E1. Problem 7. Derivative of Bandlimited signal

Let $x(\cdot)$ be a signal with $\hat{x}(f)=\begin{cases}1 & \text{if } |f|<1\\ 0 & \text{otherwise}\end{cases}$

$$x(t)=\int_{-\infty}^{\infty}\hat{x}(f)\,e^{j2\pi ft}\,dt=\int_{-1}^{1}e^{j2\pi ft}\,df=\left(\frac{e^{j2\pi ft}}{j2\pi t}\right)\Bigg|_{f=-1}^{1}=\frac{e^{j2\pi t}-e^{-j2\pi t}}{j2\pi t}=\frac{\sin 2\pi t}{\pi t}=2\,\mathrm{sinc}(2t)$$

Let $y(\cdot)$ be $y(t)=\frac{d^2}{dt^2}x(t)$, $t\in\mathbb{R}$

What is the value of $\int_{-\infty}^{\infty}|y(t)|^2\,dt$.

$$y(t)=\frac{d^2}{dt^2}x(t)\;\circ\!\!-\!\!-\!\!\bullet\;\hat{y}(f)=(j2\pi f)^2\,\hat{x}(f)=\begin{cases}-4\pi^2f^2 & \text{if } |f|<1\\ 0 & \text{otherwise}\end{cases}$$

$$\int_{-\infty}^{\infty}|y(t)|^2\,dt\;\circ\!\!-\!\!-\!\!\bullet\;\int_{-\infty}^{\infty}|\hat{y}(f)|^2\,df=\int_{-1}^{1}(-4\pi^2f^2)^2\,df=16\pi^4\int_{-1}^{1}f^4\,df=16\pi^4\left(\frac{f^5}{5}\right)\Bigg|_{f=-1}^{1}=\boxed{\frac{32}{5}\pi^4}$$

$$\cancel{y(t)=\int_{-\infty}^{\infty}\hat{y}(f)\,e^{j2\pi ft}\,df=\int_{-1}^{1}-4\pi^2f^2e^{j2\pi ft}\,df=\left[\begin{matrix}u=f^2\Rightarrow du=2f\,df\\ dv=e^{j2\pi ft}df\Rightarrow v=\frac{e^{j2\pi ft}}{j2\pi t}\end{matrix}\right]=}$$

$$\cancel{=-4\pi^2\left(f^2\frac{e^{j2\pi ft}}{j2\pi t}\Bigg|_{-1}^{1}-\int_{-1}^{1}f\,\frac{e^{j2\pi ft}}{j2\pi t}\,df\right)=-4\pi^2}$$