(3) E3. Problem 2.

c) $N=5 \Rightarrow T_s = \frac{2\pi}{5} \Rightarrow x[n] = x[n] = \frac{1}{2}\cos\left(\frac{6\pi}{5}n\right) + 2\sin\left(\frac{8\pi}{5}n\right)$

$$a_k = \frac{1}{5}\sum_{n=0}^{4}\left(\frac{1}{2}\cos\left(\frac{6\pi}{5}n\right) + 2\sin\left(\frac{8\pi}{5}n\right)\right)e^{-j\frac{2\pi}{5}kn} =$$

$$= \frac{1}{5}\left(\frac{1}{2}\left(\sum_{n=0}^{4}\frac{1}{2}e^{j\frac{2\pi}{5}(3-k)n} + \sum_{n=0}^{4}\frac{1}{2}e^{-j\frac{2\pi}{5}(3+k)n}\right) + 2\left(\sum_{n=0}^{4}\frac{1}{2}e^{j\frac{2\pi}{5}(4-k)n} - \sum_{n=0}^{4}\frac{1}{2}e^{-j\frac{2\pi}{5}(4+k)n}\right)\right) =$$

$$= \frac{1}{5}\left(\frac{1}{4}\left(\frac{1-e^{j2\pi(3-k)}}{1-e^{j\frac{2\pi}{5}(3-k)}} + \frac{1-e^{-j2\pi(3+k)}}{1-e^{-j\frac{2\pi}{5}(3+k)}}\right) + \frac{1-e^{j2\pi(4-k)}}{1-e^{j\frac{2\pi}{5}(4-k)}} - \frac{1-e^{-j2\pi(4+k)}}{1-e^{-j\frac{2\pi}{5}(4+k)}}\right) =$$

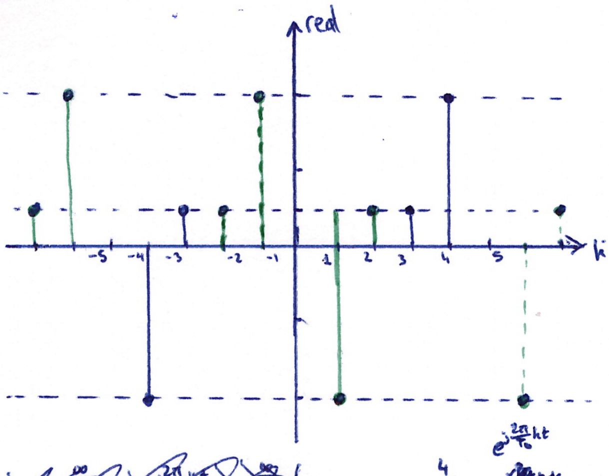
$$\left[\frac{1-e^{j2\pi(\overbrace{x(k)}^{\in\mathbb{Z}})}}{1-e^{j\frac{2\pi}{N}(x(k))}} = \frac{\overbrace{1-\cos(2\pi(x(k)))}^{0}+j\sin(2\pi(x(k)))}{1-e^{j\frac{2\pi}{N}(x(k))}} = \frac{-j\sin(2\pi x(k))}{1-e^{j\frac{2\pi}{N}x(k)}} = \begin{cases}\underbrace{\lim_{x(k)\to 0}\frac{-j\sin(2\pi x(k))}{-j\sin(\frac{2\pi}{N}x(k))}}_{N}, & x(k)\in N\ell,\ \ell\in\mathbb{Z}\\ 0, & \text{others}\end{cases}\right] =$$

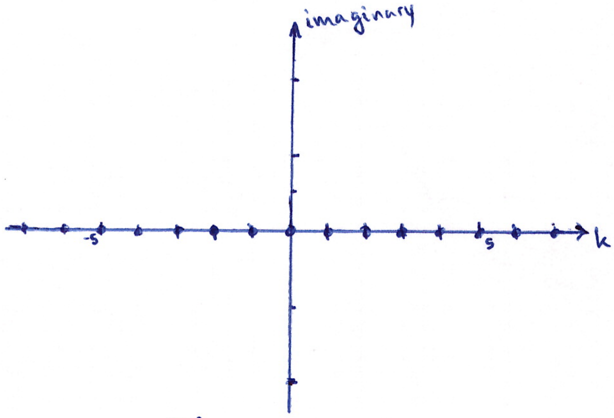
$$\cancel{= \frac{1}{5}\left(\frac{1}{4}\left(5\delta[3-k]+5\delta[3+k]\right)\right)}$$

$$= \frac{1}{5}\left(\frac{1}{4}\left(5\,I\{3-k=5\ell : \ell\in\mathbb{Z}\} + 5\,I\{3+k=5\ell : \ell\in\mathbb{Z}\}\right) + 5\,I\{4-k=5\ell : \ell\in\mathbb{Z}\} - 5\,I\{4+k=5\ell : \ell\in\mathbb{Z}\}\right) =$$

$$= \frac{1}{4}I\{k\in\ldots,-2,3,8,\ldots\} + \frac{1}{4}I\{k=\ldots,-8,-3,2,\ldots\} + I\{k=\ldots,-1,4,9,\ldots\} - I\{k=\ldots,-9,-4,1,\ldots\} =$$

$$= \frac{1}{4}\delta[k-3] + \frac{1}{4}\delta[\underbrace{k-2}_{k+3}] + \delta[k-4] - \delta[\underbrace{k-1}_{k+4}], \quad 0\le k<5$$





$$\cancel{x(t) = \sum_{k=-\infty}^{\infty}a_k e^{j\frac{2\pi}{T_0}kt} = \sum_{k=-\infty}^{\infty}} \qquad \tilde{x}(t) = \sum_{k=0}^{4}a_k \cancel{e^{j\frac{2\pi}{T_0}kt}} = \cancel{x(t)} = \sum_{k=-2}^{2}a_k e^{j\frac{2\pi}{2\pi}kt} = \underbrace{\frac{1}{4}e^{-j\frac{2\pi}{2\pi}2t}}_{k=-2} + \underbrace{e^{-j\frac{2\pi}{2\pi}t}}_{k=-1} - \underbrace{e^{j\frac{2\pi}{2\pi}t}}_{k=1} + \underbrace{\frac{1}{4}e^{j\frac{2\pi}{2\pi}2t}}_{k=2} =$$

$$= \boxed{\frac{1}{2}\cos(2t) - 2\sin(t)} = \tilde{x}(t)$$