(2) E2. Problem 1.

b) $x[n] = \cos\left(\frac{11\pi}{4}n - \frac{\pi}{3}\right), \quad n \in \mathbb{Z}$

$$\frac{11\pi}{4}(n+N) - \frac{\pi}{3} = \frac{11\pi}{4}n - \frac{\pi}{3} + 2\pi k \Leftrightarrow \frac{11}{4}(n+N) = \frac{11}{4}n + 2k \Leftrightarrow \frac{11}{4}N = 2k \Leftrightarrow N = \frac{8}{11}k \Rightarrow N_0 = 8$$

$$a_k = \frac{1}{8}\sum_{n=0}^{7}\cos\left(\frac{11\pi}{4}n - \frac{\pi}{3}\right)e^{-j\frac{2\pi}{8}kn} \overset{\text{euler's, } F_s \text{ linearity}}{=} \frac{1}{8\cdot 2}\left(\sum_{n=0}^{7} e^{j\left(\frac{11\pi}{4}n - \frac{\pi}{3} - \frac{2\pi}{8}kn\right)} + \sum_{n=0}^{7} e^{-j\left(\frac{11\pi}{4}n - \frac{\pi}{3} + \frac{2\pi}{8}kn\right)}\right) = \left[\sum_{n=0}^{N-1} r^n = \dots\right] =$$

$$= \frac{1}{16}\left(e^{-j\frac{\pi}{3}}\frac{1 - e^{j8\left(\frac{11\pi}{4} - \frac{\pi}{4}k\right)}}{1 - e^{j\left(\frac{11\pi}{4} - \frac{\pi}{4}k\right)}} + e^{j\frac{\pi}{3}}\frac{1 - e^{-j8\left(\frac{11\pi}{4} + \frac{\pi}{4}k\right)}}{1 - e^{-j\left(\frac{11\pi}{4} + \frac{\pi}{4}k\right)}}\right) = \frac{1}{16}\left(e^{-j\frac{\pi}{3}}\frac{1 - e^{-j(2\pi k)}}{1 - e^{j\left(\frac{11-k}{4}\pi\right)}} + e^{j\frac{\pi}{3}}\frac{1 - e^{-j2\pi k}}{1 - e^{-j\left(\frac{11+k}{4}\pi\right)}}\right) =$$

(with $22\pi - 2\pi k$ and $22\pi + 2\pi k$ marked over the exponents)

$$= \frac{1}{16}\left(e^{-j\frac{\pi}{3}}\frac{+j\sin(2\pi k)}{\underbrace{1 - e^{j\left(\frac{11-k}{4}\pi\right)}}_{=0 \Leftrightarrow k=11}} + e^{j\frac{\pi}{3}}\frac{+j\sin(2\pi k)}{\underbrace{1 - e^{-j\left(\frac{11+k}{4}\pi\right)}}_{=0 \Leftrightarrow k=-11}}\right) =$$

$11-k = 8\kappa \Leftrightarrow k = 8\kappa + 11$ $\qquad$ $11+k = 8\kappa \Leftrightarrow k = 8\kappa - 11$

$$= j\frac{1}{16}\left(e^{-j\frac{\pi}{3}}\begin{cases}\lim_{k\to 11}\frac{\sin(2\pi k)}{1 - \cos\left(\frac{\pi}{4}(11-k)\right) - j\sin\left(\frac{\pi}{4}(11-k)\right)} & \text{if } \frac{11-k}{4}\pi = 2\pi\kappa \\ 0 & \text{otherwise}\end{cases} + e^{j\frac{\pi}{3}}\begin{cases}\lim_{k\to -11}\frac{\sin(2\pi k)}{1 - \cos\left(\frac{11+k}{4}\pi\right) + j\sin\left(\frac{11+k}{4}\pi\right)} & \text{if } \frac{11+k}{4}\pi = 2\pi\kappa \\ & \text{otherwise}\end{cases}\right) =$$

$$= \begin{cases} j\frac{1}{16}e^{-j\frac{\pi}{3}}\cdot j\lim_{k\to 11}\frac{\overbrace{\sin(2\pi k)}^{2\pi(k-11)+o(k)}}{\underbrace{\sin\left(\frac{11-k}{4}\pi\right)}_{-\frac{\pi}{4}(k-11)}} & \text{if } k = 8\kappa + 11,\ \kappa \in \mathbb{Z} \\ j\frac{1}{16}e^{j\frac{\pi}{3}}\cdot(-j)\lim_{k\to -11}\frac{\sin(2\pi k)}{\underbrace{\sin\left(\frac{11+k}{4}\pi\right)}_{+\frac{\pi}{4}(k+11)}} & \text{if } k = 8\kappa - 11,\ \kappa \in \mathbb{Z} \\ 0 & \text{otherwise}\end{cases} = \begin{cases} -\frac{1}{16}e^{-j\frac{\pi}{3}}\lim_{k\to 11}\frac{2\pi(k-11)}{-\frac{\pi}{4}(k-11)} & \text{if } k = 8\kappa + 11,\ \kappa \in \mathbb{Z} \\ +\frac{1}{16}e^{j\frac{\pi}{3}}\lim_{k\to -11}\frac{2\pi(k+11)}{\frac{\pi}{4}(k+11)} & \text{if } k = 8\kappa - 11,\ \kappa \in \mathbb{Z} \\ 0 & \text{otherwise}\end{cases} =$$

$$= \begin{cases} -\frac{1}{16}e^{-j\frac{\pi}{3}}\cdot(-8) & \text{if } k \in \{8\kappa + 11\} \\ \frac{1}{16}e^{j\frac{\pi}{3}}\cdot(8) & \text{if } k \in \{8\kappa - 11\} \\ 0 & \text{other}\end{cases} = \boxed{\begin{cases} \frac{1}{2}e^{-j\frac{\pi}{3}} & \text{if } k \in \{8\kappa + 11,\ \kappa \in \mathbb{Z} \\ \frac{1}{2}e^{j\frac{\pi}{3}} & \text{if } k \in \{8\kappa - 11,\ \kappa \in \mathbb{Z} \\ 0 & \text{otherwise}\end{cases}}$$