2.44. a) If $x(t)=0$, $|t|>T_1$ and $h(t)=0$, $|t|>T_2$ then $x(t)*h(t)=0$, $|t|>T$ for some positive number T_3. Express T_3 in terms of T_1 and T_2

$$x(t)*h(t) = \int_{-\infty}^{\infty} x(\tau)\cdot h(t-\tau)\,d\tau = \int_{-T_1}^{T_1} x(\tau)h(t-\tau)\,d\tau = \int_{-T_1}^{T_1} x(\tau)\cdot \begin{cases} h(t-\tau) & \text{if } -T_2 \le t-\tau \le T_2 \\ 0 & \text{otherwise} \end{cases} =$$

$$= \int_{-T_1}^{T_1} \begin{cases} x(\tau)h(t-\tau) & \text{if } t-T_2 \le \tau \le t+T_2 \\ 0 & \text{if } t-T_2 > \tau \text{ or } \tau > t+T_2 \end{cases} \Rightarrow x(t)*h(t)=0\ \forall: [-T_1,T_1]\cap[t-T_2,t+T_2]=\emptyset \Rightarrow$$

$$\Rightarrow x(t)*h(t)=0\ \forall t: \{t+T_2<-T_1\}\cup\{t-T_2>T_1\} \Rightarrow x(t)*h(t)=0\ \forall t: \{-t>T_1+T_2\}\cup\{t>T_1+T_2\} \Rightarrow$$

$$\Rightarrow x(t)*h(t)=0\ \forall |t|>|T_1+T_2| \Rightarrow \boxed{T_3=T_1+T_2}$$

$N_0 \le n-k \le N_1 \Rightarrow n-N_1 \le k \le n-N_0$

b) A discrete-time LTI system has input $x[n]$, impulse response $h[n]$ and output $y[n]$. If $h[n]=0\ \forall n \notin [N_0,N_1]$ and $x[n]=0\ \forall n \notin [N_2,N_3]$, then $y[n]=0\ \forall n \notin [N_4,N_5]$

i) Determine N_4 and N_5 in terms of the other N_i.

$$x[n]*h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=N_2}^{N_3} \begin{cases} x[k]h[n-k] & \text{if } \cancel{N_0 \le n-k \le N_1} \\ 0 & \text{otherwise} \end{cases} \Rightarrow n-N_1 \le k \le n-N_0$$

$$\Rightarrow x[n]*h[n]=0\ \forall\ n: [N_2,N_3]\cap[n-N_1, n-N_0] \Rightarrow$$

$$\Rightarrow x[n]*h[n]=0\ \forall\ \{n-N_0<N_2\}\cup\{n-N_1>N_3\} \Rightarrow x[n]*h[n]=0\ \forall\ n: \{n<N_2+N_0\}\cup\{n>N_1+N_3\} \Rightarrow$$

$$\Rightarrow x[n]*h[n]=0\ \forall\ n\notin [N_2+N_0, N_1+N_3] \Rightarrow \boxed{N_4=N_0+N_2,\ N_5=N_1+N_3}$$

ii) If the length of $N_0\le n\le N_1$ is M_h, the length of $N_2\le n\le N_3$ is M_x and the length of $N_4\le n\le N_5$ is M_y, express M_y in terms of M_x and M_h.

$M_h=N_1-N_0+1$, $M_x=N_3-N_2+1$, $M_y=N_5-N_4+1=N_1+N_3-N_0-N_2+1=$

$=N_1-N_0+1+N_3-N_2+1-1=\boxed{M_h+M_x-1=M_y}$

c) Consider a discrete-time LTI system such that if an input $x[n]=0\ \forall n\ge 10$ then $y[n]=0\ \forall n\ge 15$. What condition must $h[n]$ satisfy?

~~Here~~ ~~Same as~~ Similar to previous case, now $N_3=9$, $N_5=14$.

$N_5=N_1+N_3 \Rightarrow N_1=N_5-N_3=14-9=5=N_1 \Rightarrow$

$\Rightarrow \boxed{h[n]=0\ \ \forall\ n>5} \Leftrightarrow h[n]=0\ \forall\ n\ge 6$