1.40.

a) Show that if a system is either additive or homogeneous, it has the property that ~~the output for ... zero~~ if the input is identically zero, the output is also zero.

~~Additive~~ Let $S: x_1 \longmapsto y_1$
$x_2 \longmapsto y_2$
$x' = x_1 + x_2 \longmapsto y'$

Additive $\Leftrightarrow y_1 + y_2 = y'$

If $x_1 \equiv 0 \Rightarrow x' = x_1 + x_2 = x_2 \Rightarrow y_2 = y' = y_1 + y_2 \Rightarrow y_2 = y_1 + y_2 \Rightarrow \boxed{y_1 = 0}$

Let $S: x \longmapsto y$
$x' = \alpha x \longmapsto y'$

Homogeneous $\Leftrightarrow y' = \alpha y \quad \forall \alpha \in$ scalar

$x \equiv 0 \Rightarrow x' = \alpha x = 0 = x \Rightarrow x' = x \Rightarrow y = y' = \alpha y \Rightarrow y = \alpha y \Rightarrow \begin{cases} \alpha = 1, \ y \text{ any} & \leftarrow \text{can't restrict } \alpha \\ \boxed{y = 0}, & \alpha \text{ any} \end{cases}$

b) Determine a system which is neither additive or homogeneous ~~that~~ but which has a zero output if the input is identically zero. (discrete or continuous)

$$\boxed{y(t) = \begin{cases} 0 & \text{if } x(t) = 0 \\ 1 & \text{if } x(t) \neq 0 \end{cases}}$$

For $x(t) \equiv 0$:

$$y(t) = \begin{cases} 0 & \text{if } 0 = 0 \\ 1 & \text{if } 0 \neq 0 \end{cases} = 0$$

For any $x(t)$:

$x'(t) = \alpha x(t) \Rightarrow y'(t) = \begin{cases} 0 & \text{if } x'(t) = 0 \\ 1 & \text{if } x'(t) \neq 0 \end{cases} = \begin{cases} 0 & \text{if } x(t) = 0 \\ 0 & \text{if } x(t) \neq 0 \end{cases}$

$\alpha y(t) = \alpha \begin{cases} 0 & \text{if } x(t) = 0 \\ 1 & \text{if } x(t) \neq 0 \end{cases} = \begin{cases} 0 & \text{if } x(t) = 0 \\ \alpha & \text{if } x(t) \neq 0 \end{cases} \neq y'(t) \Rightarrow$ not homogeneous

$x_3(t) = x_1(t) + x_2(t) \Rightarrow y_3(t) = \begin{cases} 0 & \text{if } x_3(t) = 0 \\ 1 & \text{if } x_3(t) \neq 0 \end{cases} = \begin{cases} 0 & \text{if } x_1(t) + x_2(t) = 0 \\ 1 & \text{if } x(t) + x_2(t) \neq 0 \end{cases}$

$y_1(t) + y_2(t) = \begin{cases} 0 & \text{if } x_1(t) = 0 \\ 1 & \text{if } x_1(t) \neq 0 \end{cases} + \begin{cases} 0 & \text{if } x_2(t) = 0 \\ 1 & \text{if } x_2(t) \neq 0 \end{cases} = \begin{cases} 0 & \text{if } x_1(t) = x_2(t) = 0 \\ 1 & \text{if } x_1(t) = 0 \neq x_2(t) \text{ or } x_1(t) \neq 0 = x_2(t) \\ 2 & \text{if } x_1(t) \neq 0 \neq x_2(t) \end{cases}$

$y_1(t) + y_2(t) \neq y_3(t) \Rightarrow$ not additive