...1.33. (2)

3) If $x[n]$ is periodic, then $y_2[n]$ is periodic.

$$\left.\begin{aligned} & x[n] \text{ periodic w/ fundamental period } N_x \Rightarrow x[n]=x[N+n] \ \forall N\in\{k\cdot N_x : k\in\mathbb{Z}\} \\ & y_2[n]=\begin{cases} x[\frac{n}{2}] & \text{if } n \text{ even} \\ 0 & \text{if } n \text{ odd}\end{cases} \xRightarrow{\frac{n}{2}\to n} x[n]=y_2[2n] \ \forall n\in\mathbb{Z} \Rightarrow x[n+N]=y_2[2n+2N] \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow y_2[2n]=y_2[2n+2N] \ \forall N\in\{k\cdot N_x : k\in\mathbb{Z}\} \xRightarrow{2n\to n} y_2[n]=y_2[n+N] \ \forall N\in\{k\cdot N_{y_2} : k\in\mathbb{Z},\ N_{y_2}=2N_x\} \Rightarrow$$

$$\Rightarrow \boxed{y_2[n] \text{ is periodic with } \cancel{\text{period}} \text{ fundamental period } N_{y_2}=2N_x}$$

4) If $y_2[n]$ is periodic, then $x[n]$ is periodic.

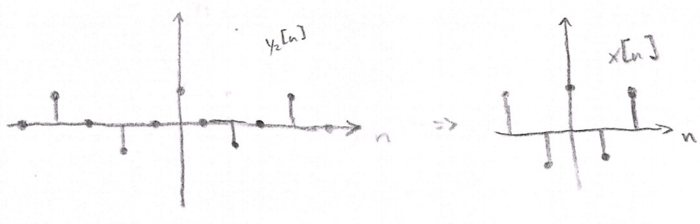
$$\left.\begin{aligned} & y_2[n] \text{ periodic w/ fundamental period } N_{y_2} \Rightarrow y_2[n]=y_2[n+N] \ \forall N\in\{k\cdot N_{y_2} : k\in\mathbb{Z}\} \\ & y_2[n]=\begin{cases} x[\frac{n}{2}] & \text{if } n \text{ even} \\ 0 & \text{if } n \text{ odd}\end{cases} \Rightarrow y_2[n+N]=\begin{cases} x[\frac{n+N}{2}] & \text{if } n+N \text{ even} \\ 0 & \text{if } n+N \text{ odd}\end{cases} \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow \begin{cases} x[\frac{n}{2}] & \text{if } n \text{ even} \\ 0 & \text{if } n \text{ odd}\end{cases} = \begin{cases} x[\frac{n+N}{2}] & \text{if } n+N \text{ even} \\ 0 & \text{if } n+N \text{ odd}\end{cases} \quad \forall N\in\{k\cdot N_{y_2} : k\in\mathbb{Z}\} \Rightarrow$$

$$\Rightarrow \begin{cases} x[\frac{n}{2}]=x[\frac{n}{2}+\frac{N}{2}] & \text{if } n, N \text{ even} \\ x[\frac{n}{2}]=0 & \text{if } n \text{ even, } N \text{ odd} \\ 0 \quad =x[\frac{n}{2}+\frac{N}{2}] & \text{if } n, N \text{ odd} \\ 0 \quad =0 & \text{if } n \text{ odd, } N \text{ even}\end{cases} \quad \forall N\in\{k\cdot N_{y_2} : k\in\mathbb{Z}\}$$

$$\Rightarrow \left\{\begin{aligned} & \text{If } N_{y_2} \text{ even} \Rightarrow k\cdot N_{y_2} \text{ even} \Rightarrow N \text{ even} \Rightarrow x[\tfrac{n}{2}]=x[\tfrac{n}{2}+\tfrac{N}{2}] \text{ if } n \text{ even } \forall N\in\{k\cdot N_{y_2} : k\in\mathbb{Z}\} \Rightarrow \\ & \qquad \Rightarrow x[n]=x[n+N] \ \forall N\in\{k\cdot \tfrac{N_{y_2}}{2} : k\in\mathbb{Z}\},\ n\in\mathbb{Z} \\ & \text{If } N_{y_2} \text{ odd} \Rightarrow \begin{cases} \dots \\ x[\frac{n}{2}]=0 \text{ if } n \text{ even } \forall N\in\{k\cdot N_{y_2} : k\in\mathbb{Z}\} \\ \dots \end{cases} \Rightarrow x[n]=0 \ \forall n\in\mathbb{Z} \end{aligned}\right. \Rightarrow$$

$$\Rightarrow \boxed{\begin{aligned} & x[n] \text{ is periodic with fundamental period } N_x=\frac{N_{y_2}}{2} \text{ if } N_{y_2} \text{ is even (if } y_2[n]\neq 0 \text{ constant)} \\ & x[n] \text{ is constantly } 0 \text{ if } N_{y_2} \text{ is odd } (y_2[n]=0 \ \forall n\in\mathbb{Z}). \text{ Fundamental period } 1. \end{aligned}}$$



Odd period in $y_2[n] \Rightarrow$ constant 0 because

$$\left.\begin{aligned} & \forall n \text{ even},\ \exists N\in\{k\cdot N_{y_2} : k\in\mathbb{Z}\} : y_2[n]= \\ & \quad = y_2[n+N] : n+N \text{ odd} \\ & y_2[n]=0 \ \forall n \text{ odd} \end{aligned}\right\} \Rightarrow y_2[n]=0$$