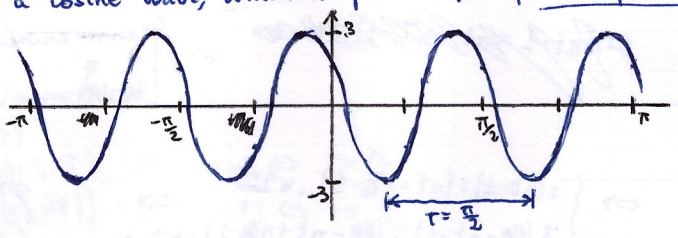
1.25. Determine whether or not each of the following continuous time signals is periodic. If the signal is periodic, determine its fundamental period.

a) $x(t) = 3\cos\left(4t + \frac{\pi}{3}\right)$

This signal is a linear transformation of a cosine wave, which is periodic, so it is periodic

$\omega = 4\pi$, $T = \frac{2\pi}{\omega}$ $\Rightarrow$ $T = \frac{2}{4}\pi = \frac{\pi}{2}$



c) $x(t) = \left[\cos\left(2t - \frac{\pi}{3}\right)\right]^2$ $\quad * k, n, m, c \in \mathbb{Z}$

Find T: $\cos^2\left(2t - \frac{\pi}{3}\right) = \cos^2\left(2(t+kT) - \frac{\pi}{3}\right) \Leftrightarrow \left|\cos\left(2t - \frac{\pi}{3}\right)\right| = \left|\cos\left(2(t+kT) - \frac{\pi}{3}\right)\right| \Leftrightarrow$

~~$\Leftrightarrow 2t - \frac{\pi}{3} = \pm\arccos\left(\cos\left(2t - \frac{\pi}{3}\right)\right) + 2\pi n = \pm\arccos\left(\cos\left(2(t+kT) - \frac{\pi}{3}\right)\right) + 2\pi m \Leftrightarrow$~~

~~$\Leftrightarrow \pm\left(2t - \frac{\pi}{3}\right) + 2\pi n = \pm\left(2t + 2kT - \frac{\pi}{3}\right) + 2\pi m \Leftrightarrow \pm\left(2t - \frac{\pi}{3}\right) = \pm\left(2t + 2kT - \frac{\pi}{3}\right) + 2\pi(m-n) \Leftrightarrow$~~

~~$\Leftrightarrow \pm\left(2t - \frac{\pi}{3}\right) \mp \left(2t - \frac{\pi}{3} + 2kT\right) = 2\pi \cdot c \Leftrightarrow \pm\left(2t - \frac{\pi}{3} - 2t + \frac{\pi}{3} - 2kT\right) = 2\pi c \Leftrightarrow -2kT = 2\pi c \Leftrightarrow$~~

~~$\Leftrightarrow \begin{cases} 2t - \frac{\pi}{3} - 2t + \frac{\pi}{3} + 2kT = 2\pi c \Leftrightarrow T = \pi \\ 2t - \frac{\pi}{3} + 2t - \frac{\pi}{3} + 2kT = 2\pi c \Leftrightarrow 4t - \frac{2}{3}\pi + 2kT = 2\pi c \end{cases}$~~

~~$\Leftrightarrow -Tk = \pi c \Leftrightarrow T = \pi$~~

~~This is a periodic signal with period $T = \pi$~~

$\Leftrightarrow \begin{cases} \cos\left(2t - \frac{\pi}{3}\right) = \cos\left(2t + 2Tk - \frac{\pi}{3}\right) \\ \text{or} \\ \cos\left(2t - \frac{\pi}{3}\right) = -\cos\left(2t + 2Tk - \frac{\pi}{3}\right) = \cos\left(2t + 2Tk - \frac{\pi}{3} + \pi\right) = \cos\left(2t + 2Tk + \frac{2\pi}{3}\right) \end{cases} \Leftrightarrow \begin{cases} \pm\left(2t - \frac{\pi}{3}\right) + 2\pi n = \pm\left(2t + 2Tk - \frac{\pi}{3}\right) + 2\pi m \\ \text{or} \\ \pm\left(2t - \frac{\pi}{3}\right) + 2\pi n = \pm\left(2t + 2Tk + \frac{2\pi}{3}\right) + 2\pi m \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} \pm\left(2t - \frac{\pi}{3} - 2t + \frac{\pi}{3} - 2Tk\right) = 2\pi(m-n) = 2\pi c \\ \text{or} \\ \pm\left(2t - \frac{\pi}{3} - 2t - \frac{2\pi}{3} - 2Tk\right) = 2\pi(m-n) = 2\pi c \end{cases} \Leftrightarrow \begin{cases} \mp 2Tk = 2\pi c \\ \text{or} \\ \mp 2Tk - \pi = 2\pi c \end{cases}$ w.l.o.g. $c = -c$ $\Leftrightarrow \begin{cases} Tk = \pi c \\ \text{or} \\ Tk = \pi c + \frac{\pi}{2} \end{cases}$ w.l.o.g $\Leftrightarrow$ $T = \pi$ or $T = \pi + \frac{\pi}{2k}$

$\Leftrightarrow T = \frac{\pi c}{k}$ or $\frac{\pi c}{k} + \frac{\pi}{2k}$ $\quad Tk = \frac{\pi}{2} \cdot c \Leftrightarrow T = \frac{\pi}{2}$

This is a periodic signal with period $T = \frac{\pi}{2}$