ECE-442/542 Digital Control Systems
Homework #2
Due: March 02, 2022

Instructions

1. This page must be signed and stapled to your assignment. Homework handed in without this signed page will not be graded.
2. Your signature indicates your assertion of the truth of the following statement

I acknowledge that this homework is solely my effort. I have done this work by myself. I have not consulted with others about this homework beyond the allowed level of verbal (non-written) exchanges of thoughts and opinions with my classmates. I have not received outside aid (out- side of my own brain) on this homework. I understand that violation of these rules contradicts the class policy on academic integrity.

Name: ______________________________

Signature: ______________________________

Date: ______________________________

Required Problems for the Homework

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Problem #1

Consider a normalized (frequency and gain) model of a DC motor, $G(s)$. Suppose a closed-loop bandwidth of $\omega = 10$ radians/second is desired.

$$G(s) = \frac{1}{s(s+1)} \tag{1}$$

a) Explain why a sample period of $T = 0.02$ seconds is acceptable.

b) Find a Zero-Order Hold (ZOH) equivalent (by hand) of $G(s)$. Call your ZOH equivalent $G(z)$.

c) Compare your results in part (b) with results using MATLAB. (You should be able to use the `c2d()` function in MATLAB.)

d) Find the poles and zeroes of $G(s)$ and $G(z)$. You may use MATLAB.

Problem #2

Compute, by hand, the discrete equivalents using the following numerical integration techniques for a second-order low-pass Butterworth filter having a bandwidth of about 10 Hz. Assume the sample period is T = 0.0035 seconds.

$$G(s) = \frac{3600}{s^2 + 84.853s + 3600} \tag{2}$$

a) Forward rule;

b) Backward rule;

c) Bilinear transform

d) Check the trapezoidal discrete equivalent using MATLAB's `c2d()` function. [e)] Compute the poles and zeroes of all of the different transfer functions. You may use MATLAB.

Problem #3

Find a discrete equivalent for the low-pass filter in Problem 2 using a pole/zero mapping technique. Assume the sample period is $T = 0.0035$ seconds. Check your work using MATLAB's `c2d()` function with the "matched" argument.

Problem #4

Verify the formulas for the discrete-time state space matrices for the backward numerical integration technique in terms of the continuous-time state-space matrices.

$$\Phi = (1 - AT)^{-1}, \quad \Gamma = (1 - AT)^{-1}BT, \quad H = C(1 - AT)^{-1}, \quad J = D + C(1 - AT)^{-1}BT$$

Problem #5

Consider a mass-spring-damper system with $M = 2.0\text{kg}$, $B = 5\text{N/(m/sec)}$, $K = 350\text{N/m}$, and a scaling factor between the applied input force and the system equal to 160.

a) Using a state-space approach, find the formulas in terms of T of a ZOH equivalent of a state space representation of the system.

b) With $T = 0.003\text{seconds}$, does T seem reasonable? Why or why not?

c) Use MATLAB to find numerical values for the discrete-time system matrix, Φ, and the discrete-time input matrix, Γ.

d) Use MATLAB to find the eigenvalues of the continuous-time and discrete-time systems.