

2.46. Consider an LTI system  $S$  and a signal  $x(t) = 2e^{-3t}u(t-1)$ . If  $x(t) \rightarrow y(t)$  and  $\frac{dx(t)}{dt} \rightarrow -3y(t) + e^{2t}u(t)$ , determine the impulse response.

$$\frac{dx(t)}{dt} = \frac{d}{dt}(2e^{-3t}u(t-1)) = -3 \cdot 2e^{-3t}u(t-1) + 2e^{-3t} \cdot \delta(t-1) = -3x(t) + 2e^{-3t}\delta(t-1)$$

$$\left. \begin{array}{l} x(t) \rightarrow y(t) \\ -3x(t) + 2e^{-3t}\delta(t-1) \rightarrow -3y(t) + e^{2t}u(t) \end{array} \right\} \Rightarrow 2e^{-3t}\delta(t-1) \rightarrow e^{2t}u(t) \Rightarrow$$

$$\Rightarrow 2e^{-3t}\delta(t-1) \cdot h(t) \stackrel{=}{=} e^{2t}u(t) \Rightarrow \cancel{h(t-1)} = \frac{e^{2t}}{2e^{-3t}}u(t) \Rightarrow$$

$$\Rightarrow h(t) = \frac{1}{2} e^{-2t-2+3t} u(t) = \boxed{\frac{1}{2} e^{-t+1} u(t) = h(t)}$$