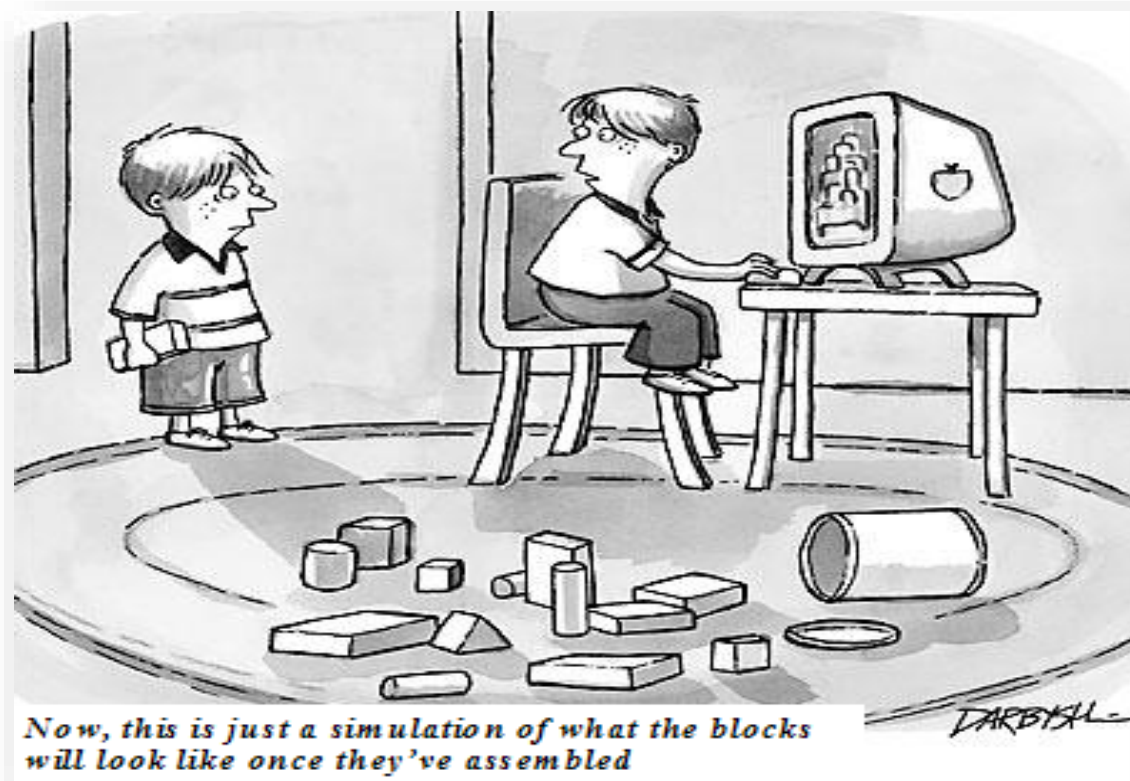


ECE569

Module 25



- Matrix Multiplication – Boundary Conditions

Tiled Matrix Multiplication Kernel

```
__global__ void MatrixMulKernel(float* M, float* N, float* P, int Width)
{
    __shared__ float ds_M[TILE_WIDTH][TILE_WIDTH];
    __shared__ float ds_N[TILE_WIDTH][TILE_WIDTH];
    int bx = blockIdx.x;  int by = blockIdx.y;
    int tx = threadIdx.x; int ty = threadIdx.y;

    int Row = by * TILE_WIDTH + ty;
    int Col = bx * TILE_WIDTH + tx;
    float Pvalue = 0;

    // Loop over the M and N tiles required to compute the P element
    for (int p = 0; p < Width/TILE_WIDTH; ++p) {
        // Collaborative loading of M and N tiles into shared memory
        ds_M[ty][tx] = M[Row*Width + p*TILE_WIDTH+tx];
        ds_N[ty][tx] = N[(p*TILE_WIDTH+ty)*Width + Col];
        __syncthreads();

        for (int i = 0; i < TILE_WIDTH; ++i)
            Pvalue += ds_M[ty][i] * ds_N[i][tx];
        __syncthreads();
    }
    P[Row*Width+Col] = Pvalue;
}
```

Handling Matrix of Arbitrary Size

- **Real applications need to handle arbitrary sized matrices.**
 - One could pad (add elements to) the rows and columns into multiples of the tile size, but would have significant space and data transfer time overhead.
- **We will take a different approach.**
 - Boundary condition checking
 - Regularizing tile contents

Phase 0 Loads for Block (0,0) for a 3x3 Example

$N_{0,0}$	$N_{0,1}$	$N_{0,2}$	
$N_{1,0}$	$N_{1,1}$	$N_{1,2}$	
$N_{2,0}$	$N_{2,1}$	$N_{2,2}$	

$N_{0,0}$	$N_{0,1}$
$N_{1,0}$	$N_{1,1}$

$M_{0,0}$	$M_{0,1}$	$M_{0,2}$	
$M_{1,0}$	$M_{1,1}$	$M_{1,2}$	
$M_{2,0}$	$M_{2,1}$	$M_{2,2}$	

$M_{0,0}$	$M_{0,1}$
$M_{1,0}$	$M_{1,1}$

$P_{0,0}$	$P_{0,1}$	$P_{0,2}$	
$P_{1,0}$	$P_{1,1}$	$P_{1,2}$	
$P_{2,0}$	$P_{2,1}$	$P_{2,2}$	

Phase 0 is ok: all threads access a legitimate address

Phase 1 Loads for Block (0,0) for a 3x3 Example

$N_{0,0}$	$N_{0,1}$	$N_{0,2}$	
$N_{1,0}$	$N_{1,1}$	$N_{1,2}$	
$N_{2,0}$	$N_{2,1}$	$N_{2,2}$	

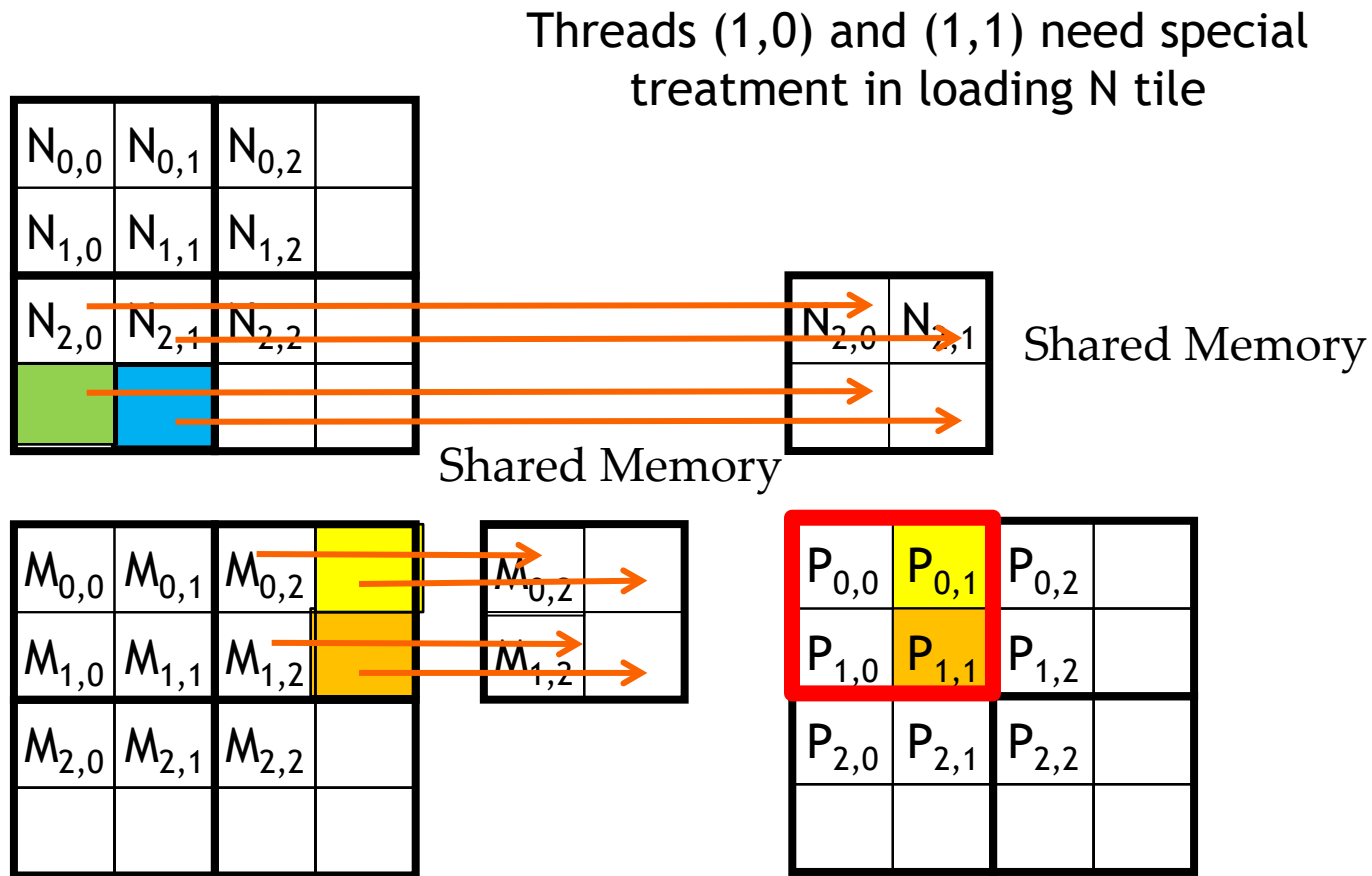
Shared Memory

Shared Memory

$M_{0,0}$	$M_{0,1}$	$M_{0,2}$	
$M_{1,0}$	$M_{1,1}$	$M_{1,2}$	
$M_{2,0}$	$M_{2,1}$	$M_{2,2}$	

$P_{0,0}$	$P_{0,1}$	$P_{0,2}$	
$P_{1,0}$	$P_{1,1}$	$P_{1,2}$	
$P_{2,0}$	$P_{2,1}$	$P_{2,2}$	

Phase 1 Loads for Block (0,0) for a 3x3 Example



If we let thread(0,1) read from M(0,3), what will actually be read into SM(0,1)?

Phase 0 Use for Block (0,0) (iterations 0 and 1)

Which threads participate in computations?

$N_{0,0}$	$N_{0,1}$	$N_{0,2}$	
$N_{1,0}$	$N_{1,1}$	$N_{1,2}$	
$N_{2,0}$	$N_{2,1}$	$N_{2,2}$	

$N_{0,0}$	$N_{0,1}$
$N_{1,0}$	$N_{1,1}$

Shared Memory

Shared Memory

$M_{0,0}$	$M_{0,1}$	$M_{0,2}$	
$M_{1,0}$	$M_{1,1}$	$M_{1,2}$	
$M_{2,0}$	$M_{2,1}$	$M_{2,2}$	

$M_{0,0}$	$M_{0,1}$
$M_{1,0}$	$M_{1,1}$

$P_{0,0}$	$P_{0,1}$	$P_{0,2}$	
$P_{1,0}$	$P_{1,1}$	$P_{1,2}$	
$P_{2,0}$	$P_{2,1}$	$P_{2,2}$	

Phase 1 Use for Block (0,0) (iteration 0)

Which threads participate in computations?

$N_{0,0}$	$N_{0,1}$	$N_{0,2}$	
$N_{1,0}$	$N_{1,1}$	$N_{1,2}$	
$N_{2,0}$	$N_{2,1}$	$N_{2,2}$	

$N_{2,0}$	$N_{2,1}$

Shared Memory

Shared Memory

$M_{0,0}$	$M_{0,1}$	$M_{0,2}$	
$M_{1,0}$	$M_{1,1}$	$M_{1,2}$	
$M_{2,0}$	$M_{2,1}$	$M_{2,2}$	

$M_{0,2}$	
$M_{1,2}$	

$P_{0,0}$	$P_{0,1}$	$P_{0,2}$	
$P_{1,0}$	$P_{1,1}$	$P_{1,2}$	
$P_{2,0}$	$P_{2,1}$	$P_{2,2}$	

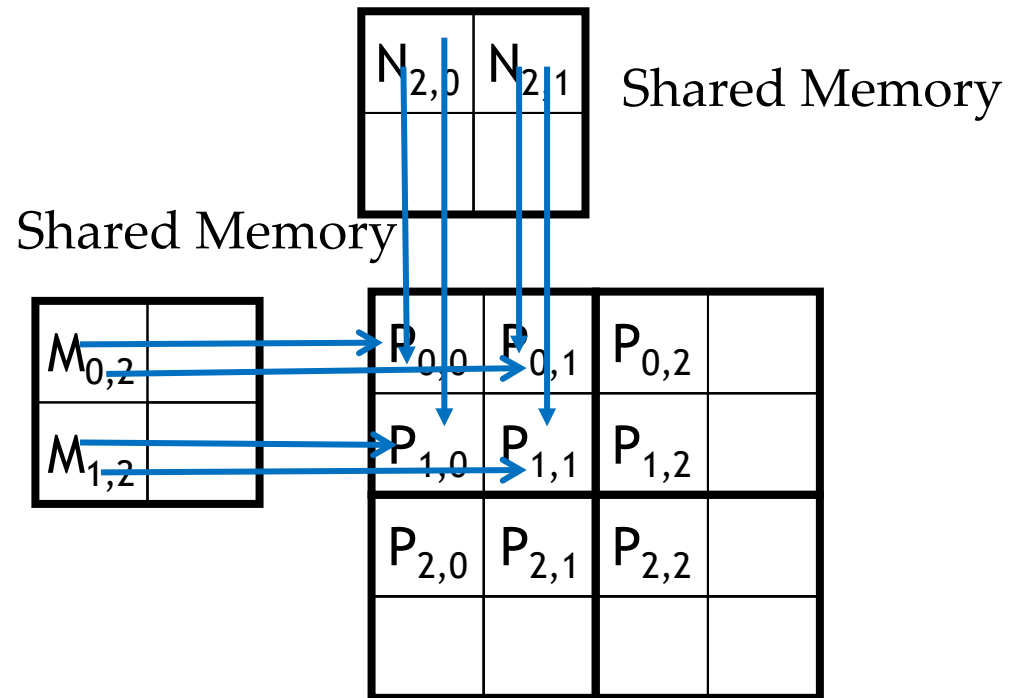
Phase 1 Use for Block (0,0) (iteration 0)

$N_{0,0}$	$N_{0,1}$	$N_{0,2}$	
$N_{1,0}$	$N_{1,1}$	$N_{1,2}$	
$N_{2,0}$	$N_{2,1}$	$N_{2,2}$	

$M_{0,0}$	$M_{0,1}$	$M_{0,2}$	
$M_{1,0}$	$M_{1,1}$	$M_{1,2}$	
$M_{2,0}$	$M_{2,1}$	$M_{2,2}$	

Participation in Computations!

All threads operate on legitimate data



Thread(0,1) participates in $[M(0,2)*N(2,1)]$ but should not read $M(0,3)$!
Thread(1,1) participates in $(M(1,2)*N(2,1))$ but should not read $M(1,3)$!
Similar for Thread(1,0) and Thread(1,1) on $N(3,0)$ and $N(3,1)$

Phase 1 Use for Block (0,0) (iteration 1)

Which threads participate in computations?

$N_{0,0}$	$N_{0,1}$	$N_{0,2}$	
$N_{1,0}$	$N_{1,1}$	$N_{1,2}$	
$N_{2,0}$	$N_{2,1}$	$N_{2,2}$	

$N_{2,0}$	$N_{2,1}$

Shared Memory

Shared Memory

$M_{0,0}$	$M_{0,1}$	$M_{0,2}$	
$M_{1,0}$	$M_{1,1}$	$M_{1,2}$	
$M_{2,0}$	$M_{2,1}$	$M_{2,2}$	

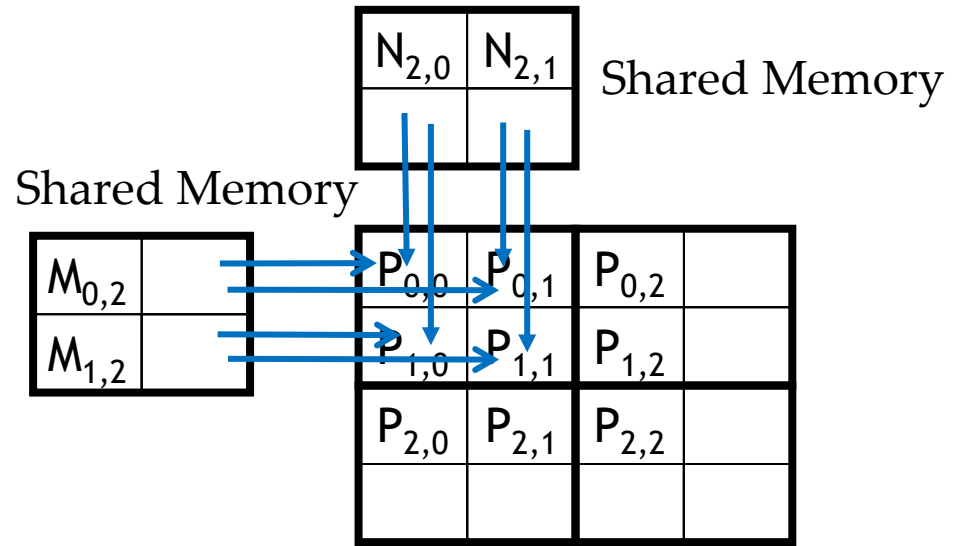
$M_{0,2}$	
$M_{1,2}$	

$P_{0,0}$	$P_{0,1}$	$P_{0,2}$	
$P_{1,0}$	$P_{1,1}$	$P_{1,2}$	
$P_{2,0}$	$P_{2,1}$	$P_{2,2}$	

Phase 1 Use for Block (0,0) (iteration 1)

$N_{0,0}$	$N_{0,1}$	$N_{0,2}$	
$N_{1,0}$	$N_{1,1}$	$N_{1,2}$	
$N_{2,0}$	$N_{2,1}$	$N_{2,2}$	

$M_{0,0}$	$M_{0,1}$	$M_{0,2}$	
$M_{1,0}$	$M_{1,1}$	$M_{1,2}$	
$M_{2,0}$	$M_{2,1}$	$M_{2,2}$	



All Threads need special treatment. None of them should introduce invalidate contributions to their P elements.

Load problems can occur even during phase 0!

$N_{0,0}$	$N_{0,1}$	$N_{0,2}$	
$N_{1,0}$	$N_{1,1}$	$N_{1,2}$	
$N_{2,0}$	$N_{2,1}$	$N_{2,2}$	

Shared Memory

Shared Memory

$M_{0,0}$	$M_{0,1}$	$M_{0,2}$	
$M_{1,0}$	$M_{1,1}$	$M_{1,2}$	
$M_{2,0}$	$M_{2,1}$	$M_{2,2}$	

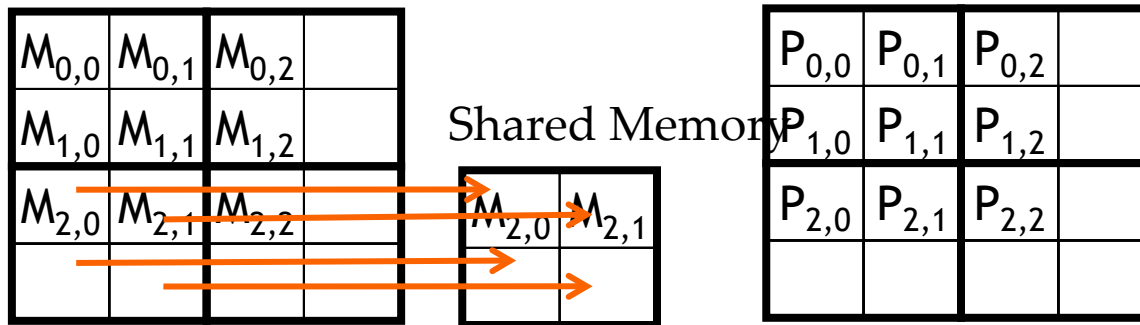
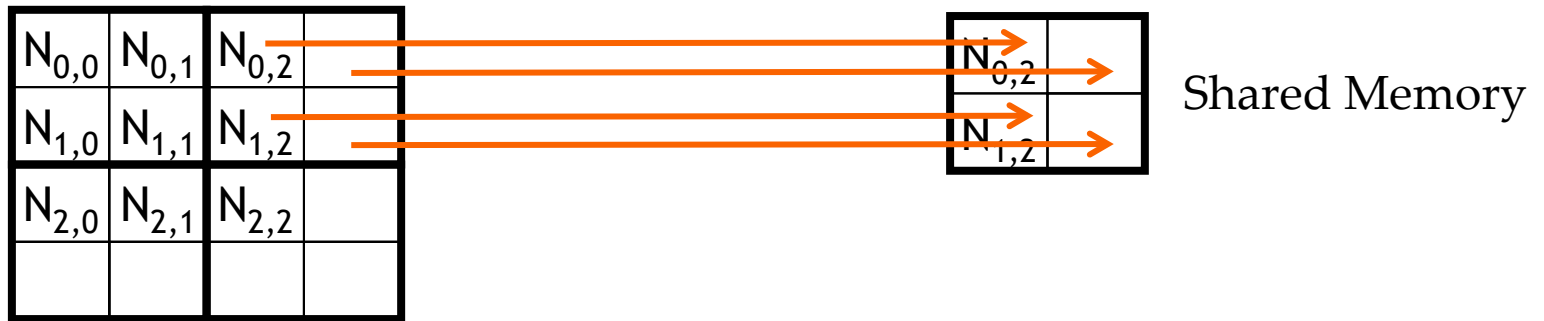
$P_{0,0}$	$P_{0,1}$	$P_{0,2}$	
$P_{1,0}$	$P_{1,1}$	$P_{1,2}$	
$P_{2,0}$	$P_{2,1}$	$P_{2,2}$	

Which thread block will have threads not participating in loads during Phase 0?

Phase 0 Loads for Block (1,1) for a 3x3 Example

**Problems can occur
even during phase 0!**

Threads (0,1) and (1,1) need special
treatment in loading N tile



Threads (1,0) and (1,1) need special
treatment in loading M tile

Threads not participating in computations in phase 0 may participate in loads. Really???

$N_{0,0}$	$N_{0,1}$	$N_{0,2}$	
$N_{1,0}$	$N_{1,1}$	$N_{1,2}$	
$N_{2,0}$	$N_{2,1}$	$N_{2,2}$	

Shared Memory

Shared Memory

$M_{0,0}$	$M_{0,1}$	$M_{0,2}$	
$M_{1,0}$	$M_{1,1}$	$M_{1,2}$	
$M_{2,0}$	$M_{2,1}$	$M_{2,2}$	

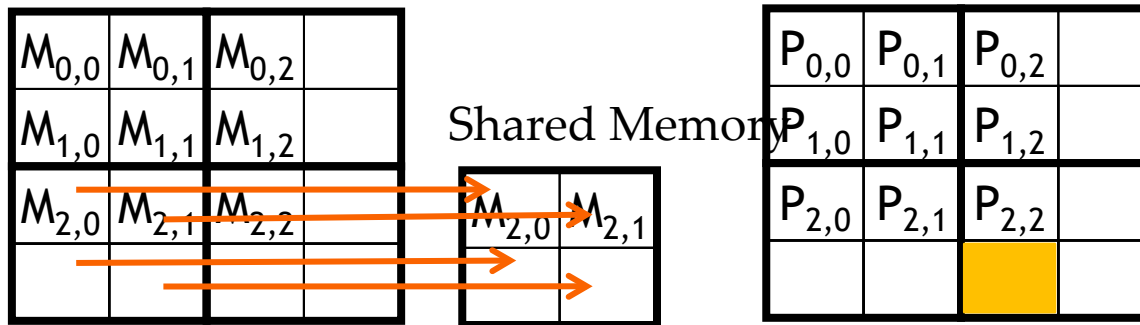
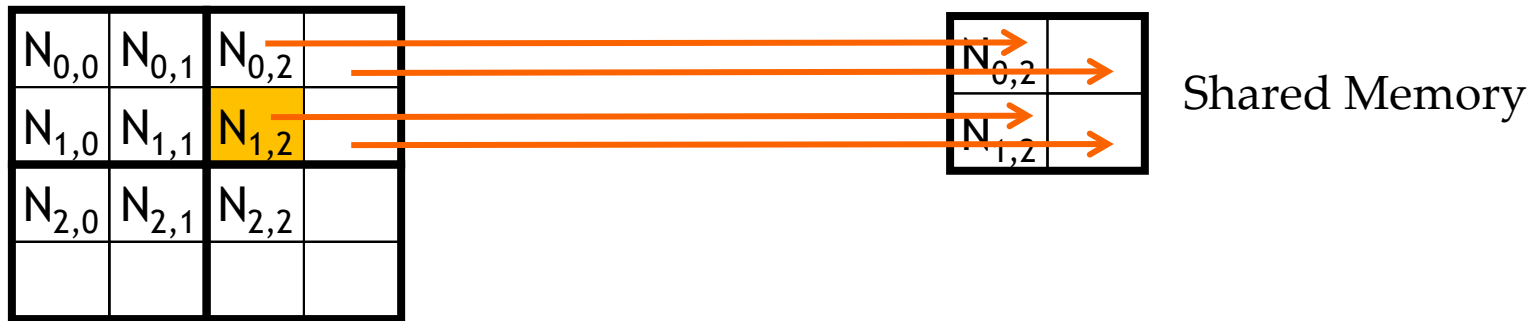
$P_{0,0}$	$P_{0,1}$	$P_{0,2}$	
$P_{1,0}$	$P_{1,1}$	$P_{1,2}$	
$P_{2,0}$	$P_{2,1}$	$P_{2,2}$	

Which thread block?

Phase 0 Loads for Block (1,1) for a 3x3 Example

**Problems can occur
even during phase 0!**

Threads (0,1) and (1,1) need special
treatment in loading N tile



Threads (1,0) and (1,1) need special
treatment in loading M tile

Phase 0 of Block(1,1), Thread(1,0), assigned to calculate non-existent $P[3,2]$ but need to participate in loading tile element $N[1,2]$

Major Cases in Toy Example

- **We can not just simply exclude the threads that do not calculate valid P elements**
 - they still need to participate in loading the input tiles
 - Phase 0 of Block(1,1), Thread(1,0), assigned to calculate non-existent $P[3,2]$ but need to participate in loading tile element $N[1,2]$
- **Threads that calculate valid P elements may attempt to load non-existing input elements when loading input tiles**
 - Phase 0 of Block(0,0), Thread(1,0), assigned to calculate valid $P[1,0]$ but attempts to load non-existing $N[3,0]$

Next

- **Writing boundary conditions for matrix multiplication**