(12) E4. Problem 9.

c) $y[n] = (h_2 * x)[n] = s[n] \iff \hat{y}(z) = \hat{h}_2(z)\hat{x}(z) = \hat{s}(z) \iff \hat{h}_2(z) = \frac{\hat{s}(z)}{\hat{x}(z)} =$

$= \frac{1}{1 - e^{-\alpha\beta} z^{-\beta}}$, ROC: $|z| < e^{-\alpha}$ or $|z| > e^{-\alpha}$

$R_a = e^{\alpha} R_{h_2} = |z| < 1$ or $|z| > 1$

$\hat{h}_2(z) = \frac{1}{1 - (e^{\alpha} z)^{-\beta}} = \frac{1}{1 - z^{-\beta}}\Big|_{z = e^{\alpha} z} = \hat{a}(e^{\alpha} z)$;

$\hat{a}(z) = \frac{1}{1 - z^{-\beta}} = \hat{b}(z^{\beta})$, $\hat{b}(z) = \frac{1}{1 - z^{-1}} \Rightarrow b[n] = \begin{cases} u[n] & \text{if } R_b \text{ is } |z| > e^{-\alpha} \\ u[-n-1] & \text{if } R_b \text{ is } |z| < e^{-\alpha} \end{cases}$

$R_b = R_a^{1/\beta} = |z|^{\frac{1}{\beta}} < 1$ or $|z|^{\frac{1}{\beta}} > 1 = R_a$

$a[n] = b_{(\beta)}[n] = \begin{cases} b[\frac{n}{\beta}] & \text{if } \frac{n}{\beta} \in \mathbb{Z} \\ 0 & \text{otherwise} \end{cases}$

$h_2[n] = (e^{-\alpha})^n a[n] = \begin{cases} e^{-\alpha n} b[\frac{n}{\beta}] & \text{if } \frac{n}{\beta} \in \mathbb{Z} \\ 0 & \text{otherwise} \end{cases}$

For $R_{h_2} = \{|z| < e^{-\alpha}\}$: $R_a = R_b = \{|z| < 1\} \Rightarrow b[n] = u[n] \Rightarrow$ $\boxed{h_2[n] = \begin{cases} e^{-\alpha n} & \text{if } n \geq 0, \frac{n}{\beta} \in \mathbb{Z} \\ 0 & \text{otherwise} \end{cases}}$

For $R_{h_2} = \{|z| > e^{-\alpha}\}$: $R_a = R_b = \{|z| > 1\} \Rightarrow b[n] = u[-n-1] \Rightarrow$ $\boxed{h_2[n] = \begin{cases} e^{-\alpha n} & \text{if } n < 0, \frac{n}{\beta} \in \mathbb{Z} \\ 0 & \text{otherwise} \end{cases}}$