Problem 1 *Fourier Series Representations*

(a) $T_0 = \pi$.

$$a_k = \begin{cases} \frac{1}{2}, & k = 2, -2 \\ \frac{1}{2j}, & k = 3 \\ -\frac{1}{2j}, & k = -3 \\ 0, & \text{otherwise.} \end{cases}$$

(b) $T_0 = 8$.
$a_k = \frac{1}{2}\big(\operatorname{sinc}(1-k) + \operatorname{sinc}(1+k)\big)e^{-j\frac{\pi}{4}k}$, $k \in \mathbb{Z}$.

(c) $a_k = (-1)^k \frac{e - e^{-1}}{2 + j2\pi k}$, $k \in \mathbb{Z}$.

(d) $a_k = \begin{cases} 0, & k = 0 \\ j\frac{(-1)^k}{\pi k}, & k \neq 0. \end{cases}$

(e) $a_k = \begin{cases} 0, & k = 0 \\ -j\frac{1-\cos(\pi k)}{\pi k}, & k \neq 0. \end{cases}$

Problem 2 *Periodic Signal*

$$a_k = \begin{cases} \frac{1}{2}, & k = 3, -3 \\ 0, & \text{otherwise.} \end{cases}$$

Problem 3 *Properties of the Fourier Series*

(a) $b_k = a_k e^{-j\frac{2\pi}{T_0}kt_0}$, $k \in \mathbb{Z}$.

(b) $b_k = a_{-k}$, $k \in \mathbb{Z}$.

(c) $b_k = a_k + a_{-k}^*$, $k \in \mathbb{Z}$.

(d) $b_k = a_k + j\frac{2\pi}{T_0}ka_k$, $k \in \mathbb{Z}$.

(e) $b_k = a_k$, $k \in \mathbb{Z}$ and period changes to T_0/α.

Problem 4 *Modulation Property of Fourier Series*

(a) ...

(b) $c_k = \begin{cases} \frac{1}{2}, & k = 0 \\ \frac{1}{4}, & k = -2, 2 \\ 0, & \text{otherwise.} \end{cases}$

Problem 5 *Fourier Transforms*

(a) $\hat{x}(f) = \frac{2}{1+(2\pi f)^2}$, $f \in \mathbb{R}$.

(b) $\hat{x}(f) = \frac{2j}{(j2\pi f)^2}\sin(4\pi f) - \frac{4}{j2\pi f}\cos(4\pi f)$, $f \in \mathbb{R}$.

(c) $\hat{x}(f) = \begin{cases} 1, & |f| \leq \frac{1}{2} \\ 0, & \text{otherwise.} \end{cases}$

(d) $\hat{x}(f) = \begin{cases} \frac{1}{2} - \frac{|f|}{4}, & |f| \leq 2 \\ 0, & \text{otherwise.} \end{cases}$

(e) $\hat{x}(f) = \frac{1}{1+j2\pi f}$, $f \in \mathbb{R}$.

(f) $\hat{x}(f) = 4\,\mathrm{sinc}^2(2t)$, $f \in \mathbb{R}$.

(g) $\hat{x}(f) = \frac{\left(1-e^{-j2\pi f}\right)^2}{j2\pi f}$, $f \in \mathbb{R}$.

Problem 6 *Properties of the Fourier Transform of Real-Valued Signals*

(a) ...

(b) ...

Problem 7 *Derivative of Bandlimited Signal*

$\int_{-\infty}^{\infty} |y(t)|^2 \, \mathrm{d}t = \frac{32}{5}\pi^4$.

Problem 8 *Inverse Filter*

(a) $\hat{y}(f) = \begin{cases} \frac{1}{4}\frac{1}{1+(2\pi f)^2}, & |f| \leq 4 \\ 0, & |f| > 4. \end{cases}$

(b) $\hat{g}(f) = \begin{cases} \frac{1}{2}\left(1+(2\pi f)^2\right), & |f| \leq 4 \\ 0, & |f| > 4. \end{cases}$

$g(t) = 4\,\mathrm{sinc}(8t) + 512\pi^2\,\mathrm{sinc}(8t) + 8\frac{\cos(8\pi t)}{t^2} - 8\frac{\mathrm{sinc}(8t)}{t^2}$, $t \in \mathbb{R}$.

Problem 9 *LTI System 1*

(a) $\hat{h}(f) = \frac{1}{2+j2\pi f}$, $f \in \mathbb{R}$.

(b) $\hat{y}(f) = \frac{1}{1+j2\pi f}\frac{1}{2+j2\pi f}$, $f \in \mathbb{R}$.

(c) $y(t) = e^{-t} - e^{-2t}$, $t \geq 0$.

Problem 10 *LTI System 2*

(a) $\hat{h}(f) = \frac{2+j2\pi f}{3+j8\pi f+(j2\pi f)^2}$, $f \in \mathbb{R}$.

(b) $\hat{y}(f) = \frac{2+j2\pi f}{(3+j8\pi f+(j2\pi f)^2)(1+j2\pi f)}$, $f \in \mathbb{R}$.

(c) $y(t) = \frac{1}{2}te^{-t} + \frac{1}{4}e^{-t} - \frac{1}{4}e^{-3t}$, $t \geq 0$.