... 1.42 (2)

b) Is this statement true or false?

The series interconnection of two nonlinear systems is nonlinear.

Let $S_1: x(t) \longrightarrow y(t) = t + x(t)$, $S_2: y(t) \longrightarrow z(t) = 2y(t) - 2t$

- System 1 linear check:

Let $x'(t) = \alpha x_1(t) + \beta x_2(t) \rightarrow y'(t) = t + x'(t) = t + \alpha x_1(t) + \beta x_2(t)$

$\alpha y_1(t) + \beta y_2(t) = \alpha(t + x_1(t)) + \beta(t + x_2(t)) = (\alpha+\beta)t + \alpha x_1(t) + \beta x_2(t) \neq y'(t) \Rightarrow S_1$ is not linear

- System 2 linearity check:

~~y(t)~~ Let $y(t) = \alpha y_1(t) + \beta y_2(t) \longrightarrow z'(t) = 2y(t) - 2t = 2\alpha y_1(t) + 2\beta y_2(t) - 2t$

$\alpha z_1(t) + \beta z_2(t) = \alpha(2y_1(t) - 2t) + \beta(2y_2(t) - 2t) = -2(\alpha+\beta)t + 2\alpha y_1(t) + 2\beta y_2(t) \neq z'(t) \Rightarrow S_2$ is not linear

~~System 3 linearity check:~~

- Compound system $S_3: x(t) \longrightarrow z(t)$ formed by series interconnection of S_1 and S_2 in the following way: $S_3: x(t) \xrightarrow{S_1} y(t) = t + x(t) \xrightarrow{S_2} z(t) = 2y(t) - 2t = 2(t + x(t)) - 2t = 2x(t)$

Let $x'(t) = \alpha x_1(t) + \beta x_2(t) \longrightarrow z'(t) = 2y'(t) - 2t = 2(t + x'(t)) - 2t = 2x'(t) = 2(\alpha x_1(t) + \beta x_2(t))$

$\alpha z_1(t) + \beta z_2(t) = \alpha(2x_1(t)) + \beta(2x_2(t)) = 2(\alpha x_1(t) + \beta x_2(t)) = z'(t) \Rightarrow S_3$ is linear.

In this case, the series interconnection of two nonlinear systems is linear, disproving the statement.

$\boxed{\text{The statement is false}}$

c) Consider the following systems:

System 1: $y[n] = \begin{cases} x[n/2], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$; System 2: $y[n] = x[n] + \frac{1}{2}x[n-1] + \frac{1}{4}x[n-2]$

System 3: $y[n] = x[2n]$

Suppose they are connected in series. Find the input-output relationship. Is the system linear? Is it time invariant?

$$x[n] \longrightarrow \boxed{\text{Sys. 1}} \xrightarrow{a[n]} \boxed{\text{Sys. 2}} \xrightarrow{b[n]} \boxed{\text{Sys. 3}} \longrightarrow y[n]$$

$y[n] = b[2n] = a[2n] + \frac{1}{2}a[2n-1] + \frac{1}{4}a[2n-2]$

$= \begin{cases} x[n], & 2n \text{ even} \\ 0, & 2n \text{ odd} \end{cases} + \frac{1}{2}\begin{cases} x[\frac{2n-1}{2}], & 2n-1 \text{ even} \\ 0, & 2n-1 \text{ odd} \end{cases} + \frac{1}{4}\begin{cases} x[n-1], & 2n-2 \text{ even} \\ 0, & 2n-2 \text{ odd} \end{cases} = \boxed{x[n] + 0 + \frac{x[n-1]}{4} = y[n]}$

- $x'[n] = \alpha x_1[n] + \beta x_2[n] \rightarrow y'[n] = x'[n] + \frac{1}{4}x'[n-1] = \alpha x_1[n] + \beta x_2[n] + \frac{1}{4}(\alpha x_1[n-1] + \beta x_2[n-1])$

$\alpha y_1[n] + \beta y_2[n] = \alpha(x_1[n] + \frac{1}{4}x_1[n-1]) + \beta(x_2[n] + \frac{1}{4}x_2[n-1]) = y'[n] \Rightarrow \boxed{\text{linear}}$

- $x'[n] = x[n+k] \rightarrow y'[n] = x'[n] + \frac{1}{4}x'[n-1] = x[n+k] + \frac{1}{4}x[n-1+k]$

$y[n+k] = x[n+k] + \frac{1}{4}x[n+k-1] = y'[n] \Rightarrow \boxed{\text{time invariant}}$