ECE-442/542 Digital Control Systems
Homework #2
Due: February 14, 2022

Instructions

1. This page must be signed and stapled to your assignment. Homework handed in without this signed page will not be graded.
2. Your signature indicates your assertion of the truth of the following statement

I acknowledge that this homework is solely my effort. I have done this work by myself. I have not consulted with others about this homework beyond the allowed level of verbal (non-written) exchanges of thoughts and opinions with my classmates. I have not received outside aid (out- side of my own brain) on this homework. I understand that violation of these rules contradicts the class policy on academic integrity.

Name: ______________________________

Signature: ______________________________

Date: ______________________________

Required Problems for the Homework

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2. Your signature indicates your assertion of the truth of the following statement

Problem #1

For the given first-order system in (1), linear difference equation with $a = 0.43$ and $b = 0.47$: (a) iteratively evaluate the response, $y[k]$, when $y[0] = 0$, and $u[k] = 10$ for $k = 0, 1, 2, 3, 4, 5$. That is, compute numerical values for $y[0], y[1], \ldots, y[5]$. (b) With $u[k] = 0$ and $y[0] = 1$, find the number of sample periods, N, needed for an initial condition, e.g., $y[0] = 1$, to decay to 1% of its original value. Or, stated another way, find the integer value N in the 1% settling time expression, $T_s = NT$, where T is the sample period. TT wasn't explicitly given in the problem statement. T is already accounted for in the values of $a = 0.43$ and $b = 0.47$. One doesn't need T to find N in this problem. In fact, you are unable to find a numerical value for T_s , since T has not been given.

$$y[k+1] = ay[k] + bu[k] \tag{1}$$

Problem #2 [Code]

Use Matlab to obtain insight into the system behavior for first-order systems and second-order systems when the system has (a) one real pole and no finite zero (first-order system response), (generate one step response); (b) real and equal poles and no finite zero (critically damped system), (plot one step response); (c) real and unequal poles and no finite zero (overdamped system), (plot one step response); (d) complex poles and no finite zero (underdamped), (plot three different step responses);

$$G(s) = \frac{n(s)}{d(s)}$$

a) Let $n(s) = 1$ and $d(s) = (s + 1) = s + 1$

b) Let $n(s) = 1$ and $d(s) = (s + 1)(s + 1) = s^2 + 2s + 1$

c) Let $n(s) = 4$ and $d(s) = (s + 1)(s + 4) = s^2 + 5s + 4$.

d) Let $n(s) = 1$ and $d(s) = s^2 + 2\zeta s + 1$, with (i) $\zeta = 0.707$, (ii) $\zeta = 0.45$, and (iii) $\zeta = 0.1$.

Problem #3

Part I (a) Find the transfer function description associated with the following difference equation. This difference equation could be associated with the dynamics of a servomotor that has been sampled to produce a discrete-time difference equation.

$$y[n] - 1.905y[n-1] + 0.9048y[n-2] = 0.04837u[n-1] + 0.04679u[n-2]$$

(b) Find the poles and zeroes of the transfer function found in part (a) and comment on the stability of this open-loop system.

Part II Suppose constant output feedback is applied to the system in Part I. That is let $U(z) = R(z) - Y(z)$, where $R(z)$ is a reference input that will be applied to the closed-loop system. (a) Find the transfer function between the reference input, $R(z)$, and the output signal, $Y(z)$. (b) Find the poles and zeroes of this closed-loop transfer function and comment on the stability of the closed-loop system.

Part III Derive a state space representation for the difference equation in Part II. Notice that this difference equation also contains delays in the input terms. Even with the delays in the input terms, the state space representation is still only second order. Thus, if one draws a block diagram representation with the smallest number of delay blocks, then the block diagram will only contain two delay blocks.