

...2.25.(2)

b) Determine  $y[n]$  using the distributive property of convolution.

$$y[n] = x[n] * h[n] = 3^n u[n-1] * \left(\frac{1}{4}\right)^n u[n+3] + \left(\frac{1}{3}\right)^n u[n] * \left(\frac{1}{4}\right)^n u[n+3] =$$

$$= \sum_{k=-\infty}^{\infty} 3^k u[k-1] \cdot \left(\frac{1}{4}\right)^{n-k} u[n-k+3] + \sum_{k=-\infty}^{\infty} \left(\frac{1}{3}\right)^k u[k] \cdot \left(\frac{1}{4}\right)^{n-k} u[n-k+3] =$$

$$= \sum_{k=-\infty}^{\min(-1, n+3)} 3^k \left(\frac{1}{4}\right)^{n-k} + \sum_{k=0}^{n+3} \left(\frac{1}{3}\right)^k \left(\frac{1}{4}\right)^{n-k}$$

From here, proceed in the same way as in part a, since this is the same expression we reached at line 3.

Finally:

$$y[n] = \frac{1}{4^n} \begin{cases} \frac{12^{n+4}}{11} & \text{if } n \leq -4 \\ \frac{1}{11} + 3 \left( \left( \frac{4}{3} \right)^{n+4} - 1 \right) & \text{if } n \geq -3 \end{cases}$$