ECE 441a/541a **Laboratory #2**

Exercise

Record Number of Masses and Spring Number:

Number of Masses = ________ , Total Mass = ________

Spring Number = ________

a) Write down the numerical values for your transfer function expression from Laboratory #1 and write down the gain K_{pf} that was used for the open-loop response in Laboratory #1.

$$G[s] = \frac{\frac{K_{hw}}{M}}{s^2 + \frac{B}{M}s + \frac{K}{M}} = \qquad\qquad , K_{pf} = \_\_\_\_\_\_$$

b) Determine the pole locations of your system model. (Roots of $s^2 + \frac{B}{M}s + \frac{K}{M}$.)

$$p_1 = -\sigma + j\omega_d = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

$$p_2 = -\sigma - j\omega_d = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

c) Find the angle, θ , and the damping ratio, ζ , associated with these open-loop poles.

$$\theta = \_\_\_\_\_\_\_ , \qquad \zeta = \_\_\_\_\_\_\_$$

d) From the above roots that are parameterized by σ , ω_d, and ζ , i.e., the damping, damped frequency, and damping ratio, respectively, calculate the percent overshoot, P.O., the period of oscillation, T, the peak time, T_p, and the 2% settling time, T_s .

P.O. = __________ , $T = \frac{2\pi}{\omega_d} = \_\_\_\_\_\_\_\_\_\_$

$T_p = \frac{1}{2}T = \frac{\pi}{\omega_d} = \_\_\_\_\_\_\_\_\_\_$, $T_s = 4\tau = \frac{4}{\sigma} = \_\_\_\_\_\_\_\_\_\_\_\_$

The differential equation describing the open-loop system is given by

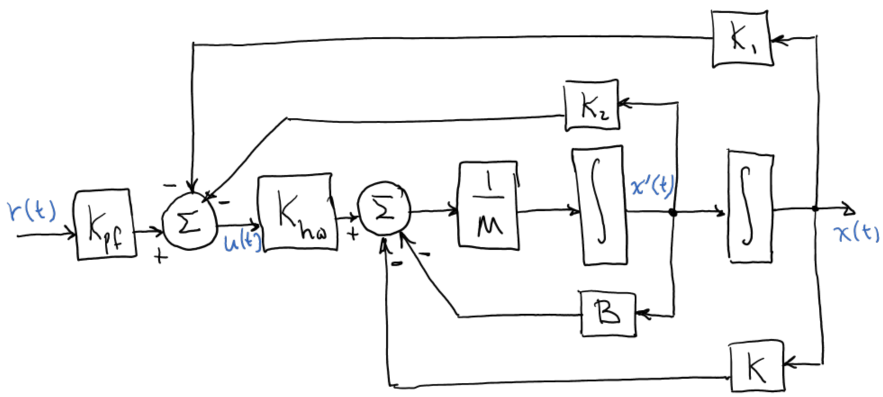
$$M\ddot{x}(t) + B\dot{x}(t) + Kx(t) = K_{hw}u(t)$$

As discussed in the video for Lab #2, Laboratory #2 will be using different feedback signals to help change the response characteristics of your system. Before examining the particular feedback signals that will be used individually in Laboratory #2, let's recall the most general feedback structure for this lab that incorporates the feedback of the position signal, the velocity signal, and incorporates the reference input term to create input. Note that in laboratory #2, these signals are scaled by constants. The structure of the input signal applied to your system is given by

$$u(t) = K_{pf}r(t) - K_1x(t) - K_2\dot{x}(t) \ ,$$

where K_{pf} is the reference input constant scaling factor, K_1 is the position feedback constant gain, and K_2 is the velocity feedback constant gain.

A block diagram of the mass-spring-damper system using this input structure is provided below.



e) Find the closed-loop transfer function, in terms of $M, B, K, K_{hw}, K_{pf}, K_1$, and K_2 .

$$T[s] =$$

f) Find an expression for the D.C. gain of your system in terms of the system parameter.

$$T[0] =$$

g) Find an expression, in terms of the system parameters, for K_{pf} that will result in a D.C. gain of 1.0.

$$K_{pf} =$$

h) Using the information above, apply constant velocity feedback to double the linear term in the denominator polynomial by the choice of K_2 . In other words, find a value for

K_2 such that $2\left(\frac{B}{M}\right) = \left(\frac{B+K_{hw}K_2}{M}\right)$. Let K_1 be zero and find K_{pf} to produce a DC gain of one.

$K_2 =$ $\qquad$ $K_{pf} =$

i) For your choice of K_{pf} and K_2 in Step (h), compute the new closed-loop poles.

$\bar{p}_1 =$ ________________

$\bar{p}_2 =$ ________________

j) Using the complex poles in Step (i), find the angle, $\bar{\theta}$, and the damping ratio, $\bar{\zeta}$.

$\bar{\theta} =$ _______ , $\bar{\zeta} =$ _______

k) Compute the Percent Overshoot, $\overline{P.O.}$, peak time, $\bar{T}_p$, and the 2% settling time, $\bar{T}_s$, associated with the closed-loop poles in Step (i).

$\overline{P.O.} =$ ____________ , $\bar{T}_p =$ ________________ , $\bar{T}_s =$ ____________

l) In this step, assume you feedback position and velocity independently, i.e., assume you can select K_1 and K_2 independently. Find values of K_1, K_2, and K_{pf} to produce a closed-loop transfer function in your system that yields a percent overshoot of 10%, a natural frequency of 35 radians/second ($\hat{\omega}_n = 35$) and a DC gain of one.

$\hat{K}_1 =$ $\qquad$ $\hat{K}_2 =$ $\qquad$ $\hat{K}_{pf} =$

m) Describe how velocity feedback, i.e., K_2, influences the time domain response characteristics.

n) In step (l), you used two, independent feedback gain values, K_1 and K_2 to independently scale position and velocity signals. This feedback approach, where you have access to all of the state variable components, is called full-state feedback. Compare how the feedback in Step (l) (full-state feedback) differs from the feedback in Step (h) (the feedback of a single state variable component).