Fundamentals of Information & Network Security
ECE 471/571

Lecture #16: Pseudorandom number generation

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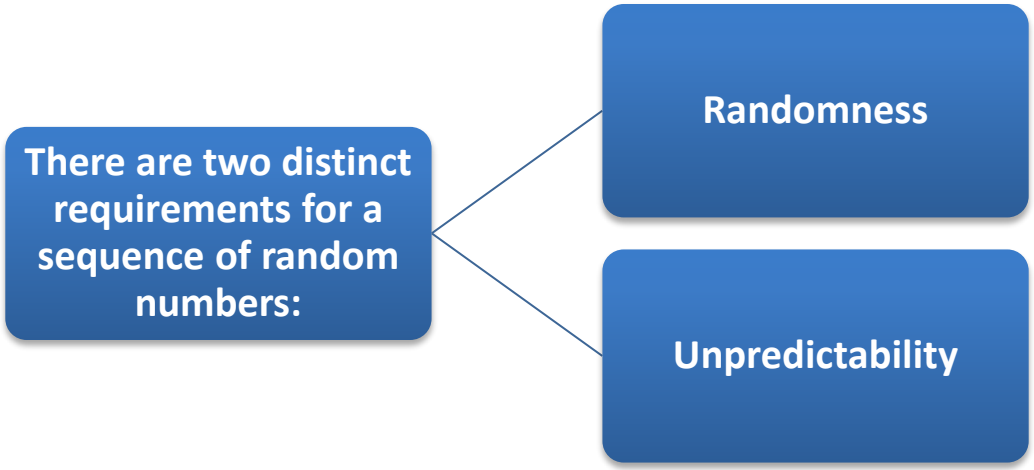
Random Numbers

- A number of network security algorithms and protocols based on cryptography make use of random binary numbers:
  - Key distribution and reciprocal authentication schemes
  - Session key generation
  - Generation of keys for the RSA public-key encryption algorithm
  - Generation of a bit stream for symmetric stream encryption



Randomness

- The generation of a sequence of allegedly random numbers being random in some well-defined statistical sense has been a concern

Two criteria are used to validate that a sequence of numbers is random:

Uniform distribution

- The frequency of occurrence of ones and zeros should be approximately equal

Independence

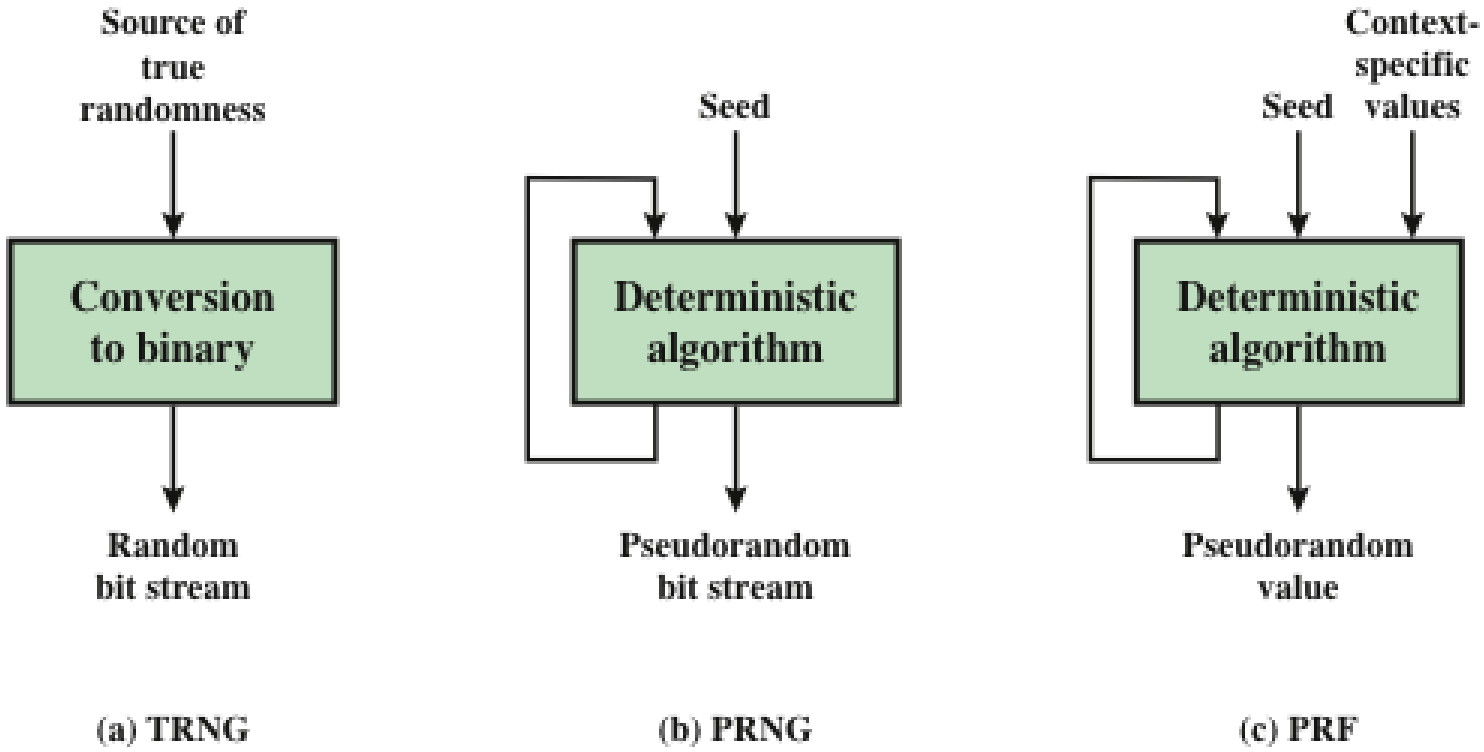
- No one subsequence in the sequence can be inferred from the others

Unpredictability

- The requirement is not just that the sequence of numbers be statistically random, but that the successive members of the sequence are unpredictable
- With “true” random sequences each number is statistically independent of other numbers in the sequence and therefore unpredictable
  - True random numbers have their limitations, such as inefficiency, so it is more common to implement algorithms that generate sequences of numbers that appear to be random
  - Care must be taken that an opponent not be able to predict future elements of the sequence on the basis of earlier elements

Pseudorandom Numbers

- Cryptographic applications typically make use of algorithmic techniques for random number generation
- These algorithms are deterministic and therefore produce sequences of numbers that are not statistically random
- If the algorithm is good, the resulting sequences will pass many tests of randomness and are referred to as *pseudorandom numbers*



TRNG = true random number generator
PRNG = pseudorandom number generator
PRF = pseudorandom function

Figure 8.1 Random and Pseudorandom Number Generators

True Random Number Generator (TRNG)

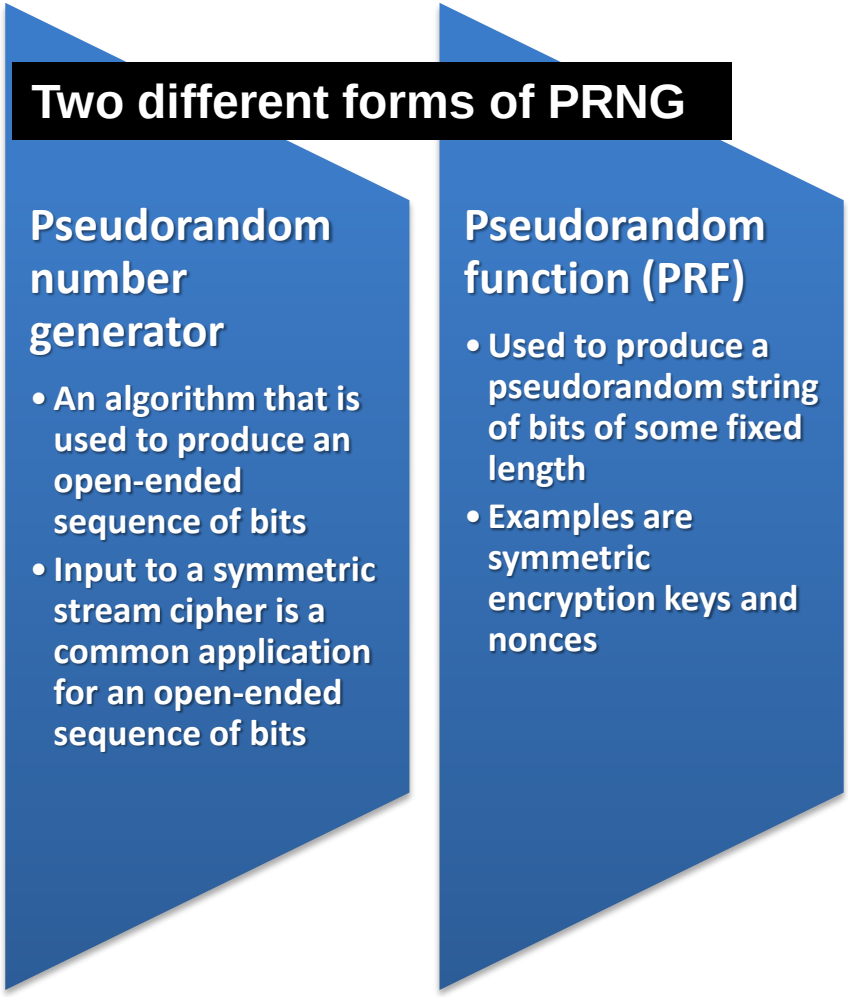
- Takes as input a source that is effectively random
- The source is referred to as an *entropy source* and is drawn from the physical environment of the computer
  - Includes things such as keystroke timing patterns, disk electrical activity, mouse movements, and instantaneous values of the system clock
  - The source, or combination of sources, serve as input to an algorithm that produces random binary output

- The TRNG may simply involve conversion of an analog source to a binary output

- The TRNG may involve additional processing to overcome any bias in the source

Pseudorandom Number Generator (PRNG)

- Takes as input a fixed value, called the *seed,* and produces a sequence of output bits using a deterministic algorithm
  - Quite often the seed is generated by a TRNG
- The output bit stream is determined solely by the input value or values, so an adversary who knows the algorithm and the seed can reproduce the entire bit stream
- Other than the number of bits produced there is no difference between a PRNG and a PRF

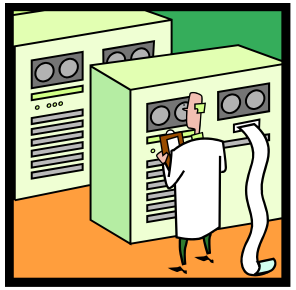


PRNG Requirements

- The basic requirement when a PRNG or PRF is used for a cryptographic application is that an adversary who does not know the seed is unable to determine the pseudorandom string
- The requirement for secrecy of the output of a PRNG or PRF leads to specific requirements in the areas of:
  - Randomness
  - Unpredictability
  - Characteristics of the seed

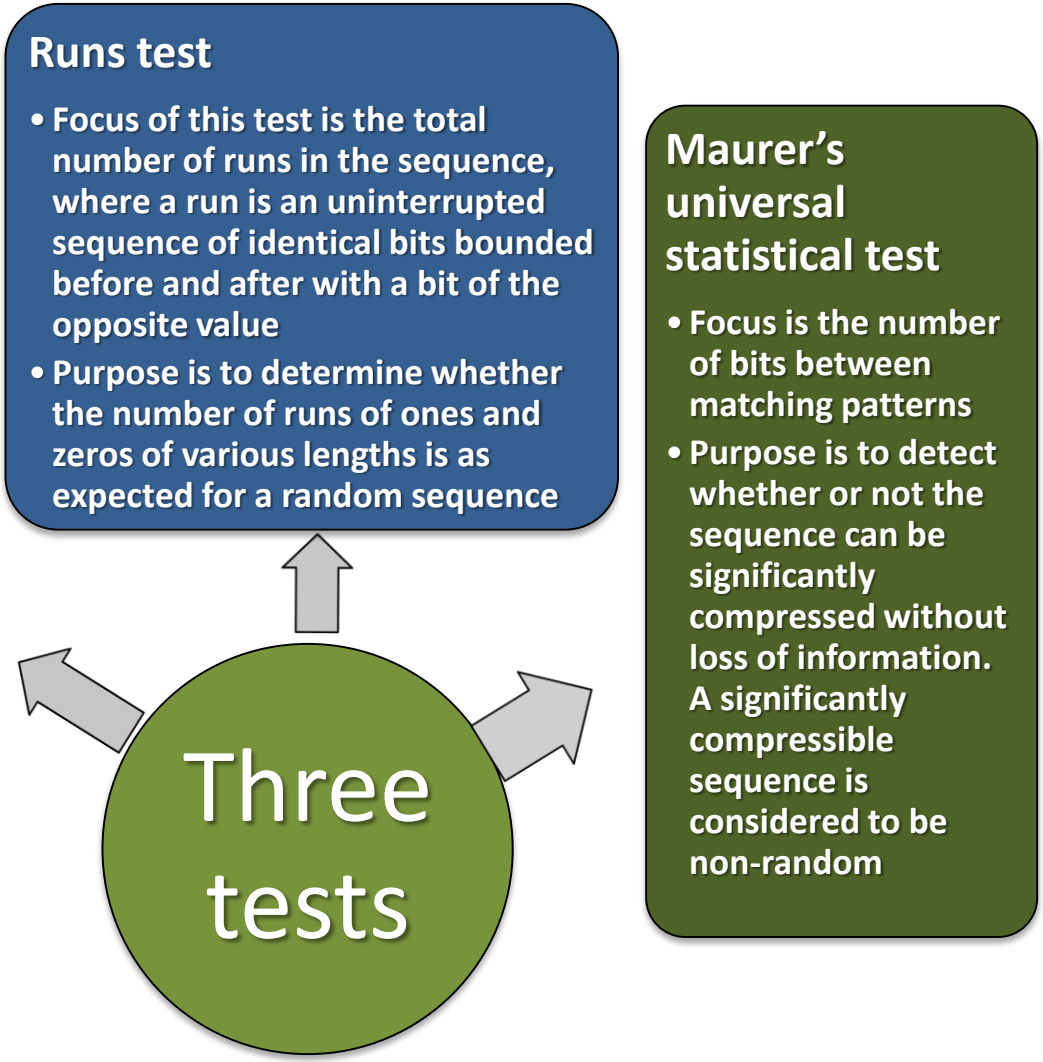
Randomness

- The generated bit stream needs to appear random even though it is deterministic
- There is no single test that can determine if a PRNG generates numbers that have the characteristic of randomness
  - If the PRNG exhibits randomness on the basis of multiple tests, then it can be assumed to satisfy the randomness requirement
- NIST SP 800-22 specifies that the tests should seek to establish three characteristics:
  - Uniformity
  - Scalability
  - Consistency



Randomness Tests

- SP 800-22 lists 15 separate tests of randomness



Unpredictability

- A stream of pseudorandom numbers should exhibit two forms of unpredictability:
  - Forward unpredictability
    - If the seed is unknown, the next output bit in the sequence should be unpredictable in spite of any knowledge of previous bits in the sequence
  - Backward unpredictability
    - It should not be feasible to determine the seed from knowledge of any generated values
    - No correlation between a seed and any value generated from that seed should be evident
    - Each element of the sequence should appear to be the outcome of an independent random event whose probability is 1/2
- The same set of tests for randomness also provides a test of unpredictability
  - A random sequence will have no correlation with a fixed value (the seed)

Seed Requirements

- The seed that serves as input to the PRNG must be secure and unpredictable
- The seed itself must be a random or pseudorandom number
- Typically the seed is generated by TRNG

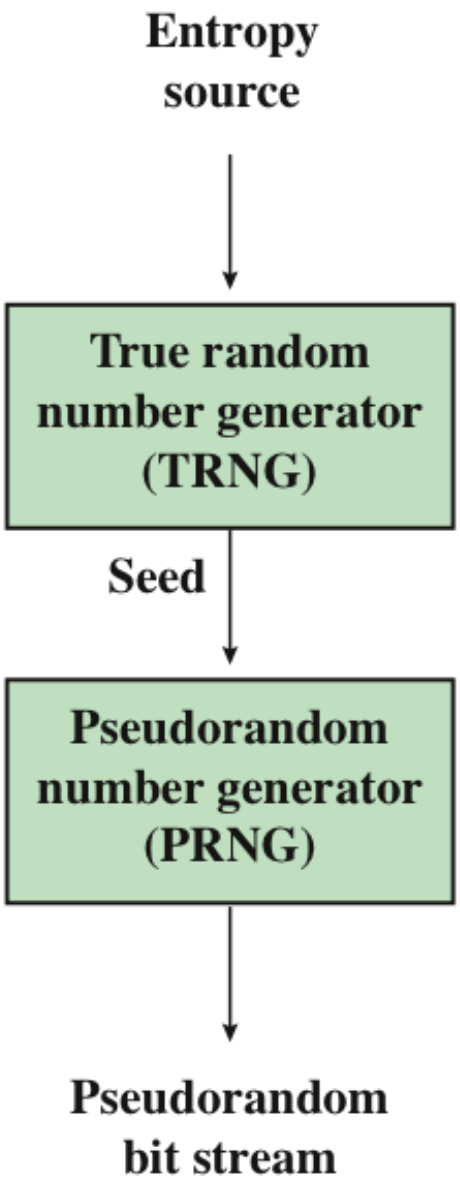


Figure 8.2 Generation of Seed Input to PRNG

Algorithm Design

- Algorithms fall into two categories:
  - Purpose-built algorithms
    - Algorithms designed specifically and solely for the purpose of generating pseudorandom bit streams
  - Algorithms based on existing cryptographic algorithms
    - Have the effect of randomizing input data

Three broad categories of cryptographic algorithms are commonly used to create PRNGs:

- Symmetric block ciphers
- Asymmetric ciphers
- Hash functions and message authentication codes

Linear Feedback Shift Registers

- Generating a keystream: $z_{i+m} = \sum_{j=0}^{m-1} c_j z_{i+j} \mod 2,$
- Example: let m=4, and $z_{i+4} = (z_i + z_{i+1}) \mod 2.$ Seed K=(1,0,0,0)
- Problem: not secure!
  - Knowledge of any consecutive 2m bits can recover the seed, due to the linearity of LFSR

Linear Congruential Generator

- An algorithm first proposed by Lehmer that is parameterized with four numbers:

m	the modulus	$m > 0$
a	the multiplier	$0 < a < m$
c	the increment	$0 \leq c < m$
X_0	the starting value, or seed	$0 \leq X_0 < m$

- The sequence of random numbers $\{X_n\}$ is obtained via the following iterative equation:

$$X_{n+1} = (aX_n + c) \bmod m$$

- If m , a , c , and X_0 are integers, then this technique will produce a sequence of integers with each integer in the range $0 \leq X_n < m$
- The selection of values for a , c , and m is critical in developing a good random number generator

Blum Blum Shub (BBS) Generator

- Has perhaps the strongest public proof of its cryptographic strength of any purpose-built algorithm
- Referred to as a *cryptographically secure pseudorandom bit generator* (CSPRBG)
  - A CSPRBG is defined as one that passes the *next-bit-test* if there is not a polynomial-time algorithm that, on input of the first k bits of an output sequence, can predict the (k + 1)st bit with probability significantly greater than 1/2
- The security of BBS is based on the difficulty of factoring n

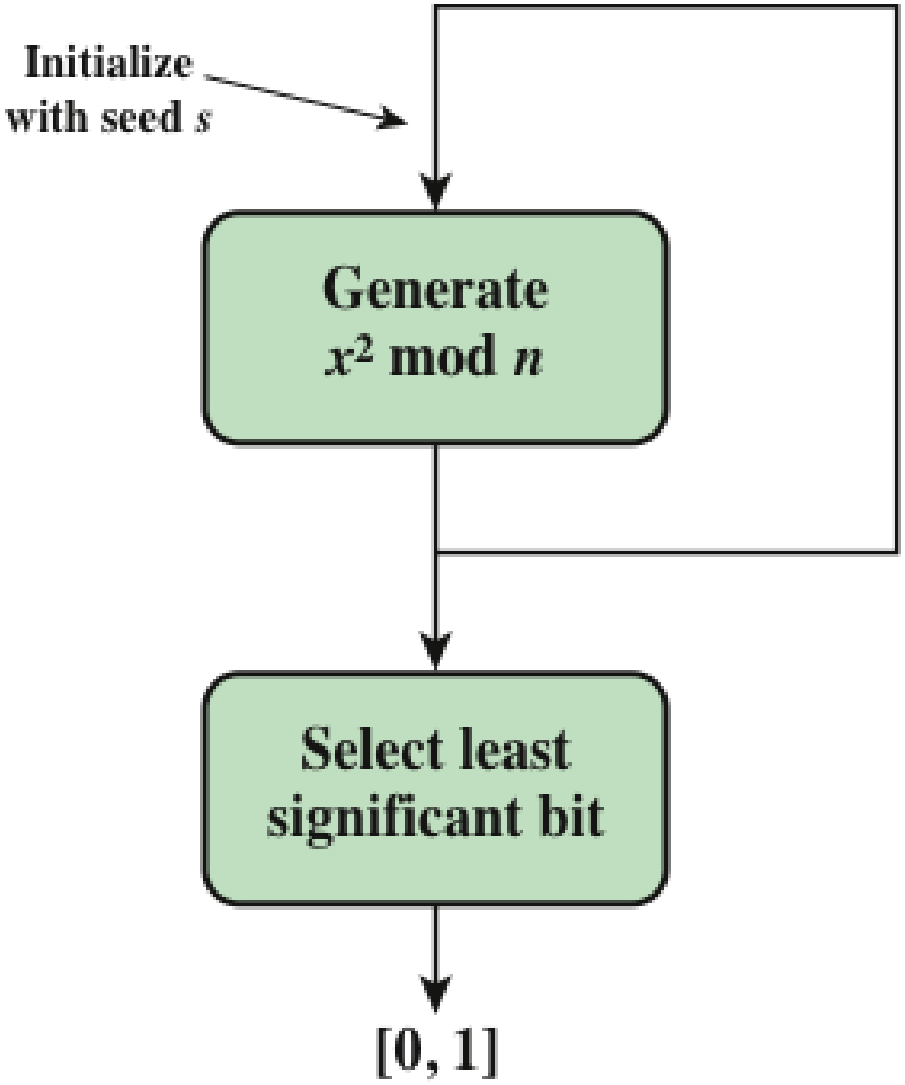


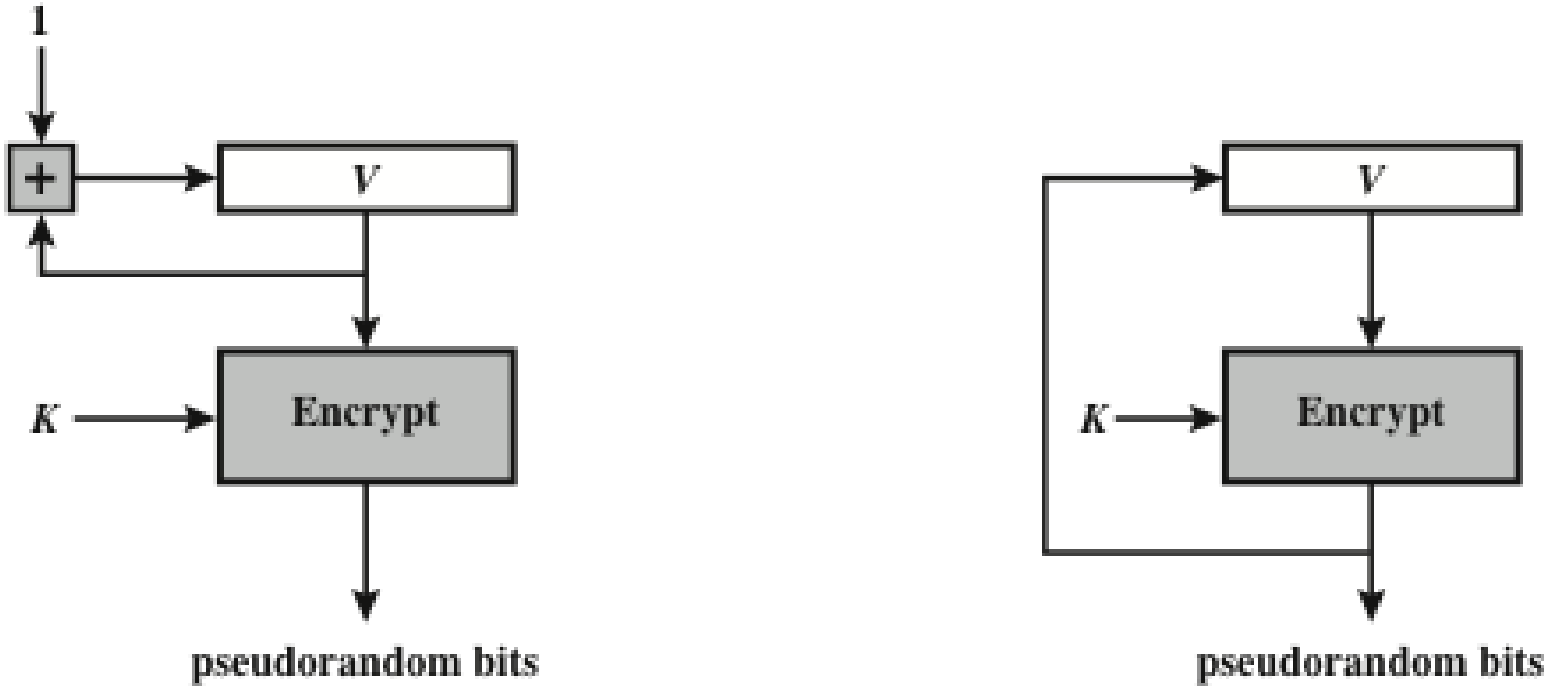
Figure 8.3 Blum Blum Shub Block Diagram

i	X_i	B_i
0	20749	
1	143135	1
2	177671	1
3	97048	0
4	89992	0
5	174051	1
6	80649	1
7	45663	1
8	69442	0
9	186894	0
10	177046	0
11	137922	0
12	123175	1
13	8630	0
14	114386	0
15	14863	1
16	133015	1
17	106065	1
18	45870	0
19	137171	1
20	48060	0

Table 8.1
Example Operation of BBS Generator

PRNG Using Block Cipher Modes of Operation

- Two approaches that use a block cipher to build a PNRG have gained widespread acceptance:
  - CTR mode
    - Recommended in NIST SP 800-90, ANSI standard X.82, and RFC 4086
  - OFB mode
    - Recommended in X9.82 and RFC 4086



(a) CTR Mode

(b) OFB Mode

Figure 8.4 PRNG Mechanisms Based on Block Ciphers