

**Note: Due Wednesday, Mar. 13th at midnight;
Each problem is worth equal points (10 points).**

7 common problems for everyone, 10 problems in total (3 additional ones):

Problems 12.2, 12.3, and 13.8 from the textbook;

Note: for 13.8, there's a typo: $V_i = E(K_i, U_i)$, instead of $V_i = E(k_i, U_i)$.

Problem 4: The following two sub problems involve Fermat's Theorem (same as problems 2.20 and 2.22 from the textbook).

- (a) Using Fermat's Theorem, find $3^{201} \bmod 11$.
- (b) Using Fermat's Theorem, find a number x between 0 and 28 with x^{85} congruent to 6 modulo 29.
(you should not use any brute-force searching)

Problem 5: The following two sub problems involve Euler's Theorem (same as problems 2.23 and 2.24 from the textbook).

- (a) Using Euler's Theorem, find a number a between 0 and 9 such that a is congruent to $7^{1000} \bmod 10$. (note: this is the same as the last digit of the decimal expansion of 7^{1000})
- (b) Using Euler's Theorem, find a number x between 0 and 28 with x^{85} congruent to 6 modulo 35.
(you should not use any brute-force searching)

Problem 6: Suppose Fred sees your RSA signature on m_1 and on m_2 (i.e. he sees $m_1^d \bmod n$ and $m_2^d \bmod n$). How does he compute the signature on each of these messages: $m_1^j \bmod n$ (for positive integer j), $m_1^{-1} \bmod n$, $m_1 \cdot m_2 \bmod n$, and in general $m_1^j \cdot m_2^k \bmod n$ (for arbitrary integers j and k)?

Problem 7: Suppose Alice and Bob know each other's public key. Alice sends a message to Bob. How can she encrypt the message so that, when Bob receives it, he is sure about all of the following?

- (1) Nobody else can view the content (confidentiality),
- (2) The message is from Alice and no one has modified it (authentication, integrity).
- (3) Nobody else (Eve) could trick Bob into thinking that Eve also generated the same message.

Additional problems for 471 students only:

Problems 9.3, 9.8, and 11.1, all from textbook.

Additional problems for 571 students only:

Problems 9.18, 11.3, and 12.9, all from the textbook;

Hint: for 11.3 (b), you can use the quadratic residue problem:

https://en.wikipedia.org/wiki/Quadratic_residuosity_problem

(Or, taking square roots modulo a large composite integer n is considered to be infeasible)