Assignment #3

This assignment is due on March 9th right BEFORE class starts – hand in your solutions in person to me.

This assignment includes non-Matlab questions, that is, questions which you should answer in **hand-written form (I will not accept computer print-outs!!!)**. These are usually small derivations of equations and problem solutions. Please write your answers to these questions on paper (try to write legibly ☺) and hand them to me before class. For your derivations, include **all necessary steps** that got you to the solution!

There is one Matlab-related question here: for this, I do require you to create a plot, print it out, and attach it to your assignment!

Part1 Taylor series. (10 points):

Note, that we had defined the Taylor approximation of order *n* of a function f(x) around x=*a* to be:

$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n,$ where $f^{(n)}$ denotes the n-th derivative of f(x).

a) Determine the Taylor expansion of order **n=3** for $f(x) = \frac{1}{x^2}$ at x=-1

b) Based on a), write the expansion for **arbitrary** n in a closed form with $f_{Taylor} = \Sigma$ ...

Part2 (20 points):

One of the common kinds of neurons are so-called integrate-and-fire neurons. After such a neuron has fired, its membrane potential V(t) (a voltage as function of time) returns to its so-called resting potential V_R with the following functional relationship:

$V(t) = V_R * (1 - \exp(-t/\tau))$

In the following, let V_R = -50 mV, and τ = 50ms.

a) What is V(t) at time t=0?

b) Show that V(t) satisfies a differential equation

$dV(t) / dt = 1 / \tau * (V_R - V(t))$

Note, that this is the equation from which V(t) was derived in the first place based on physical principles, that is, the change in membrane potential at time t (dV(t) / dt) is related to the membrane potential at that time (V(t)). Here you have solved the problem in reverse.

c) Calculate the total current flow across the membrane up to time t, which is simply given by the integral over the membrane potential from time t=0 to time t.

$I(t) = 1/R * \int_0^t (V(s) - V_R)\, ds$, where R = 5MΩ = 5 * 10^6 Volt / Ampere is the resistance of the membrane.

What is I(t)? How does the current behave, when time goes to infinity?

Hint: Think about how you can express ($V(s) - V_R$) in terms of dV(s) / ds. Using the trick $V(t) = \int_0^t (dV(s)/ds)\, ds$, you can actually solve this task **without integration**!

d) Calculate I(t) if V(t) is given instead by $V(t) = V_R (1 + 1 / (1+t^2))$.

Hint: $f'(t) = 1 / (1+t^2)$ for $f(t) = \text{atan}(t)$

Part3 Psychometric function (20 points):

In psychophysical experiments, the goal is derive a functional relationship between a stimulus dimension and a behavioral measure. A typical task is a detection task, in which the detection difficulty is varied by changing the stimulus parameter x. In this – and other tasks – the relationship between x and the probability that an observer correctly detects the stimulus can be modeled by a so-called logistic function:

$f(x) = 1 / (1+\exp(a*(b - x)))$. In this context, a is called the "slope-parameter" of the function, and b is referred to as the "offset".

a) Who coined the term Psychophysics? Use your Naver- or Google-skills to find out.

b) Show that the derivative of the logistic function f(x) is given by $f'(x) = a*f(x)*(1 - f(x))$.

c) **MATLAB**: Using $a = 2$ and $b = 0$, plot both f(x) and f'(x) into the same plot.

d) At which stimulus value is the observer with 50% correct? What is the value of the derivative at that point?

e) Find the line that approximates f(x) around the point $x_0 = b$. **MATLAB**: Add this line into the plot from c), **print out this plot** and attach it to your assignment.

f) At which point is f'(x) maximal? (**Don't forget to prove that it is a maximum!**)

Part4 Basis vectors (10 points):

Consider the vectors $\mathbf{v_1} = (\cos(a); -\sin(a))^T$ and $\mathbf{v_2} = (\sin(a); \cos(a))^T$, where a is an angle.

a) Show that ($\mathbf{v_1}, \mathbf{v_2}$) is an orthonormal basis, i.e. show that

$$<\mathbf{v_1}, \mathbf{v_2}> = 0, \text{ and } <\mathbf{v_1}, \mathbf{v_1}> = <\mathbf{v_2}, \mathbf{v_2}> = 1$$

b) Compute the formula for the projections y_1 and y_2 of an arbitrary vector $\mathbf{x} = (x_1, x_2)^T$ onto v_1 and v_2, respectively. What is $y_1 * \mathbf{v_1} + y_2 * \mathbf{v_2}$? (Note, that y_1; y_2 are of course numbers!)