... Problem 1.

d) $x(t)$ periodic with $T=4$ and: $x(t)=\frac{t}{2}$, $-2<t\leq 2$

$$a_k = \frac{1}{T_0}\int_{-T/2}^{T/2} x(t)\, e^{-j\frac{2\pi}{T_0}kt}\,dt = \frac{1}{4}\int_{-2}^{2}\frac{t}{2}\, e^{-j\frac{\pi}{2}kt}\,dt \overset{\text{IBP}}{=} \left[\begin{array}{l} u=t \Rightarrow du=dt \\ dv=e^{-j\frac{\pi}{2}kt}dt \Rightarrow v=\frac{e^{-j\frac{\pi}{2}kt}}{-j\frac{\pi}{2}k}\end{array}\right] =$$

$$= \frac{1}{8}\left(t\,\frac{e^{-j\frac{\pi}{2}kt}}{-j\frac{\pi}{2}k}\Big|_{t=-2}^{2} - \int_{-2}^{2}\frac{e^{-j\frac{\pi}{2}kt}}{-j\frac{\pi}{2}k}\,dt\right) = \frac{1}{8}\left(\frac{2e^{-j\pi k} - (-2e^{j\pi k})}{-j\frac{\pi k}{2}} - \left(\frac{e^{-j\frac{\pi}{2}kt}}{(-j\frac{\pi}{2}k)^2}\right)\Big|_{t=-2}^{2}\right) =$$

$$= \frac{e^{j\pi k}+e^{-j\pi k}}{2}\cdot\frac{1}{-j\pi k} + \frac{e^{-j\pi k}-e^{j\pi k}}{2j^2\pi^2k^2} = \frac{j\cos\pi k}{\pi k} + \frac{\sin\pi k}{j\pi^2k^2} = j\left(\underbrace{\frac{\cos\pi k}{\pi k}}_{\frac{(-1)^k}{\pi k}} - \underbrace{\frac{\operatorname{sinc} k}{\pi k}}_{\frac{\delta[k]}{\pi k}}\right) =$$

$$= j\,\frac{(-1)^k-\delta[k]}{\pi k} = \begin{cases} 0 & \text{if } k=0 \\ j\frac{(-1)^k}{\pi k} & \text{if } k\neq 0\end{cases}$$