Bachelor in Mobile and Space Communications Engineering
Bachelor in Telematics Engineering
Bachelor in Sound and Image Engineering
Bachelor in Telecommunication Technologies Engineering

Notation:

- $\widehat{S}_{\text{MMSE}}$: Minimum Mean Square Error estimator.
- $\widehat{S}_{\text{MAD}}$: Mininimum Mean Absolute Deviation Error estimator.
- $\widehat{S}_{\text{MAP}}$: Maximum a posteriori estimator.
- $\widehat{S}_{\text{ML}}$: Maximum likelihood estimator.
- $\widehat{S}_{\text{LMSE}}$: Linear Minimum Mean Square Error estimator.

1. The random variables S and X are jointly distributed according to:

$$p_{S,X}(s,x) = G\left(\begin{bmatrix} s \\ x \end{bmatrix} \,\middle|\, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}\right)$$

with $|\rho| < 1$.

(a) Determine the maximum a posteriori estimator of S given X, $\widehat{S}_{\text{MAP}}$.

(b) Is $\widehat{S}_{\text{MAP}}$ an unbiased estimator? Justify your answer.

(c) Calculate the mean square error of the estimator $\widehat{S}_{\text{MAP}}$.

Solution:

(a) $\widehat{S}_{\text{MAP}} = \rho X$

(b) The estimator is unbiased

(c) $\mathrm{E}\left\{\left(S - \widehat{S}_{\text{MAP}}(X)\right)^2\right\} = 1 - \rho^2$

2. Consider a random variable X with p.d.f.:

$$p_{X|M,C}(x|m,c) = \frac{1}{m} \exp\left[-\frac{1}{m}(x-c)\right] \qquad x \geq c$$

where $m \geq 0$ and c are two parameters.

(a) Find the maximum likelihood estimator of the m parameter, $\widehat{M}_{\text{ML}}$, as a function of K samples of X independently drawn, $\left\{X^{(k)}\right\}_{k=1}^{K}$.

(b) Compute the bias and the variance of the $\widehat{M}_{\text{ML}}$ estimator.

Solution:

(a) $\widehat{M}_{\mathrm{ML}} = \dfrac{1}{K} \displaystyle\sum_{k=1}^{K} \left(x^{(k)} - c \right)$

(b) The estimator is unbiased.

$\mathrm{Var}\left\{ \widehat{M}_{\mathrm{ML}} \right\} = \dfrac{m^2}{K}$