Bachelor in Mobile and Space Communications Engineering
Bachelor in Telematics Engineering
Bachelor in Sound and Image Engineering
Bachelor in Telecommunication Technologies Engineering

Notation:

- $\widehat{S}_{\text{MMSE}}$: Minimum Mean Square Error estimator.
- $\widehat{S}_{\text{MAD}}$: Mininimum Mean Absolute Deviation Error estimator.
- $\widehat{S}_{\text{MAP}}$: Maximum a posteriori estimator.
- $\widehat{S}_{\text{ML}}$: Maximum likelihood estimator.
- $\widehat{S}_{\text{LMSE}}$: Linear Minimum Mean Square Error estimator.

1. Let S and X be two random variables with joint p.d.f.

$$p_{S,X}(s,x) = \frac{1}{x}, \qquad 0 \leq s \leq x; \qquad 0 \leq x \leq 1.$$

We want to estimate s based on the observation of x using an estimator $\hat{s}$ that minimizes the following cost function:

$$c(s,\hat{s}) = \frac{1}{3}\hat{s}^3 - \frac{1}{2}s\hat{s}^2, \qquad s, \hat{s} \in [0,1].$$

   (a) Find the minimum mean cost estimator.
   (b) Find the linear estimator with analytical shape $\hat{s} = wx$ which provides the smallest mean cost.

2. The r.v. S and X are jointly Gaussian. Variable S is known to be zero-mean with $\mathbb{E}\{S^2\} = 1$. Variable X is known to have the form $X = T + N$, where T and N are independent zero-mean gaussian random variables with variances 1 and v, respectively. Noise N is independent from S and T, and $\mathbb{E}\{ST\} = \rho$.

Determine the MMSE estimator of S given observation X.

3. It is known that r.v. X follows pdf

$$p_{X|S}(x|s) = s^2 x \exp(-sx) \quad x > 0 \tag{1}$$

   (a) Compute the ML estimate of s given x
   (b) 4 independent values of this r.v. are observed: $x^{(1)} = 0.4$, $x^{(2)} = 0.8$, $x^{(3)} = 0.3$ and $x^{(4)} = 0.5$. Compute the ML estimator of s given $(x^{(1)}, x^{(2)}, x^{(3)}, x^{(4)})$.

4. A temperature sensor obtains its output as the sum of the actual temperature and a noise component,

$$X = S + N,$$

where S and N are independent variables representing the temperature and noise components, with reciprocal and uniform pdfs, respectively,

$$p_S(s) = \frac{1}{s\log(3)}, \qquad 1 < s < 3$$

$$p_N(n) \sim U(0,2)$$

   (a) Determine the pdf of X conditioned on the value of S, $p_{X|S}(x|s)$.
   (b) Obtain the joint pdf of random variables S and X, and represent the domain of definition of such pdf.
   (c) Obtain the maximum a posteriori estimator of S given X, $\hat{S}_{\text{MAP}}$.
   (d) Obtain the minimum mean square estimator of S given X, $\hat{S}_{\text{MMSE}}$.