

CHAPTER 2 PROBLEMS

ADVANCED ~~BASIC~~ PROBLEMS

40 41 42 43 44 45 46 47 48 49 50
 51 52 53 54 55 56 57 58 59 60

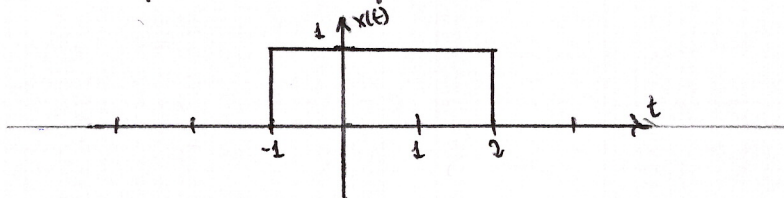
240.

a) Consider an LTI system with input and output related by $y(t) = \int_{-\infty}^t e^{-(t-\tau)} x(\tau-2) d\tau$.
 What is the input impulse response?

$$\int_{-\infty}^t x(\tau) e^{-(t-\tau)} d\tau = \int_{-\infty}^{t-2} x(\tau) e^{-(t-\tau-2)} d\tau = \int_{-\infty}^{\infty} x(\tau) e^{-(t-\tau-2)} u(t-\tau-2) d\tau \Rightarrow$$

$$\Rightarrow \boxed{h(t) = e^{2-t} u(t-2)}$$

b) Determine the response to this ~~output~~ signal.



$$x(t) = u(t+1) u(2-t)$$

$$y(t) = \int_{-\infty}^t e^{-(t-\tau)} u(\tau-2+1) u(2-\tau+2) d\tau = \begin{cases} 0 & \text{if } t \leq -1 \\ \int_{-1}^t e^{-t+\tau} d\tau & \text{if } -1 < t < 4 \\ \int_{-1}^4 e^{-t+\tau} d\tau & \text{if } 4 \leq t \end{cases}$$

$$= \begin{cases} 0 & \text{if } t \leq -1 \\ +e^{-t}(e^{\tau}) \Big|_{-1}^t & \text{if } -1 < t < 4 \\ +e^{-t}(e^{\tau}) \Big|_{-1}^4 & \text{if } 4 \leq t \end{cases} = \begin{cases} 0 & \text{if } t \leq -1 \\ +e^{-t+t} - e^{-t+1} & \text{if } -1 < t < 4 \\ +e^{-t+4} - e^{-t+1} & \text{if } 4 \leq t \end{cases}$$

$$= \begin{cases} 0 & \text{if } t \leq -1 \\ 1 - e^{1-t} & \text{if } -1 < t < 4 \\ e^{4-t} - e^{1-t} & \text{if } 4 \leq t \end{cases}$$