... Problem 1.

c) $x(t)$ periodic with $T_0=2$ and: $x(t)=e^{-t}$, $-1<t\leq 1$

$$a_k=\frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}}x(t)e^{-j\frac{2\pi}{T_0}kt}\,dt=\frac{1}{2}\int_{-1}^{1}e^{-t}e^{-j\frac{2\pi}{2}kt}=\frac{1}{2}\int_{-1}^{1}e^{-j(\pi kt)-t}dt=$$

$$=j\frac{1}{2}\frac{\sin(-\pi k-1)}{\pi k-1}=\frac{-j}{2}\operatorname{sinc}\left(-k-\frac{1}{\pi}\right)$$

$$=\frac{1}{2}\left(\frac{e^{-t(j\pi k+1)}}{-1-j\pi k}\right)\Bigg|_{t=-1}^{1}=\frac{-1}{2+j2\pi k}\cdot\left(e^{-j\pi k-1}-e^{j\pi k+1}\right)=$$

$$=-\frac{e^{-1}(\cos\pi k-j\sin\pi k)-e(\cos\pi k+j\sin\pi k)}{2+j2\pi k}=\frac{e\cos\pi k-e^{-1}\cos\pi k}{2+j2\pi k}=$$

$$=\cos\pi k\,\frac{e-e^{-1}}{2+j2\pi k}=\boxed{(-1)^k\frac{e-e^{-1}}{2+j2\pi k}}$$