

... Problem 1.

n	f	f(0)
0	$\cos nx$	1
1	$-\sin nx$	0
2	$-\cos nx$	-1
3	$\sin nx$	0

e) $x(t)$ is periodic with $T=1$ and: $x(t) = \begin{cases} -1 & \text{if } -\frac{1}{2} \leq t \leq 0 \\ 1 & \text{if } 0 < t \leq \frac{1}{2} \end{cases}$

$$a_k = \frac{1}{T_0} \int_{-T_0}^{T_0} x(t) e^{-j \frac{2\pi}{T_0} kt} dt = \frac{1}{1} \left(\int_{-\frac{1}{2}}^0 -e^{-j \frac{2\pi}{1} kt} dt + \int_0^{\frac{1}{2}} e^{-j \frac{2\pi}{1} kt} dt \right) = - \left(\frac{e^{-j 2\pi k t}}{-j 2\pi k} \right) \Big|_{t=-\frac{1}{2}}^0 + \left(\frac{e^{-j \frac{2\pi}{1} kt}}{-j 2\pi k} \right) \Big|_{t=0}^{\frac{1}{2}} =$$

$$= j \frac{(e^{-0 + j 2\pi k} - e^{-j \pi k})}{2\pi k} + j \frac{(e^{-j \pi k} - e^{-0})}{2\pi k} = j \frac{-1 + e^{j \pi k} + e^{-j \pi k} - 1}{2\pi k} = \cancel{j \frac{-2 + e^{j \pi k} + e^{-j \pi k}}{2\pi k}}$$

$$\cancel{j \frac{-2 + \cos \pi k + j \sin \pi k + \cos \pi k - j \sin \pi k}{2\pi k}} = j \frac{-2 + \cos \pi k + \cos \pi k}{2\pi k} = j \frac{-2 + 2 \cos \pi k}{2\pi k} = j \frac{\cos \pi k - 1}{\pi k}$$

at $k=0$: $a_0 = \lim_{k \rightarrow 0} j \frac{\cos \pi k - 1}{\pi k} = j \frac{1}{\pi} \lim_{k \rightarrow 0} \frac{1 - \frac{k^2}{2} - 1}{k} = \cancel{j \frac{1}{\pi} \lim_{k \rightarrow 0} \frac{-\frac{k^2}{2}}{k}} = \cancel{j \frac{1}{\pi} \lim_{k \rightarrow 0} -\frac{k}{2}} = \lim_{k \rightarrow 0} j \frac{k^2}{2\pi} = 0$

$$a_k = \begin{cases} j \frac{\cos \pi k - 1}{\pi k} & \text{if } k \neq 0 \\ 0 & \text{if } k = 0 \end{cases}$$