2.12 Let $y(t) = e^{-t}u(t) * \sum_{k=-\infty}^{\infty} \delta(t-3k)$

$$x(t) = e^{-t}u(t) = \begin{cases} e^{-t}, & t \geq 0 \\ 0, & \text{elsewhere} \end{cases}$$

$$h(t) = \sum_k \delta(t-3k) = \begin{cases} 1, & t = 3k,\ k \in \mathbb{Z} \\ 0, & \text{elsewhere} \end{cases}$$

$$\cancel{y(t) = \int_{-\infty}^{+\infty} e^{-\tau} u(\tau) \cdot \sum_k \delta(t-\tau-3k)\, d\tau = \int_{-\infty}^{+\infty} \begin{cases} e^{-\tau}, & \tau \geq 0,\ \exists k: t-\tau-3k=0 \\ 0, & \text{elsewhere} \end{cases} d\tau =}$$

$$\cancel{= \int_{0}^{+\infty} \begin{cases} e^{-\tau}, & \tau = t-3k,\ k \in \mathbb{Z} \\ 0, & \text{elsewhere} \end{cases} d\tau}$$

$$y(t) = e^{-t}u(t) * \sum_{k=-\infty}^{\infty} \delta(t-3k) = \sum_k e^{-(t-3k)} u(t-3k)$$

a) Show that $y(t) = Ae^{-t}$ for $0 \leq t < 3$ and determine A.

$$\underbrace{k \leq \tfrac{t}{3} < \tfrac{3}{3} = 1}$$

$$y(t) = \sum_k e^{-(t-3k)} u(t-3k) = \sum_k \begin{cases} e^{-(t-3k)} & \text{if } t-3k \geq 0 \\ 0 & \text{elsewhere} \end{cases} = \sum_{k=-\infty}^{0} e^{-(t-3k)} = \sum_{k=-\infty}^{0} e^{-t+3k} =$$

$$= \left(\sum_{k=-\infty}^{0} e^{3k}\right) \cdot e^{-t} \Rightarrow \begin{cases} y(t) = Ae^{-t} \\ A = \sum_{k=-\infty}^{0} e^{3k} = \sum_{k=0}^{\infty} \frac{1}{e^{3k}} = \sum_{k=0}^{\infty} \left(\frac{1}{e^3}\right)^k = \boxed{\frac{1}{1-\frac{1}{e^3}} = 1.052} \end{cases}$$