... E1. Problem 5.

c) $\operatorname{sinc}(t), \quad t \in \mathbb{R} \quad = x(t)$

$$\hat{x}(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t}\, dt = \int_{-\infty}^{\infty} \operatorname{sinc}(t) e^{-j2\pi f t}\, dt \ldots \Rightarrow \boxed{\hat{x}(f) = \begin{cases} 1 & \text{if } |f| \le \frac{1}{2} \\ 0 & \text{otherwise} \end{cases} = \Pi(f)}$$

~~d) sinc²(t), t ∈ ℝ~~

$$|t-\tau| \le \tfrac{1}{2} \iff -\tfrac{1}{2} \le t-\tau \le \tfrac{1}{2} \iff t-\tfrac{1}{2} \le \tau \le t+\tfrac{1}{2}$$

$$x(t) = \operatorname{sinc}(2t)\cdot\operatorname{sinc}(2t) = y(t)\cdot y(t) \Rightarrow y(t) = \operatorname{sinc}(2t)$$

$$\hat{y}(f) = \operatorname{rect}(f) \ldots \quad \widehat{\operatorname{sinc}}(2t) = \tfrac{1}{2}\Pi\left(\tfrac{f}{2}\right)$$

$$\hat{x}(f) = \hat{y}(f) * \hat{y}(f) = \tfrac{1}{2}\operatorname{rect}\left(\tfrac{f}{2}\right) * \tfrac{1}{2}\operatorname{rect}\left(\tfrac{f}{2}\right) = \tfrac{1}{2}\operatorname{tri}\left(\tfrac{f}{2}\right) = \tfrac{1}{2}\Lambda\left(\tfrac{f}{2}\right) = \begin{cases} \frac{1}{2} - \frac{|f|}{4} & \text{if } |f| \le 2 \\ 0 & \text{otherwise} \end{cases}$$

$$\operatorname{rect}(t) * \operatorname{rect}(t) = \int_{-\infty}^{\infty} \operatorname{rect}(\tau)\operatorname{rect}(t-\tau)\, d\tau = \ldots$$

$$= \left[\begin{matrix} t-\frac{1}{2} \le \tau \le t+\frac{1}{2} \\ -\frac{1}{2} \le \tau \le \frac{1}{2} \end{matrix}\right] \Rightarrow \begin{cases} -\frac{1}{2} \le \tau \le t+\frac{1}{2} & \text{if } -1 \le t \le 0 \\ t-\frac{1}{2} \le \tau \le \frac{1}{2} & \text{if } 0 \le t \le 1 \end{cases} =$$

$$= \begin{cases} \int_{-1/2}^{t+1/2} 1\, d\tau & \text{if } -1 \le t \le 0 \\ \int_{t-1/2}^{1/2} 1\, d\tau & \text{if } 0 < t \le 1 \\ 0 & \text{otherwise} \end{cases} = \begin{cases} (\tau)\big|_{-1/2}^{t+1/2} & \text{if } -1 \le t \le 0 \\ (\tau)\big|_{t-1/2}^{1/2} & \text{if } 0 < t \le 1 \\ 0 & \text{otherwise} \end{cases} = \begin{cases} t+\frac{1}{2}-\left(-\frac{1}{2}\right) & \text{if } -1 \le t \le 0 \\ \frac{1}{2}-\left(t-\frac{1}{2}\right) & \text{if } 0 < t \le 1 \\ 0 & \text{otherwise} \end{cases} =$$

$$= \begin{cases} 1+t & \text{if } -1 \le t \le 0 \\ 1-t & \text{if } 0 < t \le 1 \\ 0 & \text{otherwise} \end{cases} = \boxed{\begin{cases} 1-|t| & \text{if } |t| \le 1 \\ 0 & \text{otherwise} \end{cases} = \Lambda(t)}$$