**2.11.** Let $x(t) = u(t-3) - u(t-5)$ and $h(t) = e^{-3t} u(t)$

a) Compute $y(t) = x(t) * h(t)$

$$y(t) = x(t) * h(t) = \cancel{\int_{-\infty}^{\infty} \left(u(t-3) \cdot e^{-3(t-\tau)} u(t-\tau) - u(\tau-5) \cdot e^{-3(t-\tau)} u(t-\tau)\right) d\tau} =$$

$$= \cancel{\int_{-\infty}^{\infty} \begin{cases} e^{-3t+3\tau}, & \tau-3 \geq 0,\ t-\tau \geq 0 \\ 0, & \text{elsewhere} \end{cases} d\tau + \int_{-\infty}^{\infty} \begin{cases} e^{-3t+3\tau}, & \tau-5 \geq 0,\ t-\tau \geq 0 \\ 0, & \text{elsewhere} \end{cases} d\tau} =$$

$$= \int_{-\infty}^{\infty} \left(u(\tau-3) - u(\tau-5)\right) \cdot e^{-3(t-\tau)} u(t-\tau)\, d\tau = \int_{-\infty}^{\infty} \left( \begin{cases} e^{-3(t-\tau)}, & 3 \leq \tau < 5,\ \tau \leq t \\ 0, & \text{elsewhere} \end{cases} \right) d\tau =$$

$$= \begin{cases} \int_{3}^{\min(5,t)} e^{-3t+3\tau}\, d\tau, & 3 \leq t \\ 0, & \text{elsewhere} \end{cases} = \begin{cases} e^{-3t} \cdot \frac{e^{3\tau}}{3} \Big|_{\tau=3}^{\min(5,t)}, & 3 \leq t \\ 0, & \text{elsewhere} \end{cases} =$$

$$= \boxed{\begin{cases} \frac{1}{3}\left(1 - e^{-3t+9}\right), & 3 \leq t < 5 \\ \frac{1}{3}\left(e^{-3t+15} - e^{-3t+9}\right), & 5 \leq t \\ 0, & \text{elsewhere} \end{cases}}$$

b) Compute $g(t) = \frac{d\,x(t)}{dt} * h(t)$

$$\frac{dx(t)}{dt} = \frac{d}{dt}\left(u(t-3) - u(t-5)\right) = \delta(t-3) - \delta(t-5)$$

$$g(t) = \left(\delta(t-3) - \delta(t-5)\right) * h(t) = h(t-3) - h(t-5) = \boxed{e^{-3(t-3)} u(t-3) - e^{-3(t-5)} u(t-5)}$$

c) How is $g(t)$ related to $y(t)$?

$g(t)$ is the derivative of $y(t)$.

$$\frac{d}{dt}\left(x(t) * h(t)\right) = \left(\frac{d}{dt} x(t)\right) * h(t)$$ (if the functions behave properly).