2.32. Consider the difference equation $y[n] - \frac{1}{2} y[n-1] = x[n]$ and suppose that $x[n] = (\frac{1}{3})^n u[n]$. ~~Assu~~ Assume that the solution $y[n]$ is the sum of a particular solution $y_p[n]$ ~~to~~ to the first equation and a homogeneous solution $y_h[n]$ satisfying $y_h[n] - \frac{1}{2} y_h[n] = 0$.

a) Verify that the homogeneous solution is ~~A~~ $y_h[n] = A(\frac{1}{2})^n$

$y_h[n] = A(\frac{1}{2})^n \Rightarrow y_h[n] - \frac{1}{2} y_h[n-1] = \cancel{A} A(\frac{1}{2})^n - \frac{1}{2} A(\frac{1}{2})^{n-1} = 0 \Leftrightarrow 0 = 0 \Rightarrow$

$\Rightarrow$ Verified

b) Let us consider finding $y_p[n]$ such that $y_p[n] - \frac{1}{2} y_p[n] = (\frac{1}{3})^n u[n]$. By ~~as~~ assuming $y_p[n] = A \cdot (\frac{1}{3})^n \ \forall n \geq 0$ and substituting this value in the previous ~~diff~~ ~~eq~~ difference equation, find B.

$B(\frac{1}{3})^n - \frac{1}{2} B(\frac{1}{3})^{n-1} = (\frac{1}{3})^n \ \forall n \geq 0 \Leftrightarrow B\left((\frac{1}{3})^n - \frac{3}{2}(\frac{1}{3})^n\right) = (\frac{1}{3})^n \ \forall n \geq 0 \Leftrightarrow$

$\Leftrightarrow B \cdot (-\frac{1}{2})(\frac{1}{3})^n = (\frac{1}{3})^n \ \forall n \geq 0 \Leftrightarrow \boxed{B = -2.}$

c) Suppose the LTI system described by the proposed difference equation and initially at rest has the input signal $x[n]$. $x[n] = 0 \ \forall n < 0 \Rightarrow y[n] = 0 \ \forall n < 0$. We know $y[n] = A(\frac{1}{2})^n + B(\frac{1}{3})^n \ \forall n \geq 0$. To find A, we must specify a value of $y[n]$ for some $n \geq 0$. Use the condition of initial rest and the given equations to find $y[0]$. With this, determine A. This yields the solution $y[n]$.

~~$y[0] - \frac{1}{2} y[-1] = x[0]$~~

$\left.\begin{array}{l} y[0] - \frac{1}{2} y[-1] = x[0] \\ y[n] = 0 \ \forall n < 0 \end{array}\right\} \Rightarrow y[0] = x[0] = (\frac{1}{3})^0 u[0] = \boxed{1 = y[0]}$

$y[0] = A(\frac{1}{2})^0 + B(\frac{1}{3})^0 = A + B \Rightarrow A = y[0] - B = 1 - (-2) = \boxed{3 = A}$

$y[n] = 3(\frac{1}{2})^n - 2(\frac{1}{3})^n$