

# ECE 471/571. Digital Signatures

Alice  $\xrightarrow{m, \sigma = \text{sign}_{K_{\text{priv}_A}}(m)}$  Bob

$K_{\text{priv}_A}$

non-repudiation

$K_{\text{pub}_A}$

Verify  $(\sigma, m, K_{\text{pub}_A})$

$\begin{cases} \text{true} \\ \text{false} \end{cases}$

RSA signature

$\langle \underline{d}, n \rangle$  private key

$p, q$

$\langle e, n \rangle$  pub key.

$n = p \cdot q$

sign.

$$\sigma = m^d \bmod n$$

$e$  co-prime to

$$d = e^{-1} \bmod \phi(n)$$

ver

$$\sigma^e \bmod n \stackrel{?}{=} m \bmod n$$

✓ Existential forgery w/ key only attack

①.  $m = 1. \quad \sigma = 1. \quad 1^d = 1 \bmod n$

②. randomly pick #  $y$   $1^e = 1 \bmod n$

$$m = y^e \bmod n$$

$$\begin{aligned} \underline{m^d} &\equiv (y^e)^d \bmod n \\ &\equiv y^{e \cdot d \bmod \phi(n)} \bmod n \\ &\equiv y \bmod n \end{aligned}$$

$$\langle \underline{m}, y \rangle = \langle y^e, y \rangle = \sigma$$

solution: existential forgery      publishing only

hash-then sign

$$\langle m, \sigma = (H(m))^d \bmod n \rangle$$

$$\sigma^e = H(m)$$

pre-image  
resistance

✓ Existential forgery w/ known message attack

$$\underline{\langle m_1, \sigma_1 \rangle}$$

$$\sigma_1 = m_1^d \bmod n$$

$$\underline{\langle m_2, \sigma_2 \rangle}$$

$$\sigma_2 = m_2^d \bmod n$$

$$\exists \langle m', \sigma \rangle \quad \sigma = \sigma_1 \cdot \sigma_2 = (m_1 \cdot m_2)^d$$

$$c = (c_1, c_2) \pmod{n}$$

$$\langle m' = m_1 \cdot m_2, \sigma = \sigma_1 \cdot \sigma_2 \rangle$$

✓ selective forgery w/ chosen message attack  
 choose  $m_1$ , gets  $\sigma_1$   
 choose  $m_2$ , gets  $\sigma_2$   
 choose  $m/m_1 = m_2$ , gets  $\sigma_2$  for  $m_2$   
 $m = m_1 \cdot m_2$

$$\sigma_1 = \sigma / \sigma_2 \pmod{n}$$

Hash-then sign

- Exist. forgery w/ known msg. attack.

X ? exist  $(m, \sigma)$  given  $(m', \sigma')$   
 known

adv: find  $m \neq m'$

$$\text{s.t. } h(m) = h(m')$$

$$h(m) \pmod{n} = h(m') \pmod{n}$$

## second - preimage resistance

- Exist. forgery w/ chosen msg attack?

X

choose  $\langle m, \sigma \rangle$

find any  $m'$ , forge  $\langle m', \sigma' \rangle$

find collision, s.t.  $m \neq m'$

$$h(m') = h(m)$$

↓

obtain  $\sigma'$

$$h(m)^d \bmod n = h(m')^d \bmod n$$