

2.27. We define the area under a continuous-time signal  $v(t)$  as  $A_v = \int_{-\infty}^{\infty} v(t) dt$ .

Show that if  $y(t) = x(t) * h(t)$  then  $A_y = A_x A_h$ .

$$A_y = \int_{-\infty}^{\infty} y(t) dt = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau dt = \int_{-\infty}^{\infty} x(\tau) \left( \int_{-\infty}^{\infty} h(t-\tau) dt \right) d\tau =$$

$$\begin{cases} u = t - \tau \\ v = \tau \end{cases} \Rightarrow \begin{cases} t = u + v \\ \tau = v \end{cases} \Rightarrow |J| = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = 1$$

$$A_y = \int_{-\infty}^{\infty} x(v) \int_{-\infty}^{\infty} h(u) \cdot 1 \cdot du dv = \int_{-\infty}^{\infty} x(v) dv \cdot \int_{-\infty}^{\infty} h(u) du = A_x A_h$$