TELECOMMUNICATIONS ENGINEERING

STATISTICS

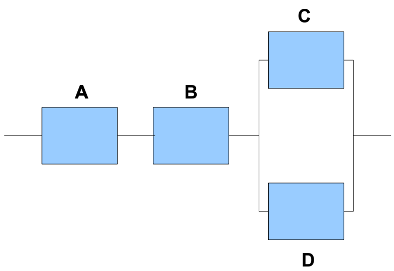
2022-2023

ASSIGNMENT 2. PROBABILITY AND RANDOM VARIABLES

Probability

Exercise 1 (3 points)

Given the circuit shown in the figure, consisting of components A, B, C y D where all of them work independently:



a) Compute by simulation using MATLAB/Octave, the probability that the system works, assuming that $P(A) = 0{,}80$, $P(B) = 0{,}85$ y $P(C) = P(D) = 0{,}75$. What is the theoretical result that the simulation should be approximate? Justify your answer.

b) Compute by simulation using MATLAB/Octave, the **increase** in the system reliability (in percentage) when we add two components E y F (that work independently) with working probability equal to 0,75 in the final subsystem formed by the components C and D. What is the theoretical result that the simulation should be approximate? Justify your answer.

   Note: The increase in the reliability is calculated as: $\frac{\text{Prob. new syst.}-\text{Prob. original syst.}}{\text{Prob. original syst.}} \times 100\,\%$.

c) Compute by simulation using MATLAB/Octave, the change in the original reliability of the system when:

   - the working probabilities of components A and B are increased by 0,1;
   - the working probabilities of components C and D are increased by 0,2.

   In which of these cases we observe a major impact on the original reliability of the system?

Random Variables

Exercise 2 (2 points)

Let $p = P(S = s)$ be a probability function defined in the sample space $S = \{1, 2, 3, 4, 5\}$ with the following probabilities:

S	1	2	3	4	5
p	0.3	0.2	0.2	0.2	0.1

a) Generate in MATLAB/Octave two independent random variables (X_1 and X_2) with probability function p. What are the approximate values that the mean and variance of X_1 and X_2 should have?

b) Check with a frequency table and a bar diagram that the generation of X_1 and X_2 is correct.

c) Compute using simulation in MATLAB/Octave the probability $P(2 \times X_1 = X_2)$. What is the theoretical value for the obtained result? Justify your answer.

Exercise 3 (3 points)

Use the inverse transformation method of the distribution function to generate a continuous random variable with distribution function $F_X(x)$ given by

$$F_X(x) = \begin{cases} 0 & x < 0 \\ \frac{x^2}{2} & 0 \leq x \leq 1 \\ 1 - \frac{(2-x)^2}{2} & 1 < x \leq 2 \\ 1 & x > 2 \end{cases}$$

a) Determine analytically the density function $f_X(x)$.

b) Write the **pseudocode** to generate random numbers from the random variable X using the inverse transformation method.

c) Write the MATLAB/Octave code to generate random numbers from the random variable X.

d) Obtain analytically $\mathbb{E}[X]$ and $\mathbb{V}\text{ar}[X]$. How can you check if the simulation in c) is performing correctly?

Exercise 4 (2 points)

Let X be a random variable with density function $f(x)$ defined as

$$f(x) = \begin{cases} \alpha\beta x^{\beta-1} \exp\left(-\alpha x^{\beta}\right) & x > 0 \\ 0 & x \leq 0 \end{cases}$$

a) Write a MATLAB/Octave code to generate $n = 100000$ values of X with $\alpha = 0{,}2$ y $\beta = 0{,}5$. Justify your answer.

b) Given that X has mean and variance defined as:

$$\mu = \mathbb{E}[X] = \alpha^{-\frac{1}{\beta}}\Gamma\left(1 + \frac{1}{\beta}\right) \quad \text{and} \quad \sigma^2 = \mathbb{V}\text{ar}[X] = \alpha^{-\frac{2}{\beta}}\left\{\Gamma\left(1 + \frac{2}{\beta}\right) - \left[\Gamma\left(1 + \frac{1}{\beta}\right)\right]^2\right\}$$

How would you check that the simulation of X is correct?

Note: The Gamma function $\Gamma(\cdot)$ for positive integer values is defined as $\Gamma(n) = (n-1)!$