2.10. Suppose that $x(t) = \begin{cases} 1, & 0 \le t \le 1 \\ 0, & \text{elsewhere} \end{cases}$ and $h(t) = x\left(\frac{t}{\alpha}\right)$, $0 < \alpha \le 1$

a) determine and sketch $y(t) = x(t) * h(t)$

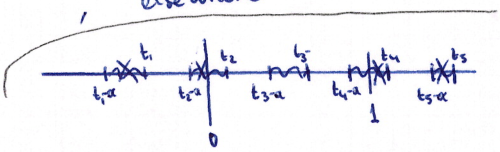
$$h(t) = x\left(\frac{t}{\alpha}\right) = \begin{cases} 1, & 0 \le \frac{t}{\alpha} \le 1 \iff 0 \le t \le \alpha \\ 0, & \text{elsewhere} \end{cases}$$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau)\, d\tau = \int_{-\infty}^{\infty} \left(\begin{cases} 1, & 0 \le \tau \le 1 \\ 0, & \text{elsewhere} \end{cases} \right) \cdot \left(\begin{cases} 1, & \overbrace{0 \le t-\tau \le \alpha}^{t-\alpha \le \tau \le t} \\ 0, & \text{elsewhere} \end{cases} \right) d\tau =$$

$$= \int_{-\infty}^{\infty} \left(\begin{cases} 1, & 0 \le \tau \le 1 \text{ and } t-\alpha \le \tau \le t \\ 0, & \text{elsewhere} \end{cases} \right) d\tau = \int_{-\infty}^{\infty} \left(\begin{cases} 1, & \max(0, t-\alpha) \le \tau \le \min(1, t) \\ 0, & \text{elsewhere} \end{cases} \right) d\tau =$$

$$= \int_{\max(0,t-\alpha)}^{\min(1,t)} \left(\begin{cases} 1, & \max(0, t-\alpha) \le \min(1,t) \\ 0, & \text{elsewhere} \end{cases} \right) d\tau = \begin{cases} \int_{\max(0,t-\alpha)}^{\min(1,t)} d\tau & \text{if } \max(0,t-\alpha) \le \min(1,t) \\ 0 & \text{elsewhere} \end{cases} =$$

$$= \begin{cases} \tau \Big|_{\max(0,t-\alpha)}^{\min(1,t)} & \text{if } \max(0,t-\alpha) \le \min(1,t) \\ 0 & \text{elsewhere} \end{cases} = \begin{cases} \min(1,t) - \max(0,t-\alpha), & \text{if } \max(0,t-\alpha) \le \min(1,t) \\ 0, & \text{elsewhere} \end{cases} =$$



$$= \begin{cases} t, & \overbrace{t-\alpha < 0 < t}^{0<t<\alpha} \\ t-(t-\alpha), & 0 \le t-\alpha,\ t \le 1 \\ 1-(t-\alpha), & t-\alpha < 1 < t \\ 0, & \text{elsewhere} \end{cases} = \boxed{\begin{cases} t, & 0 < t < \alpha \\ \alpha, & \alpha \le t \le 1 \\ 1+\alpha-t, & 1 < t < 1+\alpha \\ 0, & \text{elsewhere} \end{cases}}$$