Controls Examples

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Abstract

This paper documents the formulation of the example controllers.

1 Goals

Compare the performance of different controllers

- LQR
- LQT
- MPC

in different dynamic environments

- Linear HCW
- Nonlinear HCW

And in different situations

- Full state knowledge
- Noisy relative sensor estimate, but orbit radius of the client known
- Noisy relative sensor estimate and noisy estimate of orbit radius of the client (meaning the A and B matrices are time-varying)

Record metrics for each controller

- Computational cost (this needs a metric)
- Fuel consumption
- Stability
- Time to rendezvous
- Success of rendezvous in the presence of noisy sensor measurments

2 Dynamics Models

2.1 Hill-Clohessy-Wiltshire Equations

The linearized discrete-time Hill-Clohessy-Wiltshire dynamic equations are used as described in [1] and reprinted below: What are the orientation dynamics in space?

What other dynamics models are out there that handle J2 perturbations and longer time stretches? LTI HCW is not valid for long time stretches.

3 Controllers

3.1 Linear Quadratic Regulator

The cost function is

$$V(x(0), u) = \sum_{k=0}^{N-1} \frac{1}{2}[x_k^T Q x_k + u_k^T R u_k] + \frac{1}{2} x_N^T Q_f x_N$$

Where l is the stage cost $l(k) = \frac{1}{2}(x_k^T Q x_k + u_k^T R u_k)$ for $k = 0, 1, ...N-1$. There is no action we can take at the final stage, so the terminal stage cost is $l(N) = \frac{1}{2} x_N^T Q_f x_N$. However, there is an action we can take at the penultimate state, or with horizon $N-1$. The problem to be solved at the penultimate stage is

$$V_{N-1}^0(x_{N-1}, u_{N-1}) = \min_{u_{N-1}, x_N} \frac{1}{2}(x_{N-1}^T Q x_{N-1} + u_{N-1}^T R u_{N-1} + x_N^T Q_f x_N)$$

subject to the system dynamics

$$x_N = A x_{N-1} + B u_{N-1}$$

Where the optimal cost at the penultimate stage is V_{N-1}^0 and the optimal input is u_{N-1}^0. For the sake of compacting the notation, we re-write V_{N-1}^0 as

$$V_{N-1}^0 = \min_{u_{N-1}, x_N} \frac{1}{2}(\|x_{N-1}\|_Q^2 + \|u_{N-1}\|_R^2 + \|x_N\|_{Q_f}^2)$$

And replace x_N with the model

$$V_{N-1}^0 = \min_{u_{N-1}, x_N} \frac{1}{2}(\|x_{N-1}\|_Q^2 + \|u_{N-1}\|_R^2 + \|A x_{N-1} + B u_{N-1}\|_{Q_f}^2)$$

After some manipulation, we can re-write this as

$$V_{N-1}^0 = \min_{u_{N-1}, x_N} \frac{1}{2}(\|x_{N-1}\|_Q^2 + \|u_{N-1} - v\|_H^2 + d)$$

Where

$$H = R + B^T Q_f V$$

$$v = -(B^T Q_f B + R)^{-1} B^T Q_f A x_{N-1}$$
$$d = x_{N-1}^T [A^T Q_f A - A^T Q_f^T B (B^T Q_f B + R)^{-1} B^T Q_f A] x_{N-1}$$

The optimal u_{N-1}^0 is then

$$u_{N-1}^0 = v = -(B^T Q_f B + R)^{-1} B^T Q_f A x_{N-1}$$
$$u_{N-1}^0 = K_{N-1} x_{N-1}$$
$$K_{N-1} = -(B^T Q_f B + R)^{-1} B^T Q_f A$$
$$x_N^0 = (A + B K_{N-1}) x_{N-1}$$

And

$$V_{N-1}^0 = \min_{u_{N-1}, x_N} \frac{1}{2} (\|x_{N-1}\|_Q^2 + d)$$
$$V_{N-1}^0 = \frac{1}{2} x_{N-1}^T P_{N-1} x_{N-1}$$

And

$$P_{N-1} = Q + A^T Q_f A - A^T Q_f^T B (B^T Q_f B + R)^{-1} B^T Q_f A$$

Starting from

$$P_N = Q_f$$

At the next stage we solve the optimization

$$\min_{u_{N-1}, x_{N-1}} \frac{1}{2} (x_{N-2}^T Q x_{N-2} + u_{N-2}^T R u_{N-2} + V_{N-1}^0)$$

This problem is identical in structure so the solution is too

$$u_{N-2}^0 = K_{N-2} x_{N-2}$$
$$x_{N-1}^0 = (A + B K_{N-2}) x_{N-2}$$
$$V_{N-2}^0 = \frac{1}{2} x_{N-2}^T P_{N-2} x_{N-2}$$
$$K_{N-2} = -(B^T P_{N-1} B + R)^{-1} B^T P_{N-1} A$$
$$P_{N-2} = Q + A^T P_{N-1} A - A^T P_{N-1}^T B (B^T P_{N-1} B + R)^{-1} B^T P_{N-1} A$$

3.2 Model Predictive Control with Constraints Tightening

Iskender [2] provides the original source for this controller.

References

[1] Christopher Michael Jewison. "Guidance and Control for Multi-stage Rendezvous and Docking Operations in the Presence of Uncertainty". In: Massachusetts: Massachusetts Institute of Technology, 2017.

[2] Iskender O.B., Ling K.V., and Dubanchet V. "Constraints Tightening Approach Towards Model Predictive Control Based Rendezvous and Docking with Uncooperative Targets". In: Limassol, Cyprus: IEEE, 2018.