

Ex. Problem 2 Properties of Fourier Series

Let $x[n]$ be periodic with period N and Fourier series repr. $x[n] = \sum_{k=-\infty}^{\infty} a_k e^{j \frac{2\pi}{N} kn}$
For each of these signals, derive their Fourier series coefficients as a function of $\{a_k\}$.

a) $y[n] = x[n - n_0]$, $n \in \mathbb{Z}$

$$b_k = a_k e^{j \frac{2\pi}{N} kn_0}, \quad k \in \mathbb{Z}$$

b) $y[n] = x[n] - x[n-1]$, $n \in \mathbb{Z}$

$$b_k = a_k - a_k e^{j \frac{2\pi}{N} k}, \quad k \in \mathbb{Z}$$

c) $y[n] = x[n] + x^*[n]$, $n \in \mathbb{Z}$

$$b_k = a_k + a_{-k}^*, \quad k \in \mathbb{Z}$$

d) $y[n] = (-1)^n x[n]$, $n \in \mathbb{Z}$, N even

~~$y[n] = (-1)^n x[n] = \cos(\pi n) x[n]$~~

$$(-1)^n = \cos(\pi n) = \cos(\pi n) + j \sin(\pi n) = e^{j\pi n} = e^{j \frac{2\pi}{N} \cdot \frac{N}{2} n} \Rightarrow y[n] = e^{j \frac{2\pi}{N} \cdot \frac{N}{2} n} x[n] \Rightarrow$$

$$\Rightarrow b_k = a_{k - \frac{N}{2}}, \quad k \in \mathbb{Z}$$

e) $y[n] = \begin{cases} x[\frac{n}{m}] & \text{if } n \text{ multiple of } m \\ 0 & \text{otherwise} \end{cases}$

$$b_k = \frac{a_k}{m}, \quad \text{period changes to } mN$$