2.29. The following are impulse responses for continuous-time LTI systems. Determine whether each one is causal and/or stable.

a) $h(t) = e^{-4t}u(t-2)$

$h(t) = 0 \;\; \forall\; t<2 \Rightarrow h(t)=0 \;\forall\; t<0 \Rightarrow \boxed{\text{causal}}$

$$\int_{-\infty}^{\infty}|h(t)|\,dt = \int_{2}^{\infty}|e^{-4t}|\,dt = \int_{2}^{\infty}e^{-4t}\,dt = \frac{e^{-4t}}{-4}\Bigg|_{t=2}^{\infty} = e^{-\infty} + \frac{1}{4}e^{-4\cdot 2} = \frac{e^{-8}}{4} < \infty \Rightarrow \boxed{\text{stable}}$$

b) $h(t) = e^{-6t}u(3-t)$

$h(-1) = e^{6}u(3+1) = e^{6} \neq 0 \Rightarrow \boxed{\text{not causal}}$

$$\int_{-\infty}^{\infty}|h(t)|\,dt = \int_{-\infty}^{3}e^{-6t}\,dt = -\frac{1}{6}e^{-6t}\Bigg|_{t=-\infty}^{3} = -\frac{e^{-18}}{6} + e^{+\infty} = \infty \Rightarrow \boxed{\text{not stable}}$$

c) $h(t) = e^{-2t}u(t+50)$

$h(-1) = e^{2}u(-1+50) = e^{2} \neq 0 \Rightarrow \boxed{\text{not causal}}$

$$\int_{-\infty}^{\infty}|h(t)|\,dt = \int_{-50}^{\infty}e^{-2t}\,dt = \frac{-e^{-2t}}{2}\Bigg|_{t=-50}^{\infty} = -e^{-\infty} + \frac{e^{-100}}{2} = \frac{1}{2}e^{-100} < \infty \Rightarrow \boxed{\text{stable}}$$

d) $h(t) = e^{2t}u(-1-t)$

$h(-1) = e^{-2}u(-1+1) = e^{-2} \neq 0 \Rightarrow \boxed{\text{not causal}}$

$$\int_{-\infty}^{\infty}|h(t)|\,dt = \int_{-\infty}^{-1}e^{2t}\,dt = \frac{e^{2t}}{2}\Bigg|_{t=-\infty}^{-1} = \frac{e^{-2}}{2} + e^{-\infty} = \frac{e^{-2}}{2} < \infty \Rightarrow \boxed{\text{stable}}$$

e) $h(t) = \cancel{e^{-6|t|}}\; e^{-6|t|}$

$h(-1) = e^{-6} \neq 0 \Rightarrow \boxed{\text{not causal}}$

$$\int_{-\infty}^{\infty}|h(t)| = \int_{-\infty}^{0}e^{-6(-t)}\,dt + \int_{0}^{\infty}e^{-6t}\,dt = \left(\frac{e^{6t}}{6}\right)\Bigg|_{t=-\infty}^{0} + \left(\frac{-e^{-6t}}{6}\right)\Bigg|_{t=0}^{\infty} = \frac{e^{0}}{6} - e^{-\infty} + \left(-e^{-\infty} + \frac{e^{0}}{6}\right) = \frac{1}{3} < \infty \Rightarrow$$

$\Rightarrow \boxed{\text{stable}}$

f) $h(t) = te^{-t}u(t)$

$h(t) = 0 \;\; \forall\; t<0 \Rightarrow \boxed{\text{causal}}$

$$\int_{-\infty}^{\infty}|h(t)|\,dt = \int_{0}^{\infty}te^{-t}\,dt \rightarrow \left[\begin{matrix} u=t \Rightarrow dt=du \\ dv=e^{-t}dt \Rightarrow v=\int e^{-t}dt = -e^{-t}\end{matrix}\right] \Rightarrow -te^{-t} - \int -e^{-t}dt\,\Bigg|_{t=0}^{\infty} = -te^{-t} - e^{-t}\Bigg|_{t=0}^{\infty} =$$

$$= -\left(\lim_{t\to\infty} te^{-t}\right) - e^{-\infty} - \left(-0 - e^{-0}\right) = 0 - 0 + 0 + 1 = 1 < \infty \Rightarrow \boxed{\text{stable}}$$