1.41. Consider a system S with ~~input~~ input $x[n]$ and output $y[n] = x[n]\{g[n]+g[n-1]\}$

a) If $g[n]=1 \ \forall n$, show that S is time invariant

$g[n]=1 \ \forall n \Rightarrow y[n]= x[n]\cdot(g[n]+g[n-1]) = x[n]\cdot(1+1) \quad \forall n \quad \Rightarrow y[n]=2x[n]$

$x'[n]= x[n+k] \rightarrow y'[n]=2x'[n]= 2x[n+k]$

$y[n+k]=2x[n+k] = y'[n] \Rightarrow$ time-invariant

b) If $g[n]=n$, show that S is not time invariant

$g[n]=n \Rightarrow y[n]= x[n]\cdot(n+(n-1)) = (2n-1)\,x[n]$

$x'[n]= x[n+k] \rightarrow y'[n]= (2n-1)x'[n]= (2n-1)\cdot x[n+k]$

$y[n+k]= (2\cdot(n+k)-1)\,x[n+k] = (2n+2k-1)\,x[n+k] \neq y'[n] \Rightarrow$ not time-invariant

c) If $g[n] = 1+(-1)^n$, show that S is time-invariant.

$\cancel{g[n]=(-1)^n+1 \Rightarrow y[n]=x[n]\cdot((-1)^n+1+(-1)^{n-1}+1-1) = x[n]\cdot((-1)^n+(-1)^n\cdot(-1)+1) =}$

$\cancel{= x[n]\cdot(-(-1)^n\cdot 2+1) = ((-1)^{n-1}\cdot 2+1)\,x[n]}$

$\cancel{x'[n]=x[n+k] \rightarrow y'[n]=(2(-1)^{n-1}+1)\,x'[n]=(2(-1)^{n-1}+1)\,x[n+k]}$

$\cancel{y[n+k]}=$

$g[n]= (-1)^n+1 \Rightarrow y[n]= x[n]\cdot((-1)^n+1+(-1)^{n-1}+1) = x[n]\cdot((-1)^n+(-1)^{-1}(-1)^n+2) =$

$= ((-1)^n-(-1)^n+2)\,x[n] = 2x[n]$

$x'[n]= x[n+k] \rightarrow y'[n] = 2x'[n]= 2x[n+k]$

$y[n+k] = 2x[n+k = y'[n] \Rightarrow$ time invariant