... 1.28. (4)

f) $y[n] = \begin{cases} x[n], & n \geq 1 \\ 0, & n = 0 \\ x[n], & n \leq -1 \end{cases}$

1) $y[n_0]$ only depends on x at n_0 $\Rightarrow$ **memoryless**

2) $x'[n] = x[n+k] \Rightarrow y[n] = \begin{cases} x[n+k], & n \geq 1 \\ 0, & n = 0 \\ x[n+k], & n \leq -1 \end{cases}$

$y[n+k] = \begin{cases} x[n+k], & n+k \geq 1 \\ 0, & n = 0 \\ x[n+k], & n \leq 1 \end{cases} \neq y'[n]$ while $k \neq 0$ $\Rightarrow$ **not time invariant**

3) $x'[n] = \alpha x_1[n] + \beta x_2[n] \Rightarrow y'[n] = \begin{cases} \alpha x_1[n] + \beta x_2[n], & n \geq 1 \\ 0, & n = 0 \\ \alpha x_1[n] + \beta x_2[n], & n \leq -1 \end{cases}$

$\alpha y_2[n] + \beta y_2[n] = \alpha \begin{cases} x_1[n], & n \geq 1 \\ 0, & n = 0 \\ x_1[n], & n \leq -1 \end{cases} + \beta \begin{cases} x_2[n], & n \geq 1 \\ 0, & n = 0 \\ x_2[n], & n \leq -1 \end{cases} = \begin{cases} \alpha x_1[n] + \beta x_2[n], & n \geq 1 \\ 0, & n = 0 \\ \alpha x_1[n] + \beta x_2[n], & n \leq -1 \end{cases} = y'[n] \Rightarrow$

$\Rightarrow$ **linear**

4) memoryless $\Rightarrow$ **causal**

5) $|x[n]| \leq B \quad \forall n \in \mathbb{Z}$

$|y[n]| \leq |x[n]| \leq B \Rightarrow$ **stable**

g) $y[n] = x[4n+1]$

1) $y[n_0]$ depends on x at $4n_0+1 \neq n_0$ $\Rightarrow$ **not memoryless**

2) $x'[n] = x[4n+k] \Rightarrow y'[n] = x'[4n+1] = x[4n+1+k]$,

$y[n+k] = x[4(n+k)+1)] = x[4n+4k+1] \neq y'[n] \Rightarrow$ **not time invariant**

3) $x'[n] = \alpha x_1[n] + \beta x_2[n] \Rightarrow y'[n] = \alpha x_1[4n+1] + \beta x_2[4n+1]$,

$\alpha y_1[n] + \beta y_2[n] = \alpha x_1[4n+1] + \beta x_2[4n+1] = y'[n] \Rightarrow$ **linear**

4) $y[n_0]$ depends on x at $4n_0+1 > n_0 \quad \forall n_0 \geq 0$ $\Rightarrow$ **not causal**

5) $|x[n]| \leq B \quad \forall n \in \mathbb{Z}$

$|y[n]| = |x[4n+1]| \leq B \Rightarrow$ **stable**