Universidad
Carlos III de Madrid
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Escuela Politécnica Superior

Asignatura

Nombre del Alumno

Fecha   Curso   Grupo

PROBLEMA 17

Se dispone de K muestras tomadas indep. $\{x^{(k)}\}_{k=1}^{K}$

$$p_X(x) = \frac{1}{bx^2} \cdot \exp\left[\frac{-1}{bx}\right] u(x) \quad \text{con } b>0.$$

a) Determinar $\hat{B}_{ML}$ en función de dichas muestras.

$$p_X(x) \Rightarrow p_{X|B}(x|b) = \frac{1}{bx^2} \cdot \exp\left[\frac{-1}{bx}\right] \cdot u(x) \Rightarrow \{x^{(k)}\}_{k=1}^{K}$$

$$p(\{x^{(k)}\}|b) = \prod_{k=1}^{K} \frac{1}{b(x^{(k)})^2} \cdot \exp\left[\frac{-1}{bx^{(k)}}\right]$$

$$\hat{B}_{ML} = \arg\max_b \; p(\{x^{(k)}\}|b) = \arg\max_b \left[\ln p(\{x^{(k)}\}|b)\right]$$

$$\ln\left[p(\{x^{(k)}\}|b)\right] = \ln\left[\prod_{k=1}^{K} \frac{1}{b(x^{(k)})^2} \cdot \exp\left[\frac{-1}{b \cdot x^{(k)}}\right]\right]$$

$$= \underbrace{\sum_{k=1}^{K} \ln\left[\frac{1}{b(x^{(k)})^2}\right]} - \frac{1}{b} \cdot \sum_{k=1}^{K} \frac{1}{x^{(k)}}$$

$$-\sum_{k=1}^{K} \ln\left[b \cdot (x^{(k)})^2\right] - \frac{1}{b} \cdot \sum_{k=1}^{K} \frac{1}{x^{(k)}}$$

$$= -K \cdot \ln b - \sum_{k=1}^{K} \ln[x^{(k)}]^2 - \frac{1}{b} \cdot \sum_{k=1}^{K} \frac{1}{x^{(k)}}$$

$$\left.\frac{\partial \ln[p(\{x^{(k)}\}|b)]}{\partial b}\right|_{b=\hat{b}_{ML}} = 0 \Rightarrow \frac{-K}{\hat{b}_{ML}} + \frac{1}{\hat{b}_{ML}^2} \cdot \sum_{k=1}^{K} \frac{1}{x^{(k)}} = 0$$

$$\boxed{\hat{B}_{ML} = \frac{1}{K} \cdot \sum_{k=1}^{K} \frac{1}{x^{(k)}}}$$

Papel ecológico.

b) Verificar que $Y = 1/X$ sigue una d.d.p. exponencial unilateral y establecer la media de la distribución.

Cambio de v.a. unidimensional:

$$p_Y(y) = p_X\left(x = f^{-1}(y)\right)\cdot\left|\frac{dx}{dy}\right|$$

La v.a. X tiene una $p_X(x)$ conocida $\Rightarrow p_X(x) = \frac{1}{bx^2}\cdot\exp\left(\frac{-1}{bx}\right)\cdot u(x)$

$Y = 1/X \Rightarrow X = 1/Y$

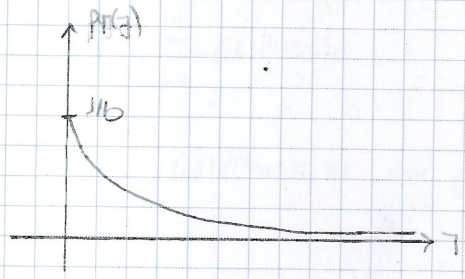
$$\frac{dx}{dy} = \frac{-1}{y^2} \Rightarrow \text{como } 0<x<\infty \Rightarrow 0<y<\infty$$

$$\left|\frac{-1}{y^2}\right| = \frac{1}{y^2}$$

Con esto:

$$p_Y(y) = \frac{1}{b\cdot\frac{1}{y^2}}\cdot\exp\left(\frac{-1}{b\cdot 1/y}\right)\cdot\frac{1}{y^2} \qquad 0<y<\infty$$

$$= \frac{\cancel{y^2}}{b}\cdot\exp\left(-\frac{y}{b}\right)\frac{1}{\cancel{y^2}} = \frac{1}{b}\cdot\exp\left[\frac{-y}{b}\right] \quad 0<y<\infty$$



$$p_Y(y) = \frac{1}{b}\cdot\exp\left[\frac{-y}{b}\right] \quad 0<y<\infty$$

$$E\{y\} = \int_{(y)} y\cdot p_Y(y)\cdot dy = \int_{0\,(y)}^{+\infty}\frac{y}{b}\exp\left[\frac{-y}{b}\right]\cdot dy$$

$$= \frac{1}{b}\cdot\left[-y\cdot b\exp(-y/b)\Big|_0^{+\infty} + b\cdot\int_0^{+\infty}\exp(-y/b)\cdot dy\right]$$

$u = y \quad du = dy$

$dv = \int_0^{+\infty}\exp(-y/b)\,dy \quad v = -b\exp(-y/b)$

$$= -b\cdot\exp(-y/b)\Big|_0^{+\infty} = +b$$

$$\boxed{E\{y\} = +b}$$

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c) ¿es $\hat{B}_{ML}$ un estimador insesgado?

$$E\{\hat{B}_{ML} - b \mid b\} = E\{\hat{B}_{ML} \mid b\} - E\{b \mid b\}$$
$$= E\{\hat{B}_{ML} \mid b\} - b$$

Sabiendo que: $Y = 1/X \Rightarrow \hat{B}_{ML} = \frac{1}{k} \cdot \sum_{k=1}^{k} \frac{1}{x(k)} = \frac{1}{k} \cdot \sum_{k=1}^{k} y(k)$

Con esto:

$$E\{\hat{B}_{ML} \mid b\} = E\{\frac{1}{k} \cdot \sum_{k=1}^{k} y(k) \mid b\} = \frac{1}{\cancel{k}} \cdot \cancel{k} \cdot \underbrace{E\{y \mid b\}}_{b} = b$$

$$\downarrow$$

$$\boxed{E\{\hat{B}_{ML} - b \mid b\}} = E\{\hat{B}_{ML} \mid b\} - b$$
$$= b - b$$
$$\boxed{= 0}$$

El estimador es insesgado!