CHAPTER 1 PROBLEMS:

ADVANCED PROBLEMS

32 33 34 35 36 37 38 39

40 41 42 43 44 45 46 47

1.32. Let $x(t)$ be a continuous-time signal, $y_1(t) = x(2t)$, $y_2(t) = x\left(\frac{t}{2}\right)$. The signal $y_1(t)$ represents a sped up version of $x(t)$, while $y_2(t)$ represents a slowed down version. Consider the following ~~statements~~ statements, determine whether each one is true. If so, determine the relation between the ~~1) If $x(t)$ is periodic, then $y_1(t)$ is periodic~~ fundamental periods of the signals considered. If not, produce a counterexample.

1) If $x(t)$ is periodic, then $y_1(t)$ is periodic

$x(t)$ periodic $\Leftrightarrow \exists\, T_x : x(t) = x(t+T) \;\forall\; T \in \{k \cdot T_x ;\, k \in \mathbb{Z}\}$

$y_1(t) = x(2t) \Leftrightarrow y_1\left(\frac{t}{2}\right) = x(t) \Rightarrow x(t+T) = y_1\left(\frac{t}{2} + \frac{T}{2}\right) = y_1\left(\frac{t}{2} + \frac{T_x}{2}\right)$ } $\Rightarrow$

C.V. $\left[\frac{t}{2} \to t\right]$

$\Rightarrow y(t) = y\left(t + \frac{T_x}{2}\right) \;\forall\; T \in \{k \cdot T_x ;\, k \in \mathbb{Z}\} \Rightarrow y(t) = y(t+T) \;\forall\; T \in \left\{k \cdot \frac{T_x}{2} : k \in \mathbb{Z}\right\} \Rightarrow$

$T_x \in \{k T_{x_0} ;\, k \in \mathbb{Z}\} \Rightarrow \nexists\, T_x : |T_x| < T_{x_0}$

T_{x_0} is the fundamental period for $x(t)$

$T_y \in \{k \cdot T_{y_0}$ $T_{y_0} = \frac{T_{x_0}}{2}$

$T_y \in \{k \cdot T_{y_0} ;\, k \in \mathbb{Z}\}$ } $\Rightarrow |T_{y_0}| \leq |T_y| \;\forall\; T_y \in \{k \cdot T_{y_0} ;\, k \in \mathbb{Z}\} \Rightarrow$

$\Rightarrow$ The fundamental period for $y_1(t)$ is $T_{y_0} = \frac{T_{x_0}}{2}$ ($y_1(t)$ is periodic)

2) If $y_1(t)$ is periodic, then $x(t)$ is periodic

$y_1(t)$ periodic $\Leftrightarrow \exists\, T_{y_1}$ (w/ fundamental period T_{y_1}) : $y_1(t) = y_1(t+T) \;\forall\; T \in \{k \cdot T_{y_1} ;\, k \in \mathbb{Z}\}$ } $\Rightarrow$

$y_1(t) = x(2t) \Rightarrow y_1(t + T_{y_1}) = x(2(t + T_{y_1})) = x(2t + 2T_{y_1})$

C.V. $[2t \to t]$

$\Rightarrow x(t) = x(t + 2T_{y_1}) \;\forall\; T_{y_1} \in \{k \cdot T_{y_1} ;\, k \in \mathbb{Z}\} \Rightarrow x(t) = x(t + T_x) \;\forall\; T_x \in \{2 \cdot k \cdot T_{y_1 0} ;\, k \in \mathbb{Z}\} \Rightarrow$

$\Rightarrow x(t)$ is periodic with period T_x

$\Rightarrow$ $x(t)$ is periodic with fundamental period $T_x = 2T_{y_1}$