2.45. (2)

c) An LTI system has the response $y(t) = \sin \omega_0 t$ to the input $x(t) = e^{-5t} u(t)$. Use the result of part (a) to determine the impulse response.

$$x'(t) = -5e^{-5t} u(t) + e^{-5t} \delta(t) = e^{0}\delta(t) - 5e^{-5t} u(t) = \delta(t) - 5e^{-5t} u(t) = \delta(t) - 5x(t)$$

$$y'(t) = \omega_0 \cos(\omega_0 t) = x'(t) * h(t) = (\delta(t) - 5e^{-5t}) * h(t) = \delta(t) * h(t) - 5x(t) * h(t) = h(t) - 5y(t) =$$

$$= h(t) - 5\sin\omega_0 t \Rightarrow \boxed{h(t) = \omega_0 \cos(\omega_0 t) + 5\sin(\omega_0 t)}$$

d) Let $s(t)$ be the unit step response of a continuous-time LTI system. Use part b to prove that $y(t) = \int_{-\infty}^{+\infty} x'(\tau) * s(t-\tau)\, d\tau$ and that $x(t) = \int_{-\infty}^{+\infty} x'(t)\, u(t-\tau)\, d\tau$

~~let this,~~ I will assume it's $y(t) = \int_{-\infty}^{\infty} x'(\tau) \cdot s(t-\tau)\, d\tau$. let $u_1(t)$: $x(t) * u_1(t) = \frac{dx(t)}{dt}$

$$y(t) = x(t) * h(t) = x(t) * \delta(t) * h(t) = x(t) * (u_1(t) * u(t)) = x'(t) * s(t) = \int_{-\infty}^{+\infty} x(\tau) s(t-\tau)\, d\tau$$

$$x(t) = x(t) * \delta(t) = x(t) * (u_1(t) * u(t)) = x'(t) * u(t) = \boxed{\int_{-\infty}^{\infty} x'(\tau)\, u(t-\tau)\, d\tau}$$

e) Use the equation $y(t) = \int_{-\infty}^{\infty} x'(\tau) \cdot s(t-\tau)\, d\tau$ to determine the response to $x(t) = e^{t} u(t)$ of a system with step response $s(t) = (e^{-3t} - 2e^{-2t} + 1) u(t)$

$$x'(t) = e^{t} u(t) + \delta(t)$$

$$y(t) = \int_{-\infty}^{\infty} x'(\tau) \cdot s(t-\tau)\, d\tau = \int_{-\infty}^{\infty} (e^{\tau} u(\tau) + \delta(\tau)) (e^{-3(t-\tau)} - 2e^{-2(t-\tau)} + 1) u(t-\tau)\, d\tau =$$

$$= (e^{-3t} - 2e^{-2t} + 1) u(t) + \int_0^t (e^{-3t+4\tau} - 2e^{-2t+3\tau} + e^{\tau})\, d\tau =$$

$$= (e^{-3t} - 2e^{-2t} + 1) u(t) + \begin{cases} \frac{1}{4}e^{t} - \frac{2}{3}e^{t} + e^{t} - \frac{1}{4}e^{-3t} + \frac{2}{3}e^{-2t} - 1 & \text{if } t \geq 0 \\ 0 & \text{otherwise} \end{cases} =$$

$$= \boxed{\left(\frac{3}{4}e^{-3t} - \frac{4}{3}e^{-2t} + \frac{7}{12}e^{t}\right) u(t)}$$

f) Let $s[n]$ be the unit step response of a ~~cont-time~~ discr.-time LTI system. What are the discrete-time counterparts to the equations of part d?

The equivalent to differentiation is a function $u_1[n]$: $u[n] * u_1[n] = \delta[n]$

$$\delta[n] = u[n] - u[n-1] = u[n] * \delta[n] - u[n] * \delta[n-1] = u[n] * (\delta[n] - \delta[n-1]) \Rightarrow u_1[n] = \delta[n] * \delta[n-1]$$

The equivalent to $y(t) = \int_{-\infty}^{\infty} x'(\tau) \cdot s(t-\tau)\, d\tau$ is $y[n] = \sum_{k=-\infty}^{\infty} (x[k] * u_1[k]) \cdot s[n-k] = \boxed{\sum_{k=-\infty}^{\infty} (x[k] - x[k-1]) \cdot s[n-k]}$

The equivalent to $x(t) = \int_{-\infty}^{\infty} x'(\tau)\, u(t-\tau)\, d\tau$ is $x[n] = \boxed{\sum_{k=-\infty}^{\infty} (x[k] - x[k-1]) u[n-k]}$