

... 1.27. (2)

c)  $y(t) = \int_{-\infty}^{2t} x(\tau) d\tau$

1)  $y(t_0)$  depends on terms  $x(\tau)$ , where  $\tau \in (-\infty, 2t_0]$  may be different from  $t_0 \Rightarrow$  not memoryless

2)  $x(t) = x(t+k) \Rightarrow y(t) = \int_{-\infty}^{2t} x(\tau) d\tau = \int_{-\infty}^{2t} x(\tau+k) d\tau = \left[ \tau' = \tau+k \right] = \int_{-\infty}^{2t+k} x(\tau') d\tau'$

$y(t+k) = \int_{-\infty}^{2t+2k} x(\tau) d\tau \neq y(t) \Rightarrow$  not time-invariant

3)  $x(t) = \alpha x_1(t) + \beta x_2(t) \Rightarrow y(t) = \int_{-\infty}^{2t} x(\tau) d\tau = \int_{-\infty}^{2t} (\alpha x_1(\tau) + \beta x_2(\tau)) d\tau = \alpha \int_{-\infty}^{2t} x_1(\tau) d\tau + \beta \int_{-\infty}^{2t} x_2(\tau) d\tau$

$\alpha y_1(t) + \beta y_2(t) = \alpha \int_{-\infty}^{2t} x_1(\tau) d\tau + \beta \int_{-\infty}^{2t} x_2(\tau) d\tau = y(t) \Rightarrow$  linear

4)  $y(t)$  depends on terms  $x(\tau)$ , where  $\tau \in (-\infty, 2t_0]$  may be greater than  $t_0 \Rightarrow$  not causal

5)  $|x(t)| \leq B \quad \forall t \in \mathbb{R}$

(let  $x(t) = c \Rightarrow y(t) = \left| \int_{-\infty}^0 c d\tau \right| = \left| [c\tau]_{-\infty}^0 \right| = |-\infty| = \infty$ )

$|y(t)| = \left| \int_{-\infty}^{2t} x(\tau) d\tau \right| \leq \left| \int_{-\infty}^{2t} B d\tau \right| = \infty \Rightarrow$  not stable

d)  $y(t) = \begin{cases} 0 & t < 0 \\ x(t) + x(t-2) & t \geq 0 \end{cases}$

1)  $y(t_0)$  depends on terms  $x(t_0-2)$ , where  $t_0-2 \neq t_0 \Rightarrow$  not memoryless

2)  $x(t) = x(t+k) \Rightarrow y(t) = \begin{cases} 0 & t < 0 \\ x(t) + x(t-2) & t \geq 0 \end{cases} = \begin{cases} 0 & t < 0 \\ x(t+k) + x(t+k-2) & t \geq 0 \end{cases}$

$y(t+k) = \begin{cases} 0 & t < 0 \\ x(t+k) + x(t+k-2) & t+k \geq 0 \end{cases} \neq y(t) \Rightarrow$  not time-invariant

3)  $x(t) = \alpha x_1(t) + \beta x_2(t) \Rightarrow y(t) = \begin{cases} 0 & t < 0 \\ x(t) + x(t-2) & t \geq 0 \end{cases} = \begin{cases} 0 & t < 0 \\ \alpha x_1(t) + \beta x_2(t) + \alpha x_1(t-2) + \beta x_2(t-2) & t \geq 0 \end{cases}$

$\alpha y_1(t) + \beta y_2(t) = \alpha \begin{cases} 0 & t < 0 \\ x_1(t) + x_1(t-2) & t \geq 0 \end{cases} + \beta \begin{cases} 0 & t < 0 \\ x_2(t) + x_2(t-2) & t \geq 0 \end{cases} = \begin{cases} 0 & t < 0 \\ \alpha x_1(t) + \alpha x_1(t-2) + \beta x_2(t) + \beta x_2(t-2) & t \geq 0 \end{cases} = y(t) \Rightarrow$  linear

4)  $y(t_0)$  depends on  $x(t_0)$  and  $x(t_0-2)$ , where  $t_0 \neq t_0-2$  and  $t_0-2 \neq t_0 \Rightarrow$  causal

5)  $|x(t)| \leq B \Rightarrow |x(t) + x(t-2)| \leq 2B \quad \forall t \in \mathbb{R} \Rightarrow y(t) \leq 2B \quad \forall t \in \mathbb{R} \Rightarrow$  stable