19

GROUP SYNCHRONIZING OF BINARY DIGITAL SYSTEMS

R. H. BARKER

Signals Research and Development Establishment, Christchurch England

SUMMARY

A SEQUENCE of binary digits has very little meaning unless the significance of the individual digits is known. If one digit can be labelled in some way others may be identified by counting. It is shown that transmission of a prearranged synchronizing pattern can perform such a labelling operation and guidance is given as to the length of pattern necessary under various specified conditions of operation. In particular, the probabilities are calculated of failure to synchronize and of false synchronizing when some of the digits are received wrongly by reason of noise or interference. Certain patterns are shown to be specially suitable for the purpose.

INTRODUCTION

All digital systems for the transmission of information depend to a greater or less extent for their efficiency upon correct synchronization. In general two distinct synchronizing operations must be performed. First of all there is digit synchronizing which can be said to establish equal time scales at the two ends of the link. In practice it means the generation at the receiver of a regular train of pulses having the same frequency as and a known phase relationship to a similar train of pulses at the transmitter. It enables a distorted signal to be regenerated with a low probability of error.

Secondly there is group synchronization which effectively pin-points an origin of time. If full advantage is to be taken of the binary codes, each digit of a group is differently weighted, and it is essential to be able to assign these weights correctly. It is equivalent to tying a label to one digit, after which any other may be related to it by counting. In a system where the number of digits in the group is small and constant, a convenient method of synchronizing is to provide circuits which can decide on a time average basis whether or not synchronization is correct. If not, the group phase at the receiver is altered by one digit, the process being repeated as many times as is necessary.

An alternative method—the one to be considered in more detail—is to transmit a special pattern of digits which is unambiguously recognized at the receiver. It is undoubtedly the better method when the group is a long one. In the extreme the whole message may constitute the group, in which case each message would be prefixed by the synchronizing pattern and the system would be of the start–stop variety. One might perhaps suppose that transmission of the first digit is an adequate indication of the beginning of a message, but a good receiver, intended to operate on weak and perhaps noisy signals, will do its best to generate binary digits on all occasions, and there may be no obvious way to distinguish between a true message and a random binary sequence produced by a noise only input. The present approach assumes that the synchronizing signal will consist of a pattern of digits which is sufficiently unlikely to occur by chance in a random sequence or in a previous message.

A logical starting point in assessing the necessary length of the synchronizing signal is to decide how much information is conveyed by the act of synchronizing. This quantity depends upon one's *a priori* knowledge of when the labelled digit is most likely to be received. For example, there may be an agreement between the operators concerned to commence working at 9.00 a.m. and to allow for errors in timing, a tolerance of $\pm$ 30 sec is permitted. If the digit rate is r per sec, then the labelled digit is one of a set of $60r$. It is of course most likely that this one will be nearer to the middle of the possible set than the edge, but to simplify the problem we shall for the moment assume that all positions of the set are equally likely. The synchronizing operation is then equivalent to specifying one of the $60r$ equally likely numbers and the information conveyed is

$$H = \log_2 m \text{ bits} \qquad \ldots .(1)$$

where m is the number of equally likely choices, in this case $60r$.

It might appear, therefore, that the necessary information could be conveyed by H binary digits. This is false, however, since it assumes that the digits have a weighted significance whereas at the receiver all digits must initially have equal significance.

PROBABILITY OF SYNCHRONIZING CORRECTLY

Suppose a synchronizing pattern n digits long is to be used to label one out of a possible choice of m digits. That is to say, the pattern may be sent so as to terminate at any one of m different instants. To identify the synchronizing instant correctly the receiver must be able to examine a total of $m + n - 1$ digits. If $n \ll m$ the probability of the pattern occurring by chance within a random set $m + n - 1$ is $(m + n - 1)2^{-n}$. When the pattern is received, therefore, the probability that it is actually being transmitted is $1 - (m + n - 1)2^{-n}$. It is possible to ensure that this is sufficiently near to unity for practical purposes, even though n may be very much smaller than m. When n is comparable with m the true expression becomes very complicated and does in fact depend upon the particular pattern chosen for synchronizing. Certainty is of course realized when $n = m$.

The situation is more complicated if the communication link is noisy so that the regenerated digits are not always correct. There is then no certainty of being able to recognize the synchronizing pattern when it is sent, and the receiver circuits must be designed to permit a suitable margin for errors.

The best that can be done is to determine the probabilities of the various alternatives.

P_1 = probability that synchronizing pattern is sent and is recognized;

P_2 = probability that synchronizing pattern is sent but not recognized;

P_3 = probability that synchronizing pattern is not sent but is recognized by chance coincidence;

P_4 = probability that synchronizing pattern is not sent and is not recognized.

The sum of these, $P_1 + P_2 + P_3 + P_4 = 1$.

The following symbols will be used in evaluating these probabilities:

S = the *a priori* probability that the synchronizing signal was sent;

h = probability of any digit being received in error;

n = the number of digits in the synchronizing pattern;

y = the number, out of the last n digits received, which agree with the expected synchronizing pattern;

$x = n - y$;

k = maximum number of errors permitted by the pattern recognizer;

$\varepsilon = nh$.

P_1: *Probability that pattern is sent and recognized*

If the pattern is known to be sent, the probability that it is received with x errors is $C_x^n\,(1-h)^{n-x}h^x$ and the probability of recognition is

$$\sum_{x=0}^{k} C_x^n\,(1-h)^{n-x}h^x$$

Hence

$$P_1 = S\sum_{x=0}^{k} C_x^n\,(1-h)^{n-x}h^x \qquad \ldots.(2)$$

P_2: *Probability that pattern is sent but not recognized*

If the pattern is known to be sent, the probability that it is received with more than k errors is

$$\sum_{x=k+1}^{n} C_x^n\,(1-h)^{n-x}h^x$$

$$= 1 - \sum_{x=0}^{k} C_x^n\,(1-h)^{n-x}h^x$$

Hence

$$P_2 = S\left[1 - \sum_{x=0}^{k} C_x^n\,(1-h)^{n-x}h^x\right] \qquad \ldots.(3)$$

When $h \ll 1$ the probability that the pattern, if sent, will be received with ust x errors is given approximately by the Poisson law:

$$p(x) = \frac{e^{-\varepsilon}\varepsilon^x}{x!} \qquad \text{where} \quad \varepsilon = nh$$

The probability that it is received with more than k errors is

$$\sum_{k+1}^{\infty} p(x) = \Pi(k+1, \varepsilon), \text{ say, so that}$$

$$P_2 = \Pi(k+1, \varepsilon) \qquad \ldots.(4)$$

and

$$P_1 = S[1 - \Pi(k+1, \varepsilon)] \qquad \ldots.(5)$$

The advantage of equations 4 and 5 is that the probability is now basically a function of two variables, k and ε, instead of three. Furthermore the Π function is to be found ready tabulated[1].

P_3: *Probability that pattern is recognized by chance*

Knowing that the pattern is not sent, there is still some chance of its being generated by a random series of digits. The probability of its occurring with x errors is $2^{-n}C_x^n$ and of its occurring with k errors or less is $2^{-n}\sum_{x=0}^{k} C_x^n$

Hence

$$P_3 = (1 - S)2^{-n}\sum_{x=0}^{k} C_x^n \qquad \ldots.(6)$$

P_4: *Probability that pattern was not sent or recognized*

This follows simply from equation 6

$$P_4 = (1 - S)\left[1 - 2^{-n}\sum_{x=0}^{k} C_x^n\right] \qquad \ldots.(7)$$

The situation which arises after the reception of each successive digit is as follows: Of the previous n digits, y agree with the synchronizing pattern and x do not. Is the last digit y received to be labelled as the synchronizing instant? We make the arbitrary choice to do so if $x \leq k$. The probability that this will happen is $P_1 + P_3$. If it does so happen, what is the probability that our interpretation is correct? This is the *a posteriori* probability that the synchronizing signal was sent. Of more direct interest is the probability that it was not sent, since this is the probability of a false synchronizing operation occurring. Let it be denoted by $F(k)$. By application of Bayes's theorem, we obtain

$$F(k) = \frac{P_3}{P_1 + P_3}$$

$$= \frac{(1 - S)2^{-n}\sum^{k} C_x^n}{S[1 - \Pi(k, \varepsilon)] + (1 - S)2^{-n}\sum^{k} C_x^n} \qquad \ldots.(8)$$

If a system is to work satisfactorily in practice, the conditions must be such that both the probability of a failure to synchronize to a signal which is sent, and the probability of synchronizing to a signal not sent are small. Both $F(k)$ and Π are small in these circumstances and equation 8 reduces to the approximate expression

$$SF(k) = 2^{-n}\sum_{x=0}^{k} C_x^n \qquad \ldots.(9)$$

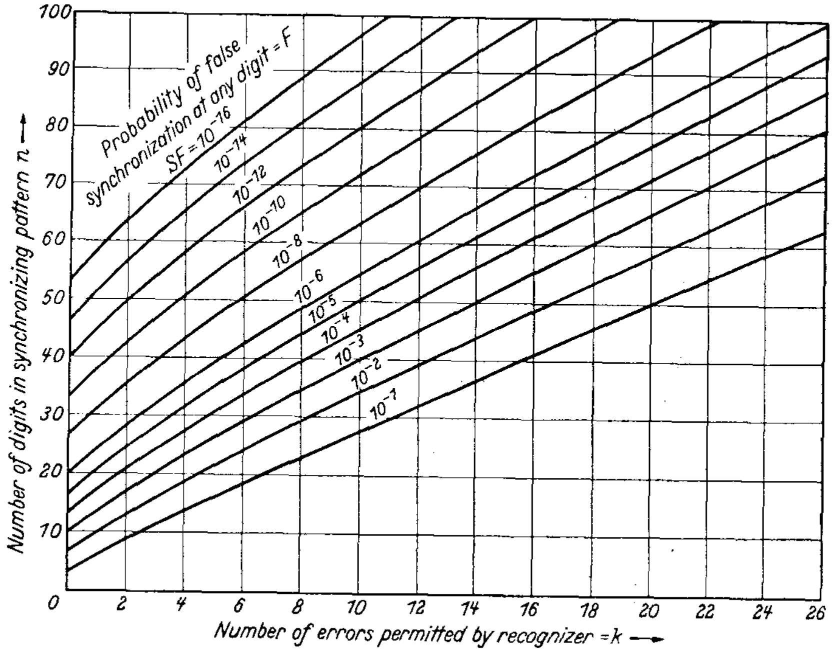

Figure 1

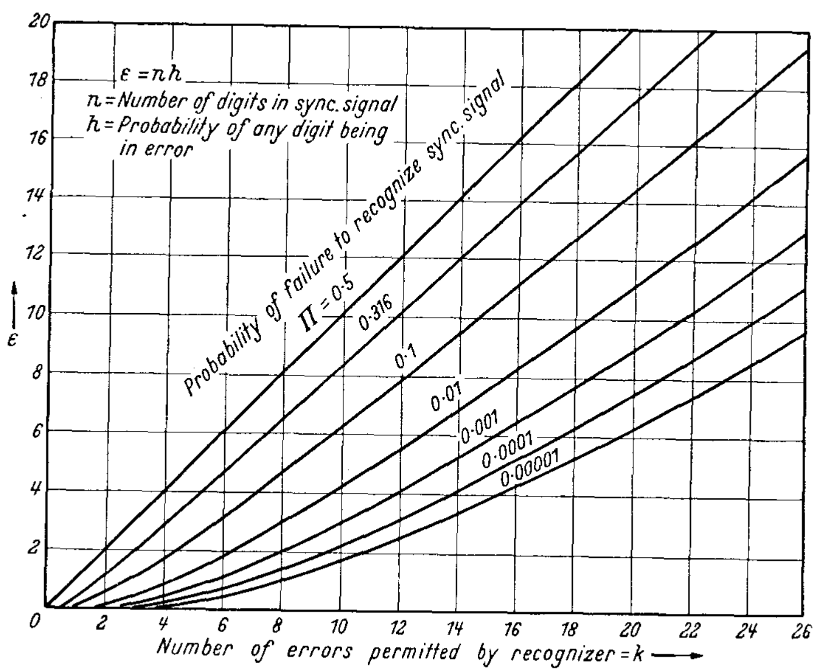

Figure 2

The right hand side is plotted in *Figure 1* for various values of k.

The probability of false synchronizing, $F(k)$, and the probability of a failure to synchronize to a signal that is actually sent are the most important factors to be considered. The latter, from equation 4, is simply Π. This is plotted in *Figure 2* as a function of k and ε.

Sufficient assumptions must be made concerning the conditions of operation of a system to enable upper limits to be set to $F(k)$ and Π. *Figures 1* and *2* and a little trial and error then enable suitable values of n and k to be determined. These are the important parameters of the synchronizing system. The longer the pattern is, the less the chance of its occurring out of a random sequence. The larger the margin of error, k, the less is the probability of a failure to recognize a synchronizing signal actually sent. A compromise must be made by balancing good performance against complexity of equipment.

Equation 9 shows that it is desirable for S to be as large as possible. It may be possible in some applications to know quite accurately when the synchronizing signal is to be expected. The probability that it is sent at that time is then quite high. For example on a 'Press to Transmit' system the first digits might be the synchronizing signal. Reception of the carrier would probably indicate to an accuracy of a fraction of a second when to expect the signal. Two examples are given.

Example 1

No. of digits per second $= 5 \times 10^5$

Uncertainty of synchronizing time $= \pm 0{\cdot}1$ sec

Hence $S = 10^{-5}$

Probability of any digit being in error $= h = 0{\cdot}05$

Desired probability of recognizing pattern when sent $= 0{\cdot}999$

$\Pi = 0{\cdot}001$

Maximum allowable probability of a false synchronizing $= F = 0{\cdot}001$ (say)

$SF = 10^{-8}$

A short trial and error process with *Figures 1* and *2* shows that approximately these probabilities are obtained when $n = 64$ and $k = 10$.

Example 2

No. of digits per second $= 28000$

Desired probability of recognizing pattern when sent $= 0{\cdot}99$

$\Pi = 0{\cdot}01$

The system is assumed to be operating continuously with random synchronizing. One false synchronization per hour is considered acceptable.

$$F(k) = \frac{1}{28 \times 10^3 \times 3600} \approx 10^{-8}$$

Suitable values of n and k, for different values of h are:

h	0·1	0·05	0·02	0·01	0
n	55	38	27	24	17
k	12	6	3	2	0

EQUIVOCATION

The equivocation, as defined by SHANNON, measures the uncertainty about the input when the output is known. In this application the output is known to the extent that y out of n digits agree with the synchronizing pattern and there is a corresponding uncertainty as to whether the synchronizing signal was sent or not. By arguments similar to those outlined above, we can calculate the *a posteriori* probability that the signal received with x errors is false. It is

$$F(x) = \frac{(1-S)2^{-n}}{(1-S)2^{-n} + Sh^{x}(1-h)^{n-x}} \qquad \ldots.(10)$$

The expectation of x is hn. Substituting this in equation 10 gives F_l, the most likely *a posteriori* probability of false synchronizing. With the simplifying assumption that $S \ll 1$, this is

$$F_l = \frac{1}{1 + Sh_0^n} \qquad \ldots.(11)$$

where

$$h_0 = 2h^h(1-h)^{1-h} \qquad \ldots.(12)$$

Since h_0 is never less than unity, increasing the number of digits, n, in the synchronizing pattern always decreases the probability of false synchronization, except in the special case of $h = 0{\cdot}5$, when no information can be transmitted.

The actual equivocation in any circumstance depends upon the amount of agreement between the digits actually received and those of the pattern expected, but equation 11 may be used to calculate the most likely value

$$R_l = -[F_l \log_2 F_l + (1-F_l)\log_2(1-F_l)] \qquad \ldots.(13)$$

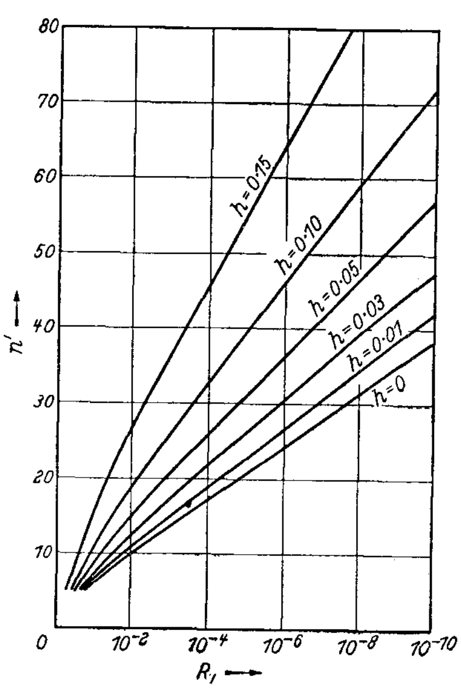

Figure 3

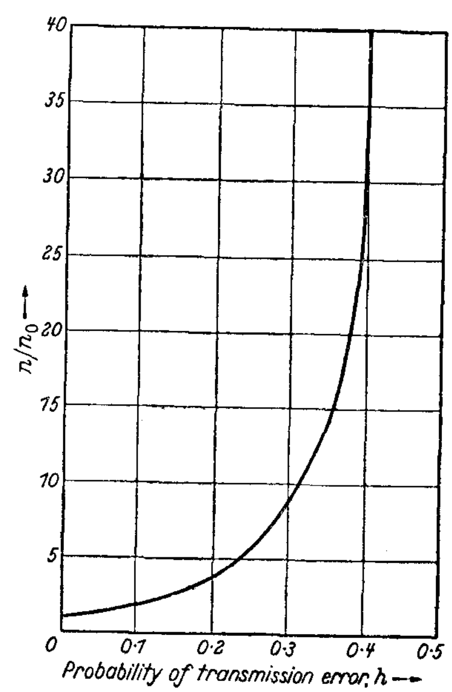

Figure 4

If F_l is small an approximate expression is

$$\log_{10} R_l = \log_{10} F_l + \log_{10} (1{\cdot}44 - 3{\cdot}32 \log_{10} F_l) \qquad \ldots.(14)$$

Figure 3 shows the way in which it depends upon h and n. The dependence upon the *a priori* probability of synchronizing, S, can be taken into account by a correction added to the ordinate. The actual number of digits, n, is given by

$$n = n' - c \log_{10} S \qquad \ldots.(15)$$

where $c = 1/\log_{10} h_0$ and is tabulated below.

h	0	0·01	0·03	0·05	0·10	0·15
c	3·32	3·61	4·15	4·90	6·25	8·77

Suppose that F_l, the most likely *a posteriori* probability of false synchronizing, and S the *a priori* probability of synchronizing are arbitrarily specified. The number of digits of the pattern can then be expressed in terms of the error rate. If $F_l \ll 1$ it follows from equation 11 that

$$\frac{n}{n_0} = \frac{1}{\log_2 h_0} \qquad \ldots.(16)$$

where n_0 is the value of n when there are no errors ($h = 0$). This function is plotted in *Figure 4* and shows the way in which the number of digits must be increased in order to convey the same amount of information in the presence of errors. The same relation, of course, holds good for the transmission of a message. There is, however, a big difference between the use of the redundancy in the two cases.

In message transmission it is theoretically possible by suitable encoding to utilize the redundancy so as to make the probability of correct interpretation as near to unity as may be desired. This encoding is accompanied by delay. The same amount of redundancy applied to the synchronizing operation results in an *a posteriori* probability of synchronizing, the most probable value of which is as given in equation 11. It is not possible by any rearrangement of the digits or other encoding to improve this probability.

For this reason the number of digits needed to secure a useful *a posteriori* probability of synchronizing increases more rapidly with transmission errors than would be the case for a message.

THE SYNCHRONIZING PATTERN

The arguments given above are concerned with the selection of a suitable length for the synchronizing pattern and providing $m \gg n$ they are not in the least affected by the nature of the pattern. A further important factor, however, is that there is a possibility that the random digits preceding the pattern will combine with the first part of the pattern to produce a synchronizing signal that is slightly too early. For example if the pattern is $++++++++++$ there is a 50 per cent chance of it being preceded by a $+$ with a consequent synchronizing error. The form of the pattern should be such that the probability of this type of error is minimized. The pattern will be represented by a sequence of digits each of which is $\pm$ unity. Digits preceding and following the pattern will be taken to have an equal probability of being plus or minus. Consider the method of pattern recognition illustrated in *Figure 5.* The incoming binary digits are fed into a chain of $n - 1$ delay units. The signals at the input of each unit and at the output from the last are added together with or without a change of phase, as indicated by the plus and minus signs. The arrangement of the plus and

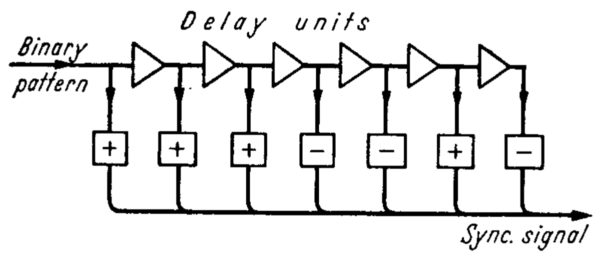

Figure 5. A simple pattern recognizer

minus signs used for this purpose is identical with the pattern used for synchronizing, but reversed in order. In the example of *Figure 5* the latter would be $-+--+++$.

Imagine that the single digit $+ 1$, preceded and followed by zeros, is fed into the recognizer. The output is the sequence

$$1, 1, 1, -1, 1, -1$$

so that the pulse transfer function[2] is

$$1 + z^{-1} + z^{-2} - z^{-3} - z^{-4} + z^{-5} - z^{-6}$$

A synchronizing signal timed to terminate at $t = 0$ can be represented by the sequence

$$- z^6 + z^5 - z^4 - z^3 + z^2 + z + 1$$

where z is the operator of the sequence transform.

The output from the recognizer with this as the input is the product of these two expressions, namely

$$- z^6 - z^4 - z^2 + 7 - z^{-2} - z^{-4} - z^{-6}$$

This is in fact the auto-correlation function of the pattern, and may be alternatively written as

$$-1, 0, -1, 0, -1, 0, 7, 0, -1, 0, -1, 0, -1$$

It is the output sequence obtained when the synchronizing signal is preceded and followed by zeros. In the binary system under consideration the two possible states are represented by $+ 1$ and $- 1$, so that this is impossible. The result does, however, have a useful meaning since random elements preceding and following cause deviations which are equally likely to be positive or negative. The output of $+ 7$ units is obtained when the pattern is exactly contained within the delay line store. The large pulse obtained at this instant is the label used for marking the instant of synchronization. It is not possible for the full amplitude to be reached by any random digits in combination with one or more of the pattern. This is illustrated in *Figure 6* in which the maximum possible amplitudes are shown around the synchronizing instant, together with the probabilities that they shall occur by chance. In this illustration the reception of the synchronizing digits has been assumed free from error. Any errors have the effect of decreasing the amplitude of the wanted pulse and of permitting an increase in that of unwanted ones.

It is possible to show that none of the 0 or $-$ 1 terms in the above auto-correlation sequence can be made more negative without some other of these

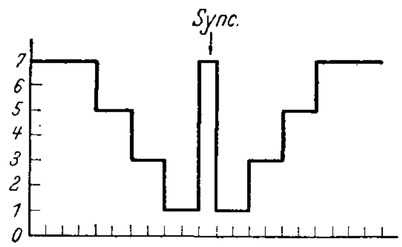

Figure 6a. Maximum possible output near Sync. digit

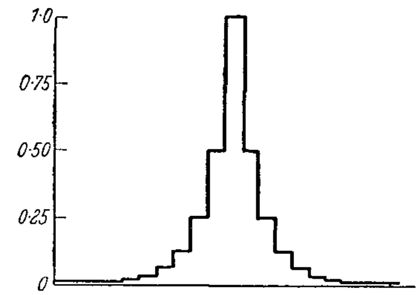

Figure 6b. Probability that maximum output is obtained

terms becoming more positive. The pattern suggested is in this sense an ideal one, and it would appear desirable to search for other patterns, preferably with more digits, having similar properties. The following conclusions have been reached as a result of attempts in this direction.

(1) The number of digits must be one less than an integral multiple of four.

(2) If digits are paired counting from both ends, the odd pairs must be different, even pairs alike.

(3) The simplest patterns are

$$n = 3 \quad + + -$$
$$n = 7 \quad + + + - - + -$$
$$n = 11 \quad + + + - - - + - - + -$$

(4) No longer patterns have been found and it is considered unlikely that they exist, although this has not been proved.

It is possible to obtain longer patterns which are very nearly ideal and many such may be found by trial and error. Patterns whose auto-correlation sequence terms do not exceed $+$ 1 (except at the centre) can, however, be constructed by combining ideal patterns in ideal groups. The meaning will be made clear by an example. The above pattern of three digits $(+ + -)$ can be written down twice in the same sense followed by once in the opposite sense, $+ + - + + - - - +$. This has the auto-correlation sequence

$$1, 0, -3, 0, 1, 0, -3, 0, 9, 0, -3, 0, 1, 0, -3, 0, 1$$

The other possible combinations using the above ideal patterns have the numbers of digits $n = 21, 33, 49, 77, 121$.

The instrumentation for the reception and recognition of these long sequences must necessarily be somewhat complicated. It may be accomplished by means of two delay lines, the first dealing with individual digits and the second with groups of digits. For the sequence of 21 digits

$$+++--+-+++--+-----++-+$$

the arrangement would be as in *Figure 7*.

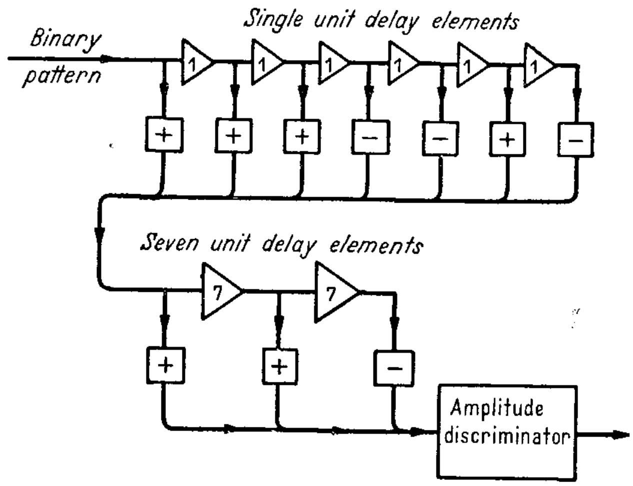

Figure 7. Recognizer for 21-digit pattern

TWO-STAGE SYNCHRONIZING

It may well be desirable to utilize patterns which fall short of those discussed so far, but which may be generated and recognized with less complicated equipment. Such patterns will have to be longer if the standard of performance is to be maintained. One method is to perform the synchronizing operation in two stages. The first stage is to label one digit of a repeated frame, and the second is to label one particular frame of the sequence.

Suppose the first stage consists of sending a repetition of an ideal pattern of n elements and that a recognition unit such as that in *Figure 5* is available. The output, if there are no errors, will be the sequence

$$------1, -1, n, -1, -1 \;---\; -1, n, -1, -1,$$
$$-----\; n, -1, ---, \textit{etc.}$$

in which the positive pulses of amplitude n are separated by $n - 1$ pulses of amplitude -1. This signal may be fed into a resonant system so that the repeated pulses build up to large amplitude. A simple device would be a delay line open at the end to yield an in-phase reflection delayed by n digits and fed from a high impedance circuit to give repeated reflections. If each reflection is attenuated by d relative to the initial input, the output pulses will build up in amplitude to yield finally

$$n(1 + d + d^2 + \ldots) = \frac{n}{1 - d}$$

if the summation is continued to infinity. Errors occur at random times and therefore do not add in phase.

The second stage, that of labelling one frame, can be undertaken when the ideal pattern has been repeated a sufficient number of times to build up a reasonable amplitude of pulse. A suitable method of labelling is to transmit a frame in which the sense (polarity) of the pattern is reversed. The regular sequence of pulses built up in the first stage may then be used to define the instants at which the output of the ideal pattern recognizer is tested for polarity. *Figure 8* shows a suitable arrangement.

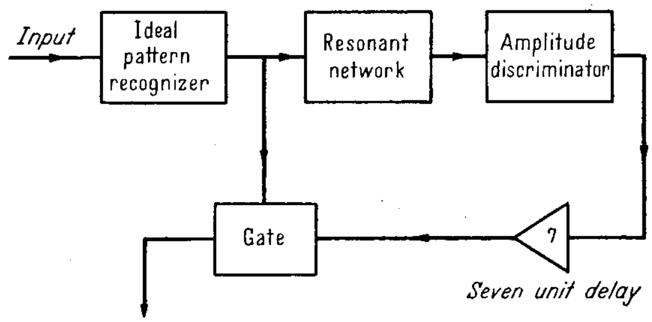

Figure 8. A two-stage recognizer

The operation may be understood more readily by reference to the waveforms of *Figure 9* based upon the use of an ideal pattern of seven digits.

At *Figure 9a* is the synchronizing signal consisting of the ideal 7-digit pattern repeated four times in the positive sense and once in the negative sense. The output of the recognizer is shown at *b* and the build up of the pulses in the resonant network at *c*. When the pulse amplitude has built up sufficiently to operate the amplitude discriminator, pulses are fed to the gate circuit with a delay of 7 digits. This delay is to ensure that a pulse is available to operate the gate when the sense of the pattern is reversed. If the decrement of the resonant network is sufficiently small the stored energy would serve this purpose and this unit could be dispensed with. The gate is of a type such that a pulse is generated by the combination of a negative pulse on its input terminal and a positive pulse on its control terminal.

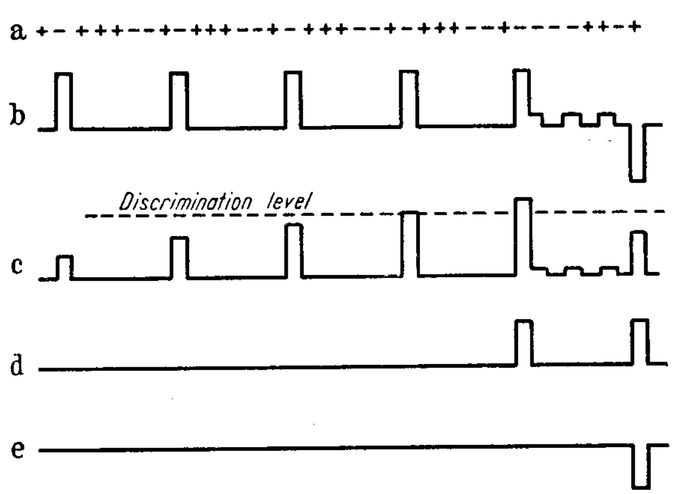

Figure 9. Two-stage recognizer waveforms

An important factor in the design of a synchronizer of this type is the response to a random sequence. Let the ideal pattern contain n digits. The output of the ideal pattern recognizer may be any of $n + 1$ levels with the probability distribution of the binomial coefficients C^n_x. This distribution approximates to the normal or gaussian distribution, particularly when n is large. After passing through a linear filter such as the resonant network, the amplitude distribution approximates much more closely to the normal. The standard deviation, or r.m.s. noise, at the output of the pattern recognizer is $\sigma = \sqrt{n}$ and the peak signal output is n. After the resonant network the peak signal output is $n/(1 - d)$ and the mean square noise is $\sigma^2/(1 - d^2)$. The signal–noise ratio is therefore improved by the factor $\sqrt{[(1 + d)/(1 - d)]}$, and for the same peak signal output the standard deviation of the noise is correspondingly reduced.

$$\frac{\text{Peak signal}}{\text{r.m.s. noise}} \approx \sqrt{\left[\frac{n(1 + d)}{1 - d}\right]}$$

and the probability that the noise builds up to exceed a fraction α of the peak signal is given approximately by the error function

$$\frac{1}{2}\,\text{erf}\sqrt{\left[\frac{n(1 + d)}{1 - d}\right]}$$

The approximation is very inaccurate when α is close to unity. The formula yields a finite probability of the noise building up to exceed the peak signal, which is, of course, physically impossible. A better approximation for this condition is probably to use the binomial distribution based upon an equivalent number of digits n_0 where $n_0 = n(1 + d)/(1 - d)$. This is the number of digits which would have to be used for a simple ideal pattern to give the same performance. *Figures 1* and *2* may be used and the value of k so obtained applied to determine the level of amplitude discrimination.

REPEATED SYNCHRONIZING

The length of the synchronizing pattern may be reduced if the *a priori* probability of synchronizing is increased. One method of doing this is to send the synchronizing pattern at regular intervals. The synchronizing signal is then only accepted as correct if it is received at the instant it is expected. This method is obviously not capable of yielding absolute synchronization, but only of marking repeatedly the beginning of a group. It is suitable for most types of time multiplex systems. *Figure 10* illustrates one way in which it may be applied.

A simple ideal pattern is inserted into the transmission at regular intervals of m digits. At the receiver an ideal pattern recognizer, such as in *Figure 5*, generates a pulse whenever this pattern is received, either because it was inserted or by chance. If no other pulse has been received for some time previously, it will pass through gate 1 and will reset the counter of modulus m. The action of resetting sends a pulse to a coincidence detector unit. Because the resetting was initiated by a pulse from the pattern recognizer, this unit will perceive coincidence and will trigger the flip-flop circuit so cutting off further counter resetting pulses. This condition is maintained until the flip-flop restores, which will happen after a time which may be controlled arbitrarily though not accurately. We will assume that this time corresponds to M digits, where $M > m$.

The counter restores automatically after m pulses. If synchronism is correct the pulse so generated will coincide with a pulse from the pattern recognizer and any interference with the counter is held up for another M digits. In other words, as long as the system stays synchronized no false pulses from the pattern recognizer can get through to upset it. If the system is not synchronized, each pulse from the recognizer is assumed to be a synchronizing pulse until subsequent lack of coincidence with the counter output pulse show it to be wrong.

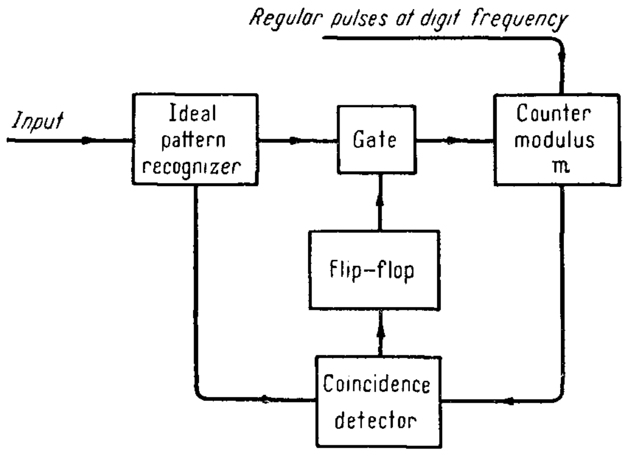

Figure 10. System for repeated synchronizing

It will be seen that the method bears some resemblance to the search method mentioned earlier, but it has the advantage that the search does not extend to all digits of the group. It includes only those in which the combination of n immediately preceding happens to be identical with the chosen pattern (or nearly so if errors are allowed for) and the chance of this is relatively small.

CONCLUSIONS

The minimum number of digits for the synchronizing pattern is a function of four variables:

(1) The *a priori* probability of synchronizing, S.
(2) The probability of a transmission error, h.
(3) The required probability of correct operation when the pattern is sent, $(1 - \Pi)$.
(4) The maximum allowable probability of false synchronization due to chance coincidence, F_l.

Graphs have been prepared which enable this number to be calculated, and also the best margin to allow for transmission errors. A study of the properties of various patterns as revealed by their auto-correlation sequences shows that a few particular patterns are specially suitable. Furthermore equipment for generating and identifying them need not be unduly complicated.

Although only binary systems have been considered it is apparent that these same 'ideal' patterns could advantageously be applied to the synchronization of multi-level systems and in fact to continuous systems in which a peak power limitation becomes effective before a mean power limitation.

ACKNOWLEDGEMENTS

Acknowledgement is made to the chief Scientist, Ministry of Supply and to the Controller of H.M. Stationery Office for permission to publish this paper.

REFERENCES

[1] FRY, T. C., *Probability and its Engineering Uses*, New York, 1928

[2] BARKER, R. H., "The Pulse Transfer Function and its Application to Sampling Servo Systems", *Inst. elect. Engrs Monograph* No. 43, July 1952.