac{1}{2} \left[ \frac{e^{-(1+j\pi k)t}}{-1-j\pi k} \right]_{t=-1}^1 = \frac{-1}{2+j2\pi k} (e^{-1-j\pi k} - e^{1-j\pi k}) = \frac{-1}{2+j2\pi k} (e^{-1} e^{-j\pi k} - e^{1} e^{-j\pi k}) = \frac{-1}{2+j2\pi k} (e^{-1} \cos \pi k - j e^{-1} \sin \pi k - e^{1} \cos \pi k + j e^{1} \sin \pi k)$$

$$= - \frac{e^{-1} (\cos \pi k - j \sin \pi k) - e^{1} (\cos \pi k + j \sin \pi k)}{2+j2\pi k} = \frac{e \cos \pi k - e^{-1} \cos \pi k}{2+j2\pi k} = \frac{(e - e^{-1}) \cos \pi k}{2+j2\pi k}$$

$$= \cos \pi k \frac{e - e^{-1}}{2+j2\pi k} = \boxed{(-1)^k \frac{e - e^{-1}}{2+j2\pi k}}$$