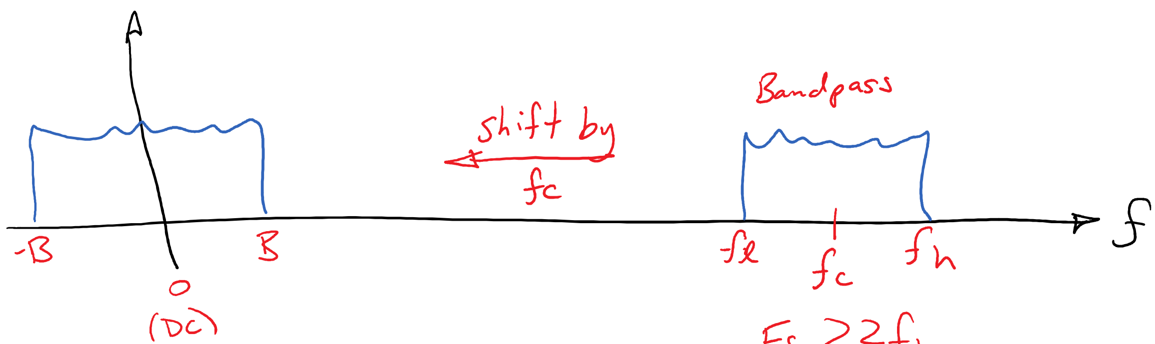
Complex Sampling (AKA Quadrature Sampling)

- Eliminates the negative "ghost" frequencies from a real bandpass signal
- Allows for baseband processing
  - significantly reduces the sampling rates required



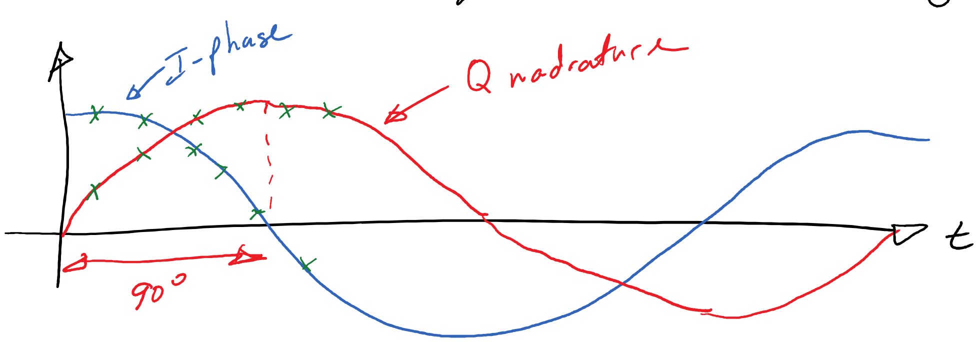
$F_s > 2f_h$

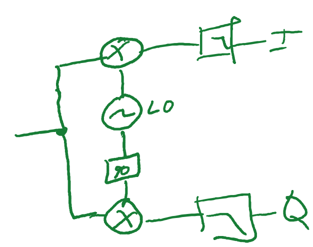
$F_s > 2B$

Nyquist ⇒ Sample at twice the bandwidth of interest

Quadrature (for DSP & SDR)

- refers to two signals that are 90° out of phase
- These signals become orthogonal



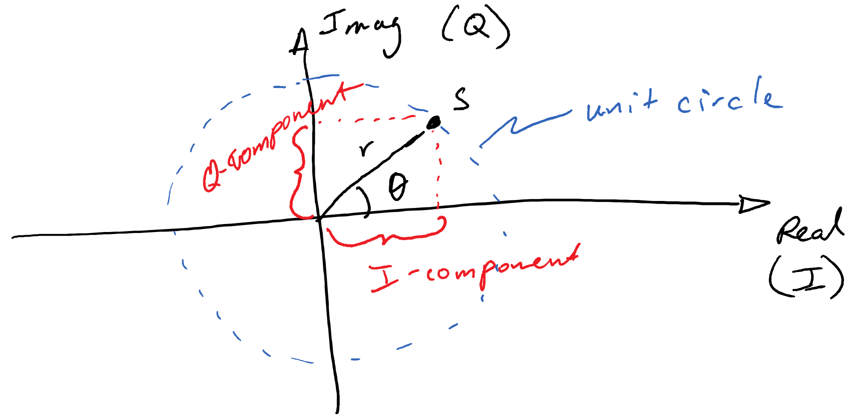


Recall $\sin(\theta + 90°) = \cos\theta$

Phasor Representation

Phase + Vector = Phasor

Complex number representing a sinusoidal function



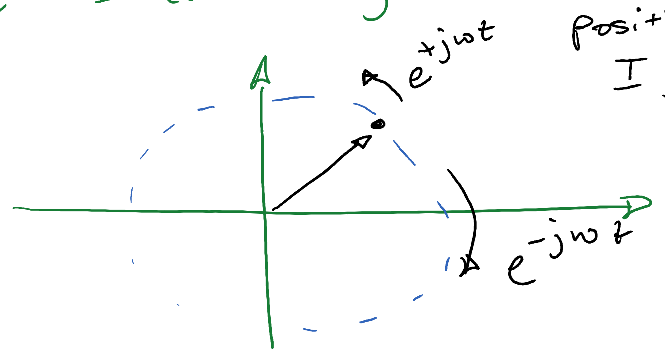
$I = \cos\theta$

$Q = \sin\theta$

$S(t) = I(t) + j\,Q(t)$

Recall Euler's Identity

$$e^{j\theta} = \cos\theta + j\sin\theta$$



positive frequency
I leads Q by 90°

Negative frequency
I lags Q by 90°

Rotational speed ~ signal frequency

Consider these alternate forms of Euler

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

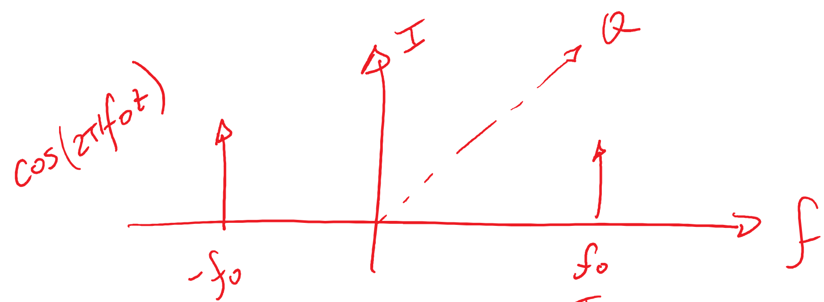
$$\begin{cases} \cos\theta = \dfrac{e + e}{2} \\ \sin\theta = \dfrac{e^{j\theta} - e^{-j\theta}}{2} \end{cases}$$

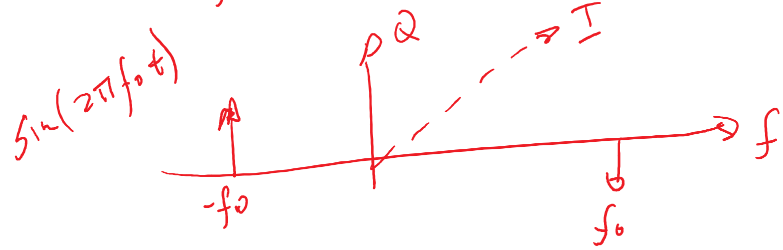
$$\cos(2\pi f_0 t) = \frac{e^{-j2\pi f_0 t}}{2} + \frac{e^{-j2\pi f_0 t}}{2}$$

negative frequency

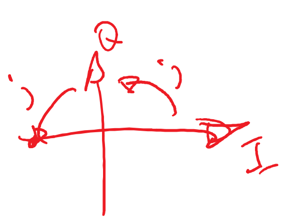
positive frequency

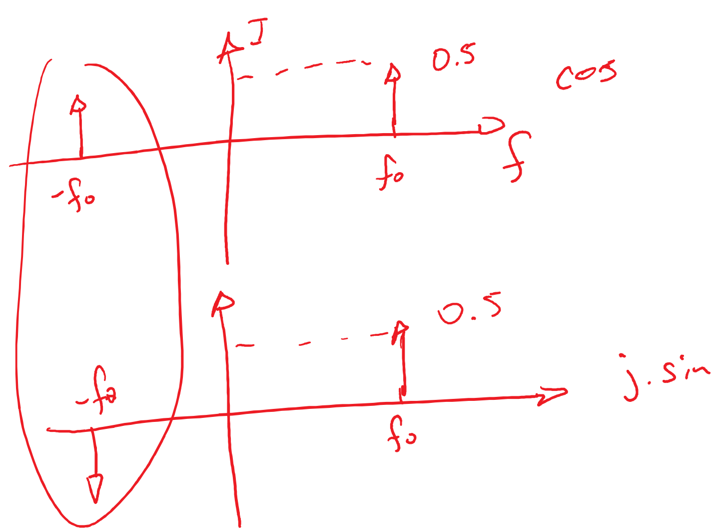
$$\sin(2\pi f_0 t) = \frac{j e^{-j2\pi f_0 t}}{2} - \frac{j e^{j2\pi f_0 t}}{2}$$

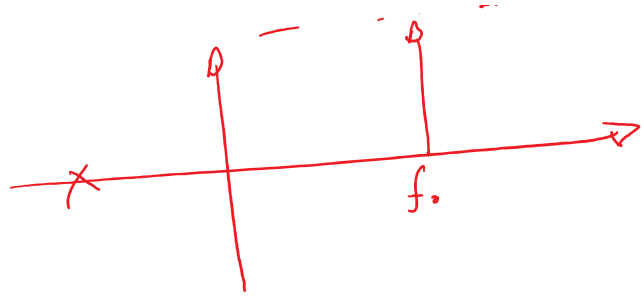




$$S(t) = I(t) + j\,Q(t)$$



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