... Problem 1.

$$e^{-j\frac{\pi}{4}} = \cos\frac{\pi}{4} - j\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}(1-j)$$

b) $x(t) = \cos\left(\frac{\pi}{4}(t-1)\right),\ t \in \mathbb{R} \quad = \cos\left(\frac{\pi}{4}t - \frac{\pi}{4}\right)$

$$e^{j\frac{\pi}{4}(7-8k)} - e^{-j\frac{\pi}{4}} = e^{-j2\pi k} \cdot e^{j\frac{7\pi}{4}} - e^{-j\frac{\pi}{4}}$$

$$T_0 = \frac{2\pi}{\frac{\pi}{4}} = 8$$

$$a_k = \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{-j\frac{2\pi}{T_0}kt}\,dt = \frac{1}{8}\int_0^8 \cos\left(\frac{\pi}{4}t - \frac{\pi}{4}\right) e^{-j\frac{2\pi}{8}kt}\,dt = \frac{1}{8}\int_0^8 \frac{e^{j(\frac{\pi}{4}t-\frac{\pi}{4})} + e^{-j(\frac{\pi}{4}t-\frac{\pi}{4})}}{2} e^{-j\frac{\pi}{4}kt}\,dt =$$

$$= \frac{1}{16}\int_{t=0}^8 \left(\underbrace{e^{j(\frac{\pi}{4}t-\frac{\pi}{4})} e^{-j(\frac{\pi}{4}kt)}}_{e^{j\frac{\pi}{4}((1-k)t-1)}} + \underbrace{e^{-j(\frac{\pi}{4}t-\frac{\pi}{4})} e^{-j\frac{\pi}{4}kt}}_{e^{j\frac{\pi}{4}(1-(1+k)t)}}\right)dt = \frac{1}{16}\left(\frac{e^{j\frac{\pi}{4}((1-k)t-1)}}{j\frac{\pi}{4}(1-k)} + \frac{e^{j\frac{\pi}{4}(1-(1+k)t)}}{-j\frac{\pi}{4}(1+k)}\right)\Bigg|_{t=0}^{8} =$$

$$= j\frac{1}{4\pi}\left(-\frac{e^{j\frac{\pi}{4}((1-k)8-1)} - e^{-j\frac{\pi}{4}}}{(1-k)} + \frac{e^{j\frac{\pi}{4}(1-(1+k)8)} - e^{j\frac{\pi}{4}}}{(1+k)}\right)$$

$$= j\frac{1}{4\pi}\left(-\frac{e^{-j\frac{\pi}{4}}\left(\cos(2\pi(1-k)) + j\sin(2\pi(1-k))\right) - e^{-j\frac{\pi}{4}}}{(1-k)} + \frac{e^{j\frac{\pi}{4}}\left(\cos(2\pi(1+k)) - j\sin(2\pi(1+k))\right) - e^{j\frac{\pi}{4}}}{(1+k)}\right) =$$

$$= j\frac{2\pi}{4\pi}\left(-\frac{e^{-j\frac{\pi}{4}}\left(1 + j\sin(2\pi(1-k)) - 1\right)}{2\pi(1-k)} + \frac{e^{j\frac{\pi}{4}}\left(1 - j\sin(2\pi(1+k)) - 1\right)}{2\pi(1+k)}\right) =$$

$$= \frac{1}{2}\left(e^{-j\frac{\pi}{4}} \overbrace{\operatorname{sinc}(2-2k)}^{\delta[1-k]} + e^{j\frac{\pi}{4}} \overbrace{\operatorname{sinc}(2+2k)}^{\delta[1+k]}\right) = \frac{e^{j\frac{\pi}{4}}}{2} \overbrace{\delta[1+k]}^{k=-1} + \frac{e^{-j\frac{\pi}{4}}}{2} \overbrace{\delta[1-k]}^{k=1} =$$

$$= \boxed{\frac{1}{2} e^{-j\frac{\pi}{4}k}\left(\delta[1+k] + \delta[1-k]\right)}$$