1.43.

a) Consider a time-invariant system with input $x(t)$ and output $y(t)$. Show that if $x(t)$ is periodic with period T, then so is $y(t)$. Show that it also holds in discrete time.

Periodic $x(t)$ with period $T \Rightarrow x(t) = x(t+T)$

Time invariant $\Rightarrow$ For $x'(t) = x(t+k) \longrightarrow y'(t) = y(t+k)$ (with $x(t) \rightarrow y(t)$)

$x(t+T) \longrightarrow y(t+T)$

$x(t) \longrightarrow y(t)$

$x(t+T) = x(t) \underset{\text{Deterministic system}}{\Rightarrow} y(t) = y(t+T) \Rightarrow y(t)$ periodic with period T

$x[n+N] = x[n]$

$x[n] \longrightarrow y[n]$

$x[n+N] \underset{\text{Time invariant}}{\longrightarrow} y[n+N]$

$\Rightarrow y[n] = y[n+N] \Rightarrow y[n]$ periodic with period N

b) Give an example of a time-invariant system and a non-periodic input signal $x(t)$ such that its corresponding output $y(t)$ is periodic.

One such example is the system S: $x(t) \longrightarrow y(t) = \cos(x(t))$

- $x'(t) = x(t+k) \longrightarrow y'(t) = \cos(x'(t)) = \cos(x(t+k))$
  $y(t+k) = \cos(x(t+k)) = y'(t) \Rightarrow$ ~~time~~ time invariant

Let the ~~signa~~ non-periodic signal $x(t) = t$ be the input to this system.

Proof that it isn't periodic:

$x(t) = x(t+T) \Leftrightarrow t = t+T \Leftrightarrow T = 0 \} \Rightarrow$ non periodic

The output signal is $y(t) = \cos(x(t)) = \cos(t)$, which is known to be periodic with a period of $T = 2\pi$:

$y(t) = y(t+T) \Leftrightarrow \cos(x(t)) = \cos(x(t+T)) \Leftrightarrow \pm t = \pm(t+T) + 2\pi k$, $k \in \mathbb{Z} \Leftrightarrow$

$= \begin{cases} t - t - T = 2\pi k, & k \in \mathbb{Z} \\ \text{or} \\ t + t + T = 2\pi k, & k \in \mathbb{Z} \end{cases} \Leftrightarrow \begin{cases} T = 2\pi k, & k \in \mathbb{Z} \leftarrow T \in \{2\pi k, k \in \mathbb{Z}\} \\ T = 2\pi k - 2t, & k \in \mathbb{Z} \leftarrow \text{not independent of } t \end{cases} \Rightarrow$

$\Rightarrow$ fundamental period $T_0 = 2\pi$