1.27. (3)

e) $y(t) = \begin{cases} 0 & x(t) < 0 \\ x(t) + x(t-2) & x(t) \geq 0 \end{cases}$

1) $y(t_0)$ depends on $x(t_0-2)$, where $t_0 - 2 \neq t_0$ $\Rightarrow$ **not memoryless**

2) $x'(t) = x(t+k) \Rightarrow y'(t) = \begin{cases} 0 & x'(t) < 0 \\ x(t) + x(t-2) & x'(t) \geq 0 \end{cases} = \begin{cases} 0 & x(t+k) < 0 \\ x(t+k) + x(t+k-2) & x(t+k) \geq 0 \end{cases}$

$y(t+k) = \begin{cases} 0 & x(t+k) < 0 \\ x(t+k) + x(t+k-2) & x(t+k) \geq 0 \end{cases} = y'(t) \Rightarrow$ **time invariant**

3) $x(t) = \alpha x_1(t) + \beta x_2(t) \Rightarrow y'(t) = \begin{cases} 0 & x'(t) < 0 \\ \alpha x'(t) + \beta x'(t-2) & x'(t) \geq 0 \end{cases} = \begin{cases} 0 & \alpha x_1(t) + \beta x_2(t) < 0 \\ \alpha x_1(t) + \beta x_2(t) + \alpha x_1(t-2) + \beta x_2(t-2) & \alpha x_1(t) + \beta x_2(t) \geq 0 \end{cases}$

$\alpha y_1(t) + \beta y_2(t) = \alpha \begin{cases} 0 & x_1(t) < 0 \\ x_1(t) + x_1(t-2) & x_1(t) \geq 0 \end{cases} + \beta \begin{cases} 0 & x_2(t) < 0 \\ x_2(t) + x_2(t-2) & x_2(t) \geq 0 \end{cases} \neq y'(t) \Rightarrow$ **not linear**

4) $y(t_0)$ depends on $x(t_0)$ and $x(t_0-2)$, where $t_0 \neq t$, $t_0 - 2 \neq t_0$ $\Rightarrow$ **causal**

5) $|x(t)| \leq B \quad \forall t$ (bounded in)

$|y(t)| \leq |x(t) + x(t-2)| \leq |x(t)| + |x(t-2)| \leq 2B \Rightarrow$ **stable**

f) $y(t) = x\left(\frac{t}{3}\right)$

1) $y(t_0)$ depends on $x\left(\frac{t_0}{3}\right)$, where $\frac{t_0}{3} \neq t_0$ sometimes $\Rightarrow$ **not memoryless**

2) $x'(t) = x(t+k) \Rightarrow y'(t) = x\left(\frac{t}{3}\right) = x\left(\frac{t}{3} + k\right)$

$y(t+k) = x\left(\frac{t+k}{3}\right) \neq y'(t) \Rightarrow$ **not time invariant**

3) $x(t) = \alpha x_1(t) + \beta x_2(t) \Rightarrow y(t) = x'\left(\frac{t}{3}\right) = \alpha x_1\left(\frac{t}{3}\right) + \beta x_2\left(\frac{t}{3}\right)$

$\alpha y_1(t) + \beta y_2(t) = \alpha x_1\left(\frac{t}{3}\right) + \beta x_2\left(\frac{t}{3}\right) = y'(t) \Rightarrow$ **linear**

4) $y(t_0)$ depends on $x\left(\frac{t_0}{3}\right)$, where sometimes $\frac{t_0}{3} > t \Rightarrow$ **not causal**

5) $|x(t)| \leq B \quad \forall t$ (bounded in)

$|y(t)| = |x\left(\frac{t}{3}\right)| \leq B \Rightarrow$ **stable**