...2.21. (2)

c) $x[n] = \left(-\frac{1}{2}\right)^n u[n-4]$

$h[n] = 4^n u[2-n]$

$$x[n] * h[n] = \sum_{k=-\infty}^{\infty} \left(-\frac{1}{2}\right)^k u[k-4] \cdot 4^{n-k} u[2-n+k] = 4^n \sum_{k=-\infty}^{\infty} \begin{cases} (-1)^k \left(\frac{1}{8}\right)^k & \text{if } k-4 \geq 0 \ \& \ 2-n+k \geq 0 \\ 0 & \text{otherwise} \end{cases} =$$

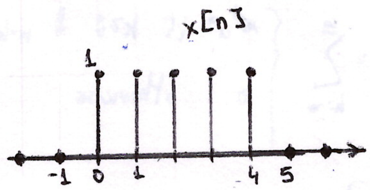
$$= 4^n \sum_{k=\max(4,n-2)}^{\infty} \left(-\frac{1}{8}\right)^k = 4^n \cdot \left(\sum_{k=0}^{\infty} \left(-\frac{1}{8}\right)^k - \sum_{k=0}^{\max(4,n-2)} \left(-\frac{1}{8}\right)^k \right) =$$

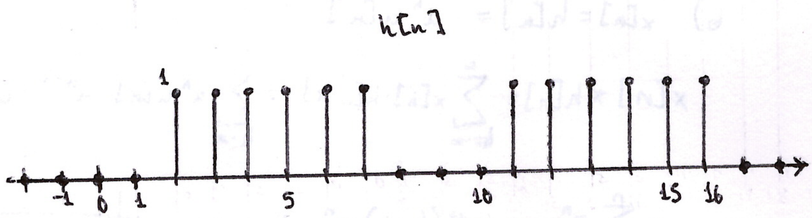
$$= 4^n \cdot \left(\lim_{m\to\infty} \left(\frac{1-\underbrace{\left(-\frac{1}{8}\right)^m}_{0}}{1+\frac{1}{8}} \right) - \frac{1-\left(-\frac{1}{8}\right)^{\max(4,n-2)}}{1+\frac{1}{8}} \right) = 4^n \cdot \frac{8}{9} \cdot \left(1 - \left(1 - \left(-\frac{1}{8}\right)^{\max(4,n-2)}\right)\right) =$$

$$= \frac{2^{2n+3}}{9} \cdot \left(-\frac{1}{8}\right)^{\max(4,n-2)} = \begin{cases} \frac{2^{2n+3}}{9} \cdot \left(\frac{-1}{8}\right)^4 & \text{if } n-2 \leq 4 \\ \frac{2^{2n+3}}{9} \cdot \left(\frac{-1}{8}\right)^n & \text{if } n-2 > 4 \end{cases} =$$

$$= \begin{cases} \frac{2^{2n+3-12}}{9} & \text{if } n \leq 6 \\ \frac{2^{2n+3-3n}}{9} (-1)^n & \text{if } n > 6 \end{cases} = \boxed{\begin{cases} \frac{2^{2n-9}}{9} & \text{if } n \leq 6 \\ \frac{8}{9}\left(-\frac{1}{2}\right)^n & \text{if } n > 6 \end{cases}}$$

d)





$$x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k] = x[0] \cdot h[n] + x[1] \cdot h[n-1] + x[2] h[n-2] + x[3] h[n-3] + x[4] h[n-4] =$$

$$= h[n] + h[n-1] + h[n-2] + h[n-3] + h[n-4] = \sum_{k=0}^{4} h[n-k]$$