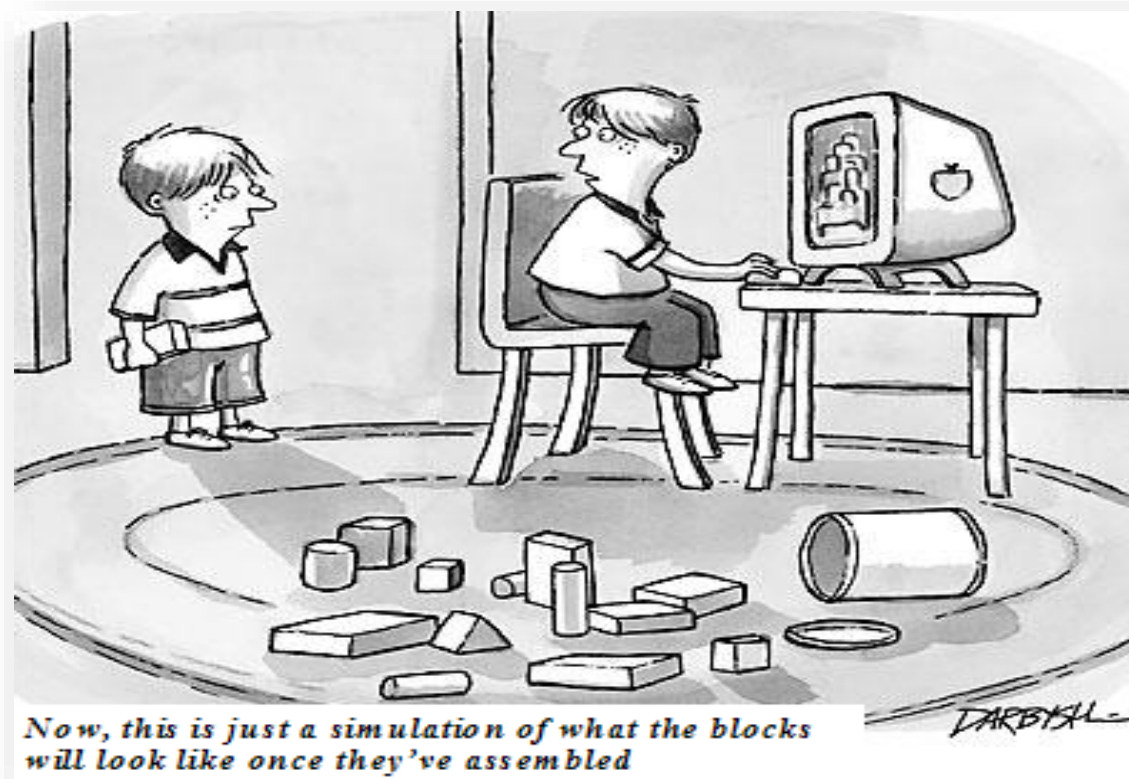


ECE569

Module 21



- Matrix Multiplication

Matrix Multiplication

Goal: *Learn ways to reduce the limiting effect of memory bandwidth on parallel kernel performance*

- **This module:**
 - Global memory usage
 - Naïve implementation
 - Expose performance bottleneck
- **Later**
 - understand the design of a tiled parallel algorithm for matrix multiplication
 - Shared memory
 - Loading a tile
 - Phased execution
 - Barrier Synchronization

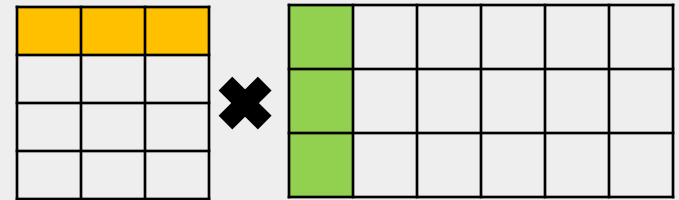
Matrix Multiplication

- **Vector multiplication**

- One Row in M with One Column in N
- Between $M(r1, c1)$ and $N(r2, c2)$, generates $P(r1, c2)$ matrix

```
// Multiplying matrices M and N and  
// storing result in P matrix
```

```
for(i=0; i<r1; ++i)  
  for(j=0; j<c2; ++j)  
    for(k=0; k<c1; ++k)  
    {  
      P[i][j]+=M[i][k]*N[k][j];  
    }
```



Matrix Multiplication (Square Matrix!)

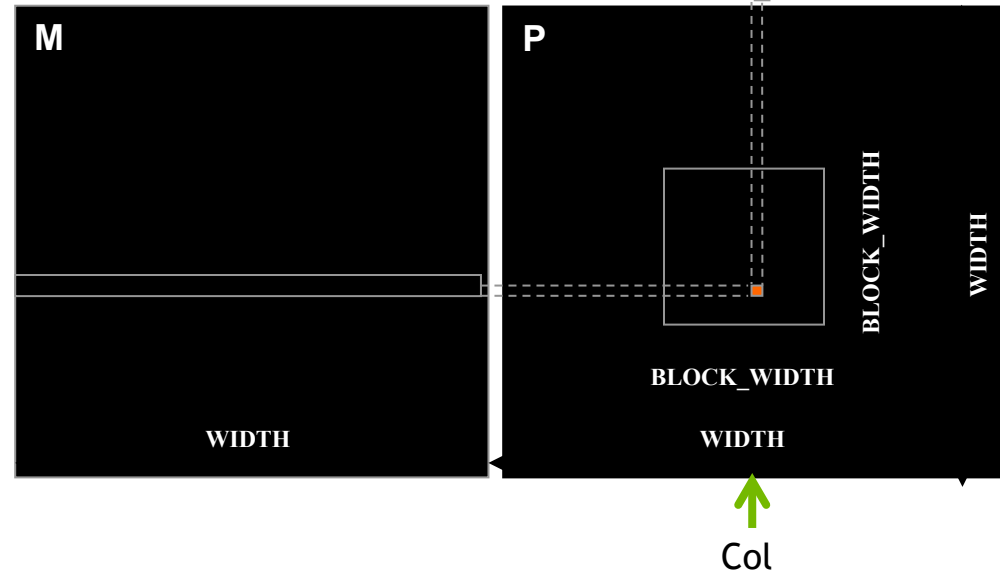
- **Vector multiplication**

- Convention: Row,Col is the element at Rowth position in the (y) vertical direction and Colth position in the (x) horizontal direction.

$$P_{\text{Row},\text{Col}} = \sum M_{\text{Row},k} * N_{k,\text{Col}}$$

Where $k = 0, 1, \dots, \text{Width}-1$

Row →



$P_{\text{Row},\text{Col}}$ is the inner product of the Rowth row of M and the Colth column of N

$$P_{1,5} = M_{1,0} * N_{0,5} + M_{1,1} * N_{1,5} + M_{1,2} * N_{2,5} + \dots + M_{1,\text{Width}-1} * N_{\text{Width}-1,5}$$

Basic Matrix Multiplication (Square Matrix!)

```
__global__ void MatrixMulKernel(float* M, float* N, float* P,
int Width) {
```

```
// Calculate the row index of the P element and M
```

```
int Row =
```

```
// Calculate the column index of P and N
```

```
int Col =
```

}

Basic Matrix Multiplication (Square Matrix!)

```
__global__ void MatrixMulKernel(float* M, float* N, float* P,  
int Width) {
```

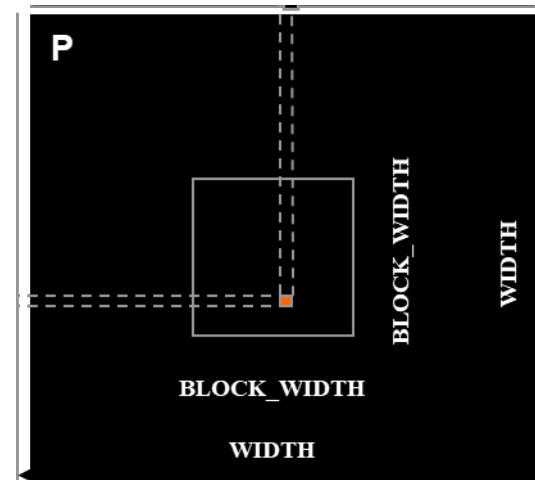
```
    // Calculate the row index of the P element and M
```

```
    int Row = blockIdx.y*blockDim.y+threadIdx.y;
```

```
    // Calculate the column index of P and N
```

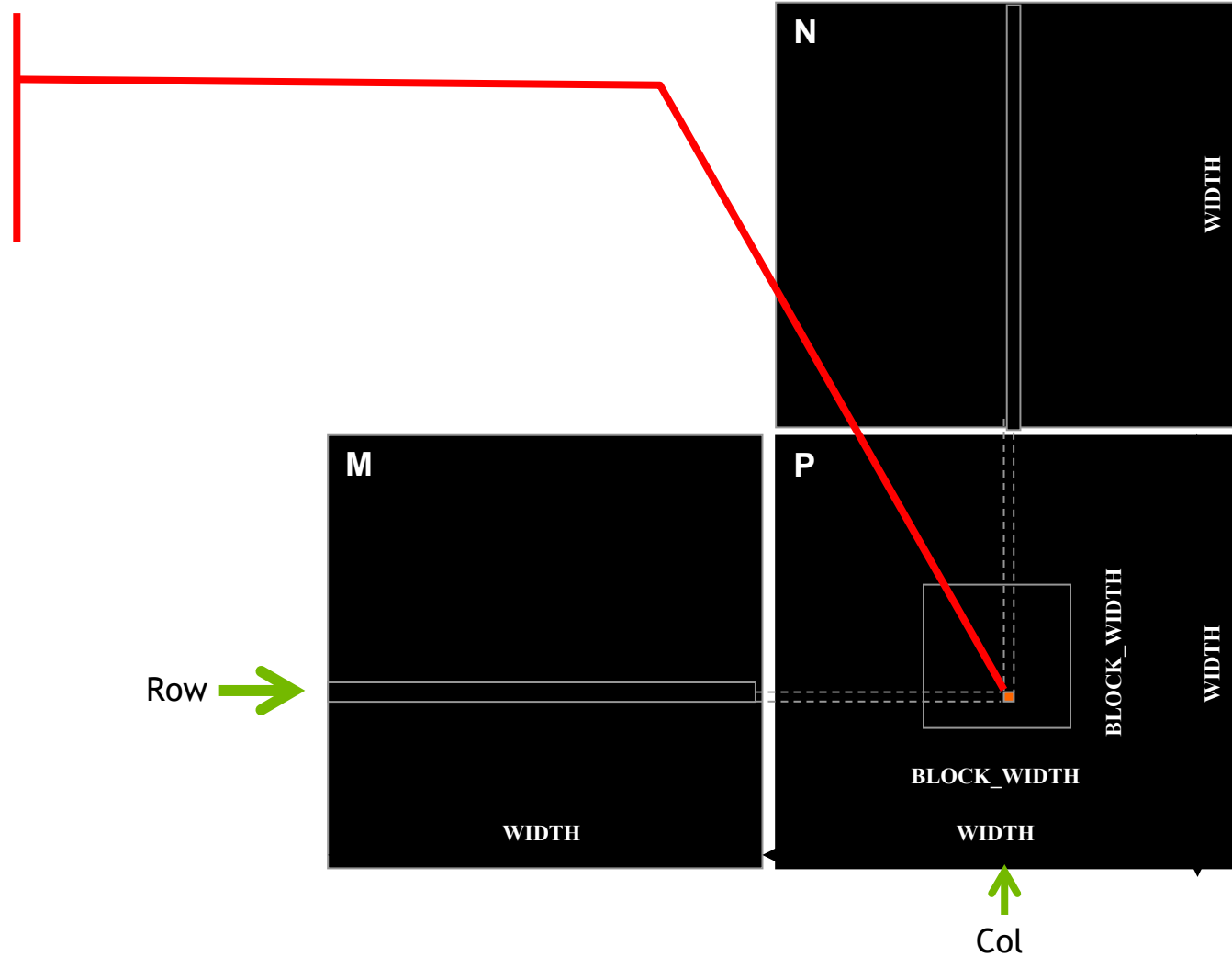
```
    int Col = blockIdx.x*blockDim.x+threadIdx.x;
```

```
}
```



Mapping thread to a P(Row,Col)

- $P[???]$

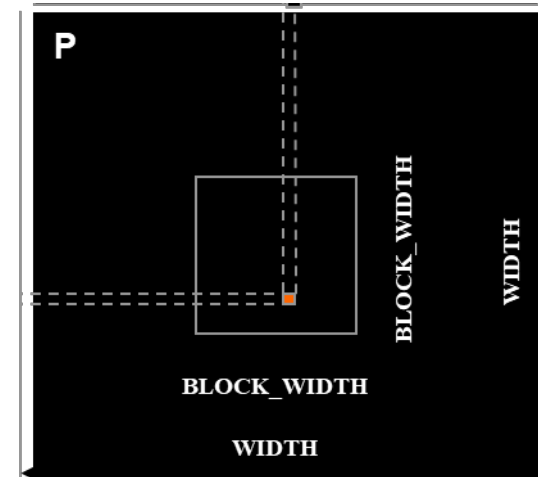


Basic Matrix Multiplication (Square Matrix!)

```
__global__ void MatrixMulKernel(float* M, float* N, float* P,
int Width) {
    // Calculate the row index of the P element and M
    int Row = blockIdx.y*blockDim.y+threadIdx.y;
    // Calculate the column index of P and N
    int Col = blockIdx.x*blockDim.x+threadIdx.x;

    // Boundary condition
    if (

        // each thread computes one element of the block sub-matrix
        // in P[Row*Width+Col]
```

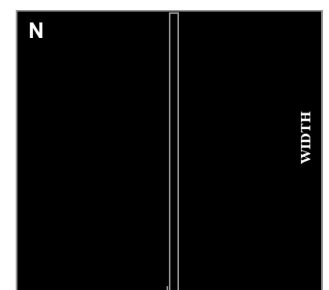
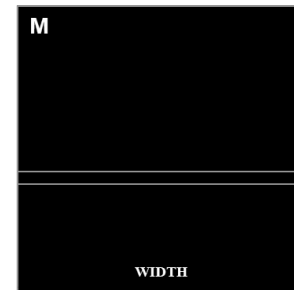


Basic Matrix Multiplication (Square Matrix!)

```
__global__ void MatrixMulKernel(float* M, float* N, float* P,
int Width) {
    // Calculate the row index of the P element and M
    int Row = blockIdx.y*blockDim.y+threadIdx.y;
    // Calculate the column index of P and N
    int Col = blockIdx.x*blockDim.x+threadIdx.x;
    if ((Row < Width) && (Col < Width)) {
        // each thread computes one element of the block sub-matrix
        // in P[Row*Width+Col]
        // Insert multiplication expression in a loop

```

```
}
```



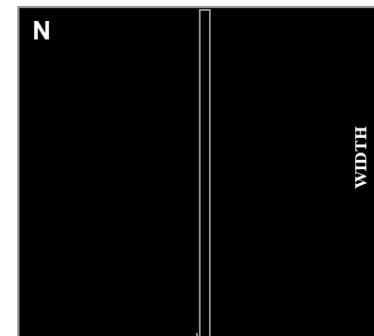
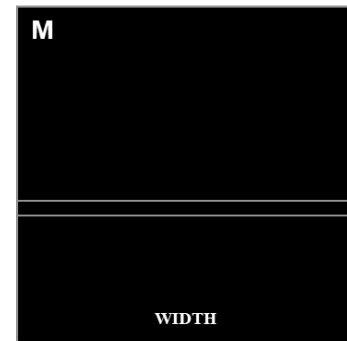
Basic Matrix Multiplication (Square Matrix!)

```
__global__ void MatrixMulKernel(float* M, float* N, float* P,
int Width) {

    // Calculate the row index of the P element and M
    int Row = blockIdx.y*blockDim.y+threadIdx.y;

    // Calculate the column index of P and N
    int Col = blockIdx.x*blockDim.x+threadIdx.x;

    if ((Row < Width) && (Col < Width)) {
        float Pvalue = 0;
        // each thread computes one element of the block sub-matrix
        for (int k = 0; k < Width; ++k) {
            Pvalue += M[Row*Width+k]*N[k*Width+Col];
        }
        P[Row*Width+Col] = Pvalue;
    }
}
```



Basic Matrix Multiplication (Square Matrix!)

```
if ((Row < Width) && (Col < Width)) {  
    float Pvalue = 0;  
    // each thread computes one element of the block sub-matrix  
    for (int k = 0; k < Width; ++k) {  
        Pvalue += M[Row*Width+k]*N[k*Width+Col];  
    }  
    P[Row*Width+Col] = Pvalue;  
}
```

P100: 9300GFLOPS, 720GB/sec

How far are we from peak FLOPS for the matrix multiplication with global memory?

Basic Matrix Multiplication (Square Matrix!)

```
if ((Row < Width) && (Col < Width)) {  
    float Pvalue = 0;  
    // each thread computes one element of the block sub-matrix  
    for (int k = 0; k < Width; ++k) {  
        Pvalue += M[Row*Width+k]*N[k*Width+Col];  
    }  
    P[Row*Width+Col] = Pvalue;  
}
```

2 Global memory accesses

1 floating point multiply operation

1 floating point addition

1 memory access per FP operation

P100: 9300GFLOPS, 720GB/sec

$(720\text{GB/sec}) / (4\text{B/FPop}) = 180\text{GFLOPS max out of } 9300$

Massive under utilization!

Next

- **Tiling method**