1.30 (3)

i) $y[n]=\sum_{k=-\infty}^{n}\left(\frac{1}{2}\right)^{n-k}x[k]=\left(\frac{1}{2}\right)^{n-n}x[n]+\sum_{k=-\infty}^{n-1}\left(\frac{1}{2}\right)^{n-k}x[k]=x[n]+\left(\frac{1}{2}\right)\sum_{k=-\infty}^{n-1}\left(\frac{1}{2}\right)^{n-1-k}x[k]=$

$=x[n]+\frac{1}{2}y[n-1] \iff x[n]=y[n]-\frac{1}{2}y[n-1]$

$\boxed{\text{Invertible with } y[n]\xrightarrow{S^{-1}}x[n]=y[n]-\frac{1}{2}y[n-1]}$

j) $y(t)=\frac{dx}{dt}(t)$

Let $x_1(t)=t$; $x_2(t)=t+1\neq x_1(t)$

$y_1(t)=\frac{d}{dt}(t)=1$; $y_2(t)=\frac{d}{dt}(t+1)=1=y_1(t)\Rightarrow \boxed{\text{not invertible}}$

k) $y[n]=\begin{cases}x[n+1], & n\geq 0\\ x[n], & n\leq -1\end{cases}$

Let $x_1[n]=u[n]$; $x_2[n]=u[n-1]\neq x_1[n]$

$y_1[n]=\begin{cases}x_1[n+1]=u[n+1], & n\geq 0\\ x_1[n]=u[n], & n\leq -1\end{cases}=\begin{cases}1 \text{ if } n\geq 0 \text{ and } \overset{n\geq -1}{n+1\geq 0}\\ 0 \text{ if } n\geq 0 \text{ and } \overset{n<-1}{n+1<0}\ \not\exists\\ 1 \text{ if } n\leq -1 \text{ and } n\geq 0\ \not\exists\\ 0 \text{ if } n\leq -1 \text{ and } n<0\end{cases}=\begin{cases}1 \text{ if } n\geq 0\\ 0 \text{ otherwise}\end{cases}=u[n]$

$y_2[n]=\begin{cases}x_2[n+1], & n\geq 0\\ x_2[n], & n\leq -1\end{cases}=\begin{cases}1 \text{ if } n\geq 0 \text{ and } n\geq 0\\ 0 \text{ if } n\geq 0 \text{ and } n<0\ \not\exists\\ 1 \text{ if } n\leq -1 \text{ and } \overset{n\geq 1}{n-1\geq 0}\ \not\exists\\ 0 \text{ if } n\leq -1 \text{ and } \overset{n<1}{n-1<0}\end{cases}=\begin{cases}1 \text{ if } n\geq 0\\ 0 \text{ otherwise}\end{cases}=y_1[n]\Rightarrow$

$\Rightarrow \boxed{\text{not invertible}}$

l) $y(t)=x(2t) = \text{C.V.}\left[u=2t\Rightarrow t=\frac{u}{2}\right]= y\left(\frac{u}{2}\right)=x(u)$

$\boxed{\text{Invertible with } y(t)\xrightarrow{S^{-1}}x(t)=y\left(\frac{t}{2}\right)}$