

# 9 – High-Level Synthesis: Resource Sharing and Binding

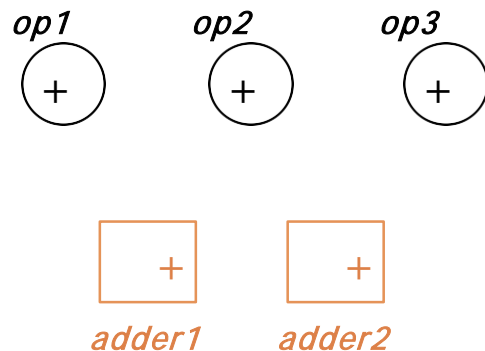
ECE 474A/574A

COMPUTER-AIDED LOGIC DESIGN

# Sharing vs. Binding

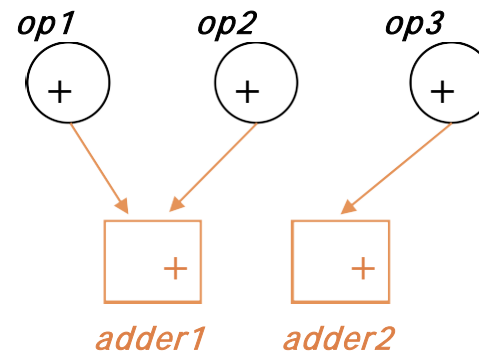
2

- Resource Sharing
  - Assignment of a resource to more than one operation
  - Goal – reduce area by allowing multiple non-concurrent operations to share the same hardware operator
- Resource Binding
  - Explicit mapping between operations and resources



Resource Sharing

- We have 3 add operations and 2 adder units



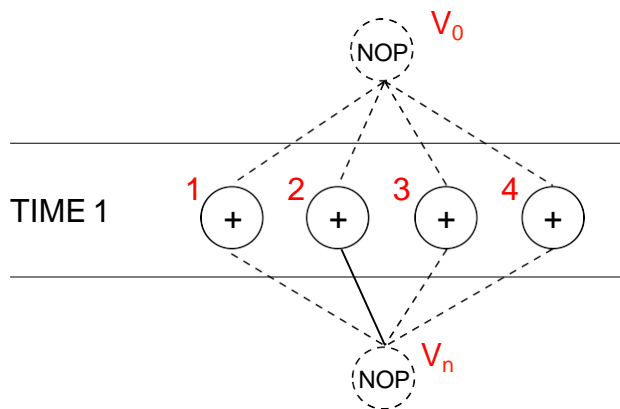
Resource Binding

- Add op1 and op2 executes on adder unit 1
- Add op3 executes on adder unit2

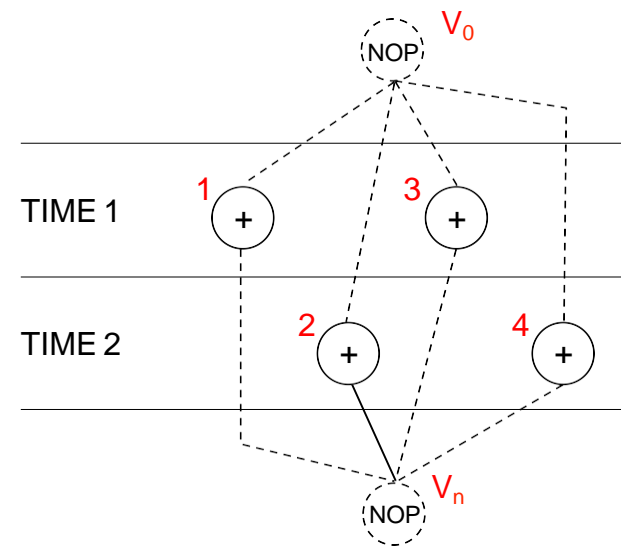
# Resource Binding

3

- Resource binding can be applied to scheduled or non-scheduled sequencing graphs
  - Scheduled sequencing graphs provides limitation on possible sharing



*Requires 4 adders to meet the time constraint  
(upper bound = 1)*

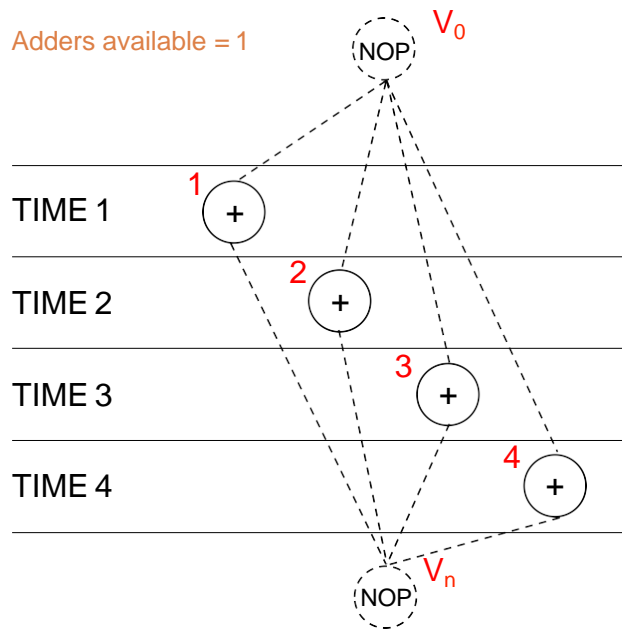


*Requires 2 adders to meet the time constraint  
(upper bound = 2)*

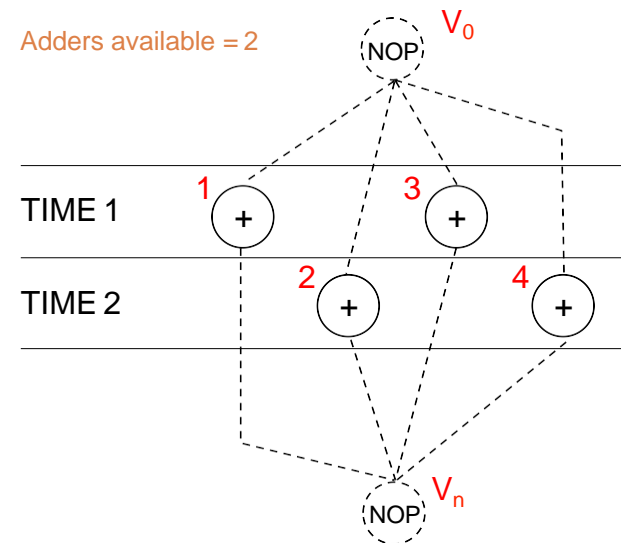
# Resource Binding

4

- Resource binding can be applied to scheduled or non-scheduled sequencing graphs
  - Scheduled sequencing graphs provides limitation on possible sharing
  - Non-scheduled sequencing graphs, the limitation of resource sharing affects the latency by limiting the concurrency of operations (LIST\_L scheduling)



*At most 1 add operation can be executed in a time slice, latency = 4*



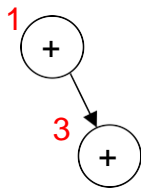
*At most 2 add operations can be executed in a time slice, latency = 2*

# Sharing and Binding for Resource Dominated Circuits

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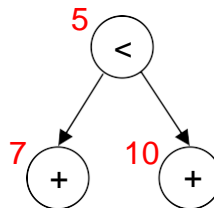
- We are interested in the set of vertices of the sequencing graph (omit source/sink nodes)
- How much sharing is possible?
- Two or more operations can be bound to the same resource if they are **compatible**
  - Not concurrent
  - Can be implemented with the same resource type

$a = b + c$   
 $e = a + 5$



v1 and v3 are not concurrent

if (  $a < b$  )  
     $c = 5 + f$   
else  
     $c = 5 + g$



v7 and v10 are not concurrent

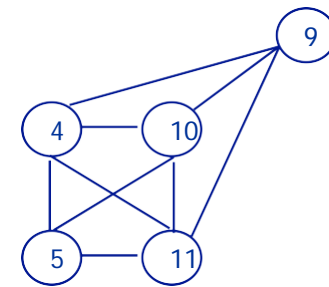
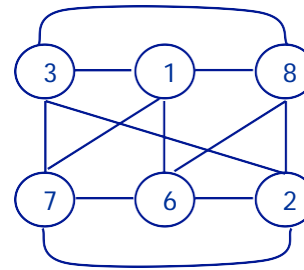
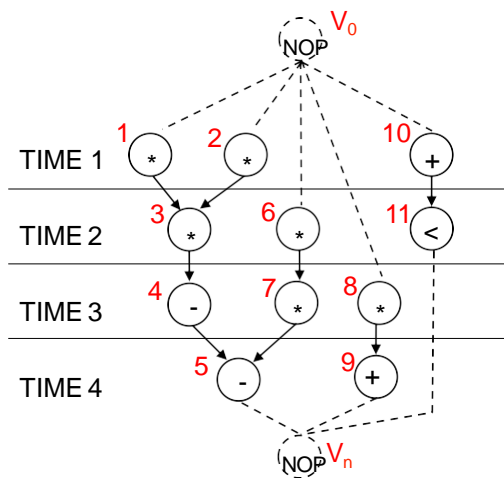
Two operations are NOT concurrent if

- Either one starts after the other has finished
- Alternative choices (mutually exclusive) of a branching decision

# Resource Compatibility Graph

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- Graph whose set of vertices is a one-to-one correspondence with operations in the sequencing graph and whose edges denotes the compatible operations pairs



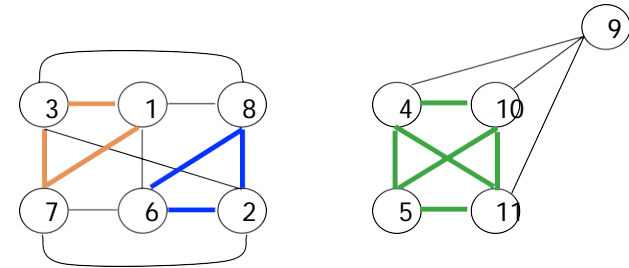
- 3, 1 – Same op, 3 starts after 1
- 3, 2 – Same op, 3 starts after 2
- 3, 4 – Different ops
- 3, 5 – Different ops
- 3, 6 – Same op, BUT neither starts after the other and not alternative choices of branch1
- 3, 7 – Same op, 3 starts in Time 2 and 7 starts in Time 3
- 3, 8 – Same op, 3 starts in Time 2 and 8 starts in Time 3
- 3, 9 – Different ops
- 3, 10 – Different ops
- 3, 11 – Different ops

*Repeat for each node*

# Compatibility Graph Shows Resource Sharing

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- As many disjoint (no common elements) components as resource types
  - A multiply operations is not compatible with an add operation
- Clique - group of mutually compatible operations correspond to subset of vertices that are mutually connected
  - Each vertex connected to every other vertices
- Maximal set of mutually compatible operations are represented by maximal clique
- The optimum resource sharing is one that minimizes the number of required resource instances
  - Resource instance relates to cliques
  - Partitioning graph into minimum number of cliques yields optimal sharing



**Maximize size of cliques, must ensure all vertices included**

{1, 3, 7}

{2, 6, 8}

{1, 8}

{4, 5, 10, 11}

{9}

**# Resources = # cliques**

**We need 2 adders, 2 multipliers**

```

CLIQUE_PARTITION( G(v, e) ){
     $\Pi = \Phi$  // initial set of partitions to empty
    while( G(v,e) not empty ) do{ // while the graph is not empty, keep iterating
        C = MAX_CLIQUE( G(v,e) ) // compute a maximal clique in graph
         $\Pi = \Pi \cup C$  // add max clique to set of partitions
        delete C from G(v,e) // remove max clique from graph
    }
}

```

```

MAX_CLIQUE( G(v, e) ){
    C = vertex with largest degree
    repeat {
        repeat {
            U = { v ∈ V : v ∉ C and adjacent to all vertices of C }
            if ( U ≠ ∅ ) { // no such vertices exist
                return C
            }
            else {
                select v ∈ U // pick one
                C = C ∪ v // add to clique
            }
        }
    }
}

```



# Clique Partitioning

## Example 1

9

$\Pi = \Phi$

// set of partitions is initially empty

Is G empty? No.

Find max clique

$C = 1$  // vertex with largest degree, anything with 4 will do

$U = \{3, 7, 6, 8\}$  // these vertices are connected to 1

$V = 3$

$C = \{1\} \cup \{3\} = \{1, 3\}$

$U = \{7, 8\}$  // these vertices are connected to 1 and 3

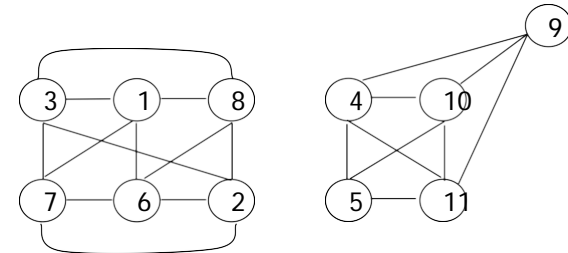
$C = \{1, 3\} \cup \{7\} = \{1, 3, 7\}$

$U = \{\Phi\}$  // no others vertices connect to 1, 3, and 7

Return  $\{1, 3, 7\}$

$\Pi = \{1, 3, 7\}$

Remove  $\{1, 3, 7\}$  from G



Vertices	Degree
1	4
2	4
3	4
4	4
5	3
6	4
7	4
8	4
9	3
10	4
11	4

# Clique Partitioning

## Example 1

10

$\Pi = \{1, 3, 7\}$

Is G empty? No.

Find max clique

$C = 4$  // vertex with largest degree, anything with 4 will do

$U = \{5, 9, 10, 11\}$  // these vertices are connected to 4

$V = 5$

$C = \{4\} \cup \{5\} = \{4, 5\}$

$U = \{10, 11\}$  // these vertices are connected to 4 and 5

$C = \{4, 5\} \cup \{10\} = \{4, 5, 10\}$

$U = \{11\}$  // these vertices are connected to 4, 5, and 10

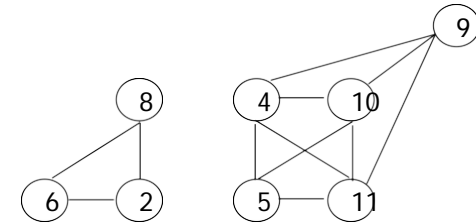
$C = \{4, 5, 10\} \cup \{11\} = \{4, 5, 10, 11\}$

$U = \{\Phi\}$  // no others vertices connect to 4, 5, 10, and 11

Return  $\{4, 5, 10, 11\}$

$\Pi = \{1, 3, 7\}, \{4, 5, 10, 11\}$

Remove  $\{4, 5, 10, 11\}$  from G



Vertices	Degree
2	2
4	4
5	3
6	2
8	2
9	3
10	4
11	4

# Clique Partitioning

## Example 1

11

$\Pi = \{1, 3, 7\}, \{4, 5, 10, 11\}$

Is  $G$  empty? No.

Find max clique

$C = 2$  // vertex with largest degree, anything with 2 will do

$U = \{6, 8\}$  // these vertices are connected to 2

$V = 6$

$C = \{2\} \cup \{6\} = \{2, 6\}$

$U = \{8\}$  // these vertices are connected to 2 and 6

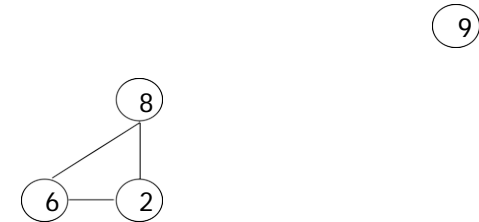
$C = \{2, 6\} \cup \{8\} = \{2, 6, 8\}$

$U = \{\Phi\}$  // no others vertices connect to 2, 6, and 8

Return  $\{2, 6, 8\}$

$\Pi = \{1, 3, 7\}, \{4, 5, 10, 11\}, \{2, 6, 8\}$

Remove  $\{2, 6, 8\}$  from  $G$



Vertices	Degree
2	2
6	2
8	2
9	0

# Clique Partitioning

## Example 1

12

$\Pi = \{1, 3, 7\}, \{4, 5, 10, 11\}, \{2, 6, 8\}$

9

Is G empty? No.

Find max clique

$C = 9$

$U = \{9\}$

$U = \{ \Phi \}$  // no others vertices connect to 9

Return  $\{9\}$

$\Pi = \{1, 3, 7\}, \{4, 5, 10, 11\}, \{2, 6, 8\}, \{9\}$

Remove  $\{9\}$  from G

Vertices	Degree
9	0

# Clique Partitioning

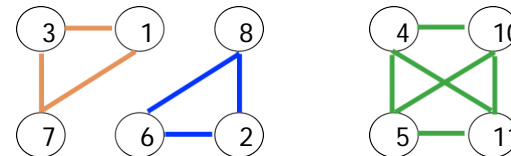
## Example 1

13

$\Pi = \{1, 3, 7\}, \{4, 5, 10, 11\}, \{2, 6, 8\}, \{9\}$

Is G empty? Yes!

- What does clique partition tell us?
  - $\{1, 3, 7\}$  – multiplier
  - §  $\{4, 5, 10, 11\}$  – alu
  - $\{2, 6, 8\}$  – multiplier
  - §  $\{9\}$  – alu
- We know how much sharing AND binding

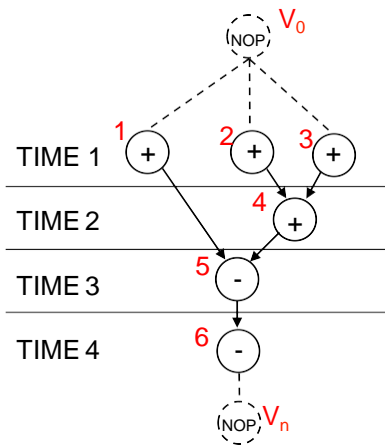


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# Clique Partitioning

## Example 2

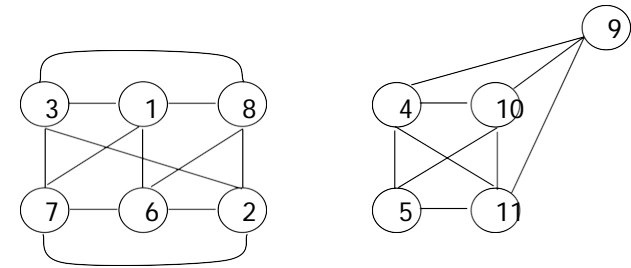
14



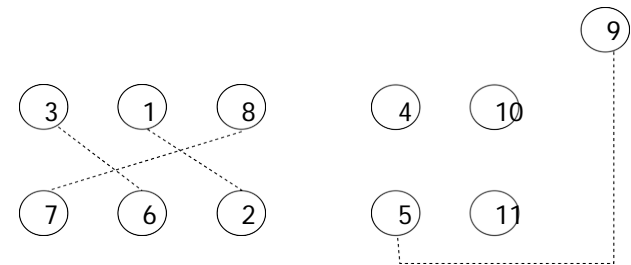
# Resource Conflict Graph

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- Instead of compatibility we can instead look at conflicts
  - May simplify the graph
- Resource conflict graph
  - Graph whose set of vertices is a one-to-one correspondence with operations in the sequencing graph and whose edges denotes the *conflicting* operations pairs
  - To simplify graph, we consider conflicts between each resource type independently



resource compatibility graph

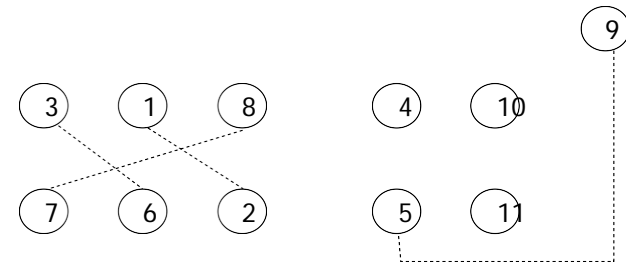
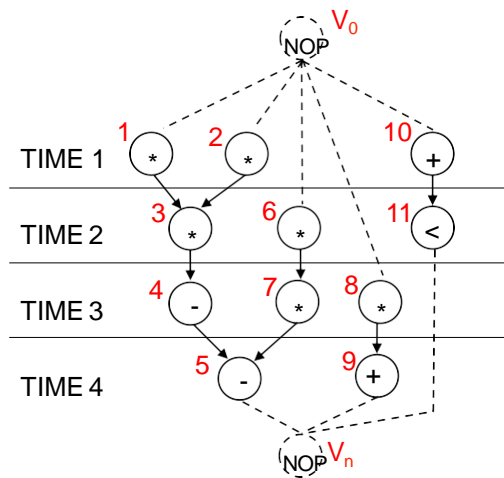


resource conflict graph

# Building Resource Conflict Graph

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- To simplify graph, we consider conflicts between each resource type independently



*Multipliers*

*ALUs*

*Consider Multipliers (1, 2, 3, 6, 7, 8)*

1, 2 – concurrent

3, 6 – concurrent

7, 8 – concurrent

*Consider ALUs (4, 5, 9, 10, 11)*

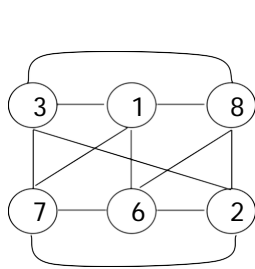
5, 9 – concurrent



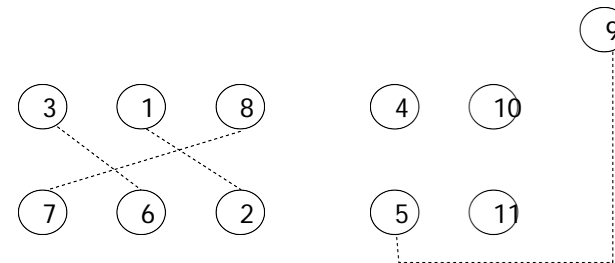
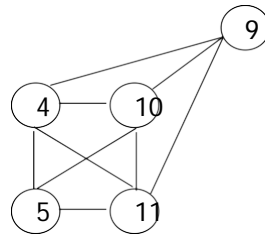
# Building Resource Conflict Graph

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- Conflict graph is the complement of the compatibility graph
- In conflict graph, looking for set of mutually compatible operations
  - Subset of vertices that are NOT connected by edges
  - Also called independent set of G



resource compatibility graph



resource conflict graph

# Graph Coloring

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```
VERTEX_COLOR ( G(v, e) ){  
  for( i=1 to | V | ){  
    C = 1                                // use number to represent color  
    while( there exists a vertex adjacent to  $v_i$  with color c) do{  
      C = C + 1  
    }  
    label  $v_i$  with C  
  }  
}
```

- Use graph coloring to find independent sets
  - Each color represents a resource instance (two adders will be represented by two different colors)
- Optimal resource sharing corresponds to vertex coloring with minimal amount of colors

# Graph Coloring

## Example 1

21

$i = 1$  // look at vertex 1

$C = c1$  // represents first color

Is there any adjacent vertices with color = 1? No.

$v_1 = c1$

$i = 2$  // look at vertex 2

$C = c1$

Is there any adjacent vertices with color = 1? Yes.

$C = c2$

Is there any adjacent vertices with color = 2? No.

$v_2 = c2$

$i = 3$  // look at vertex 3

$C = c1$  // represents first color

Is there any adjacent vertices with color = 1? No.

$v_3 = c1$

# Graph Coloring

## Example 2

23

**i = 1**

C = c1

Adjacent vertices with color = 1? No.

$v_1 = c1$

**i = 2**

C = c1

Adjacent vertices with color = 1? Yes.

C = c2

Adjacent vertices with color = 2? No.

$v_2 = c2$

**i = 3**

C = c1

Adjacent vertices with color = 1? No.

$v_3 = c1$

**i = 4**

C = c1

Adjacent vertices with color = 1? Yes.

C = c2

Adjacent vertices with color = 2? No.

$v_4 = c2$

**i = 5**

C = c1

Adjacent vertices with color = 1? Yes.

C = c2

Adjacent vertices with color = 2? Yes.

C = c3

Adjacent vertices with color = 3? No.

$v_5 = c3$

**i = 6**

C = c1

Adjacent vertices with color = 1? Yes.

C = c2

Adjacent vertices with color = 2? Yes.

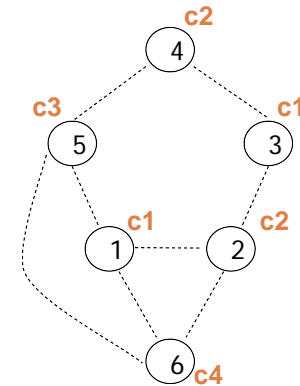
C = c3

Adjacent vertices with color = 3? Yes.

C = c4

Adjacent vertices with color = 4? No.

$v_6 = c4$



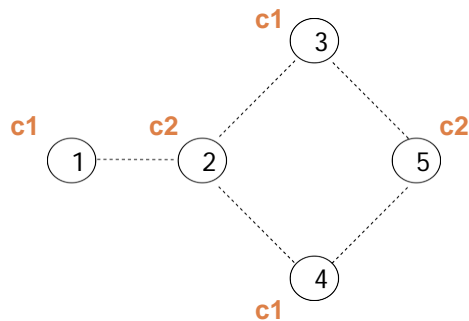
**Four colors required – need four resources**

- c1 for node 1, 3 op
- c2 for node 2, 4 op
- c3 for node 5 op
- c4 for node 6 op

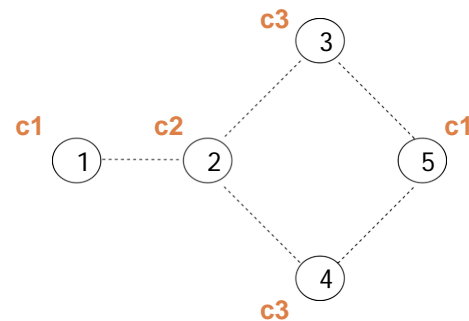
# Graph Coloring

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- VERTEX\_COLOR algorithm sensitive to ordering of vertices explored - variety of modifications available
  - Switching pair assignment of colors
  - Backtracking to switching larger number of vertices



- Node ordering 1, 2, 3, 4, 5
- Requires 2 colors



- Node ordering 1, 5, 2, 3, 4
- Requires 3 colors

# Conclusion

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- Considered several types ways to find resource sharing and binding
  - Compatibility Graph / Max Clique
  - Conflict Graph / Vertex color
  
- Again, many other methods available
  - Golumbic's algorithm
  - Left-edge algorithm
  - ILP formulation
  
- Idea of sharing and binding not limited to adders and multipliers
  - Registers
  - Determining minimal number of memory ports
  - Bus sharing