SIE/ENGR 265 Engineering Management I

Lecture 5

Chapter 4

The Time Value of Money

Section 4.6-4.7

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In-Class Assignment 4-G

You have just invested a one-time amount of $5,000 in a stock-based mutual fund. This fund should earn (on average) 9% per year over a long period of time. How much will your investment be worth in 35 years?

In-Class Assignment 4-I

You just inherited $10,000. While you plan to squander some of it away, how much should you deposit in an account earning 5% interest per year if you'd like to have $10,000 in the account in 10 years?

Finding i and N

- Finding the Interest Rate (i) Given P, F, and N

$$F = P(1+i)^N$$

$$i = \sqrt{{}^{F}/_{P}} - 1$$

- Finding N when Given P, F, and i

$$N = \frac{log(F/P)}{log(1+i)}$$

Question 4-J

In 1803, Napoleon sold the Louisiana Territory to the United States for $0.04 per acre. In 2017, the average value of an acre at this location is $10,000. What annual compounded percentage increase in value of an acre of land has been experienced?

Question 5-P

How long does it take (to the nearest whole year) for $1,000 to quadruple in value when the interest rate is 15% per year?

Question 4-R

An enterprising student invests $1,000 at an annual interest rate that will grow the original investment to $2,000 in 4 years. In 4 more years, the amount will grow to $4,000, and this pattern of doubling every 4 years repeats over a total time span of 36 years. How much money will the student gain in 36 years? What is the magical annual interest rate that the student is earning?

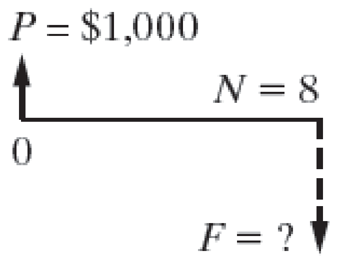
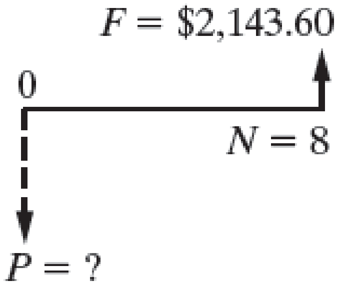
Question 4-R

Question 4-R

Question 4-R

Summary

Example Problems (All Using an Interest Rate of $i = 10\%$ per Year—See Table C-13 of Appendix C)

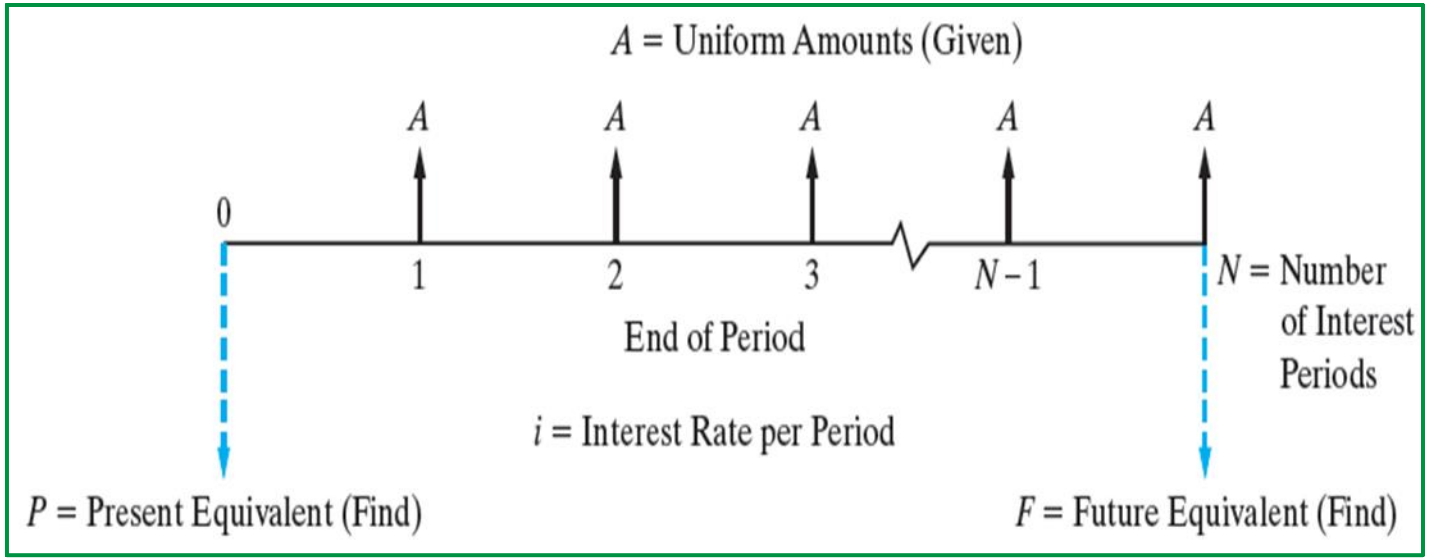
To Find:	Given:	(a) In Borrowing–Lending Terminology:	(b) In Equivalence Terminology:	Cash-Flow Diagram[a]	Solution
For single cash flows:					
F	P	A firm borrows \$1,000 for eight years. How much must it repay in a lump sum at the end of the eighth year?	What is the future equivalent at the end of eight years of \$1,000 at the beginning of those eight years?		$F = P(F/P, 10\%, 8)$ $= \$1,000(2.1436)$ $= \$2,143.60$
P	F	A firm wishes to have \$2,143.60 eight years from now. What amount should be deposited now to provide for it?	What is the present equivalent of \$2,143.60 received eight years from now?		$P = F(P/F, 10\%, 8)$ $= \$2,143.60(0.4665)$ $= \$1,000.00$

Definitions and Notation

- ***i*** = effective interest rate per interest period
- ***N*** = number of compounding (interest) periods
- ***P*** = present sum of money; equivalent value of one or more cash flows at a reference point in time; the present
- ***F*** = future sum of money; equivalent value of one or more cash flows at a reference point in time; the future
- ***A*** = end-of-period cash flows in a uniform series continuing for a certain number of periods, starting at the end of the first period and continuing through the last

Relating Uniform Series (Ordinary Annuity) to Its Present Equivalent and Future Equivalent Values

- Annuity (A) is a series of uniform (equal) receipts, each of amount A, occurring at the end of each period for N periods with interest rate i% per period.
- To relate A to P and F using formulas and tables, three rules must be followed:
    1. P (present equivalent value) occurs one interest period before the first A (uniform amount),
    2. F (future equivalent value) occurs at the same time as the last A, and N periods after P, and
    3. A (annual equivalent value) occurs at the end of periods 1 through N, inclusive.



Interest Factors Relating F and P to A

- Finding F when Given A

$$F = A\left[\frac{(1+i)^N - 1}{i}\right] = A\underbrace{\left(F/A, i\%, N\right)}_{\textit{Uniform Series Compound Amount Factor}}$$

- Finding P when Given A

$$P = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right] = A\underbrace{\left(P/A, i\%, N\right)}_{\textit{Uniform Series Present Worth Factor}}$$

Interest Factors Relating A to F and P

- Finding A when Given F

$$A = F\left[\frac{i}{(1+i)^N - 1}\right] = F\underbrace{(A/F, i\%, N)}_{\textit{Sinking Fund Factor}}$$

- Finding A when Given P

$$A = P\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right] = P\underbrace{(A/P, i\%, N)}_{\textit{Capital Recovery Factor}}$$

Finding N and i

- Finding N when given A, i, and P and/or F

  Using Excel function **NPER(*rate, pmt, pv*)**, which will compute the number of payments of magnitude *pmt* required to pay off a present amount (*pv*) at a fixed interest rate (*rate*).

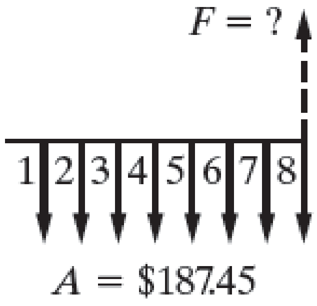
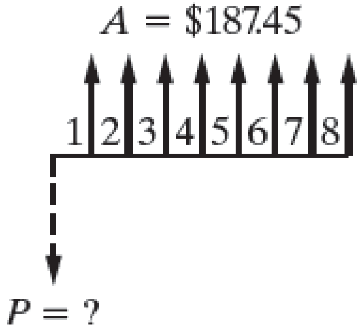
- Finding i when given A, N, and P and/or F

  Using Excel function **RATE(*nper, pmt, pv, fv*)**, which returns a fixed interest rate for an annuity of *pmt* that lasts for *nper* periods to either its present value (*pv*) or future value (*fv*).

- Alternatively, we can use linear interpolation between two factor values as explained in Example 4-13.

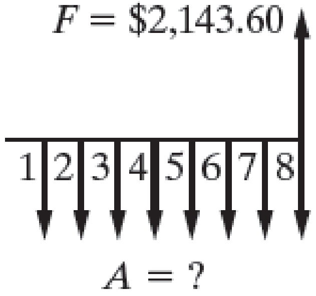
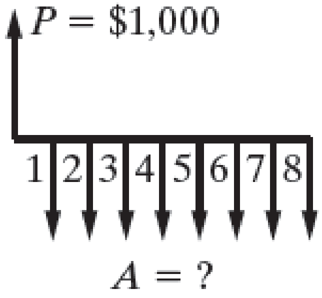
Summary

Example Problems (All Using an Interest Rate of $i = 10\%$ per Year—See Table C-13 of Appendix C)

To Find:	Given:	(a) In Borrowing–Lending Terminology:	(b) In Equivalence Terminology:	Cash-Flow Diagram[a]	Solution
For uniform series:					
F	A	If eight annual deposits of \$187.45 each are placed in an account, how much money has accumulated immediately after the last deposit?	What amount at the end of the eighth year is equivalent to eight EOY payments of \$187.45 each?		$F = A(F/A, 10\%, 8)$ $= \$187.45(11.4359)$ $= \$2{,}143.60$
P	A	How much should be deposited in a fund now to provide for eight EOY withdrawals of \$187.45 each?	What is the present equivalent of eight EOY payments of \$187.45 each?		$P = A(P/A, 10\%, 8)$ $= \$187.45(5.3349)$ $= \$1{,}000.00$

Summary

Example Problems (All Using an Interest Rate of $i = 10\%$ per Year—See Table C-13 of Appendix C)

To Find:	Given:	(a) In Borrowing–Lending Terminology:	(b) In Equivalence Terminology:	Cash-Flow Diagram[a]	Solution
A	F	What uniform annual amount should be deposited each year in order to accumulate \$2,143.60 at the time of the eighth annual deposit?	What uniform payment at the end of eight successive years is equivalent to \$2,143.60 at the end of the eighth year?		$A = F(A/F, 10\%, 8)$ $= \$2{,}143.60(0.0874)$ $= \$187.45$
A	P	What is the size of eight equal annual payments to repay a loan of \$1,000? The first payment is due one year after receiving the loan.	What uniform payment at the end of eight successive years is equivalent to \$1,000 at the beginning of the first year?		$A = P(A/P, 10\%, 8)$ $= \$1{,}000(0.18745)$ $= \$187.45$

Question 4-W

One of life's great lessons is to start early and save all the money you can! If you save $2 today and $2 each and every day thereafter until you are 60 years old (say $730 per year for 35 years), how much money will you accumulate if the annual interest rate is 7%?

Wealth Creation Reflection

One of life's great lessons is to start early and save all the money you can! If you save $2 today and $2 each and every day thereafter until you are 60 years old (say $730 per year for 35 years), how much money will you accumulate if the annual interest rate is 7%?

Wealth Creation Reflection (Cont’d)

A few words to the wise:

- Saving money early and preserving resources through frugality (avoiding waste) are extremely important ingredients of ***wealth creation*** in general.

- Often, being frugal means postponing the satisfaction of immediate material wants for the creation of a better tomorrow. In this regard, ***be very cautious*** about spending tomorrow’s cash today by undisciplined borrowing (e.g., with credit cards).

- The (F/A, i%, N) factor also demonstrates how fast your debt can accumulate!