2.26. Consider the ~~evolut~~ evaluation of $y[n] = x_1[n] * x_2[n] * x_3[n]$, where $x_1[n] = (0.5)^n u[n]$, $x_2[n] = u[n+3]$, and $x_3[n] = \delta[n] - \delta[n-1]$

a) Evaluate $x_1[n] * x_2[n]$

$$x_1[n] * x_2[n] = \sum_{k=-\infty}^{\infty} (0.5)^k u[k] \cdot u[n-k+3] = \sum_{k=0}^{n+3} (0.5)^k = \begin{cases} \frac{1-0.5^{n+4}}{1-0.5} & \text{if } n+3 \geq 0 \\ 0 & \text{if } n+3 < 0 \end{cases} = \begin{cases} 2-\left(\frac{1}{2}\right)^{n+3} & \text{if } n \geq -3 \\ 0 & \text{if } n < -3 \end{cases} = \boxed{\left(2-\left(\tfrac{1}{2}\right)^{n+3}\right) u[n+3]}$$

b) Convolve the result of (a) with $x_3[n]$ to ~~obtain the~~ evaluate $y[n]$

$$(x_1[n] * x_2[n]) * x_3[n] = \left(2-\left(\tfrac{1}{2}\right)^{n+3}\right) u[n+3] * (\delta[n] - \delta[n-1]) =$$

$$= \left(2-\left(\tfrac{1}{2}\right)^{n+3}\right) u[n+3] - \left(2-\left(\tfrac{1}{2}\right)^{n+2}\right) u[n+2] = \begin{cases} 2-\left(\frac{1}{2}\right)^{n+3} & \text{if } -3 \leq n < -2 \\ 2-\left(\frac{1}{2}\right)^{n+3} - 2 + \left(\frac{1}{2}\right)^{n+2} & \text{if } -2 \leq n \\ 0 & \text{if } n < -3 \end{cases} =$$

$$= \begin{cases} 2-\left(\frac{1}{2}\right)^{-3+3} & \text{if } n = -3 \\ -\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{n+2} + \left(\frac{1}{2}\right)^{n+2} & \text{if } -2 \leq n \\ 0 & \text{if } n < 3 \end{cases} = \begin{cases} 2-1=1 & \text{if } n = -3 \\ \left(\frac{1}{2}\right)^{n+3} & \text{if } -2 \leq n \\ 0 & \text{if } n < -3 \end{cases} = \boxed{\left(\tfrac{1}{2}\right)^{n+3} u[n+3]}$$

c) Evaluate $x_2[n] * x_3[n]$

$$x_2[n] * x_3[n] = u[n+3] * (\delta[n] - \delta[n-1]) = u[n+3] - u[n+2] = \boxed{\delta[n+3]}$$

d) Convolute the result of (c) with $x_1[n]$ to evaluate $y[n]$

$$(x_2[n] * x_3[n]) * x_1[n] = \delta[n+3] * ((0.5)^n u[n]) = \boxed{\left(\tfrac{1}{2}\right)^{n+3} u[n+3]}$$