Controls Examples

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Abstract

This paper documents the formulation of the example controllers.

1 Dynamics Models

1.1 Hill-Clohessy-Wiltshire Equations

The linearized discrete-time Hill-Clohessy-Wiltshire dynamic equations are used as described in [1] and reprinted below:

2 Controllers

2.1 Linear Quadratic Regulator

The cost function is

$$V(x(0), u) = \sum_{k=0}^{N-1} [l(x(k), u(k))] + l_N(x(N))$$

Where l is the stage cost $l(x, u) = \frac{1}{2}(x^T Q x + u^T R u)$ for $k = 0, 1, ...N-1$. There is no action we can take at the final stage, so the terminal stage cost is $l_N(x(N)) = \frac{1}{2} x^T P_f x$. However, there is an action we can take at the penultimate state, or with horizon $N-1$. The problem to be solved at the penultimate stage is

$$\min_{u(N-1), x(N)} l(x(N-1), u(N-1)) l_N(x(N))$$

subject to the system dynamics

$$x(N) = Ax(N-1) + Bu(N-1)$$

2.2 Model Predictive Control with Constraints Tightening

Iskender [2] provides the original source for this controller.

References

[1] Christopher Michael Jewison. “Guidance and Control for Multi-stage Rendezvous and Docking Operations in the Presence of Uncertainty”. In: Massachusetts: Massachusetts Institute of Technology, 2017.

[2] Iskender O.B., Ling K.V., and Dubanchet V. “Constraints Tightening Approach Towards Model Predictive Control Based Rendezvous and Docking with Uncooperative Targets”. In: Limassol, Cyprus: IEEE, 2018.