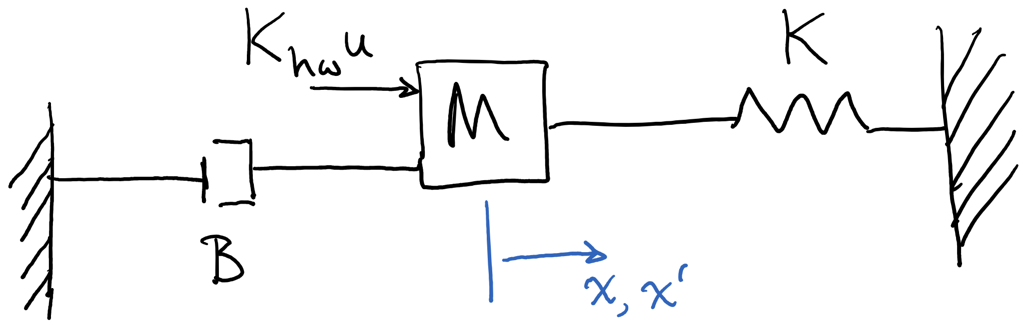
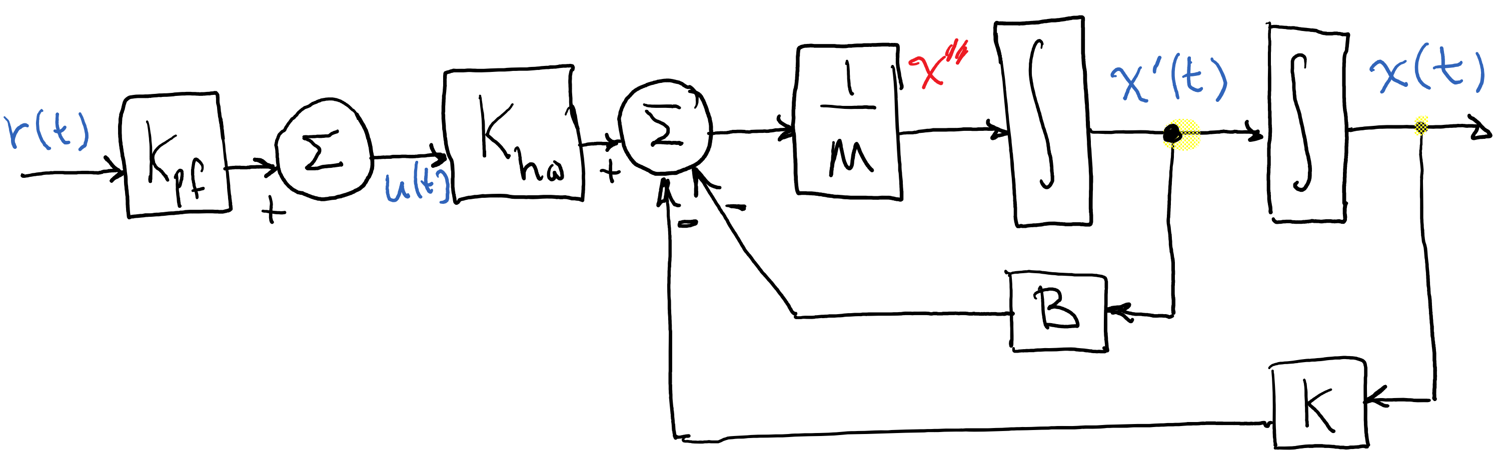
Constant Gain Feedback & Time Response



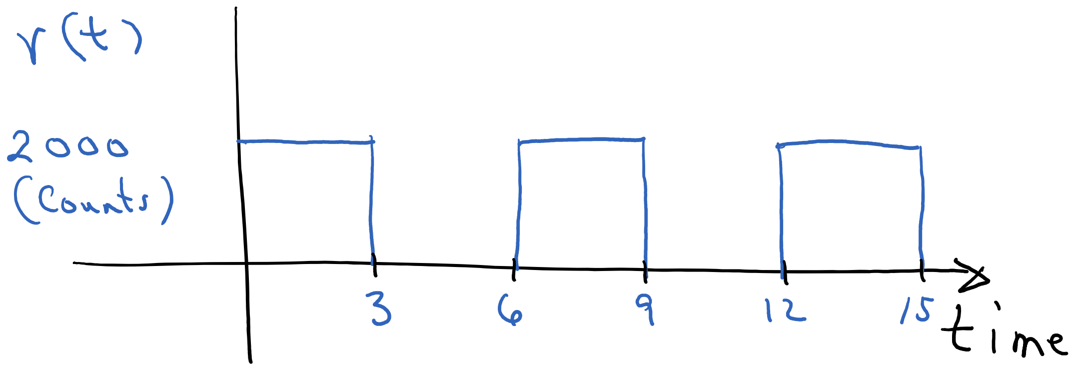
$$M x''(t) + B x'(t) + K x(t) = K_{hw} u$$

$$\Longrightarrow x''(t) = \frac{1}{M}\left(-Bx'(t) - Kx(t) + K_{hw} u(t)\right)$$

Block Diagram (Open Loop)



Note: Lab #1, $u(t) = K_{pf}\, r(t)$

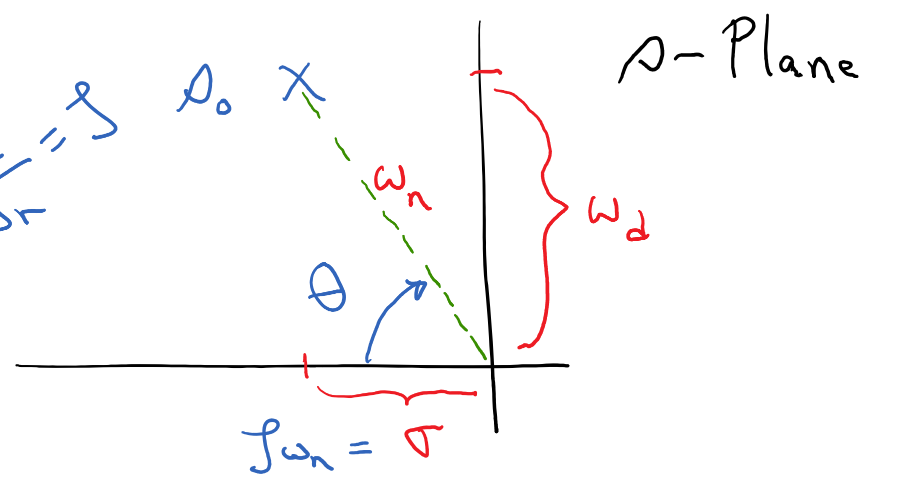


Open-loop Transfer Function:

$$T_o(s) = \frac{X(s)}{U(s)} = \frac{(K_{hw}/M)}{s^2 + \frac{B}{M}s + \frac{K}{M}}$$

Open-Loop Pole Locations:

$$\cos\theta = \frac{\zeta\omega_n}{\omega_n} = \zeta$$



$$s_0 = -\sigma + j\omega_d = -\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2}$$

How can we change the system behavior (pole locations) without changing B, K, or K_{hw}?

Feedback:

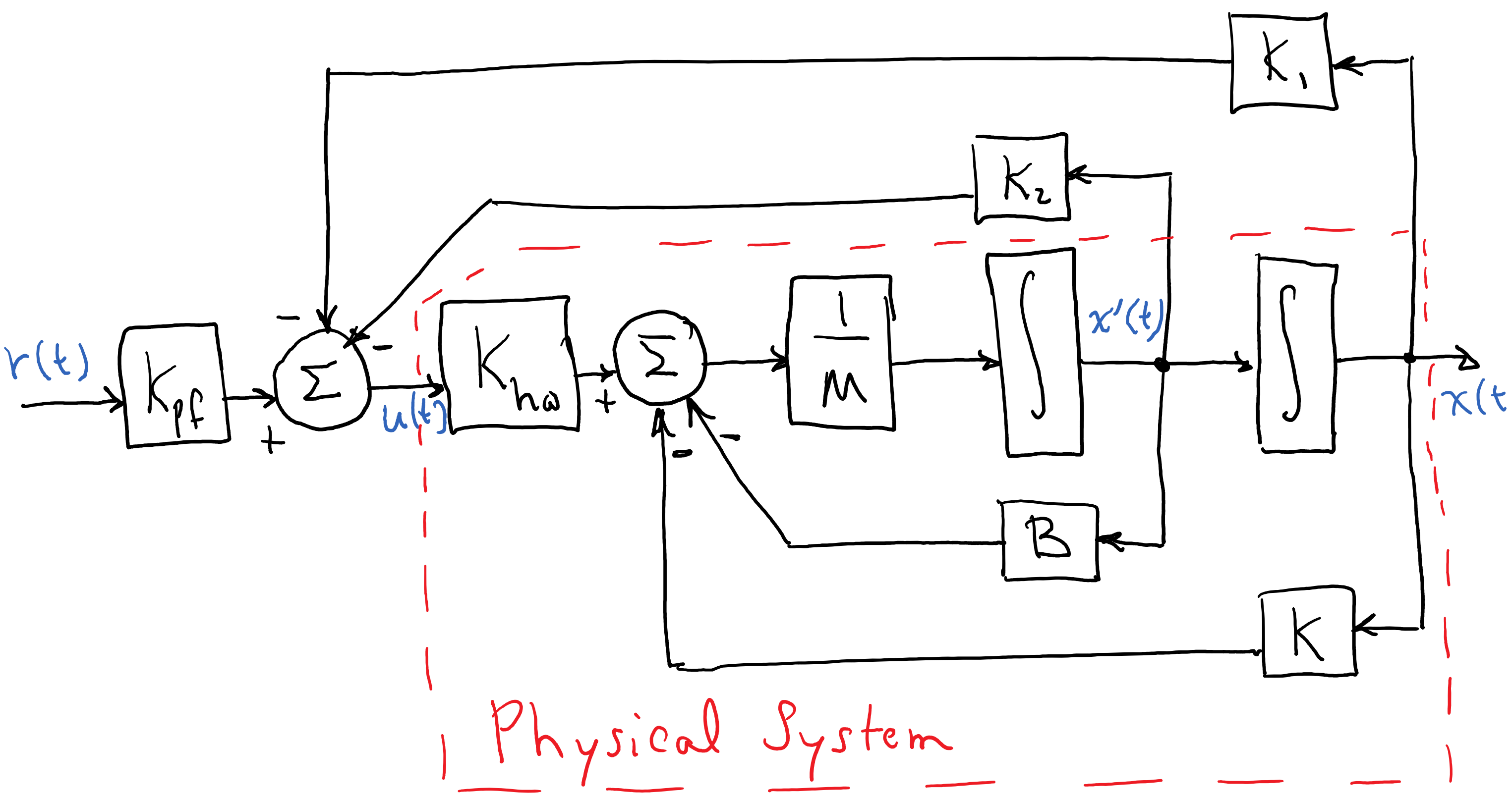
(i) Measure position, velocity, a combination, or both;

(ii) Scale the measured signals; &

(iii) Feed these scaled signals back into the input, $u(t)$.

Constant Gain Feedback

Block Diagram (Closed-loop):



Input Structure:

$$u(t) = -K_1 x(t) - K_2 x'(t) + K_{pf}\, r(t)$$

Incorporate this input into the System:

$$Mx'' = -Bx' - Kx + K_{hw}\left(-K_1 x - K_2 x' + K_{pf}\, r\right)$$

$$Mx'' + Bx' + Kx = K_{hw}(-k_1 x - k_2 x' + k_{pf} r)$$

$$Mx'' + \underbrace{(B + K_{hw}k_2)}_{\bar{B}}\, x' + \overbrace{(K + K_{hw}K_1)}^{\bar{K}}\, x = K_{hw}K_{pf}\, r$$

$\bar{B}$ = Modified damping coefficient
(from velocity feedback)

$\bar{K}$ = Modified spring constant
(from position feedback)

Closed-loop Transfer Function:

$$T_c(s) = \frac{X(s)}{R(s)} = \frac{(K_{hw}k_{pf}/M)}{s^2 + \left(\frac{B + K_{hw}k_2}{M}\right)s + \left(\frac{K + K_{hw}k_1}{M}\right)}$$

Recall the Standard Denominator Expression

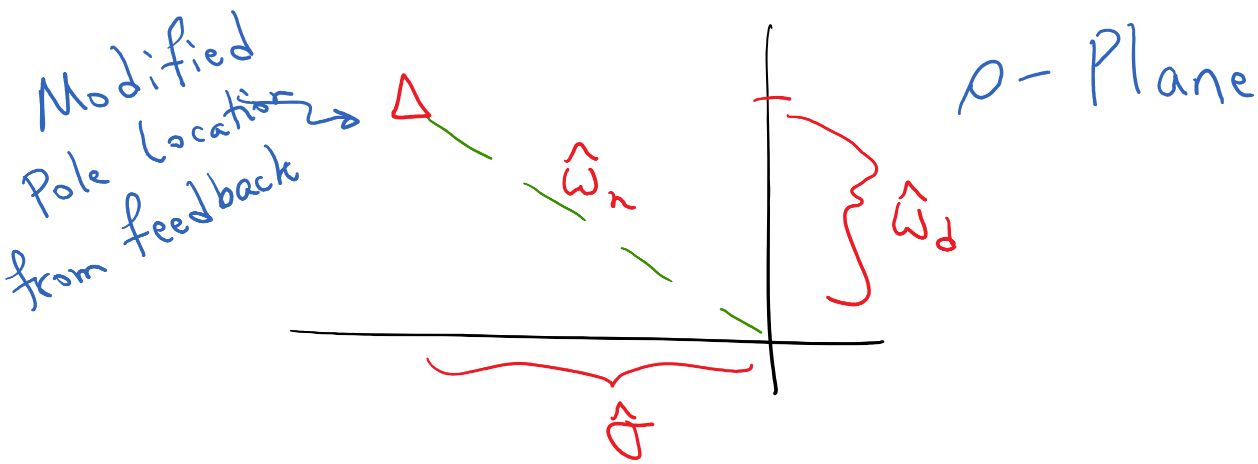
$$D[s] = (s+\sigma)^2 + (\omega_d)^2$$

$$= s^2 + 2\sigma s + (\sigma^2 + \omega_d^2)$$

- OR -

$$D[s] = s^2 + 2\zeta\omega_n s + \omega_n^2$$

$$\Longrightarrow \sigma = \zeta\omega_n \ , \quad \omega_n^2 = \sigma^2 + \omega_d^2$$



Examples:

(1) Suppose we want more damping.

$$2\left(\frac{B}{M}\right) = \left(\frac{B + K_{hw} k_2}{M}\right)$$

Solve for k_2:

$$2B = B + K_{hw} K_2$$

$$B = K_{hw} K_2$$

$$\boxed{K_2 = \frac{B}{K_{hw}}}$$

(2) Suppose we want to modify the system behavior even more.

Suppose we want the following behavior from our system:

(i) A new natural frequency, $\hat{\omega}_n$, of 35.

(ii) A new percent overshoot, $\hat{P.O.}$, of 10%. ?

(ii) A D.C. gain of one, i.e., $T_c[0] = 1$.

(Wanting the output to equal the input in the steady-state, i.e., after the transients have dissipated.)

Relating these desired performance specifications to our system with feedback.

Spec (i): $\hat{\omega}_n^2 = (35)^2 = \dfrac{K + K_{hw} k_1}{M}$

$$\therefore \boxed{k_1 = \frac{(\hat{\omega}_n)^2 M - k}{K_{hw}}}$$

Position Feedback Gain

Spec (ii) : 10% Overshoot $\Longrightarrow \zeta \approx 0.6$

$$\zeta = \left[\frac{\ln^2\left(\frac{P.O.}{100}\right)}{\pi^2 + \ln^2\left(\frac{P.O.}{100}\right)} \right]^{1/2}$$

$$2\hat{\zeta}\hat{\omega}_n = \frac{B + K_{hw}K_2}{M}$$

$$\boxed{K_2 = \frac{2\hat{\zeta}\hat{\omega}_n M - B}{K_{hw}}}$$

Velocity Feedback Gain

$\hat{\zeta} = 0.6$ and $\hat{\omega}_n = 35$.

Spec (iii) DC Gain equal to one

$$\therefore T_c[0] = 1$$

$$T_c[0] = \frac{(K_{hw}K_{pf}/M)}{(K + K_{hw}K_1)}$$

m

$$\Longrightarrow \quad \frac{K_{hw} K_{pf}}{K + K_{hw} K_1} = 1$$

$$K_{hw} K_{pf} = K + K_{hw} K_1$$

$$K_{pf} = \frac{K + K_{hw} K_1}{K_{hw}} = \frac{K}{K_{hw}} + K_1$$

$$\boxed{K_{pf} = K_1 + \frac{K}{K_{hw}}}$$

Reference Input Gain