2.35. ~~In the previous problem we saw~~ Consider a system of input $x(t)$ and output $y(t)$ satisfying $\frac{d}{dt}y(t) + 2y(t) = x(t)$ and $y(0)=0$. Determine the output for these inputs:

a) $x_1(t) = 0 \quad \forall t$

$\boxed{y_1(t) = 0 \quad \forall t}$ satisfies $\frac{d}{dt}y(t) + 2y(t) = x(t)$: $\frac{d}{dt}0 + 2 \cdot 0 = 0 \iff 0+0=0 \iff 0=0$

and $y(0)=0$

b) $x_2(t) = \begin{cases} 0, & t<-1 \\ 1, & t>-1 \end{cases}$

$y_h(t) = Ae^{-2t}$ (checked in 2.33.a.i)

Let $y_p(t) = B\,u(t+1)$: $\frac{d}{dt}B \cdot 2B = 1 \quad \forall t>-1 \Rightarrow B = \frac{1}{2}$

$= 0 \quad \forall t<-1 \Rightarrow B = 0$

$y(0) = y_h(0) + y_p(0) = Ae^0 + B \Rightarrow A = y(0) - B = 0 - \frac{1}{2} = -\frac{1}{2}$

$\Rightarrow y_2(t) = -\frac{1}{2}e^{-2t} + \frac{1}{2} \quad \forall t>-1$

For $t<-1$: $y_2(t) = Ce^{-2t}$, at -1 $Ce^{-2(-1)} = -\frac{1}{2}e^{-2(-1)} + \frac{1}{2} \Rightarrow$

$\Rightarrow Ce^2 = -\frac{1}{2}e^2 + \frac{1}{2} \Rightarrow y_2(t) = \left(\frac{1}{2} - \frac{1}{2}e^2\right)e^{-2(t+1)} \quad \forall t<-1$

$$\boxed{y_2(t) = \begin{cases} -\frac{1}{2}e^{-2t} + \frac{1}{2} & \text{if } t \geq -1 \\ \left(\frac{1}{2} - \frac{1}{2}e^2\right)e^{-2t-2} & \text{if } t<-1 \end{cases}}$$

c) Clearly, if $x_1(t) = x_2(t) = 0 \ \forall t<-1$ and their outputs are not $y_1(t) = y_2(t) = 0 \ \forall t<-1$, it's not causal because the output at -1 is seeing values after $t=-1$.