Chapter 2 Problems:

Basic Problems

21 22 23 24 25 26 27 28 29 30
31 32 33 34 35 36 37 38 39

2.21. Compute $x[n] * h[n]$

a) $x[n] = \alpha^n u[n]$
$h[n] = \beta^n u[n]$ $\Big\}$ $\alpha \neq \beta$

$$x[n]*h[n] = \sum_{k=-\infty}^{\infty} x[k]\cdot h[n-k] = \sum_{k\in\mathbb{Z}} \alpha^k u[k]\cdot \beta^{n-k}\cdot u[n-k] = \sum_{k\in\mathbb{Z}} \begin{cases} \left(\frac{\alpha}{\beta}\right)^k \beta^n & \text{if } k\geq 0 \text{ and } n-k\geq 0 \\ 0 & \text{otherwise} \end{cases} =$$

$$= \beta^n \sum_{k=0}^{n} \left(\frac{\alpha}{\beta}\right)^k \frac{1-\left(\frac{\alpha}{\beta}\right)^{n+1}}{1-\frac{\alpha}{\beta}} = \begin{cases} \beta^n \frac{1-\left(\frac{\alpha}{\beta}\right)^{n+1}}{1-\left(\frac{\alpha}{\beta}\right)} & \text{if } n\geq 0 \\ 0 & \text{if } n<0 \end{cases} =$$

$$= u[n]\frac{\beta^n - \frac{\beta^n \alpha^{n+1}}{\beta^n}}{\frac{\beta-\alpha}{\beta}} = \boxed{\frac{\beta^{n+1}-\alpha^{n+1}}{\beta-\alpha}\, u[n]}$$

b) $x[n] = h[n] = \alpha^n u[n]$

$$x[n]*h[n] = \sum_{k=-\infty}^{\infty} x[k]\cdot h[n-k] = \sum_{k=-\infty}^{\infty} \alpha^k u[k]\cdot \alpha^{n-k}\cdot u[n-k] = \sum_{k=-\infty}^{\infty} \begin{cases} \alpha^n & \text{if } k\geq 0 \ \&\ n-k\geq 0 \\ 0 & \text{otherwise} \end{cases} =$$

$$= \sum_{k=0}^{n} \alpha^n = \begin{cases} (n+1)\alpha^n & \text{if } n\geq 0 \\ 0 & \text{if } n<0 \end{cases} = \boxed{(n+1)\alpha^n \cdot u[n]}$$