1.33. Let $x[n]$ be a ~~discrete-ty~~ discrete-time signal, $y_1[n] = x[2n]$ and

$$y_2[n] = \begin{cases} x[\frac{n}{2}], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$$

. The signals $y_1[n]$ and $y_2[n]$ represent in some sense the sped up and slowed down versions of $x[n]$, respectively. Consider the following statements and determine whether each one is true. If so, determine the relationship between the fundamental periods of the signals considered. If not, produce a counterexample.

1) If $x[n]$ is periodic, then $y_1[n]$ is periodic.

$$\left.\begin{array}{l} x[n] \text{ periodic with fundamental period } N_x \Rightarrow x[n] = x[n+N] \ \forall\, N \in \{k \cdot N_x : k \in \mathbb{Z}\} \\ y_1[n] = x[2n] \overset{C.V.[2n \to n]}{\Rightarrow} y_1[\frac{n}{2}] = x[n] \ \forall n \text{ even} \Rightarrow y_1[\frac{n+N}{2}] = x[n+N] \ \forall\, n+N \text{ even} \end{array}\right\} \Rightarrow$$

$$\Rightarrow y_1[\tfrac{n}{2}] = y_1[\tfrac{n}{2} + \tfrac{N}{2}] \ \forall\, N \in \{k \cdot N_x : k \in \mathbb{Z}\}, n \text{ even}, n+N \text{ even} \Rightarrow$$

$$\overset{C.V.[\frac{n}{2} \to n]}{\Rightarrow} y_1[n] = y_1[n+N] \ \forall\, N \in \{\tfrac{k \cdot N_x}{2} : k \in \mathbb{Z}, \tfrac{k \cdot N_x}{2} \in \mathbb{Z}\} \Rightarrow$$

$$\Rightarrow \begin{cases} N_x \text{ even} \Rightarrow y_1[n] = y_1[n+N] \ \forall\, N \in \{k \cdot N_{y_1} : k \in \mathbb{Z},\ N_{y_1} = \frac{N_x}{2}\} \text{ since } \frac{N_x}{2} \in \mathbb{Z} \\ N_x \text{ odd} \Rightarrow y_1[n] = y_1[n+N] \ \forall\, N \in \{\frac{k}{2} \cdot N_{y_1} : \frac{k}{2} \in \mathbb{Z},\ N_{y_1} = N_x\} \overset{\frac{k}{2} \to k}{\Rightarrow} y_1[n] = y_1[n+N] \ \forall\, N \in \{k \cdot N_{y_1} : k \in \mathbb{Z},\ N_{y_1} = N_x\} \end{cases} \Rightarrow$$

$$\Rightarrow \boxed{y_1[n] \text{ is periodic with fundamental period } N_{y_1} = \begin{cases} \frac{N_x}{2} & \text{if } N_x \text{ even} \\ N_x & \text{if } N_x \text{ odd} \end{cases}}$$

2) If $y_1[n]$ is periodic, then $x[n]$ is periodic

Let $x[n] = \begin{cases} (-1)^{\frac{n}{2}} & \text{if } n \text{ even} \\ n & \text{if } n \text{ odd} \end{cases}$, which is not periodic

$$y_1[n] = x[2n] = \begin{cases} (-1)^{\frac{2n}{2}} & \text{if } 2n \text{ even} \\ 2n & \text{if } 2n \text{ odd} \end{cases} = (-1)^n$$

$$y_1[n] = y[n+N] \Leftrightarrow (-1)^n = (-1)^{n+N} \Leftrightarrow (-1)^n = (-1)^n \cdot (-1)^N \Leftrightarrow 1 = (-1)^N \Leftrightarrow N \text{ even} \Leftrightarrow$$

$\Leftrightarrow N \in \{2k : k \in \mathbb{Z}\} \Rightarrow y_1[n]$ is periodic with fundamental period 2, ~~while $x[n]$ is~~

However, $x[n]$ is not periodic, so the statement is false