TELECOMMUNICATIONS ENGINEERING

STATISTICS

2022-2023

ASSIGNMENT 4. RANDOM VECTORS & STOCHASTIC PROCESSES

1. Random Vectors

1.1. Exercise 1 (2 points)

Choose a point x randomly in the interval $(0, T)$ and a second point also randomly in the interval (x, T). Define the r.v. X as "the position of the first point" and the r.v. Y as "the distance of the second point respect to the first".

a) Determine, theoretically, the joint density function of (X, Y).

b) Compute by simulation using MATLAB/Octave, $P(Y - X < 0)$ for $T = 5$.

1.2. Exercise 2 (3 points)

Let X and Y be independent r.v.'s with X continuous and Y discrete, given by $X \sim U(-1, 1)$ and

$$Y \sim \begin{pmatrix} 0 & 1 \\ 1/4 & 3/4 \end{pmatrix}$$

where the notation indicates that the r.v. Y takes value 0 with probability $1/4$ and value 1 with probability $3/4$. Considering $R = X + Y$:

a) Determine $f_R(r)$.

b) Calculate theoretically $E[R]$ and $Var[R]$.

c) Check with MATLAB/Octave the results obtained in *a*) and *b*).

2. Stochastic Processes

2.1. Random walk (2 points)

A random walk is a discrete stochastic process $Y(n)$, given by the sum of n i.i.d. random variables, i.e.,

$$Y(n) = X_1 + \cdots + X_n = \sum_{i=1}^{n} X_i.$$

An example of random walk is such that $X_n \sim Ber\{-1, +1\}$ where X_n takes value $+1$ with probability p and value -1 with probability $1 - p$.

a) Determine analytically the mean and the variance of the process $Y(n)$. Are the mean and the variance of the stochastic process $Y(n)$ constant when $p = 1/4$?

b) Generate with MATLAB/Octave three realizations (overlapping the graphs) of a Bernoulli process with $p = 1/4$ and $n = 1000$. Is the obtained result consistent with *a*)?

2.2. Harmonic processes (1 point)

Let X and Y be two independent r.v.'s normally distributed with parameters $\mu_X = \mu_Y = 0$ and $\sigma_X^2 = \sigma_Y^2 = 1$. Define the harmonic process $Z(t)$ as:

$$Z(t) = X\,cos(2\pi t) + Y\,sin(2\pi t).$$

Generate with MATLAB/Octave three realizations (overlapping the graphs) of the process $Z(t)$ with $t = 0 : 0.01 : 2\pi$.

2.3. Harmonic processes with white noise (2 points)

Consider the stochastic process of sinusoid type with random phase and white noise defined by:

$$Y(t) = a sin(\varpi t + u) + W(t)$$

where $a = 1, \varpi = \pi/5$, $u \sim U(-\pi, \pi)$ and $W(t) \sim N(0, 1)$ independent from u.

a) Determine analytically the mean of the stochastic process $Y(t)$. Is the mean of the process $Y(t)$ constant?

b) Generate with MATLAB/Octave three realizations (overlapping the graphs) of the process $Y(t)$ with $t = 1 : 150$. Is the obtained result consistent with a)?