...2.33 (2)

b)

i) Determine $y_1(t)$ for $x_1(t) = Ke^{2t}u(t)$

- $y_h(t) = Ae^{-2t}$ (checked)
- Let $y_p(t) = Be^{2t}$: $\frac{d}{dt}Be^{2t} + 2Be^{2t} = Ke^{2t} \;\forall t \geq 0 \Leftrightarrow 4Be^{2t} = Ke^{2t} \Leftrightarrow B = \frac{K}{4}$

$y_1(0) = y_h(0) + y_p(0) = Ae^{-0} + Be^{+0} = A + B \Rightarrow A = y(0) - B = \underset{\uparrow \text{ initial rest}}{0} - \frac{K}{4} = -\frac{K}{4}$

$\boxed{y_1(t) = \left(-\frac{K}{4}e^{-2t} + \frac{K}{4}e^{2t}\right)u(t)}$

ii) Determine $y_2(t)$ for $x_2(t) = Ke^{2(t-T)}u(t-T)$. Show that $y_2(t) = y_1(t-T)$

- $y_h(t) = Ae^{-2t}$ (checked)
- Let $y_p(t) = Be^{2(t-T)}$: $\frac{d}{dt}Be^{2(t-T)} + 2Be^{2(t-T)} = Ke^{2(t-T)} \;\forall t-T \geq 0 \Leftrightarrow 4Be^{2(t-T)} = Ke^{2(t-T)} \;\forall t \geq T \Leftrightarrow$

$\Leftrightarrow B = \frac{K}{4} \;\forall t \geq T$

$\cancel{y_2(0) = y_h(0) + y_p(0) = Ae^{-0} + Be^{2(0-T)} = A + Be^{-2T} \Rightarrow A = y(0) - Be^{-2T} = 0 - \frac{K}{4}e^{-2T} = -\frac{K}{4}e^{-2T}}$

$y(T) = y_h(T) + y_p(T) = Ae^{-2T} + Be^{2(T-T)} = Ae^{-2T} + B \Rightarrow A = (y(T) - B)e^{2T} = (\underset{\uparrow}{0} - B)e^{2T} = -\frac{K}{4}e^{2T}$

$y_2(t) = \left(-\frac{K}{4}e^{2T}e^{-2t} + \frac{K}{4}e^{2(t-T)}\right)u(t-T) = \boxed{\left(-\frac{K}{4}e^{2(T-t)} + \frac{K}{4}e^{2(t-T)}\right)u(t-T)}$ $x(t) = 0 \;\forall t < T \Rightarrow y(t) = 0 \;\forall t < T$

Clearly, $y_2(t) = y_1(t-T)$

iii) Let $x_1(t)$ be an arbitrary signal such that $x_1(t) = 0 \;\forall t < t_0$, and $x_2(t) = x_1(t-T)$. Their respective system outputs are $y_1(t)$ and $y_2(t)$. Prove that $y_2(t) = y_1(t-T)$

$\frac{d}{dt}y_1(t) + 2y_1(t) = x_1(t)$

~~Since all operators involved are time~~

Let's make the change of variable $t = t_2 - T \Rightarrow \frac{dt}{dt_2} = \frac{d(t_2 - T)}{dt_2} \Rightarrow dt = 1 \cdot dt_2 \Rightarrow dt_2 = d(t_2 - T)$

$\frac{d}{d(t_2-T)}y_1(t_2-T) + 2y_1(t_2-T) = x_1(t_2-T) \Rightarrow \frac{d}{dt_2}y_1(t_2-T) + 2y_1(t_2-T) = x_1(t_2-T)$

Now, changing $t_2 = t$: $\frac{d}{dt}y_1(t-T) + 2y_1(t-T) = x_1(t-T) = x_2(t)$.

We can identify in the equation that $y_2(t) = y_1(t-T)$

We may conclude that this system is time invariant. ~~We also~~ Using the condition from part (a), we conclude that it is LTI. Since it satisfies initial rest, it is causal as well.