2.42. Suppose that $x(t) = u(t+0.5) - u(t-0.5)$ is convolved with $h(t) = e^{j\omega_0 t}$ $\quad \frac{1}{-j} = \frac{1}{-j}\cdot\frac{j}{j} = \frac{j}{-1}\cdot = j$

a) Determine a value of ω_0 so that $y(0) = 0$, given $y(t) = x(t) * h(t)$.

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau = \int_{-\infty}^{\infty} (u(\tau+0.5) - u(\tau-0.5))\, e^{j\omega_0 (t-\tau)}\, d\tau = \int_{-0.5}^{0.5} e^{j\omega_0(t-\tau)}\, d\tau = e^{j\omega_0 t}\int_{-0.5}^{0.5} e^{-j\omega_0 \tau}\, d\tau =$$

$$= e^{j\omega_0 t}\cdot\left(\frac{1}{-j\omega_0}\left(e^{-j\omega_0 \tau}\right)\Big|_{\tau=-0.5}^{0.5}\right) = \frac{j}{\omega_0} e^{j\omega_0 t}\cdot\left(e^{-j\omega_0 \cdot 0.5} - e^{j\omega_0 \cdot 0.5}\right) =$$

$$= \frac{j e^{j\omega_0 t}}{\omega_0}\cdot\left(\cos(-\omega_0 0.5) + j\sin(-0.5\omega_0) - \cos(\omega_0 \cdot 0.5) - j\sin(0.5\omega_0)\right) = \frac{e^{j\omega_0 t}}{\omega_0}\cdot j(0 - j\sin 0.5\omega_0 - j\sin 0.5\omega_0) =$$

$$= \frac{e^{j\omega_0 t}}{\omega_0}\cdot 2\sin 0.5\omega_0$$

$$y(0) = 0 \iff \frac{e^0}{\omega_0} 2\sin 0.5\omega_0 = 0 \iff \sin 0.5\omega_0 = 0 \iff \omega = 2\cdot\left(\frac{\pi}{2} \pm \left(\frac{\pi}{2} - \arcsin(0)\right) + 2\pi k\right) =$$

$$= \pi \pm (\pi - 0) + 2\pi k = \pi \pm \pi + 4\pi k = 2\pi k \qquad (\forall k \in \mathbb{Z})$$

$\boxed{\text{One such value is } \omega_0 = 2\pi}$

b) Is that answer unique?

No, any multiple of 2π will work, due to the periodicity of the signal $h(t)$ (which also causes the output signal $y(t)$ to be periodic). The value $\omega_0 = 0$ does not work, however.