5. Shared Access

5.1 [1 2 3 4 5 6]

5.2 [1 2 3 4]

5.1. Aloha

5.1.1.

We have a ~~100 station Aloha~~ 100 station pure Aloha network. If $T_{tx} = 1\,\mu s$, what is the number of frames/s each station can send to achieve max efficiency.

k = number of transmission attempts in a T_{tx} time frame. ; $G = \lambda \cdot T_{tx}$

$P(k=k_0) = \frac{G^{k_0} \cdot e^{-G}}{k_0!}$ (poisson) ;

$k=0 \equiv$ no transmissions in a ~~time~~ T_{tx} time frame

$P(k=0) = \frac{G^0 e^{-G}}{0!} = e^{-G}$

Successful transmission $\Leftrightarrow$ no other transmissions in $2 \cdot T_{tx}$

$P(\text{success}) = P(k=0) \cdot P(k=0) = e^{-G} \cdot e^{-G} = e^{-2G}$

Efficiency $S = G \cdot e^{-2G}$

Maximize S:

$$\frac{dS}{dG} = e^{-2G} - 2Ge^{-2G} = 0 \Leftrightarrow 1 - 2G = 0 \Leftrightarrow G = \frac{1}{2}$$

$$\left.\frac{d^2S}{dG^2}\right|_{G=\frac{1}{2}} = \left. -2e^{-2G} - 2(e^{-2G} - 2Ge^{-2G}) \right|_{G=\frac{1}{2}} = -2e^{-1} - 2(e^{-1} - e^{-1}) = -\frac{2}{e} < 0 \Rightarrow \text{maximum}$$

$$G = \frac{1}{2} = \lambda T_{tx} \Rightarrow \lambda = \frac{G}{T_{tx}} = \frac{1}{2T_{tx}} = \frac{1}{2 \cdot 1\mu s} = 5 \cdot 10^5 \ \frac{tx}{s}$$

$$\lambda_{node} = \frac{\lambda}{n} = \frac{5 \cdot 10^5 \frac{tx}{s}}{100 \text{ stations}} = \boxed{5000 \ \frac{tx/s}{station}}$$