E1. Problem 3. Problems of the Fourier Series

Let $x(\cdot)$ be a periodic signal with fundamental period T_0 and Fourier series coefficients $\{a_k\}$. For each of these signals, express their Fourier series coefficients as a function of $\{a_k\}$.

a) $y(t) = x(t-t_0)$, $t \in \mathbb{R}$

$b_k = a_k e^{-j\frac{2\pi}{T_0} k t_0}$, $k \in \mathbb{Z}$

b) $y(t) = x(t)$, $t \in \mathbb{R}$

$b_k = a_{-k}$, $k \in \mathbb{Z}$

c) $y(t) = x(t) + x^*(t)$, $t \in \mathbb{R}$

$b_k = a_k + a_{-k}^*$, $k \in \mathbb{Z}$

d) $y(t) = x(t) + \frac{d}{dt} x(t)$, $t \in \mathbb{R}$

$b_k = a_k + j\frac{2\pi}{T_0} k a_k$, $k \in \mathbb{Z}$

e) $y(t) = x(\alpha t)$, $t \in \mathbb{R}$ with $\alpha > 0$

$b_k = a_k$, $k \in \mathbb{Z}$ $\quad (T_0 \to T_0/\alpha)$