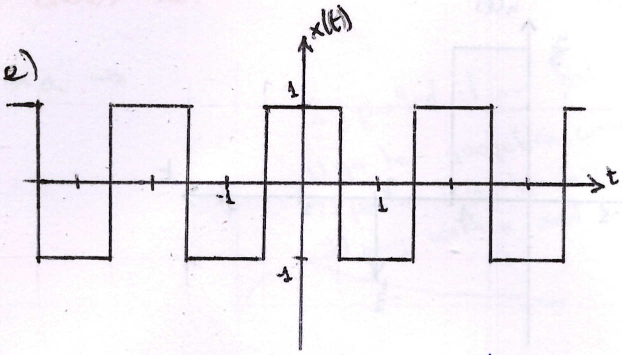
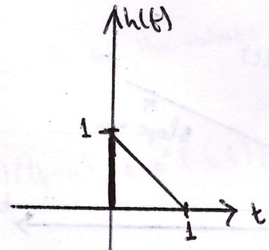
...2.22 (4)

e)



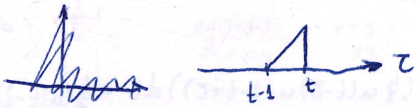


$$h(t)=\begin{cases}1-t & \text{if } 0\le t\le 1\\ 0 & \text{otherwise}\end{cases}$$

Since $x(t)$ is periodic with period 2, so must be $y(t)=x(t)*h(t)$

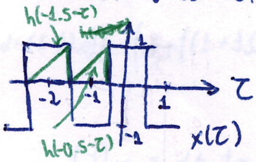
I will calculate $y(t)$ in ~~the~~ one period, choosing -1.5 to 0.5:

For any t, the support of $h(t-\tau)$ is as follows:



So we only have to calculate $\int_{t-1}^{t} x(\tau)\,h(t-\tau)$

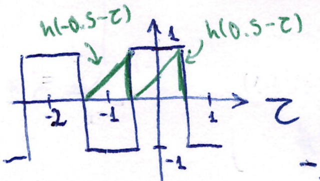
When t is between -1.5 and -0.5 we have this case:



, which can be split by $\tau=-1.5$ in two regions

$$\int_{t-1}^{-1.5} 1\cdot(1-(t-\tau))\,d\tau+\int_{-1.5}^{t}(-1)\cdot(1-t+\tau)\,d\tau=\left((1-t)\tau+\frac{\tau^2}{2}\right)\Bigg|_{t-1}^{-1.5}-\left((1-t)\tau+\frac{\tau^2}{2}\right)\Bigg|_{-1.5}^{t}$$

When t is between -0.5 and 0.5 ~~the~~ we have this:



, which again can be split in two regions, by $\tau=-0.5$

$$-\int_{t-1}^{-0.5}(1-t+\tau)\,d\tau+\int_{-0.5}^{t}(1-t+\tau)\,d\tau=-\left((1-t)\tau+\frac{\tau^2}{2}\right)\Bigg|_{t-1}^{-0.5}+\left((1-t)\tau+\frac{\tau^2}{2}\right)\Bigg|_{-0.5}^{t}$$

One period:

$$y(t)=\begin{cases}\overset{-1.5+1.5t}{\cancel{(1-t)(-1.5)}}+\frac{2.25}{2}-(1-t)(t-1)-\frac{(t-1)^2}{2}-\left(t-t^2+\frac{t^2}{2}+1.5-1.5t+\frac{2.25}{2}\right)=-\frac{3}{4}+2t+\frac{t^2}{2}+\frac{t^2-2t+1}{2}=-\frac{1}{4}+t+t^2 & \text{if } -\frac{3}{2}\le t\le-\frac{1}{2}\\ -\left(\overset{-0.5+0.5t}{\cancel{\quad}}+\frac{0.25}{2}-\left((1-t)(t-1)+\frac{(t-1)^2}{2}\right)\right)+\left(t-t^2+\frac{t^2}{2}-\left(-0.5+0.5t+\frac{0.25}{2}\right)\right)=\frac{3}{4}-\frac{t^2}{2}-\frac{t^2-2t+1}{2}=\frac{1}{4}+t-t^2 & \text{if } -\frac{1}{2}\le t\le\frac{1}{2}\end{cases}$$

$y(t)$ is defined by $\begin{cases}-\frac{1}{4}+t+t^2, & -\frac{3}{2}\le t\le-\frac{1}{2}\\ \frac{1}{4}+t-t^2, & -\frac{1}{2}\le t\le\frac{1}{2}\end{cases}$, periodically every 2 units of time.