

2.6. Compute $y[n] = x[n] * h[n]$ where $x[n] = \left(\frac{1}{3}\right)^n u[-n-1]$, $h[n] = u[n-1]$

$$x[n] = 3^n u[n-1] = 3^n u[-(n+1)]$$

$$y[n] = x[n] * h[n] = 3^n u[-(n+1)] * u[n-1] = \sum_k (3^k u[k-1] \cdot u[n-1-k]) =$$

$$= \sum_k \begin{cases} 3^k & \text{if } -k-1 \geq 0, n-1-k \geq 0 \\ 0 & \text{otherwise} \end{cases} = \sum_k \begin{cases} 3^k & \text{if } k \leq -1 \text{ and } k \leq n-1 \\ 0 & \text{otherwise} \end{cases} =$$

$$= \sum_{k=-\infty}^{\min(-1, n-1)} 3^k = \begin{cases} \sum_{k=-1}^{\infty} 3^k & \text{if } n \geq 0 \\ \sum_{k=n-1}^{-1} 3^k & \text{otherwise} \end{cases}$$

$$\sum_{k=-1}^{\infty} 3^k = \sum_{k=1}^{\infty} \frac{1}{3^k} \leftarrow \text{convergent, } \approx 0.5$$

