... 1.30. (2)

e) $y[n] = \begin{cases} x[n-1] & n \geq 1 \\ 0 & n = 0 \\ x[n] & n \leq -1 \end{cases}$

$y[n] = x[n-1]$ if $n \geq 1$ $\Leftrightarrow$ $y[n+1] = x[n]$ if $n+1 \geq 1$ $\Leftrightarrow$ $y[n+1] = x[n]$ if $n \geq 0$

$y[n] = 0$ if $n = 0$

$y[n] = x[n]$ if $n \leq -1$

$\boxed{\text{Invertible: } y(x) \xrightarrow{S^{-1}} x(t) = \begin{cases} y[n+1] & \text{if } n \geq 0 \\ y[n] & \text{if } n \leq -1 \end{cases}}$

f) $y[n] = x[n] \cdot x[n-1]$

Let $x_1[n] = \delta[n+1] + \delta[n]$, $x_2[n] = -\delta[n+1] - \delta[n] \neq x_1[n]$

$y_1[n] = x_1[n] \cdot x_1[n-1] = (\delta[n+1] + \delta[n]) \cdot (\delta[n-1+1] + \delta[n-1-1]) =$

$= \delta[n+1] \cdot \delta[n] + \delta[n+1] \cdot \delta[n-2] + \delta[n] \cdot \delta[n] + \delta[n] \cdot \delta[n-2] = \delta[n]$

$y_2[n] = x_2[n] \cdot x_2[n-1] = -(\delta[n+1] + \delta[n]) \cdot (-\delta[n-1+1] - \delta[n-1-1]) =$

$= \delta[n+1] \cdot \delta[n] + \delta[n+1] \cdot \delta[n-2] + \delta[n] \cdot \delta[n] + \delta[n] \cdot \delta[n-2] = \delta[n] = y_1[n] \Rightarrow$

$\Rightarrow$ $\boxed{\text{Not invertible}}$

g) $y[n] = x[1-n]$

$y[n] = x[1-n] \Leftrightarrow y[-n] = x[n-1] \Leftrightarrow y[1-n] = x[n]$

$\boxed{\text{Invertible: } y[n] \xrightarrow{S^{-1}} x[n] = y[1-n]}$

h) $y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau)\, d\tau = e^{-t} \int_{-\infty}^{t} e^{\tau} x(\tau)\, d\tau \Rightarrow \frac{d}{dt} y(t) = \frac{d}{dt}\left(e^{-t} \int_{-\infty}^{t} e^{\tau} x(\tau)\, d\tau\right) =$

$= -e^{-t} \int_{-\infty}^{t} e^{\tau} x(\tau)\, d\tau + e^{-t} \cdot \frac{d}{dt} \int_{-\infty}^{t} e^{\tau} x(\tau)\, d\tau = -e^{-t} \int_{-\infty}^{t} e^{\tau} x(\tau)\, d\tau + e^{-t} \cdot (e^{t} \cdot x(t)) =$

$= x(t) - \int_{-\infty}^{t} e^{-t+\tau} x(\tau)\, d\tau = x(t) - y(t) \Leftrightarrow x(t) = \frac{d}{dt} y(t) + y(t)$

$\boxed{\text{Invertible: } y(t) \xrightarrow{S^{-1}} x(t) = \frac{d}{dt} y(t) + y(t)}$