

230. Consider the first-order difference equation  $y[n] + 2y[n-1] = x[n]$ . Assuming initial rest ( $x[n] = 0 \forall n < n_0 \Rightarrow y[n] = 0 \forall n < n_0$ ), find the impulse response of a system that satisfies this.

Let  $x[n] = \delta[n]$ , then  $y[n] = x[n] * h[n] = \delta[n] * h[n] = h[n]$

$$y[n] = x[n] - 2y[n-1] = \delta[n] - 2y[n-1]$$

$$x[n] = 0 \forall n < 0 \Rightarrow y[n] = 0 \forall n < 0$$

$$y[0] = x[0] - 2y[-1] = \delta[0] - 0 = 1$$

$$y[n] = -2y[n-1] \forall n > 0 \Rightarrow \forall n > 0, y[n] = -2y[n-1] = -2 \cdot (-2) \cdot y[n-2] = \dots = (-2)(-2) \dots (-2)y[n-n] = (-2)^n \cdot y[0]$$

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$$\left. \begin{array}{l} = (-2)^n \cdot y[0] \\ \cdot y[0] = 1 = (-2)^0 \end{array} \right\} \Rightarrow$$

$$\Rightarrow y[n] = (-2)^n \forall n \geq 0 \Rightarrow y[n] = \boxed{(-2)^n u[n] = h[n]}$$