

ECE 471/571

RSA (cont'd)

$e=3$

①. $C = \underline{m^3} \bmod n$ if $\underline{m < n^{\frac{1}{3}}}$
 $m^3 < n$
 $m = \sqrt[3]{C}$ cube root problem

solution? pad m s.t. $m > n^{\frac{1}{3}}$
 $m \parallel 0010001\dots$

②. encrypt m using 3 public keys $e=3$

$$C_1 = \underline{m^3} \bmod n_1 \equiv a_1 \bmod n_1$$

$$C_2 = \underline{m^3} \bmod n_2 \equiv a_2 \bmod n_2$$

$$C_3 = \underline{m^3} \bmod n_3 \equiv a_3 \bmod n_3$$

$$m^3 > n_1$$

$$> n_2$$

$$> n_3$$

solve for

$$C \equiv \underline{m^3} \bmod \underline{n_1 n_2 n_3}$$

$$m^3 < n_1 \cdot n_2 \cdot n_3$$

CRT.

$$C^{\frac{1}{3}} = \underline{m}$$

solution? pad m w/ different numbers
for n_1, n_2, n_3 .

1st $m \parallel 000\dots 1$.
 2nd $m \parallel 100\dots 0$.
 3rd $m \parallel 110\dots 0$.

Chosen ciphertext attack for RSA.

$$C_1 = m_1^e \bmod n.$$

$$C_2 = m_2^e \bmod n.$$

$$\begin{aligned}
 C &= \underline{C_1 \cdot C_2} = m_1^e m_2^e \bmod n. \\
 &= \underline{(m_1 \cdot m_2)^e} \bmod n
 \end{aligned}$$

$$M = m_1 \cdot m_2$$

Adv. homomorphic.
given.

$$\begin{aligned}
 &\langle \underline{C}, m \rangle \\
 &\quad \xrightarrow{\text{D}} \\
 &\langle C_1, m_1 \rangle \\
 &\quad \xrightarrow{\text{D}}
 \end{aligned}$$

encryption.

$$\begin{aligned}
 &\xrightarrow{\text{encryption}} C_2, \xrightarrow{\text{recover } m} \underline{m_1}.
 \end{aligned}$$