... 2.43. (2)

... b)

ii) Now find $y[n]$ by first convolving $h_1[n]$ and $h_2[n]$: $y[n] = x[n] * (h_1[n] * h_2[n])$

$$y[n] = x[n] * \left(\sum_{k=-\infty}^{\infty} h_1[k] \cdot h_2[n-k]\right) = x[n] * \left(\sum_{k=-\infty}^{\infty} \left(-\tfrac{1}{2}\right)^k u[k] \cdot (u[n-k] + \tfrac{1}{2} u[n-k-1])\right) =$$

$$= x[n] * \left(\sum_{k=0}^{n} \left(-\tfrac{1}{2}\right)^k + \frac{1}{2}\sum_{k=0}^{n-1} \left(-\tfrac{1}{2}\right)^k\right) = x[n] * \left(\frac{1-\left(-\frac{1}{2}\right)^{n+1}}{\underbrace{1-\left(-\frac{1}{2}\right)}_{\frac{3}{2}}} + \frac{1}{2} \cdot \frac{1-\left(-\frac{1}{2}\right)^{n}}{\underbrace{1-\left(-\frac{1}{2}\right)}_{\frac{3}{2}}}\right) =$$

$$\cancel{= x[n] * \left(\frac{2}{3} + \left(-\tfrac{1}{2}\right)^n + \frac{1}{3}\right.} = x[n] * \left(\overbrace{\frac{2}{3} + \frac{2}{3} \cdot \frac{1}{2}\left(-\tfrac{1}{2}\right)^n}^{\text{if } n \geq 0 \text{, else } 0} + \overbrace{\underbrace{\frac{1}{3} - \frac{1}{3} \cdot \left(-\tfrac{1}{2}\right)^n}_{n=0 \Rightarrow \text{this} = \frac{1}{3} - \frac{1}{3} = 0}}^{\text{if } n \geq 1 \text{, else } 0}\right) = x[n] * u[n] =$$

$$= \sum_{k=-\infty}^{\infty} x[k] u[n-k] = \sum_{k=-\infty}^{\infty} u[k] u[n-k] = \sum_{k=0}^{n} 1 = \begin{cases} n+1 & \text{if } n \geq 0 \\ 0 & \text{else} \end{cases} = \boxed{(n+1) u[n] = y[n]}$$ (same as b.i)

c) Consider the same cascade with $h_1[n] = \sin 8n$ and $h_2[n] = a^n u[n]$, $|a| < 1$ and $x[n] = \delta[n] - a\delta[n-1]$. Find $y[n]$.

$$y[n] = x[n] * h_1[n] * h_2[n] = (\delta[n] - a\delta[n-1]) * \sin 8n * a^n u[n] = (\delta[n] - a\delta[n-1]) * a^n u[n] * \sin 8n =$$

$$= (a^n u[n] - a \cdot a^{n-1} u[n-1]) * \sin 8n = (a^n(u[n] - u[n-1])) * \sin 8n =$$

$$= \left(\begin{cases} a^n & \text{if } n \geq 0 \text{ \& } n < 1 \\ 0 & \text{otherwise} \end{cases}\right) * \sin 8n = \boxed{\delta[n] * \sin 8n = \sin 8n = y[n]}$$