30.

1. Determine if each of the following systems is invertible. If it is, construct the inverse. If not, find two input signals with the same output.

a) $y(t) = x(t-4)$

$x(t) \longmapsto y(t) = x(t-4)$

$y(t) \longmapsto x(t) =$

$x(t) \xrightarrow{S} y(t) = x(t-4)$

$y(t) = x(t-4) \xrightarrow{S^{-1}} x(t) = x(t-4+4) = y(t+4)$

$y(t) \xrightarrow{S^{-1}} x(t) = y(t+4)$ (Invertible)

b) $y(t) = \cos(x(t))$

let $x_1(t) = 0$ ; $x_2(t) = 2\pi \neq x_1(t)$

$y_1(t) = \cos(x_1(t)) = 1$

$y_2(t) = \cos(x_2(t)) = 1 = y_1(t) \Rightarrow$ non invertible

c) $y[n] = n x[n]$

~~let $x_1[n] = u[n]$ , $x_2[n] = u[n-1]$~~

let $x_2[n] = u[n]$; $x_2[n] = u[n-1] \neq x_1[n]$

$y_1[n] = n x_1[n] = \begin{cases} n x_1[n] & \text{if } n \neq 0 \\ 0 & \text{if } n = 0 \end{cases} = \begin{cases} n \cdot u[n] & \text{if } n \neq 0 \\ 0 & \text{if } n = 0 \end{cases} = n \cdot u[n-1]$

$y_2[n] = n x_2[n] = \begin{cases} n \cdot u[n-1] & \text{if } n \neq 0 \\ 0 & \text{if } n = 0 \end{cases} = n \cdot u[n-1] = y_1[n] \Rightarrow$ not invertible

$\begin{cases} n & \text{if } (n \neq 0 \text{ and } n \geq 0) \\ 0 & \text{otherwise} \end{cases} = \begin{cases} n & \text{if } n > 0 \\ 0 & \text{otherwise} \end{cases}$

d) $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

$x(t) \xrightarrow{S} y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau \Leftrightarrow \frac{dy}{dt}(t) = \frac{d}{dt}\left(\int_{-\infty}^{t} x(\tau)\, d\tau\right) = \frac{d}{dt}\left(\int_{-\infty}^{a} x(\tau)\, d\tau + \int_{a}^{t} x(\tau)\, d\tau\right) = \frac{d}{dt}\int_{a}^{t} x(\tau)\, d\tau = x(t)$

Invertible: $y(t) \xrightarrow{S^{-1}} x(t) = \frac{d}{dt} y(t)$