... E3. Problem 2.

(2)

b) $N=11 \Rightarrow T_s = \frac{2\pi}{11}$ ✱ $\Rightarrow x[n] = \frac{1}{2}\cos\left(\frac{6\pi}{11}n\right) + 2\sin\left(\frac{8\pi}{11}n\right)$

$\cos\left(\frac{6\pi}{11}n\right)$ p/w/ period $N_A = 11$

$\sin\left(\frac{8\pi}{11}n\right)$ w/ period $N_B = 11$ $\Big\}$ $N_0 = 11$ (okay...)

$$a_k = \frac{1}{N_0}\sum_{n=\langle N_0\rangle} x[n]\, e^{-j\frac{2\pi}{N_0}kn} = \frac{1}{11}\sum_{n=0}^{10}\left(\frac{1}{2}\cos\left(\frac{6\pi}{11}n\right) + 2\sin\left(\frac{8\pi}{11}n\right)\right)e^{-j\frac{2\pi}{11}kn} =$$

$$= \frac{1}{11}\left(\frac{1}{2}\sum_{n=0}^{10}\frac{1}{2}\underbrace{e^{j\frac{6\pi}{11}n}}_{\left(e^{j\frac{2\pi}{11}(3+k)}\right)^n} e^{-j\frac{2\pi}{11}kn} + \frac{1}{2}\sum_{n=0}^{10}\frac{1}{2}\underbrace{e^{-j\frac{6\pi}{11}n}}_{\left(e^{j\frac{2\pi}{11}(3+k)}\right)^{-n}} e^{-j\frac{2\pi}{11}kn} + 2\sum_{n=0}^{10}\frac{1}{2}\underbrace{e^{j\frac{8\pi}{11}n}}_{\left(e^{j\frac{2\pi}{11}(4-k)}\right)^n} e^{-j\frac{2\pi}{11}kn} - 2\sum_{n=0}^{10}\frac{1}{2}e^{-j\frac{8\pi}{11}n}e^{-j\frac{2\pi}{11}kn}\right) =$$

$$= \frac{1}{11}\left(\frac{1}{4}\frac{1-e^{j\frac{2\pi}{11}(3-k)\cdot 11}}{1-e^{j\frac{2\pi}{11}(3-k)}} + \frac{1}{4}\frac{1-e^{-j\frac{2\pi}{11}(3+k)\cdot 11}}{1-e^{-j\frac{2\pi}{11}(3+k)}} + \frac{1-e^{j\frac{2\pi}{11}(4-k)\cdot 11}}{1-e^{j\frac{2\pi}{11}(4-k)}} - \frac{1-e^{-j\frac{2\pi}{11}(4+k)\cdot 11}}{1-e^{-j\frac{2\pi}{11}(4+k)}}\right) =$$

$$= \frac{1}{11}\left(\frac{1}{4}\underbrace{\frac{-j\sin(2\pi(3-k))}{1-e^{j\frac{2\pi}{11}(3-k)}}}_{=0\ \forall k\in\mathbb{Z}} + \frac{1}{4}\underbrace{\frac{j\sin(2\pi(3+k))}{1-e^{-j\frac{2\pi}{11}(3+k)}}}_{=0\ \forall k\in\mathbb{Z}} + \frac{-j\sin(2\pi(4-k))}{1-e^{j\frac{2\pi}{11}(4-k)}} - \frac{j\sin(2\pi(4+k))}{1-e^{-j\frac{2\pi}{11}(4+k)}}\right) =$$

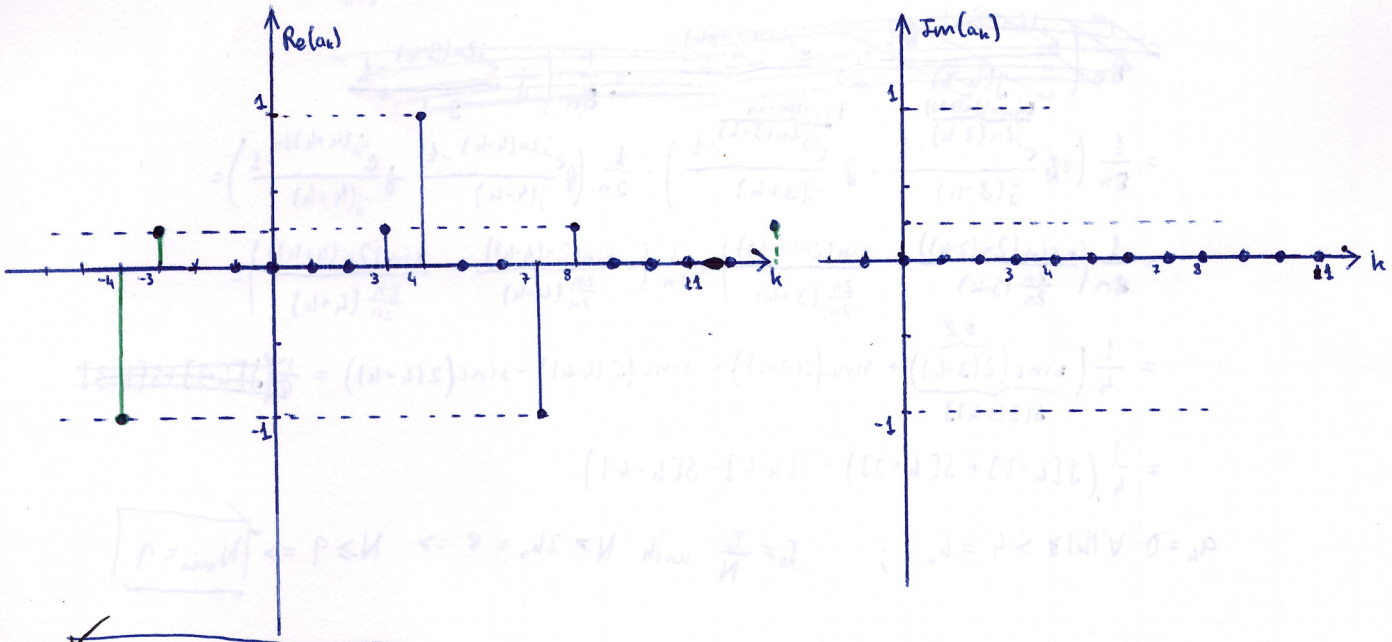
$\neq 0\ \forall\ 3-k\neq 11\ell,\ \ell\in\mathbb{Z}$ $\qquad$ $\neq 0\ \forall\ 3+k\neq 11\ell,\ \ell\in\mathbb{Z}$

$$= \frac{1}{11}\left(\frac{1}{4}\begin{cases}\lim_{3-k\to 0}\frac{-j\sin(2\pi(3-k))}{1-e^{j\frac{2\pi}{11}(3-k)}} = \lim_{3-k\to 0}\frac{-j\sin(2\pi(3-k))}{-j\sin\left(\frac{2\pi}{11}(3-k)\right)}, & 3-k\in\{\ldots,-11,0,11,\ldots\} \\ 0, & \text{otherwise}\end{cases}\right.$$

$$\left. + \frac{1}{4}\begin{cases}\lim_{3+k\to 0}\frac{j\sin(2\pi(3+k))}{1-e^{-j\frac{2\pi}{11}(3+k)}} = \lim_{3+k\to 0}\frac{j\sin(2\pi(3+k))}{j\sin\left(\frac{2\pi}{11}(3+k)\right)}, & 3+k\in\{\ldots,-11,0,11,\ldots\} \\ 0, & \text{otherwise}\end{cases} + \right.$$

$$\left. + \frac{1}{4}\begin{cases}\lim_{4-k\to 0}\frac{-j\sin(2\pi(4-k))}{1-e^{j\frac{2\pi}{11}(4-k)}}, & 4-k\in\{\ldots,-11,0,11,\ldots\} \\ 0, & \text{otherwise}\end{cases} - \frac{1}{4}\begin{cases}\lim_{4+k\to 0}\frac{j\sin(2\pi(4+k))}{1-e^{j\frac{2\pi}{11}(4+k)}}, & 4+k\in\{\ldots,-11,0,11,\ldots\} \\ 0, & \text{otherwise}\end{cases}\right) =$$

$$= \frac{1}{4}\left(\delta[k-3] + \delta[k-8]\right) + \delta[k-4] - \delta[k-7], \quad 0\le k<11$$



There is no aliasing