

... 1.30. (4)

m) $y[n] = x[2n]$

Let $x_1[n] = \delta[n+1]$; $x_2[n] = -\delta[n+1]$

$y_1[n] = x_1[2n] = \delta[2n+1] = \begin{cases} 1 & \text{if } 2n+1=0 \\ 0 & \text{otherwise} \end{cases} = \begin{cases} 1 & \text{if } n = -\frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$

$y_2[n] = x_2[2n] = -\delta[2n+1] = \begin{cases} -1 & \text{if } 2n+1=0 \\ 0 & \text{otherwise} \end{cases} = 0 = y_1[n] \Rightarrow \text{not invertible}$

n) $y[n] = \begin{cases} x[n/2], & n \text{ even} \\ 0, & n \text{ odd} \end{cases} \Rightarrow \text{C.V. } [m = \frac{n}{2} \Rightarrow n = 2m] \Rightarrow y[2m] = \begin{cases} x[m], & 2m \text{ even} \\ 0, & 2m \text{ odd} \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} y[2m] = x[m] & \text{if } (2m \text{ is even} \Leftrightarrow \frac{2m}{2} \in \mathbb{Z} \Leftrightarrow m \in \mathbb{Z}) \\ y[2m] = 0 & \text{if } (2m \text{ is odd} \Leftrightarrow \frac{2m}{2} \notin \mathbb{Z} \Leftrightarrow m \notin \mathbb{Z}) \end{cases} \Rightarrow x[m] = y[2m] \text{ if } m \in \mathbb{Z}$

Invertible with $y[n] \xrightarrow{S^{-1}} x[n] = y[2n]$