1.26. Determine whether or not the following signals are ~~periodic~~ periodic and determine its fundamental ~~periodic~~ period.

a) $x[n] = \sin\left(\frac{6}{7}\pi n + 1\right)$

$x[n] = x[n+T] \Leftrightarrow \sin\left(\frac{6}{7}\pi n + 1\right) = \sin\left(\frac{6}{7}\pi(n+T)+1\right) \Leftrightarrow$ ~~$\frac{\pi}{2} \pm \left(\frac{\pi}{2} - \arcsin(\ldots\right.$~~

$\Leftrightarrow \frac{\pi}{2} \pm \left(\frac{\pi}{2} - \left(\frac{6}{7}\pi n + 1\right)\right) + 2\pi k = \frac{\pi}{2} \pm \left(\frac{\pi}{2} - \left(\frac{6}{7}\pi(n+T)+1\right)\right) + 2\pi m \Leftrightarrow$

$\Leftrightarrow \pm\left(\frac{\pi}{2} - 1 - \frac{6}{7}\pi n\right) = \pm\left(\frac{\pi}{2} - 1 - \frac{6}{7}\pi n - \frac{6}{7}\pi T\right) + 2\pi k \Leftrightarrow$

$\Leftrightarrow \begin{cases} \frac{\pi}{2} - 1 - \frac{6}{7}\pi n - \frac{\pi}{2} + 1 + \frac{6}{7}\pi n - \frac{6}{7}\pi T = 2\pi k \Leftrightarrow T = \frac{2 \cdot 7}{6} k = \frac{7}{3} k \Rightarrow \text{smallest } T \text{ is } \boxed{7 = T} \\ \text{or} \\ \frac{\pi}{2} - 1 - \frac{6}{7}\pi n + \frac{\pi}{2} - 1 - \frac{6}{7}\pi n - \frac{6}{7}\pi T = 2\pi k \Leftrightarrow \pi - 2 - \frac{12}{7}\pi n - \frac{6}{7}\pi T = 2\pi k \leftarrow \text{not constant} \end{cases}$

must be integer

b) $x[n] = \cos\left(\frac{n}{8} - \pi\right)$

$x[n] = x[n+T] \Leftrightarrow \cos\left(\frac{n}{8} - \pi\right) = \cos\left(\frac{n-T}{8} - \pi\right) \Leftrightarrow \pm\left(\frac{n}{8} - \pi\right) = \pm\left(\frac{n-T}{8} - \pi\right) + 2\pi k \Leftrightarrow$

$\Leftrightarrow \begin{cases} \pm\left(\frac{n}{8} - \pi - \frac{n}{8} + \frac{T}{8} + \pi\right) = 2\pi k \Leftrightarrow T = 8 \cdot 2\pi k = 16\pi k \notin \mathbb{Z} \Rightarrow \text{can't define } T \\ \pm\left(\frac{n}{4} - 2\pi - \frac{T}{8}\right) = 2\pi k \leftarrow \text{not constant.} \end{cases}$

$\boxed{\text{Not periodic}}$

c) $x[n] = \cos\left(\frac{\pi}{8} n^2\right)$

$x[n] = x[n+T] \Leftrightarrow \cos\left(\frac{\pi}{8} n^2\right) = \cos\left(\frac{\pi}{8}(n+T)^2\right) \Leftrightarrow \pm\frac{\pi}{8} n^2 = \frac{\pi}{8}(n+T)^2 + 2\pi k \Leftrightarrow$

$\Leftrightarrow \begin{cases} \pm\left(\frac{\pi}{8} n^2 - \frac{\pi}{8} n^2 - \frac{\pi}{4} nT - \frac{\pi}{8} T^2\right) = 2\pi k \Leftrightarrow \begin{cases} T = 0 \\ \text{or} \\ \frac{\pi}{8} T + \frac{\pi}{4} n = 2\pi k \quad \text{(not constant)} \end{cases} \\ \pm\left(\frac{\pi}{8} n^2 + \frac{\pi}{8} n^2 + \frac{\pi}{4} nT + \frac{\pi}{8} T^2\right) = 2\pi k \leftarrow \text{not constant} \end{cases} \Rightarrow \boxed{\text{not periodic}}$

d) $x[n] = \cos\left(\frac{\pi}{2} n\right) \cos\left(\frac{\pi}{4} n\right)$

~~Compos~~ Product of two periodic functions of periods $T_1 = \frac{2\pi}{\frac{\pi}{2}} = 4$ and $T_2 = \frac{2\pi}{\frac{\pi}{4}} = 8$. (easy to prove)

~~$x[n] = x[n+T] \Leftrightarrow \cos\left(\frac{\pi}{2} n\right)\cos\left(\frac{\pi}{4} n\right) = \cos\left(\frac{\pi}{2}(n+T)\right)\cos\left(\frac{\pi}{4}(n+T)\right) \Leftrightarrow$~~

the period in this simple case is $\text{lcm}(T_1, T_2) = \text{lcm}(4, 8) = \boxed{8 = T}$

e) $x[n] = 2\cos\left(\frac{\pi}{4} n\right) + \sin\left(\frac{\pi}{8} n\right) - 2\cos\left(\frac{\pi n}{2} + \frac{\pi}{6}\right)$

Composed of periodic functions with periods $T_1 = \frac{2\pi}{\frac{\pi}{4}} = 8$, $T_2 = \frac{2\pi}{\frac{\pi}{8}} = 16$, $T_3 = \frac{2\pi}{\frac{\pi}{2}} = 4$ (can be proven using previous methods).

The period in this simple case is $\boxed{T = \text{lcm}(T_1, T_2, T_3) = \text{lcm}(4, 8, 16) = 16}$