

... C.E1. (2)

2. Change the input phase by  $\frac{\pi}{3}$  and calculate the output

This circuit, made of only resistors, capacitors and inductors, can be seen as an LTI system. A phase shift of  $\frac{\pi}{3}$  in the input means:

$$v_g' = 5 \cos(10^5 t + \frac{\pi}{3}) = \operatorname{Re}(5 e^{j10^5 t + \frac{\pi}{3}}) = \operatorname{Re}(5 e^{j10^5 t} \cdot e^{j\frac{\pi}{3}}) = \operatorname{Re}(e^{j\frac{\pi}{3}} V_g)$$

This being an LTI, if we multiply the input phasor by a constant  $\alpha = e^{j\frac{\pi}{3}}$ , the homogeneity property states that the output will be equal to the previous output, scaled by the same factor.  $\Rightarrow$

$$V_o' = e^{j\frac{\pi}{3}} V_o \Rightarrow v_o' = 3.616 \cdot \cos(10^5 t - 0.7086 + \frac{\pi}{3}) = \boxed{3.616 \cdot \cos(10^5 t + 0.3386)}$$

3. Delay the input signal by  $T=3$  sec and calculate the output.

$$v_g'' = 5 \cos(10^5 (t-3)) = \operatorname{Re}(5 e^{j10^5 t} \cdot e^{-j3 \cdot 10^5}) = \operatorname{Re}(e^{-j3 \cdot 10^5} V_g)$$

Again, taking advantage of homogeneity:

$$V_o'' = e^{-j3 \cdot 10^5} V_o \Rightarrow v_o'' = 3.616 \cdot \cos(10^5 (t-3) - 0.7086) = \boxed{3.616 \cdot \cos(10^5 t - 3 \cdot 10^5 - 0.7086)}$$

4. Change the input to  $v_2(t) = 2 \cos(10 \pi t - \pi/3) - 4 \sin(2 \pi t + 1.2)$  and calculate the output.

Again, this becomes solvable when you use linearity:

$$v_g(t) = v_{g1}(t) + v_{g2}(t) \rightarrow v_o(t) = H_1 \cdot v_{g1}(t) + H_2 \cdot v_{g2}(t), \text{ where } v_{g1}(t) \rightarrow v_{o1}(t), v_{g2}(t) \rightarrow v_{o2}(t).$$

I have written all of the I have spent too much time writing the Python code to solve this, since the numbers are quite... complex.

$$v_o(t) = 8 \cdot 10^5 \cdot \operatorname{Re} \left( \frac{e^{j(2\pi t - \pi/3 + 1.2)}}{3600\pi + 0.016j\pi^2 + 2.5 \cdot 10^8 j} + \frac{1}{10} \cdot \frac{e^{j(10\pi t - \frac{3\pi}{6})}}{3600\pi + 0.08j\pi^2 + 5 \cdot 10^7 j} \right)$$