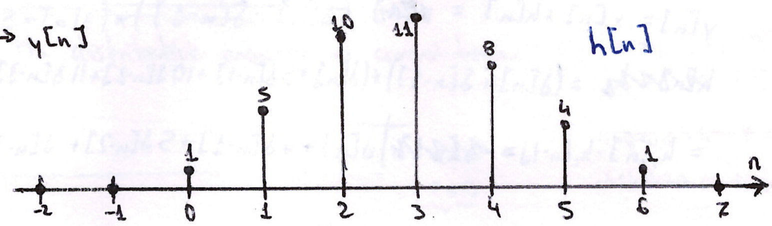
~~2.24.~~

2.24. Consider the cascade connection of three causal LTI systems as shown, where $h_2[n] = u[n] - u[n-2]$ and the overall impulse response is as depicted in the plot.

$$x[n] \rightarrow \boxed{h_1[n]} \rightarrow \boxed{h_2[n]} \rightarrow \boxed{h_2[n]} \rightarrow y[n]$$



a) Find the impulse response $h_1[n]$

$h_2[n] = u[n] \cdot u[1-n]$

$h[n] = h_1[n] * h_2[n] * h_2[n] = h_1[n] * (h_2[n] * h_2[n])$; $h_2[n]*h_2[n] = \cancel{h_1[n]*} \left(\sum_{k=-\infty}^{\infty} h_2[k]\, h_2[n-k]\right) =$

$= \cancel{h_1[n]*} \left(\sum_{k=-\infty}^{\infty} u[k]\cdot u[1-k]\cdot u[n-k]\cdot u[1-n+k]\right) = \cancel{h_1[n]*} \left(\sum_{k=0}^{1} u[n-k]\, u[1-n+k]\right) =$

$\cancel{= u[n-0]\cdot u[1-n+0] + \underbrace{u[n-1]}_{n\geq 1}\cdot \underbrace{u[3-n]}_{n\leq 3} + \underbrace{u[n-2]}_{n\geq 2}\cdot \underbrace{u[4-n]}_{n\leq 4}}$

$= (u[n]\cdot u[1-n]) + (u[n-1]\cdot u[2-n]) = (\delta[n] + \delta[n-1]) + (\delta[n-1] + \delta[n-2]) =$

$= \delta[n] + 2\delta[n-1] + \delta[n-2]$

$h[n] = h_1[n] * (\delta[n] + 2\delta[n-1] + \delta[n-2]) = h_1[n] + 2h_1[n-1] + h_1[n-2] \Rightarrow$

~~Causal ⇒ $x[n] = 0\ \forall\ n<0$ implies~~ $\Rightarrow h_1[n] = h[n] - 2h_1[n-1] - h[n-2]$

$h_1[n] = 0 \quad \forall \quad n<0$

$h_1[0] = h[0] - 2h_1[-1] - h_1[-2] = 1+0+0 = 1$

$h_1[1] = h[1] - 2h_1[0] - h_1[-1] = 5-2\cdot 1-0 = 3$

$h_1[2] = h[2] - 2h_1[1] - h_1[0] = 10-2\cdot 3-1 = 3$

$h_1[3] = h[3] - 2h_1[2] - h_1[1] = 11-2\cdot 3-3 = 2$

$h_1[4] = h[4] - 2h_1[3] - h_1[2] = 8-2\cdot 2-3 = 1$

$h_1[5] = h[5] - 2h_1[4] - h_1[3] = 4-2\cdot 1-2 = 0$

$h_1[6] = h[6] - 2h_1[5] - h_1[4] = 1-2\cdot 0-1 = 0$

$h_1[7] = h[7] - 2h_1[6] - h_1[5] = 0-2\cdot 0-0 = 0$

$h_1[n] = 0 \quad \forall \quad n>7$

With all this data: $\boxed{h_1[n] = \delta[n] + 3\delta[n-1] + 3\delta[n-2] + 2\delta[n-3] + \delta[n-4]}$