17. A system may or may not be:

1) Memoryless
2) Time invariant
3) Linear
4) Causal
5) Stable

Determine which properties hold for the following systems. Justify your answers.

a) $y(t) = x(t-2) + x(2-t)$

$T(\alpha x_1 + \beta x_2) = \alpha T(x_1) + \beta T(x_2)$

1) $y(t_0)$ depends on $x(t_0-2)$, where $t_0-2 < t_0$ $\Rightarrow$ not memoryless

2) ~~$y(t) = x(t) \Rightarrow y(t-k) = x(t-k)$~~

$x'(t) = x(t-k) \Rightarrow y'(t) = x'(t-2) + x'(2-t) = x(t-2-k) + x(2-t-k)$

$y(t-k) = x(t-k-2) + x(2-(t-k)) = x(t-2-k) + x(2-t+k) \neq y'(t)$ $\Rightarrow$ not time-invariant

3) ~~$x_1(t) + x_2(t) = x_3(t)$~~ $x'(t) = \alpha x_1(t) + \beta x_2(t) \Rightarrow y'(t) = x(t-2) + x'(2-t) = (\alpha x_1(t-2) + \beta x_2(t-2)) + (\alpha x_1(2-t) + \beta x_1(2-t))$

~~$\alpha x_1(t) + \beta x_2(t) = x_3(t)$~~ $\alpha y_1(t) + \beta y_2(t) = \alpha x_1(t-2) + \alpha x_1(2-t) + \beta x_2(t-2) + \beta x_2(2-t) = y'(t) \Rightarrow$ linear

4) $y(t)$ depends on $x(2-t_0)$, where $2-t_0$ may be greater than $t_0$ $\Rightarrow$ not causal

5) ~~$x(t)$ bounded $\forall$~~ $|x(t)| \leq B \ \forall t$

$|y(t)| = |x(t-2) + x(2-t)| \leq B + B = 2B < \infty \ \forall t \Rightarrow$ Stable

b) $y(t) = \cos(3t) \cdot x(t)$

1) $y(t_0)$ doesn't depend on ~~any~~ any $x(t)$ with $t \neq t_0$ $\Rightarrow$ memoryless

2) $x'(t) = x(t-k) \Rightarrow y'(t) = \cos(3t)\, x'(t) = \cos(3t)\, x(t-k)$

$y(t-k) = \cos(3t-3k)\, x(t-k) \neq y'(t)$ $\Rightarrow$ not time-invariant

3) $x'(t) = \alpha x_1(t) + \beta x_2(t) \Rightarrow y'(t) = \cos(3t) \cdot x(t) = \cos(3t) \cdot (\alpha x_1(t) + \beta x_2(t)) = \alpha\cos(3t) x_1(t) + \beta\cos(3t) x_2(t)$

$\alpha y_1(t) + \beta y_2(t) = \alpha\cos(3t) x_1(t) + \beta\cos(3t) x_2(t) = x'(t) \Rightarrow$ linear

4) memoryless $\Rightarrow$ causal

5) ~~$|x| \leq B$~~ $|x(t)| \leq B \ \forall t$

$|y(t)| = |\cos(3t) x(t)| \leq |x(t)| \leq B \Rightarrow$ Stable