2.28. The following are impulse responses for LTI systems. Determine whether each one is causal and/or stable.

a) $h[n] = \left(\frac{1}{5}\right)^n u[n]$

$h[n] = 0 \quad \forall\, n < 0 \Rightarrow \boxed{\text{causal}}$

$$\sum_{k=-\infty}^{\infty} |h[k]| = \sum_{k=0}^{\infty} \left(\frac{1}{5}\right)^k = \frac{1}{1-\frac{1}{5}} = \frac{1}{\frac{4}{5}} = \frac{5}{4} < \infty \Rightarrow \boxed{\text{stable}}$$

b) $h[n] = 0.8^n u[n+2]$

$h[-1] = 0.8^{-1} u[-1+2] = \frac{5}{4} \neq 0 \Rightarrow \boxed{\text{not causal}}$

$$\sum_{k=-\infty}^{\infty} |h[k]| = \sum_{k=-2}^{\infty} 0.8^k = 0.8^{-2} + 0.8^{-1} + \sum_{k=0}^{\infty} 0.8^n = \frac{5^2}{4^2} + \frac{5}{4} + \frac{1}{1-0.8} = \frac{45}{16} + \frac{5}{1} = 7.8125 < \infty \Rightarrow \boxed{\text{stable}}$$

c) $h[n] = \left(\frac{1}{2}\right)^n u[-n]$

$h[-1] = \left(\frac{1}{2}\right)^{-1} u[1] = 2 \neq 0 \Rightarrow \boxed{\text{not causal}}$

$$\sum_{k=-\infty}^{\infty} |h[k]| = \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^k u[-n] = \sum_{k=-\infty}^{0} \left(\frac{1}{2}\right)^k = \sum_{k=0}^{\infty} 2^k = \infty \Rightarrow \boxed{\text{not stable}}$$

d) $h[n] = (5)^n u[3-n]$

$h[-1] = (5)^{-1} u[3+1] = \frac{1}{5} \neq 0 \Rightarrow \boxed{\text{not causal}}$

$$\sum_{k=-\infty}^{\infty} |h[k]| = \sum_{k=-\infty}^{3} 5^k = 5^1 + 5^2 + 5^3 + \sum_{k=0}^{\infty} 5^{-k} = 155 + \frac{1}{1-\frac{1}{5}} = 155 + \frac{5}{4} = 156.25 < \infty \Rightarrow \boxed{\text{stable}}$$

e) $h[n] = \left(-\frac{1}{2}\right)^n u[n] + (1.01)^n u[n-1]$

$h[n] = 0 \quad \forall\, n < 0 \Rightarrow \boxed{\text{causal}}$

$$\sum_{k=-\infty}^{\infty} |h[k]| \leq \sum_{k=0}^{\infty} \left|\left(-\frac{1}{2}\right)^k\right| + \sum_{k=1}^{\infty} |(1.01)^n| = \sum_{k=0}^{\infty} \left(+\frac{1}{2}\right)^k + \infty = \infty \Rightarrow \boxed{\text{not stable}}$$

$$\lim_{k\to\infty} (1.01)^k = \lim_{k\to\infty} \frac{101^n}{100^n} = \infty \Rightarrow \sum_{k=1}^{\infty} 1.01^n = \infty$$

$$\sum_{k=0}^{\infty} \left(+\frac{1}{2}\right)^k = \frac{1}{1-\frac{1}{2}} = \frac{2}{1} < \infty$$