$$\mathrm{rMAE}(FH) = \bar{k}^{-1}\frac{1}{N}\sum_{i=1}^{N}\left|k^{FH}(t_i) - k(t_i)\right|. \tag{5}$$

Following the method of Marquez and Coimbra (2012), we can approximate forecast skill s as

$$s(FH) \approx 1 - \frac{\mathrm{RMSE}(FH)}{\mathrm{RMSE}_p(FH)}, \tag{6}$$

where RMSE_p is the RMSE for a clear-sky persistence forecast, described in Section 4.2. To estimate the average skill over many days, the ratio $\frac{\mathrm{RMSE}}{\mathrm{RMSE}_p}$ is estimated by the slope of the regression fit of daily RMSE vs RMSE_p. The average skill is then $\langle s\rangle = 1 - \mathrm{slope}$. Examples of these plots and regressions are presented in Fig. 11.

4. Persistence forecasts

Persistence forecasts are the simplest type of forecast to implement and are often the most accurate at very short time horizons, making them a standard to compare with other methods. In this section we describe and compare the persistence forecasts we use for irradiance forecasting.

Before describing the various types of persistence, we first define the terminology we will use. The measured quantity of sensor n (e.g. irradiance) at time t will be denoted by $y_n(t)$. The forecast of sensor n at some time $t + FH$ in the future will be denoted by $y_n^*(t + FH)$. As mentioned in Section 3, we call FH the forecast horizon. The clear-sky expectation for a particular sensor will be denoted y_n^{clr} and the value of the clear-sky expectation at time t is $y_n^{clr}(t)$.

4.1. Measurement persistence

We call one of the simplest persistence methods "measurement persistence." A measurement persistence forecast simply assumes that the irradiance at a future time will be the same as it is at the current time. Measurement persistence is defined by

$$y_n^*(t + FH) = y_n(t). \tag{7}$$

This type of persistence is useful for short time horizons, but it does not account for the diurnal cycle of irradiance due to changing solar position and this leads to large errors at longer time horizons as shown in Fig. 5.

4.2. Clear-sky index persistence

In this method, the clear-sky index is calculated at the current time and persisted into the future. A forecast of irradiance is obtained by multiplying this clear-sky index by the value of the clear-sky expectation at the forecast time. The equation for clear-sky index persistence is

$$y_n^*(t + FH) = \frac{y_n^{clr}(t + FH)}{y_n^{clr}(t)} \times y_n(t). \tag{8}$$

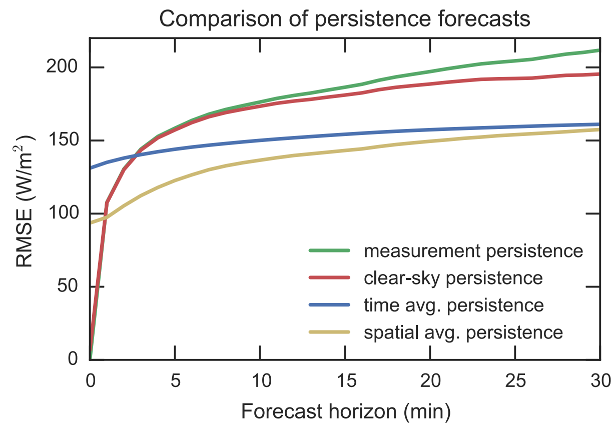

Fig. 5. Comparison of different types of persistence forecasts. RMSE, plotted as a function of forecast horizons, was computed for each type of forecast using data from the 46 cloudy days as described in Section 3. Spatially-averaged persistence has the lowest RMSE for all but the very shortest forecast horizons.

This method performs better than measurement persistence because it takes into account the diurnal cycle of irradiance, but it does require that a clear-sky expectation for the sensor, $y_n^{clr}(t)$, be known or modeled appropriately.

4.3. Time-averaged persistence

At time horizons greater than a few minutes, it can be beneficial to first average the measured clear-sky index over some time period defined by N time steps, each with period Δt, ending at some past time t_0. This average clear-sky index is then multiplied by the clear-sky expectation of the target sensor to compute a forecast. Time-averaged persistence is thus computed as

$$y_n^*(t + FH) = y_n^{clr}(t + FH) \times \frac{1}{N}\sum_{i=0}^{N-1}\frac{y_n(t - t_0 - i\Delta t)}{y_n^{clr}(t - t_0 - i\Delta t)}. \tag{9}$$

Often, a rolling averaged is used so $t_0 = 0$, Δt is the time resolution of the measured data, and N is chosen so $(N - 1)\Delta t$ gives the desired averaging time. The total averaging time does not limit the frequency with which forecasts can be made. For example, a 5 min rolling average persistence can recomputed every 1 min and still provide a useful forecast since new data is incorporated every time a forecast is made. An example of time-averaged persistence error with different averaging times using a rolling average is shown in Fig. 6.

4.4. Spatially-averaged persistence

If multiple measurements of irradiance are available in an area, one can make a persistence forecast based on the average clear-sky index of all the sensors. We refer to this method as spatially-averaged persistence. To make these forecasts, the measurements of multiple sensors are first converted to clear-sky indices using clear-sky expectations for each sensor. Then, these clear-sky indices are averaged