1.29. a) Show that the discrete time system $y[n] = Re\{x[n]\}$ is additive. Does this change if $y[n] = Re\{e^{j\pi\frac{n}{4}} x[n]\}$? (Do not assume ~~that~~ that $x[n] \in \mathbb{R}$)

Let $x[n] = a[n] + j\,b[n]$:

$y[n] = Re\{x[n]\} = a[n]$

$x'[n] = x_1[n] + x_2[n] = a_1[n] + jb_1[n] + a_2[n] + jb_2[n] = a_1[n] + a_2[n] + j(b_1[n] + b_2[n]) \Rightarrow$

$\Rightarrow y'[n] = Re\{x'[n]\} = Re\{a_1[n] + a_2[n] + j(b_1[n] + b_2[n])\} = a_1[n] + a_2[n]$

$y_1[n] + y_2[n] = Re\{x_1[n]\} + Re\{x_2[n]\} = a_1[n] + a_2[n] = y'[n] \Rightarrow \boxed{\text{additive}}$

Now, $y[n] = Re\{e^{j\pi\frac{n}{4}} x[n]\} = Re\{(\cos(\frac{n\pi}{4}) + j\sin(\frac{n\pi}{4}))\cdot(a[n] + jb[n])\} =$

$= Re\{\cos\frac{n\pi}{4}\cdot a[n] - \sin\frac{n\pi}{4}\cdot b[n] + j(\cos\frac{n\pi}{4}\cdot b[n] + \sin\frac{n\pi}{4}\cdot a[n])\} =$

$= \cos\frac{n\pi}{4}\cdot a[n] - \sin\frac{n\pi}{4}\cdot b[n]$

$x'[n] = \cancel{a_1[n] + a_2[n]}\ x_1[n] + x_2[n] = a_1[n] + a_2[n] + j(b_1[n] + b_2[n]) = a'[n] + jb'[n]$

$y'[n] = Re\{e^{j\pi\frac{n}{4}} x'[n]\} = \cos\frac{n\pi}{4}\cdot(a_1[n] + a_2[n]) - \sin\frac{n\pi}{4}\cdot(b_1[n] + b_2[n])$

$y_1[n] + y_2[n] = \cos\frac{n\pi}{4}\cdot a_1[n] - \sin\frac{n\pi}{4}\cdot b_1[n] + \cos\frac{n\pi}{4}\cdot a_2[n] - \sin\frac{n\pi}{4}\cdot b_2[n] = y'[n] \Rightarrow \boxed{\text{additive}}$

b) Determine whether each of the systems below is additive and/or homogeneous. Justify your answer with proof or a counterexample.

- i) $y(t) = \frac{1}{x(t)}\cdot\left(\frac{dx(t)}{dt}\right)^2$

$x'(t) = x_1(t) + x_2(t) \Rightarrow y'(t) = \frac{1}{x_1(t)+x_2(t)}\left(\frac{d}{dt}(x_1(t)+x_2(t))\right)^2 = \frac{1}{x_1(t)+x_2(t)}\cdot\left(\left(\frac{d}{dt}x_1(t)\right)^2 + 2\left(\frac{d}{dt}x_1(t)\cdot\frac{d}{dt}x_2(t)\right) + \left(\frac{d}{dt}x_2(t)\right)^2\right)$

this is stressing me out

$y_1(t) + y_2(t) = \frac{1}{x_1(t)}\left(\frac{d}{dt}x_1(t)\right)^2 + \frac{1}{x_2(t)}\left(\frac{d}{dt}x_2(t)\right)^2 \neq y'(t) \Rightarrow \boxed{\text{not additive}}$

Let $x_1(t) = 2x$, $x_2(t) = 3x^2 \Rightarrow$: $x'(t) = 2x + 3x^2 \Rightarrow$

$\Rightarrow y'(t) = \frac{1}{2x+3x^2}\cdot\left(\frac{d}{dt}(2x+3x^2)\right)^2 = \frac{(2+6x)^2}{2x+3x^2}$

$y_1(t) + y_2(t) = \frac{1}{2x}\left(\frac{d}{dt}(2x)\right)^2 + \frac{1}{3x^2}\left(\frac{d}{dt}(3x^2)\right)^2 = \frac{2^2}{2x} + \frac{(6x)^2}{3x^2} = \frac{2}{x} + \frac{36}{3} = \frac{2}{x} + 12 \neq y'(t) \Rightarrow \boxed{\text{not additive}}$

- $x'(t) = \alpha x(t) \Rightarrow y'(t) = \frac{1}{\alpha x(t)}\left(\frac{d}{dt}(\alpha x(t))\right)^2 = \frac{\alpha}{x(t)}\cdot\left(\frac{d}{dt}x(t)\right)^2$

$\alpha y(t) = \alpha\cdot\frac{1}{x(t)}\cdot\left(\frac{d}{dt}x(t)\right)^2 = y'(t) \Rightarrow \boxed{\text{homogeneous}}$