Note on Bayes' Rule for Monty Hall

Peter Alonzi

March 1, 2022

These notes are the full work for the Bayes' calculation of the Monty Hall Problem.

Assumptions

1. The player chooses door 1 ($D = 1$)

2. The car is behind door C, chosen from all possible doors $(1, 2, 3)$

3. Monty opens door H revealing not a car (only $H = 2$ is shown)

$$P(H = 2|C = 1) = \frac{1}{2} \tag{1}$$

$$P(H = 2|C = 2) = 0 \tag{2}$$

$$P(H = 2|C = 3) = 1 \tag{3}$$

Our strategy is to switch so to win we require $C = 3$ given that $H = 2$

$$P(C = 3|H = 2) = \frac{P(H = 2|C = 3) \times P(C = 3)}{P(H = 2)} = \frac{1 \times \frac{1}{3}}{\frac{1}{2}} = \frac{2}{3} \tag{4}$$

$$P(C = 1|H = 2) = \frac{P(H = 2|C = 1) \times P(C = 1)}{P(H = 2)} = \frac{\frac{1}{2} \times \frac{1}{3}}{\frac{1}{2}} = \frac{1}{3} \tag{5}$$