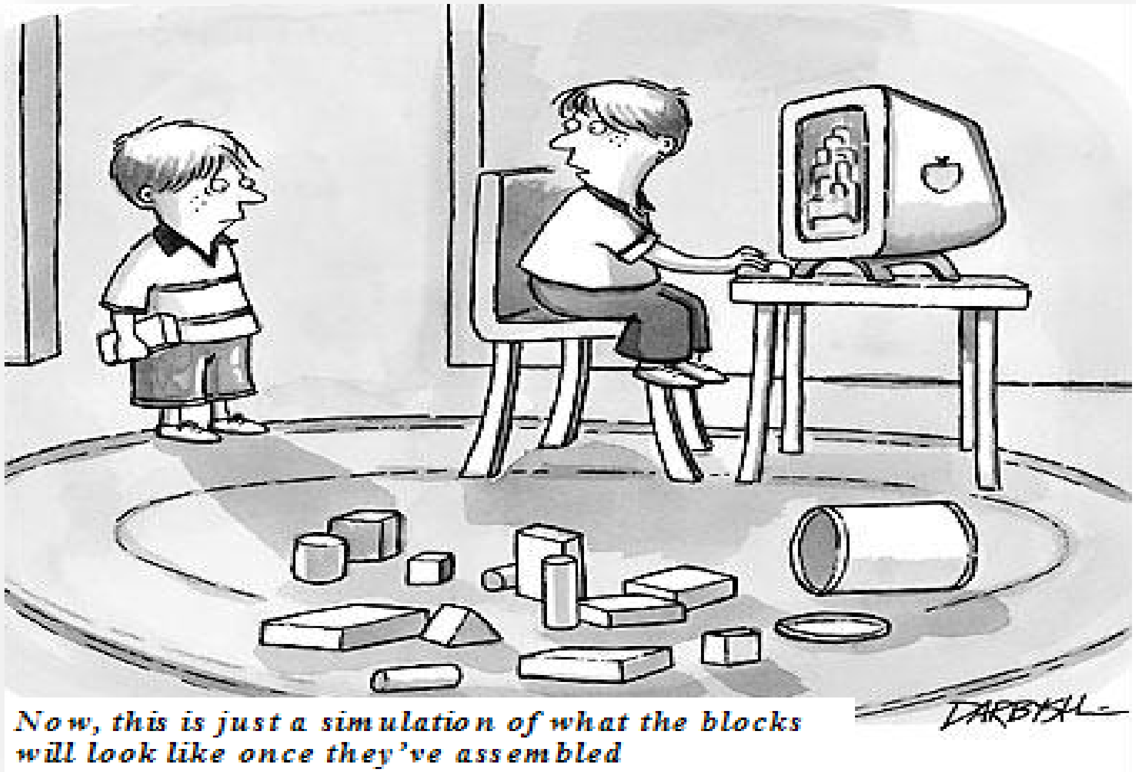
ECE569
Module 23



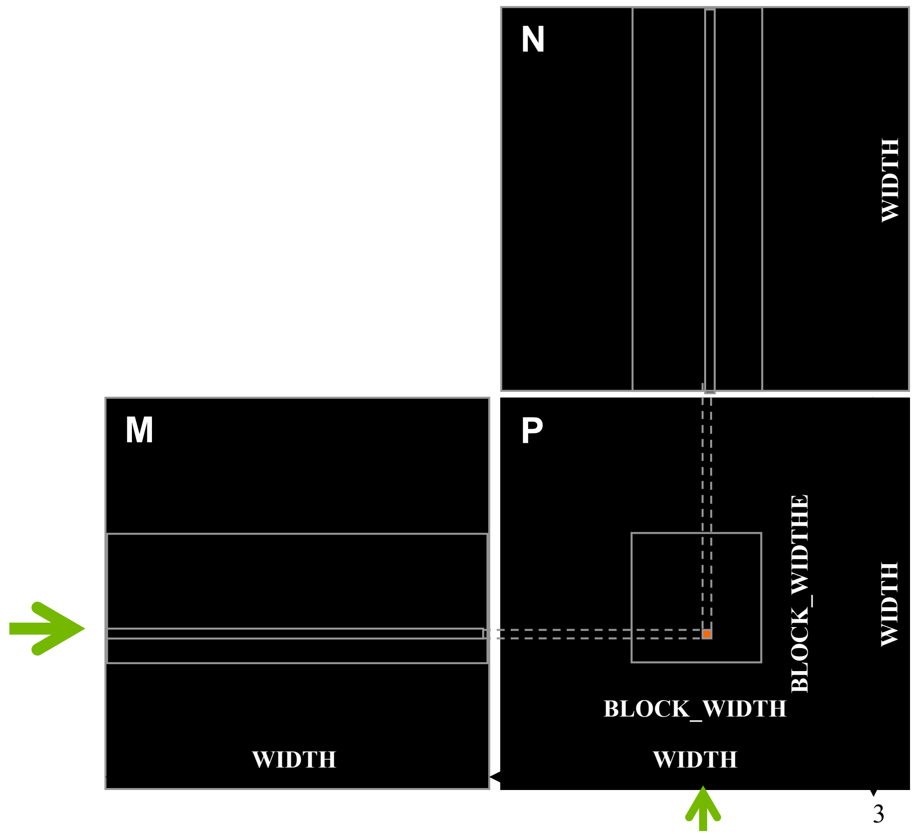
- Tiling for Matrix Multiplication

Outline of Tiling Technique

- Identify a tile of global memory contents that are accessed by multiple threads
- Load the tile from global memory into on-chip memory
- Use barrier synchronization to make sure that all threads are ready to start the phase
- Have the multiple threads to access their data from the on-chip memory
- Use barrier synchronization to make sure that all threads have completed the current phase
- Move on to the next tile

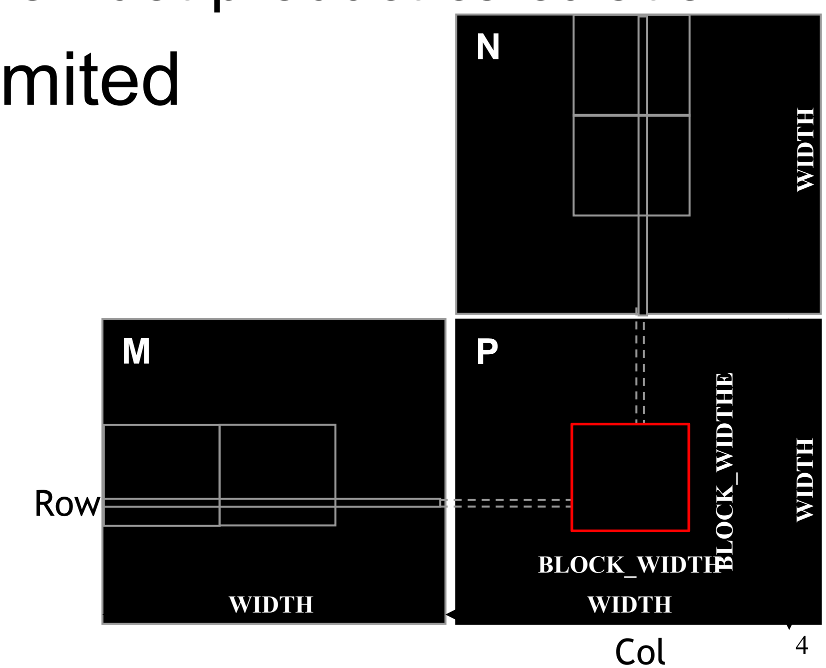
Matrix Multiplication

- **Data access pattern**
  - Each thread - a row of M and a column of N
  - Each thread block – a strip of M and a strip of N



Tiled Matrix-Multiplication

- Divide the M and N matrices into smaller tiles.
  - Breaks up the execution of each thread into phases focusing on one tile of M and one tile of N
  - Threads collaboratively load subsets of the M and N elements into the shared memory
    - The tile is of BLOCK_SIZE elements in each dimension
  - Threads use the elements in their dot product calculation.
- Size of the shared memory limited
  - Do not exceed the capacity
  - Choose tile size carefully

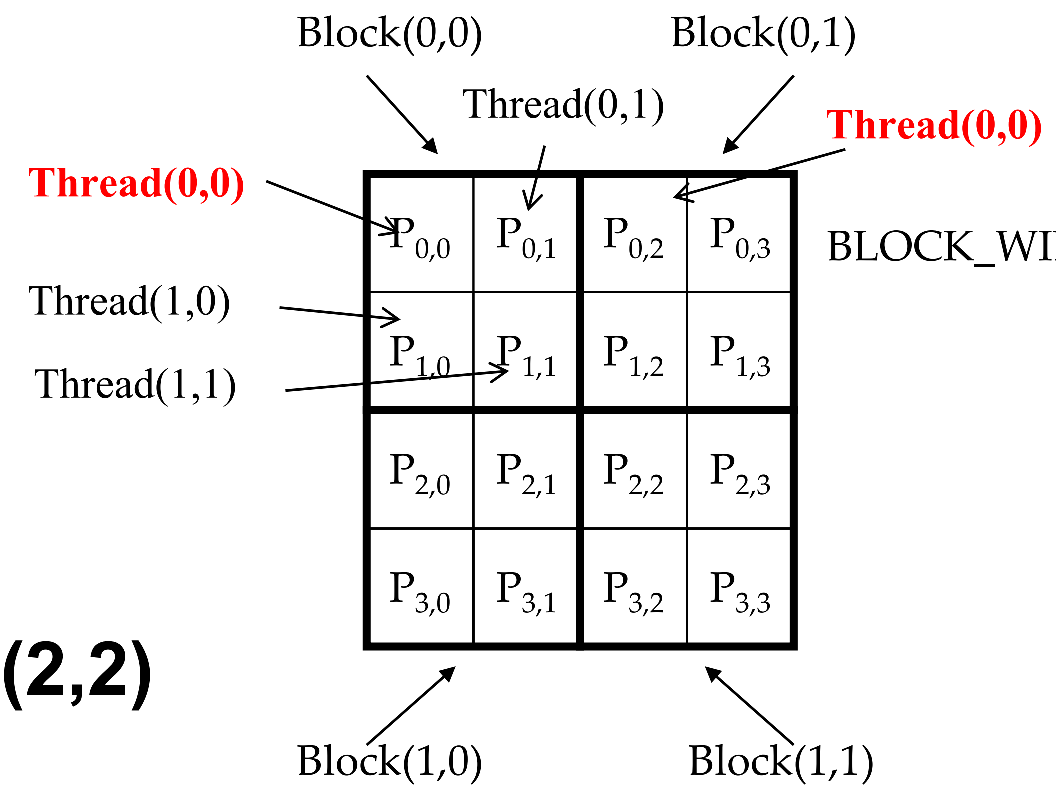


A Toy Example: Thread to P[4x4] Data Mapping

$N_{0,0}$	$N_{0,1}$	$N_{0,2}$	$N_{0,3}$
$N_{1,0}$	$N_{1,1}$	$N_{1,2}$	$N_{1,3}$
$N_{2,0}$	$N_{2,1}$	$N_{2,2}$	$N_{2,3}$
$N_{3,0}$	$N_{3,1}$	$N_{3,2}$	$N_{3,3}$

$P_{0,0}$	$P_{0,1}$	$P_{0,2}$	$P_{0,3}$
$P_{1,0}$	$P_{1,1}$	$P_{1,2}$	$P_{1,3}$
$P_{2,0}$	$P_{2,1}$	$P_{2,2}$	$P_{2,3}$
$P_{3,0}$	$P_{3,1}$	$P_{3,2}$	$P_{3,3}$

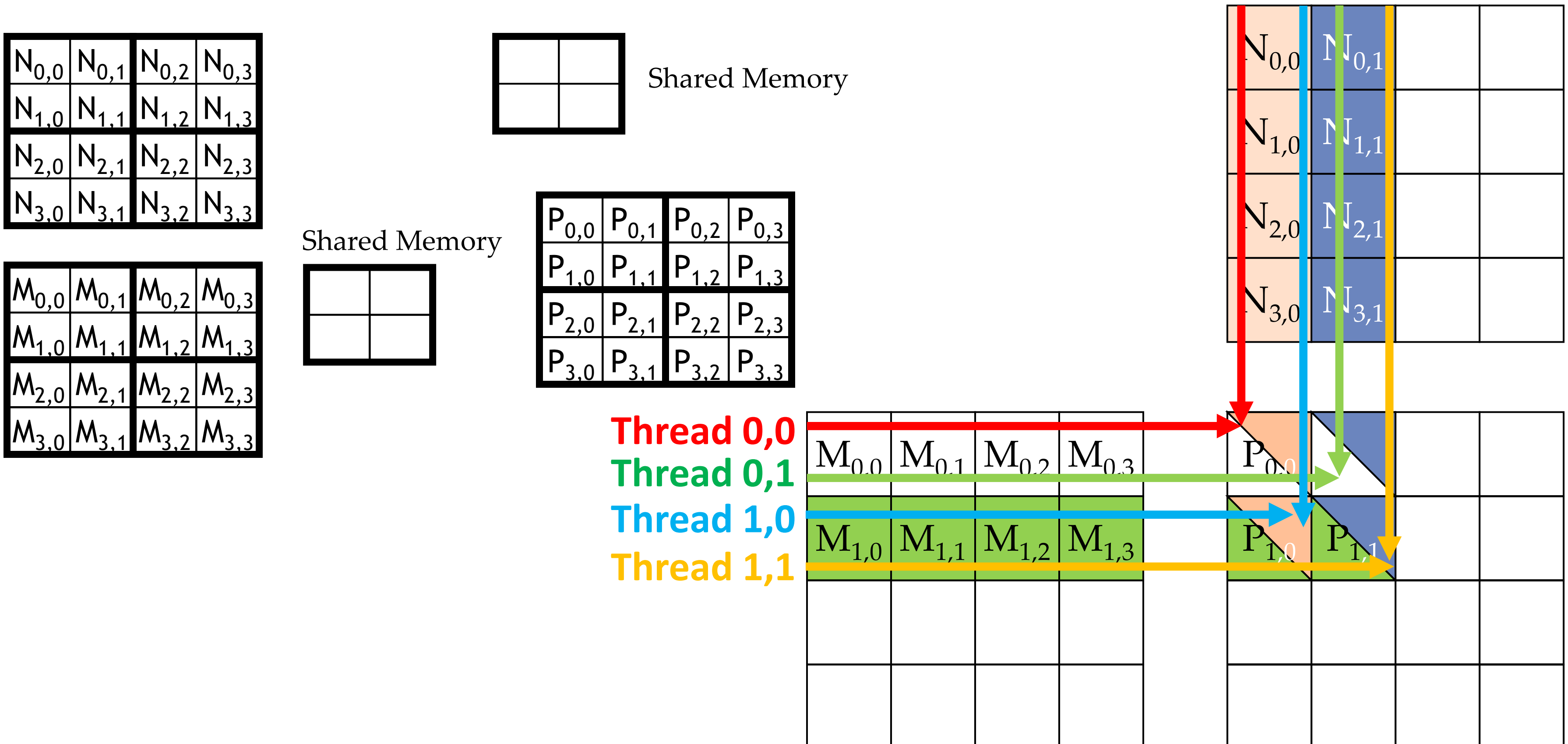
$M_{0,0}$	$M_{0,1}$	$M_{0,2}$	$M_{0,3}$
$M_{1,0}$	$M_{1,1}$	$M_{1,2}$	$M_{1,3}$
$M_{2,0}$	$M_{2,1}$	$M_{2,2}$	$M_{2,3}$
$M_{3,0}$	$M_{3,1}$	$M_{3,2}$	$M_{3,3}$



BLOCK_WIDTH = 2

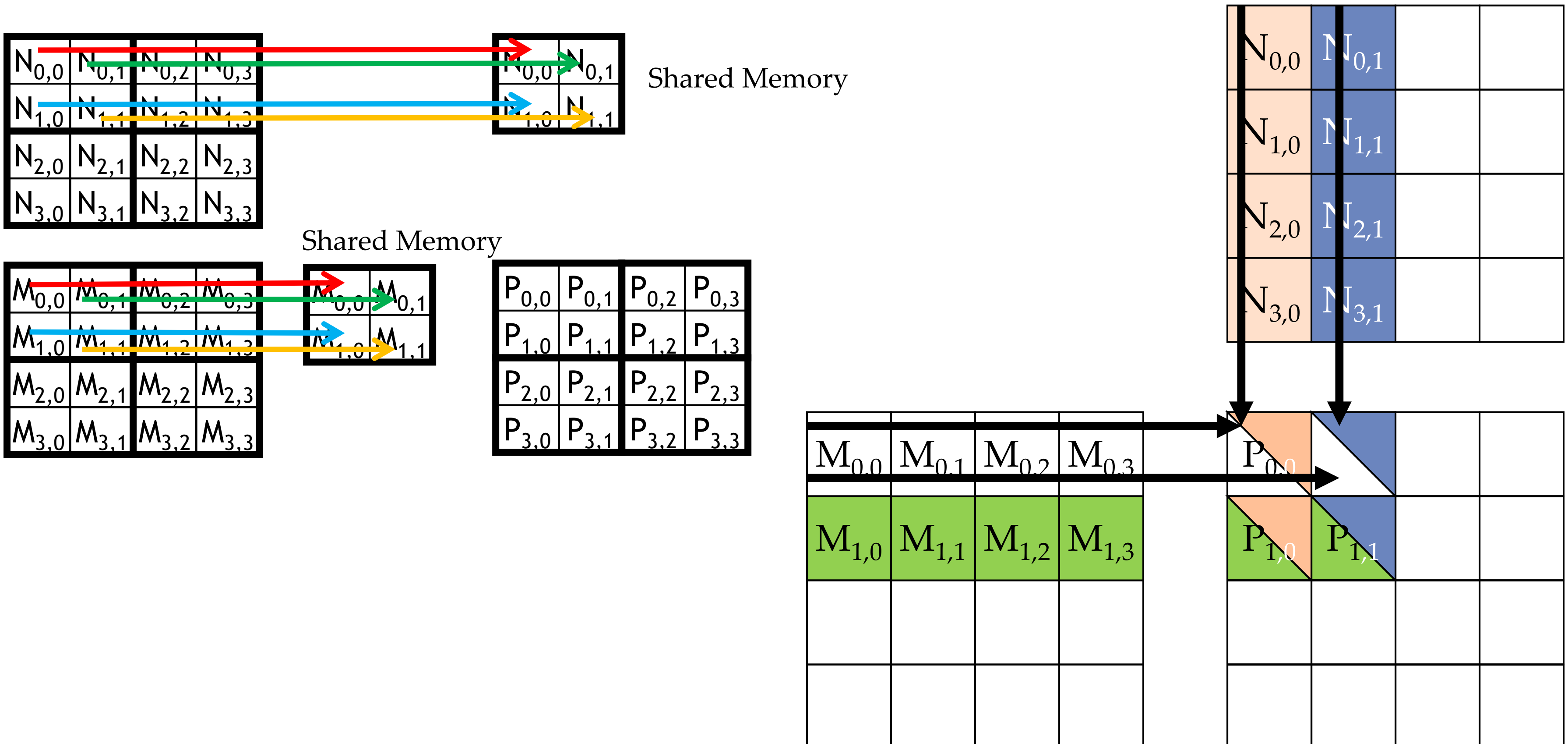
- **Grid(2,2), Block(2,2)**

Phase 0 Load for Block (0,0)



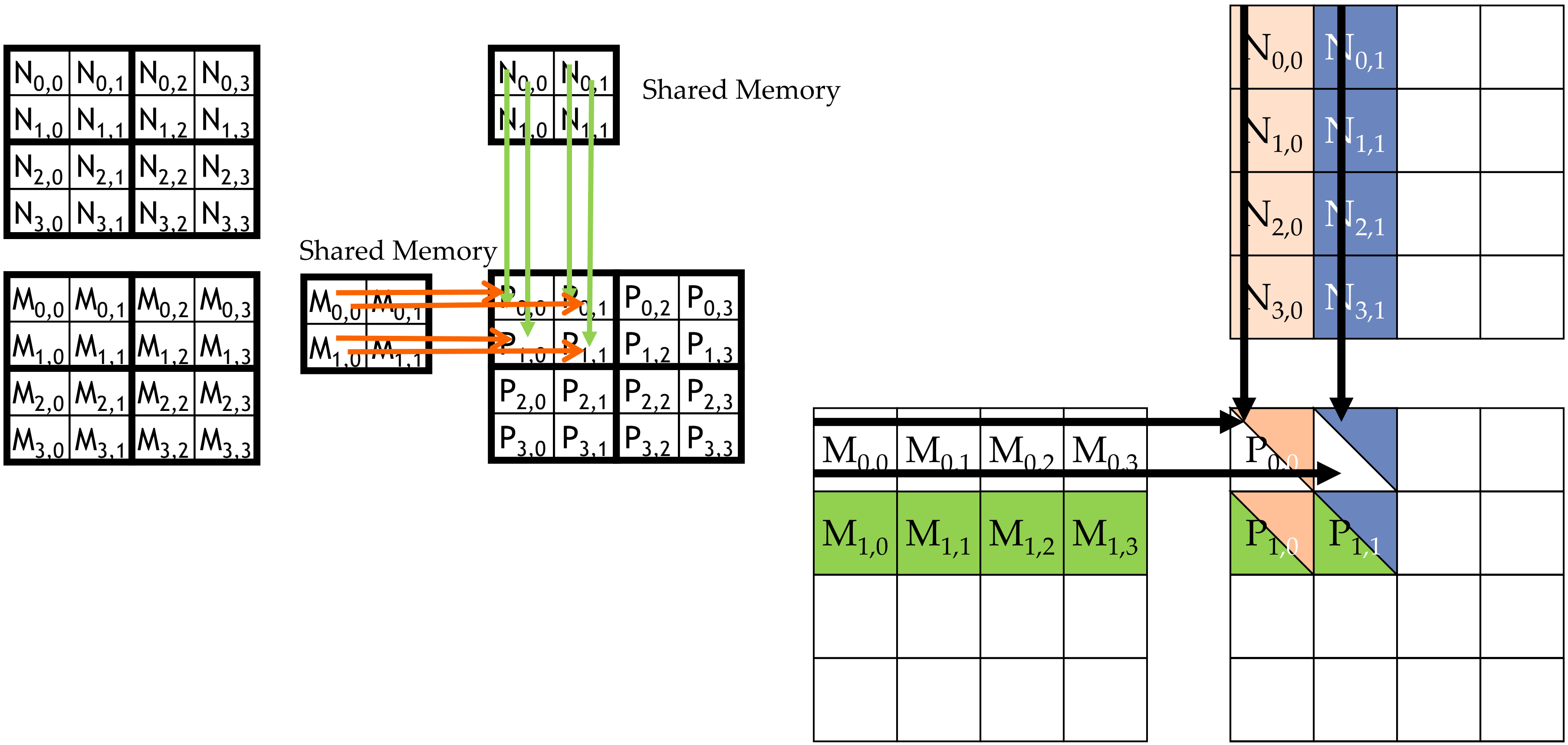
- All threads in a block participate
- Each thread loads one M element and one N element in tiled code
  - four threads of block0,0 collaboratively load a tile of M and N into shared memory

Phase 0 Load for Block (0,0)



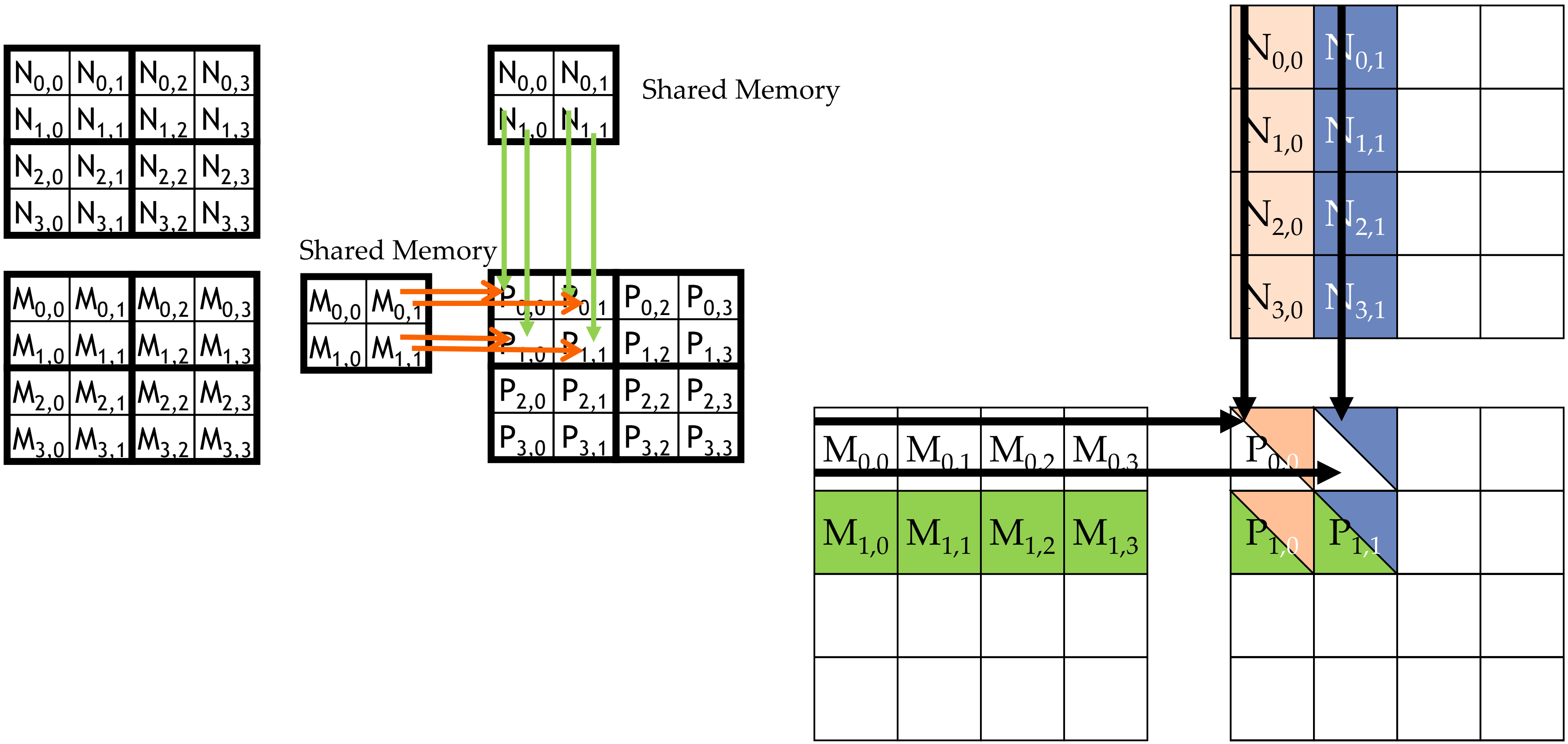
- All threads in a block participate
- Each thread loads one M element and one N element in tiled code
  - four threads of block0,0 collaboratively load a tile of M and N into shared memory

Phase 0 Use for Block (0,0) (iteration 0)



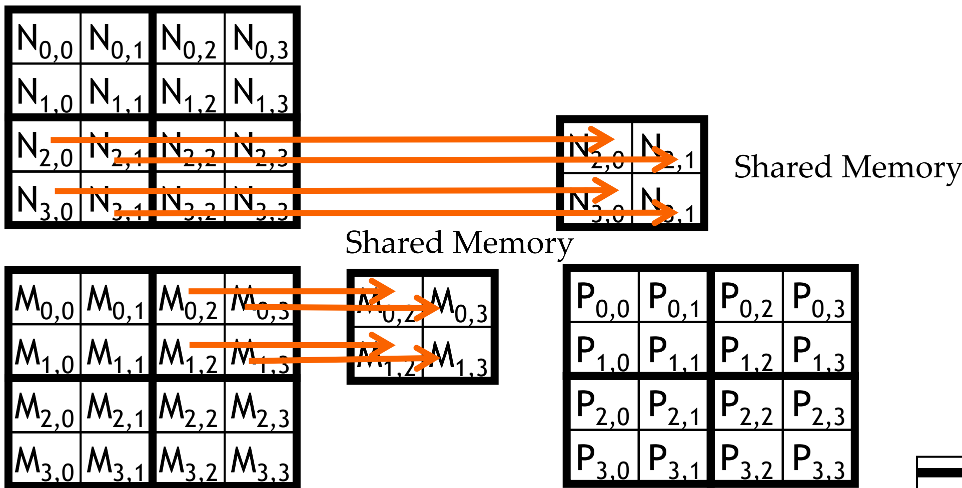
– After tiles of M and N are in the shared memory, these elements are used in the calculation of the dot product
  - Two iterations
– Note that each value in the shared memory is used twice.
  - Reduces global memory access by half

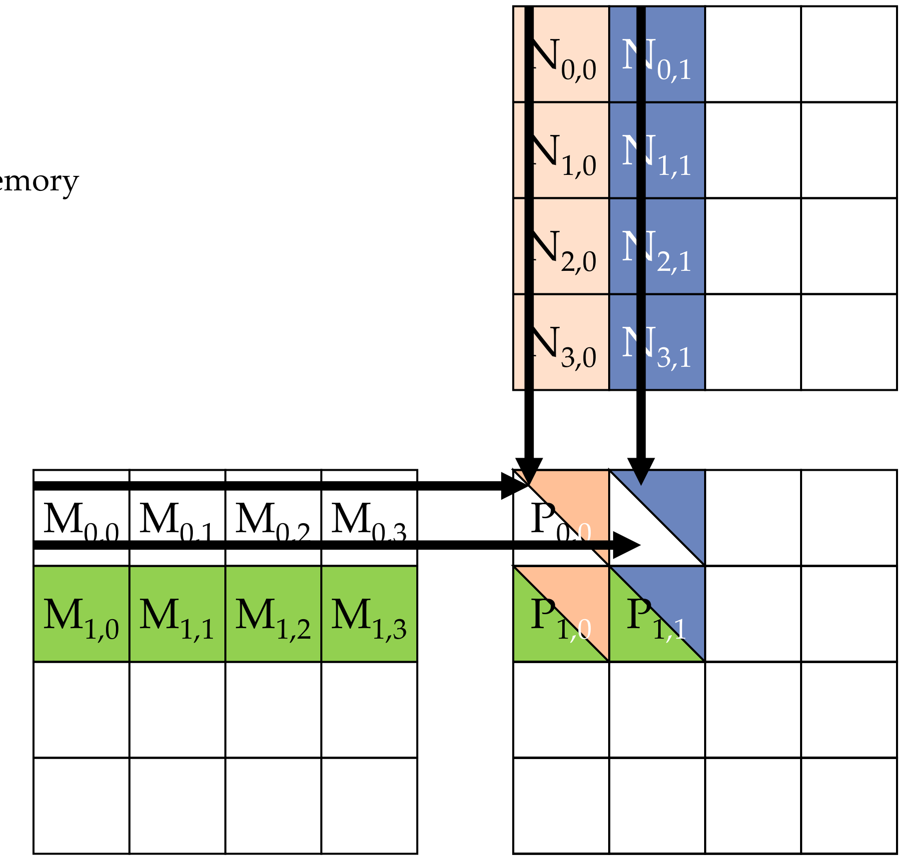
Phase 0 Use for Block (0,0) (iteration 1)



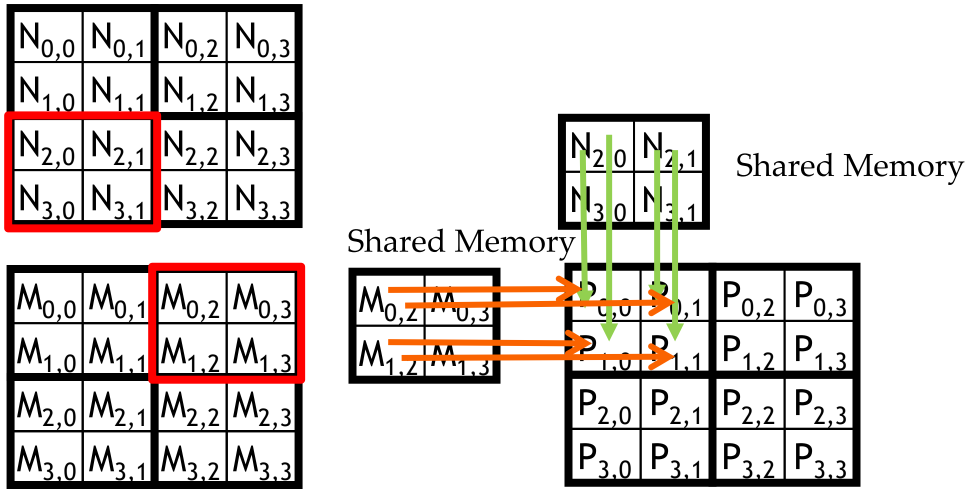
Partial accumulated dot product is a private value generated for each thread

Phase 1 Load for Block (0,0)

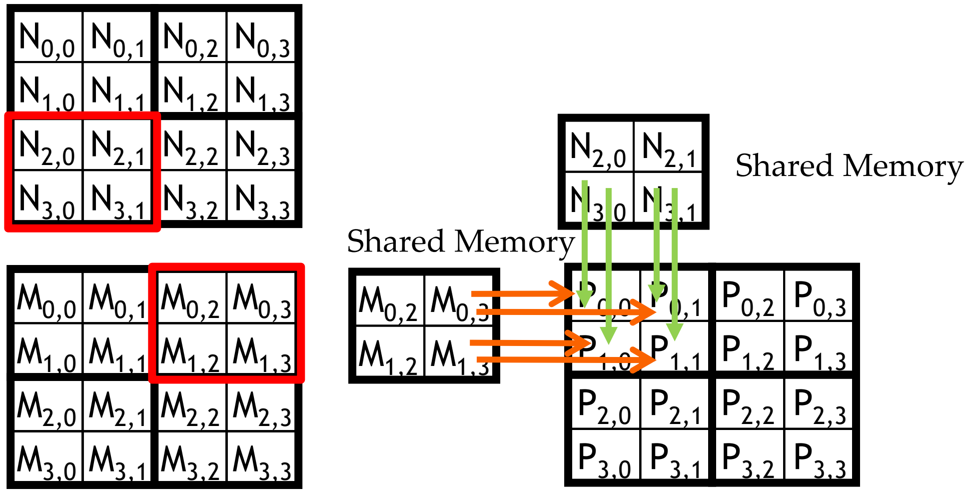




Phase 1 Use for Block (0,0) (iteration 0)



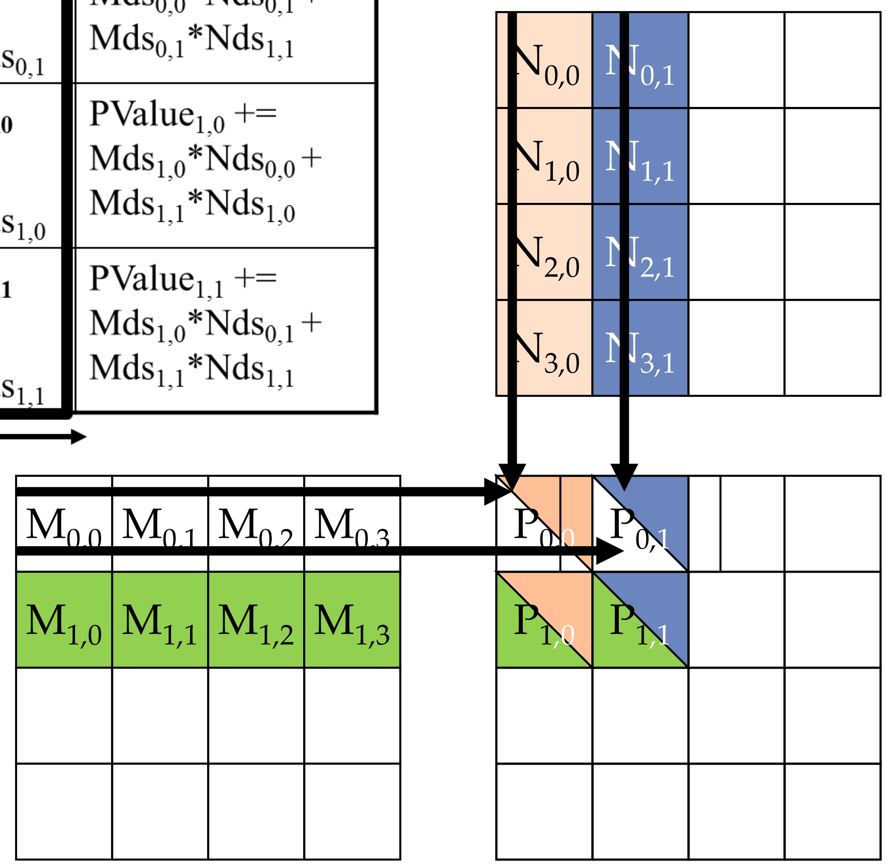
Phase 1 Use for Block (0,0) (iteration 1)



Execution Phases of Toy Example

	Phase 0			Phase 1		
$thread_{0,0}$	$\mathbf{M_{0,0}}$ ↓ $Mds_{0,0}$	$\mathbf{N_{0,0}}$ ↓ $Nds_{0,0}$	$PValue_{0,0}$ += $Mds_{0,0}$*$Nds_{0,0}$ + $Mds_{0,1}$*$Nds_{1,0}$	$\mathbf{M_{0,2}}$ ↓ $Mds_{0,0}$	$\mathbf{N_{2,0}}$ ↓ $Nds_{0,0}$	$PValue_{0,0}$ += $Mds_{0,0}$*$Nds_{0,0}$ + $Mds_{0,1}$*$Nds_{1,0}$
$thread_{0,1}$	$\mathbf{M_{0,1}}$ ↓ $Mds_{0,1}$	$\mathbf{N_{0,1}}$ ↓ $Nds_{1,0}$	$PValue_{0,1}$ += $Mds_{0,0}$*$Nds_{0,1}$ + $Mds_{0,1}$*$Nds_{1,1}$	$\mathbf{M_{0,3}}$ ↓ $Mds_{0,1}$	$\mathbf{N_{2,1}}$ ↓ $Nds_{0,1}$	$PValue_{0,1}$ += $Mds_{0,0}$*$Nds_{0,1}$ + $Mds_{0,1}$*$Nds_{1,1}$
$thread_{1,0}$	$\mathbf{M_{1,0}}$ ↓ $Mds_{1,0}$	$\mathbf{N_{1,0}}$ ↓ $Nds_{1,0}$	$PValue_{1,0}$ += $Mds_{1,0}$*$Nds_{0,0}$ + $Mds_{1,1}$*$Nds_{1,0}$	$\mathbf{M_{1,2}}$ ↓ $Mds_{1,0}$	$\mathbf{N_{3,0}}$ ↓ $Nds_{1,0}$	$PValue_{1,0}$ += $Mds_{1,0}$*$Nds_{0,0}$ + $Mds_{1,1}$*$Nds_{1,0}$
$thread_{1,1}$	$\mathbf{M_{1,1}}$ ↓ $Mds_{1,1}$	$\mathbf{N_{1,1}}$ ↓ $Nds_{1,1}$	$PValue_{1,1}$ += $Mds_{1,0}$*$Nds_{0,1}$ + $Mds_{1,1}$*$Nds_{1,1}$	$\mathbf{M_{1,3}}$ ↓ $Mds_{1,1}$	$\mathbf{N_{3,1}}$ ↓ $Nds_{1,1}$	$PValue_{1,1}$ += $Mds_{1,0}$*$Nds_{0,1}$ + $Mds_{1,1}$*$Nds_{1,1}$

time →



Mds/Nds: shared memory array for the M/N elements.

Shared memory allows each value to be accessed by multiple threads

Memory Bandwidth Utilization

What is the throughput achieved when tile size is set to 16x16 for a square matrix multiplication assuming that memory GPU device delivers 720GB/sec memory bandwidth and peak throughput of 9300GFlops?

Hint: You need to consider number of global memory accesses required in each phase and number of floating point operations are executed per tile.

- ❑ 360 GFLOPS
- ❑ 720GFLOPS
- ❑ 1440 GFLOPS
- ❑ 2880 GFLOPS

Next

- **CUDA implementation of Tiling**