

... 1.27. (4)

$$g) y(t) = \frac{dx(t)}{dt} = \lim_{h \rightarrow 0} \frac{x(t+h) - x(t)}{h}$$

1) $y(t)$ depends on $x(t, t+h)$, where $t_0 + h \neq t_0 \Rightarrow$ not memoryless

$$2) x(t) = x(t+k) \Rightarrow y(t) = \frac{dx(t)}{dt} = \lim_{h \rightarrow 0} \frac{x(t+h) - x(t)}{h} = \lim_{h \rightarrow 0} \frac{x(t+k+h) - x(t+k)}{h}$$

$$y(t+k) = \lim_{h \rightarrow 0} \frac{x(t+k+h) - x(t+k)}{h} = y(t) \Rightarrow \text{time-invariant}$$

$$3) x'(t) = \alpha x_1(t) + \beta x_2(t) \Rightarrow y(t) = \frac{dx(t)}{dt} = \frac{d}{dt} (\alpha x_1(t) + \beta x_2(t)) = \alpha \frac{dx_1(t)}{dt} + \beta \frac{dx_2(t)}{dt}$$

$$\alpha y_1(t) + \beta y_2(t) = \alpha \frac{dx_1(t)}{dt} + \beta \frac{dx_2(t)}{dt} = y(t) \Rightarrow \text{linear}$$

$$4) \exists y(t) \Leftrightarrow \exists \lim_{h \rightarrow 0} \frac{x(t+h) - x(t)}{h} \Leftrightarrow \lim_{h \rightarrow 0} \frac{x(t+h) - x(t)}{h} = \lim_{h \rightarrow 0^+} \frac{x(t+h) - x(t)}{h} = y(t) \Rightarrow$$

$$y(t) = \lim_{h \rightarrow 0} \frac{x(t+h) - x(t)}{h}, \text{ where}$$

$y(t)$ depends on $x(t+h) - x(t)$, where $t_0 + h \neq t$, $t_0 \neq t_0 \Rightarrow$ causal

$$5) \text{ let } x(t) = \begin{cases} \sqrt[3]{t} & |t| \leq 1 \\ 0 & \text{otherwise} \end{cases} \forall t$$

$$\frac{dx(t)}{dt} = \lim_{h \rightarrow 0} \frac{x(t+h) - x(t)}{h} = \lim_{h \rightarrow 0} \frac{|t+h|^{1/3} - |t|^{1/3}}{h}$$

$$\text{Let } x(t) = \begin{cases} \sqrt[3]{t} & |t| \leq 1 \\ 0 & \text{otherwise} \end{cases} \Rightarrow x(t) \leq 1 \forall t \in \mathbb{R}$$

$$x(0) = \sqrt[3]{0}$$

$$y(0) = \frac{dx}{dt}(0) = \lim_{h \rightarrow 0} \frac{x(0+h) - x(0)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt[3]{h} - 0}{h} = \lim_{h \rightarrow 0} \frac{1}{\sqrt[3]{h^2}} = +\infty \nexists B \forall B \in \mathbb{R} \Rightarrow$$

not stable