In the following **Word Match** problems, match the term with the definition by writing the correct letter in the space provided.

a. Laplace transform pair	The logarithm of the magnitude of the transfer function and the phase are plotted versus the logarithm of ω, the frequency.	______
b. Decibel (dB)	The logarithm of the magnitude of the transfer function, $20 \log_{10} \lvert G(j\omega) \rvert$.	______
c. Fourier transform	A plot of the real part of $G(j\omega)$ versus the imaginary part of $G(j\omega)$.	______
d. Bode plot	The steady-state response of a system to a sinusoidal input signal.	______
e. Transfer function in the frequency domain	All the zeros of a transfer function lie in the left-hand side of the s-plane.	______
f. Decade	The frequency at which the frequency response has declined 3 dB from its low-frequency value.	______
g. Dominant roots	The frequency at which the maximum value of the frequency response of a complex pair of poles is attained.	______
h. All-pass network	The frequency of natural oscillation that would occur for two complex poles if the damping were equal to zero.	______
i. Logarithmic magnitude	Transfer functions with zeros in the right-hand s-plane.	______
j. Natural frequency	The frequency at which the asymptotic approximation of the frequency response for a pole (or zero) changes slope.	______
k. Fourier transform pair	The transformation of a function of time into the frequency domain.	______
l. Minimum phase	The ratio of the output to the input signal where the input is a sinusoid.	______
m. Bandwidth	The units of the logarithmic gain.	______
n. Frequency response	A pair of complex poles will result in a maximum value for the frequency response occurring at the resonant frequency.	______
o. Resonant frequency	A nonminimum phase system that passes all frequencies with equal gain.	______
p. Break frequency	A factor of ten in frequency.	______
q. Polar plot	The roots of the characteristic equation that represent or dominate the closed-loop transient response.	______
r. Maximum value of the frequency response	A pair of functions, one in the time domain, and the other in the frequency domain, and both related by the Fourier transform.	______
s. Nonminimum phase	A pair of functions, one in the time domain, and the other in the frequency domain, and both related by the Laplace transform.	______

EXERCISES

E8.1 Increased track densities for computer disk drives necessitate careful design of the head positioning control [1]. The loop transfer function is

$$L(s) = G_c(s)G(s) = \frac{K}{(s+2)^2}.$$

Plot the frequency response for this system when $K = 4$. Calculate the phase and magnitude at $\omega = 0.5, 1, 2, 4,$ and ∞.

Answer: $|L(j0.5)| = 0.94$ and $\angle L(j0.5) = -28.1°$.

E8.2 A tendon-operated robotic hand can be implemented using a pneumatic actuator [8]. The actuator can be represented by

$$G(s) = \frac{1000}{(s+100)(s+10)}.$$

Plot the frequency response of $G(j\omega)$. Show that the magnitude of $G(j\omega)$ is -3 dB at $\omega = 10$ and -33 dB at $\omega = 200$. Show also that the phase is $-171°$ at $\omega = 700$.

E8.3 A robotic arm has a joint-control loop transfer function

$$L(s) = G_c(s)G(s) = \frac{300(s+100)}{s(s+10)(s+40)}.$$

Show that the frequency equals $\omega = 28.3$ rad/s when the phase angle of $L(j\omega)$ is $-180°$. Find the magnitude of $L(j\omega)$ at $\omega = 28.3$ rad/s.

Answer: $|L(j28.3)| = -2.5$ dB

E8.4 The frequency response for the system

$$G(s) = \frac{Ks}{(s+a)(s^2+20s+100)}$$

is shown in Figure E8.4. Estimate K and a by examining the frequency response curves.

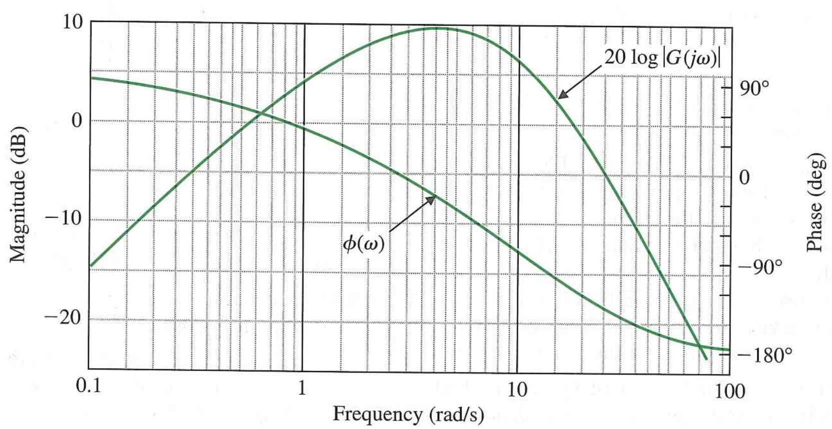

FIGURE E8.4 Bode plot.

E8.5 The magnitude plot of a transfer function

$$G(s) = \frac{K(1+0.5s)(1+as)}{s(1+s/8)(1+bs)(1+s/36)}$$

is shown in Figure E8.5. Estimate K, a, and b from the plot.

Answer: $K = 8, a = 1/4, b = 1/24$

E8.6 Several studies have proposed an extravehicular robot that could move around in a NASA space station and perform physical tasks at various worksites [9]. The arm is controlled by a unity feedback control with loop transfer function

$$L(s) = G_c(s)G(s) = \frac{K}{s(s/8+1)(s/120+1)}.$$

Sketch the Bode plot for $K = 20$ and determine the frequency when $20 \log |L(j\omega)|$ is 0 dB.

E8.7 Consider a system with a closed-loop transfer function

$$T(s) = \frac{Y(s)}{R(s)} = \frac{4}{(s^2+s+1)(s^2+0.4s+4)}.$$

This system will have no steady-state error for a step input. (a) Plot the frequency response, noting the two peaks in the magnitude response. (b) Predict the time response to a step input, noting that the system has four poles and cannot be represented as a dominant second-order system. (c) Plot the step response.

E8.8 A feedback system has a loop transfer function

$$L(s) = G_c(s)G(s) = \frac{100(s-1)}{s^2+25s+100}.$$

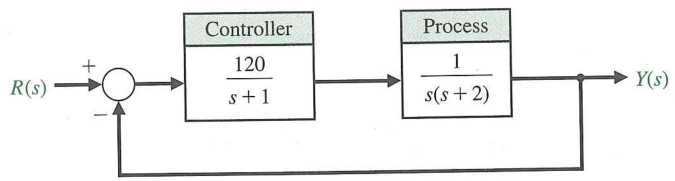

FIGURE E8.13 Third-order feedback system.

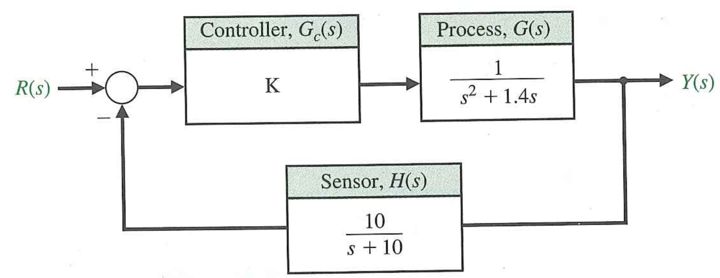

FIGURE E8.14 Nonunity feedback system with controller gain K.

the phase of the loop transfer function when the magnitude $20 \log|L(j\omega)| = 0$ dB. Recall that the loop transfer function is $L(s) = G_c(s)G(s)H(s)$.

E8.15 Consider the single-input, single-output system described by

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t)$$
$$y(t) = \mathbf{C}\mathbf{x}(t)$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -6-K & -1 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \mathbf{C} = [5 \quad 3].$$

Compute the bandwidth of the system for $K = 1, 2,$ and 10. As K increases, does the bandwidth increase or decrease?

PROBLEMS

P8.1 Sketch the polar plot for the following loop transfer functions:

(a) $L(s) = G_c(s)G(s) = \dfrac{1}{(1 + 0.25s)(1 + 3s)}$

(b) $L(s) = G_c(s)G(s) = \dfrac{5(s^2 + 1.4s + 1)}{(s - 1)^2}$

(c) $L(s) = G_c(s)G(s) = \dfrac{s - 8}{s^2 + 6s + 8}$

(d) $L(s) = G_c(s)G(s) = \dfrac{20(s + 8)}{s(s + 2)(s + 4)}$

P8.2 Sketch the Bode plot representation of the frequency response for the transfer functions given in Problem P8.1.

P8.3 A rejection network is the bridged-T network shown in Figure P8.3. The transfer function of this network is

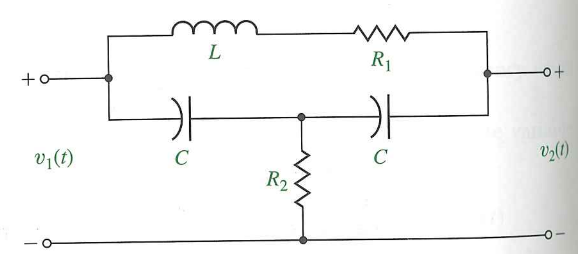

FIGURE P8.3 Bridged-T network.

$$G(s) = \frac{s^2 + \omega_n^2}{s^2 + 2(\omega_n/Q)s + \omega_n^2},$$

where $\omega_n^2 = 2/LC$, $Q = \omega_n L/R_1$, and R_2 is adjusted so that $R_2 = (\omega_n L)^2/4R_1$ [3]. (a) Determine the poles and zeros. (b) Sketch the Bode plot.

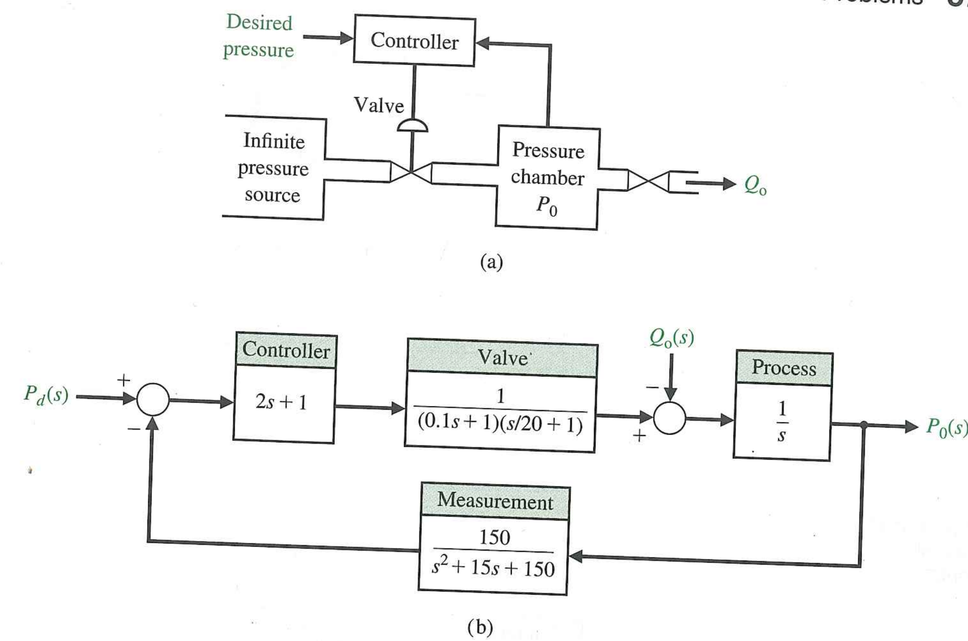

FIGURE P8.4 (a) Pressure controller. (b) Block diagram model.

P8.4 A control system for controlling the pressure in a closed chamber is shown in Figure P8.4. Sketch the Bode plot of the loop transfer function.

P8.5 The global robot industry is growing rapidly [8]. A typical industrial robot has multiple degrees of freedom. A unity feedback position control system for a force-sensing joint has a loop transfer function

$$G_c(s)G(s) = \frac{K}{(1 + s/4)(1 + s)(1 + s/20)(1 + s/80)},$$

where $K = 10$. Sketch the Bode plot of this system.

P8.6 The asymptotic log-magnitude curves for two loop transfer functions are given in Figure P8.6. Sketch the corresponding asymptotic phase shift curves for each system. Estimate the transfer function for each system. Assume that the systems have minimum phase transfer functions.

P8.7 Driverless vehicles can be used in warehouses, airports, and many other applications. These vehicles follow a wire embedded in the floor and adjust the steerable front wheels in order to maintain proper direction, as shown in Figure P8.7(a) [10]. The sensing coils, mounted on the front wheel assembly, detect an error in the direction of travel and adjust the steering. The overall control system is shown in Figure P8.7(b). The loop transfer function is

$$L(s) = \frac{K}{s(s + \pi)^2} = \frac{K_v}{s(s/\pi + 1)^2}.$$

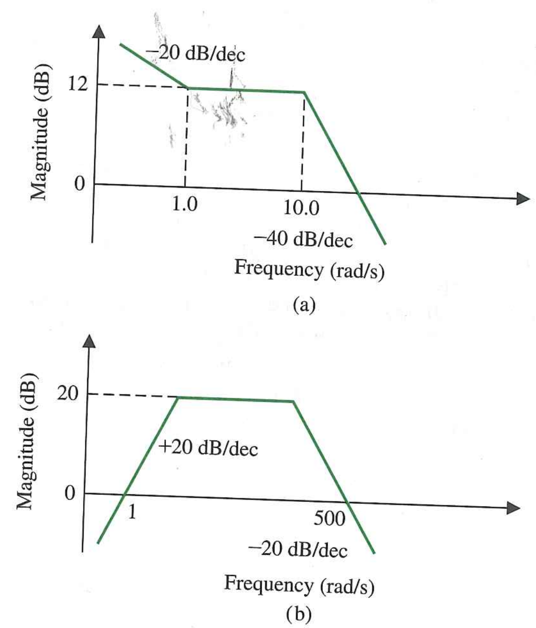

FIGURE P8.6 Log-magnitude curves.

(a) Set $K_v = \pi$ and sketch the Bode plot. (b) Using the Bode plot, determine the phase at the crossover frequency.

In the following **Word Match** problems, match the term with the definition by writing the correct letter in the space provided.

a. Deadbeat response	A system with a rapid response, minimal overshoot, and zero steady-state error for a step input.	______
b. Phase lead compensation	A network that provides a positive phase angle over the frequency range of interest.	______
c. PI controller	A compensator hat acts, in part, like an integrator.	______
d. Lead-lag compensator	A compensator with the characteristics of both a lead compensator and a lag compensator.	______
e. Design of a control system	A compensator that provides a negative phase angle and a significant attenuation over the frequency range of interest.	______
f. Phase lag compensation	An additional component or circuit that is inserted into the system to compensate for a performance deficiency.	______
g. Integration network	A compensator placed in cascade or series with the system process.	______
h. Compensator	Controller with a proportional term and an integral term.	______
i. Compensation	A transfer function, $G_p(s)$, that filters the input signal $R(s)$ prior to calculating the error signal.	______
j. Phase-lag network	The arrangement or the plan of the system structure and the selection of suitable components and parameters.	______
k. Cascade compensation network	The alteration or adjustment of a control system in order to provide a suitable performance.	______
l. Phase-lead network	A widely-used compensator that possesses one zero and one pole with the pole closer to the origin of the s-plane.	______
m. Prefilter	A widely-used compensator that possesses one zero and one pole with the zero closer to the origin of the s-plane.	______

EXERCISES

E10.1 A negative feedback control system has a transfer function

$$G(s) = \frac{K}{s+2}.$$

We select a compensator

$$G_c(s) = \frac{s+a}{s},$$

in order to achieve zero steady-state error for a step input. Select a and K so that percent the overshoot to a step is $P.O. \leq 5\%$ and the settling time (with a 2% criterion) is $T_s \leq 1$ s.

Answer: $K = 6, a = 5.6$

E10.2 A control system with negative unity feedback has a process

$$G(s) = \frac{400}{s(s+40)},$$

and we select a proportional plus integral compensation, where

$$G_c(s) = K_P + \frac{K_I}{s}.$$

Note that the steady-state error of this system for a ramp input is zero. (a) Set $K_I = 1$ and find a suitable value of K_P so the step response will have a percent overshoot of $P.O. \leq 20\%$. (b) What is the expected settling time (with a 2% criterion) of the compensated system?

Answer: $K_P = 0.5$

E10.3 A unity feedback control system in a manufacturing system has a process transfer function

$$G(s) = \frac{e^{-s}}{s+1},$$

and it is proposed to use a compensator to achieve a percent overshoot $P.O. \leq 5\%$ to a step input. The compensator is [4]

$$G_c(s) = K\left(1 + \frac{1}{\tau s}\right),$$

which provides proportional plus integral control. Show that one solution is $K = 0.5$ and $\tau = 1$.

E10.4 Consider a unity feedback system with

$$G(s) = \frac{K}{s(s+5)(s+10)},$$

where K is set equal to 100 in order to achieve a specified $K_v = 2$. We wish to add a lead-lag compensator

$$G_c(s) = \frac{(s+0.15)(s+0.7)}{(s+0.015)(s+7)}.$$

Show that the gain margin of the compensated system is $G.M. = 28.6$ dB and that the phase margin is $P.M. = 75.4°$.

E10.5 Consider a unity feedback system with the transfer function

$$G(s) = \frac{K}{s(s+2)(s+4)}.$$

We desire to obtain the dominant roots with $\omega_n = 3$, $\zeta = 0.5$, and $K_v = 2.7$. The compensator is

$$G_c(s) = \frac{7.53(s+2.2)}{(s+16.4)}.$$

Determine the value of K that should be selected.

Answer: $K = 22$

E10.6 Consider the system with the loop transfer functions

$$L(s) = G_c(s)G(s) = \frac{K(s+4)}{s(s+0.2)(s^2+15s+150)}.$$

When $K = 10$, find $T(s)$ and estimate the expected percent overshoot and settling time (with a 2% criterion). Compare your estimates with the actual percent overshoot of $P.O. = 47.5\%$ and a settling time of $T_s = 32.1$ s.

E10.7 NASA astronauts retrieved a satellite and brought it into the cargo bay of the space shuttle, as shown in Figure E10.7(a). A model of the feedback control system is shown in Figure E10.7(b). Determine the value of K that will result in a phase margin of $P.M. = 40°$ when $T = 0.6$ s.

Answer: $K = 34.15$

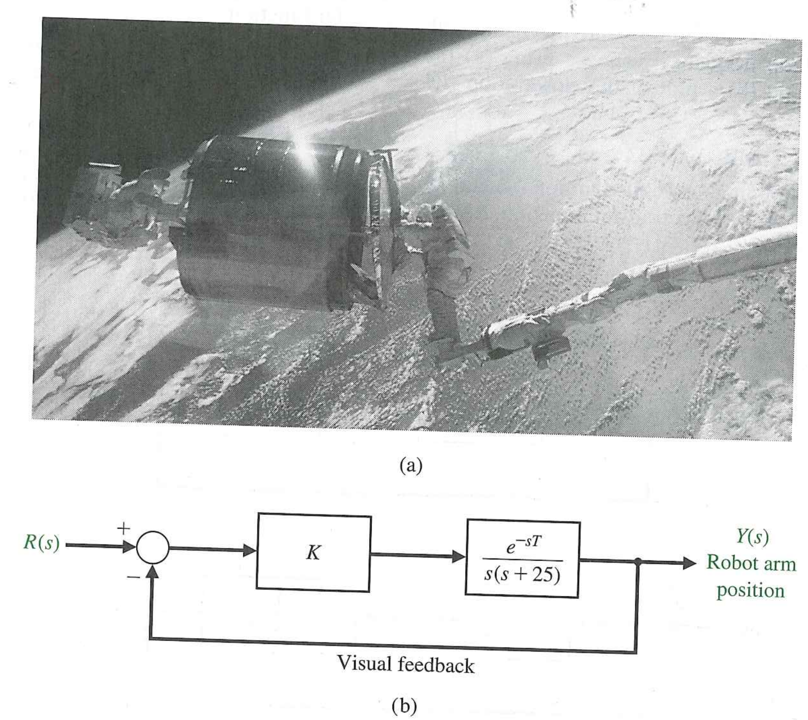

FIGURE E10.7 Retrieval of a satellite. (Photo courtesy of NASA.)

PROBLEMS

P10.1 The design of a lunar excursion module is an interesting control problem. The attitude control system for the lunar vehicle is shown in Figure P10.1. The vehicle damping is negligible, and the attitude is controlled by gas jets. The torque, as a first approximation, will be considered to be proportional to the signal $V(s)$ so that $T(s) = K_2V(s)$. The loop gain may be selected by the designer in order to provide a suitable damping. A damping ratio of $\zeta = 0.6$ with a settling time (with a 2% criterion) of less than 2.5 seconds is required. Using a lead network compensation, select the necessary compensator $G_c(s)$ by using (a) frequency response techniques and (b) root locus methods.

P10.2 A magnetic tape recorder transport for modern computers requires a high-accuracy, rapid-response control system. The requirements for a specific transport are as follows: (1) The tape must stop or start in 10 ms, and (2) it must be possible to read 45,000 characters per second. This system is illustrated in Figure P10.2. We will use a tachometer in this case and set $K_a = 50{,}000$ and $K_2 = 1$. To provide a suitable performance, a compensator $G_c(s)$ is inserted immediately following the photocell transducer. Select a compensator $G_c(s)$ so that the percent overshoot of the system for a step input is $P.O \leq 25\%$. We assume that $\tau_1 = 0.1$ ms, $\tau_a = 0.1$ ms, $K_1 = 2$, $R/L = 0.5$ ms, $K_b = 0.4$, $r = 0.2$, $K_T/LJ = 2.0$ and $K_p = 1$.

P10.3 A simplified version of the attitude rate control for a supersonic aircraft is shown in Figure P10.3. When the vehicle is flying at four times the speed of sound (Mach 4) at an altitude of 100,000 ft, the parameters are [26]

$$\tau_a = 1.0, \qquad K_1 = 0.8,$$
$$\zeta\omega_a = 1.0, \quad \text{and} \quad \omega_a = 5.$$

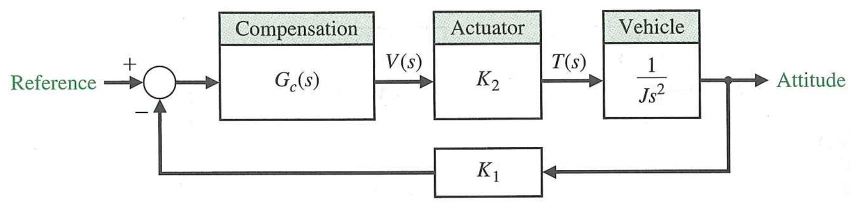

FIGURE P10.1 Attitude control system for a lunar excursion module.

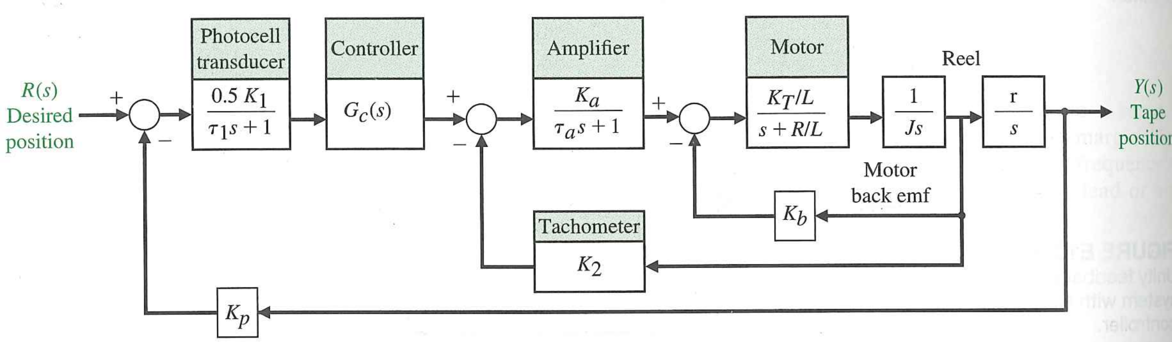

FIGURE P10.2 Block diagram of a tape control system.

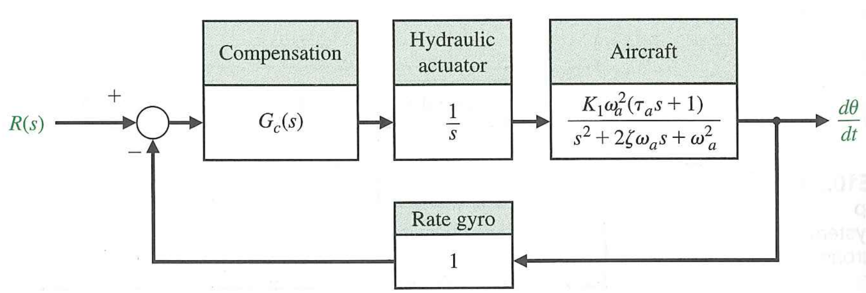

FIGURE P10.3 Aircraft attitude control.

Design a compensator $G_c(s)$ so that the response to a step input has a percent overshoot of $P.O \leq 5\%$ and a settling time (with a 2% criterion) of $T_s \leq 5$ s.

P10.4 Magnetic particle clutches are useful actuator devices for high power requirements because they can typically provide a 200-W mechanical power output. The particle clutches provide a high torque-to-inertia ratio and fast time-constant response. A particle clutch positioning system for nuclear reactor rods is shown in Figure P10.4. The motor drives two counter-rotating clutch housings. The clutch housings are geared through parallel gear trains, and the direction of the servo output is dependent on the clutch that is energized. The time constant of a 200-W clutch is $\tau = 1/40$ s. The constants are such that $K_T n/J = 1$. We want the maximum percent overshoot for a step input to be in the range of $10\% \leq P.O. \leq 20\%$. Design a compensator so that the system is adequately stabilized. The settling time (with a 2% criterion) of the system should be $T_s \leq 2$ s.

P10.5 A stabilized precision rate table uses a precision tachometer and a DC direct-drive torque motor, as shown in Figure P10.5. We want to maintain a high steady-state accuracy for the speed control. To obtain a zero steady-state error for a step command design, select a proportional plus integral compensator. Select the appropriate gain constants so that the system has a percent overshoot of $P.O. = 10\%$ and a settling time (with a 2% criterion) of $T_s \leq 1.5$ s.

P10.6 Repeat Problem P10.5 by using a phase-lead compensator and compare the results.

P10.7 A chemical reactor process whose production rate is a function of catalyst addition is shown in block diagram form in Figure P10.7 [10]. The time delay is $T = 50$ s, and the time constant τ is approximately 40 s. The gain of the process is $K = 1$. Design a compensator using Bode plot methods in order to provide a suitable system response. We want to have a steady-state error less than $0.10A$ for a step input $R(s) = A/s$. For the system with the compensation added, estimate the settling time of the system.

P10.8 A numerical path-controlled machine turret lathe is an interesting problem in attaining sufficient accuracy [2, 23]. A block diagram of a turret lathe control system is shown in Figure P10.8. The gear ratio is

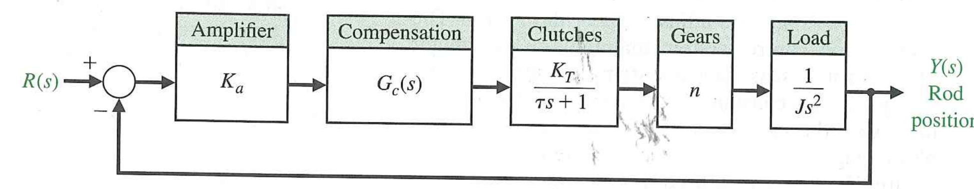

FIGURE P10.4 Nuclear reactor rod control.

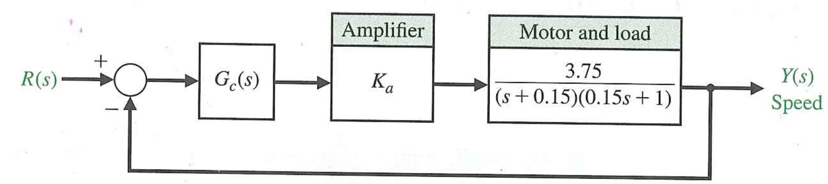

FIGURE P10.5 Stabilized rate table.

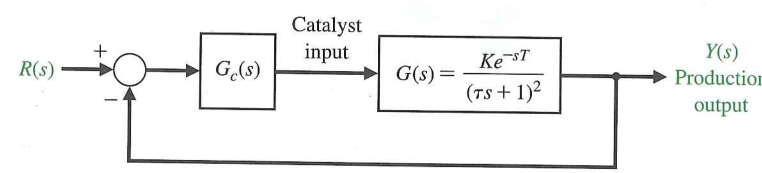

FIGURE P10.7 Chemical reactor control.

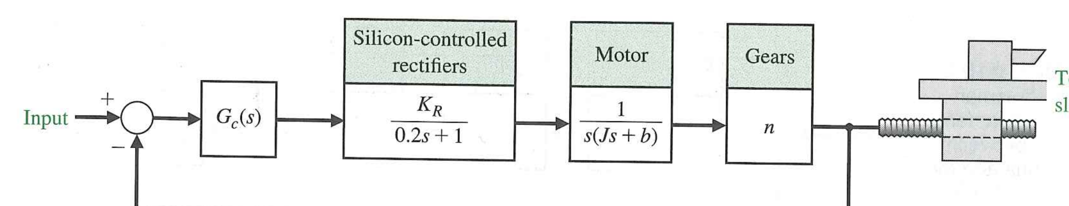

FIGURE P10.8 Path-controlled turret lathe.