1.25. (3)

e) $x(t) = \mathcal{E}v\{\sin(4\pi t)\cdot u(t)\} = \dfrac{\sin(4\pi t)\cdot u(t) + \sin(-4\pi t)\cdot u(-t)}{2} = \begin{cases} \sin(4\pi t) & \text{if } t \geq 0 \\ -\sin(4\pi t) & \text{if } t < 0 \end{cases}$

$$x(t) = x(t+T) \Leftrightarrow \begin{cases} \sin(4\pi t) & \text{if } t \geq 0 \\ -\sin(4\pi t) & \text{if } t < 0 \end{cases} = \begin{cases} \sin(4\pi(t+T)) & \text{if } t+T \geq 0 \\ -\sin(4\pi(t+T)) & \text{if } t+T < 0 \end{cases} \Leftrightarrow$$

$\Leftrightarrow$

Let $t_0 = -\frac{1}{8}$:

$$x(t_0) = x(t_0+T) \Leftrightarrow -\sin(4\pi\cdot\tfrac{1}{8}) = \begin{cases} \sin(4\pi t + 4\pi T) & \text{if } t \geq -T \Leftrightarrow T \geq \frac{1}{8} \\ +\sin(4\pi(+\frac{1}{8} - T)) & \text{if } t < -T \Leftrightarrow T < \frac{1}{8} \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \frac{\pi}{2} \pm \left(\frac{\pi}{2} - \frac{\pi}{2}\right) = \begin{cases} \frac{\pi}{2} \pm \left(\frac{\pi}{2} + \frac{\pi}{2} - 4\pi T\right) + 2\pi n, & T \geq \frac{1}{8} \\ \frac{\pi}{2} \pm \left(\frac{\pi}{2} - \frac{\pi}{2} + 4\pi T\right) + 2\pi n, & T \leq \frac{1}{8} \end{cases} \Leftrightarrow 0 = \begin{cases} \pi - 4\pi T + 2\pi n, & T \geq \frac{1}{8} \\ 4\pi T + 2\pi n, & T \leq \frac{1}{8} \end{cases} \Leftrightarrow$$

$$\Leftrightarrow 2\pi n = \begin{cases} \pi - 4\pi T, & T \geq \frac{1}{8} \\ 4\pi T, & T \leq \frac{1}{8} \end{cases} \Rightarrow \nexists T_0 : x(t_0) = x(t_0 + T_0 k)\ \forall k \in \mathbb{Z} \Rightarrow \boxed{\text{not periodic}}$$

f) $x(t) = \sum_{n=-\infty}^{\infty} e^{-(2t-n)} \cdot u(2t-n) = \dots + e^{-(2t+1)} u(2t+1) + e^{-2t}\cdot u(t) + e^{-(2t-1)}\cdot u(2t-1) + \dots$

Let $t_0 = \frac{1}{2}$:

$1-n \geq 0 \Leftrightarrow n \leq 1$ $\quad$ $1+2T-n \geq 0 \Leftrightarrow n \leq 1+2T$ $\quad$ $1-n=m$, $n \to 1 \Rightarrow m \to 1-1=0$, $n \to -\infty \Rightarrow m \to 1+\infty = \infty$

$$x(t_0) = x(t_0+T) \Leftrightarrow \sum_{n=-\infty}^{\infty} e^{-(1-n)} \cdot u(1-n) = \sum_{n=-\infty}^{\infty} e^{-(1+2T-n)} \cdot u(1+2T-n) \Leftrightarrow \sum_{n=-\infty}^{1} \frac{1}{e^{1-n}} = \sum_{n=-\infty}^{1+2T} \frac{1}{e^{1+2T-n}} \Leftrightarrow$$

$1-n = m \Leftrightarrow n = 1-m$, $n \leq 1+2T \Leftrightarrow 1-m \leq 1+2T \Leftrightarrow m \geq 1+2T$

$$\Leftrightarrow \sum_{m=0}^{\infty} \frac{1}{e^m} = \sum_{m\geq -2T}^{\infty} \frac{1}{e^m} = \sum_{m\geq -2T}^{-1} \frac{1}{e^m} + \sum_{m=0}^{\infty} \frac{1}{e^m} \Leftrightarrow 0 = \sum_{m\geq -2T}^{-1} \frac{1}{e^m} \Leftrightarrow -2T \geq -1-1 = -2 \Leftrightarrow T \leq 1 \leftarrow \text{restricted} \Rightarrow$$

$\Rightarrow \boxed{\text{not periodic}}$