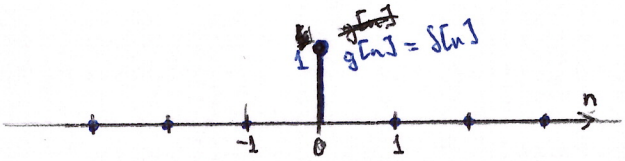
2.41. Consider the signal $x[n] = \alpha^n u[n]$

a) Sketch the signal $g[n] = x[n] - \alpha x[n-1]$

$g[n] = \alpha^n u[n] - \alpha^{n-1+1} u[n-1] = \alpha^n (u[n] - u[n-1]) = \begin{cases} \alpha^n & \text{if } n=0 \\ 0 & \text{otherwise} \end{cases} =$

$= \begin{cases} 1 & \text{if } n=0 \\ 0 & \text{otherwise} \end{cases} = \boxed{\delta[n]}$



b) Use the result of part (a) with properties of convolution in order to find $h[n]$ such that $x[n] * h[n] = \left(\frac{1}{2}\right)^n (u[n+2] - u[n-2])$

$x[n] * h[n] = \left(\frac{1}{2}\right)^n (u[n+2] - u[n-2]) = \left(\frac{1}{2}\right)^{-2} \delta[n+2] + \left(\frac{1}{2}\right)^{-1} \delta[n+1] + \left(\frac{1}{2}\right)^0 \delta[n] + \left(\frac{1}{2}\right) \delta[n-1] + \left(\frac{1}{2}\right)^2 \delta[n-2] =$

$4g[n+2] + 2\delta[n+1] + g[n] + \frac{1}{2} g[n-1] + \frac{1}{4} g[n-2] =$

$= 4(x[n+2] - \alpha x[n+1]) + 2(x[n+1] - \alpha x[n]) + (x[n] - \alpha x[n-1]) + \frac{1}{2}(x[n-1] - \alpha x[n-2]) + \frac{1}{4}(x[n-2] - \alpha x[n-3]) =$

$= 4x[n] * \delta[n+2] + (2-4\alpha) x[n] * \delta[n+1] + (1-2\alpha) x[n] * \delta[n] + \left(\frac{1}{2} - \alpha\right) x[n] * \delta[n-1] + \left(\frac{1}{4} - \frac{1}{2}\alpha\right) x[n] * \delta[n-2] - \frac{\alpha}{4} x[n-3] * \delta[n-3] =$

$= x[n] * \left(4\delta[n+2] + (2-4\alpha)\delta[n+1] + (1-2\alpha)\delta[n] + \left(\frac{1}{2} - \alpha\right)\delta[n-1] + \left(\frac{1}{4} - \frac{1}{2}\alpha\right)\delta[n-2] - \frac{\alpha}{4}\delta[n-3]\right) \Rightarrow$

$\Rightarrow \boxed{h[n] = 4\delta[n+2] + (2-4\alpha)\delta[n+1] + (1-2\alpha)\delta[n] + \left(\frac{1}{2} - \alpha\right)\delta[n-1] - \frac{1}{2}\alpha\,\delta[n-2]}$