SIE/ENGR 265 Engineering Management I

Lecture 4

Chapter 4

The Time Value of Money

Section 4.1-4.6

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Money has a time value

- ***Capital*** refers to wealth in the form of money or property that can be used to produce more wealth.

- Engineering economy studies involve the commitment of capital for extended periods of time.

- A dollar today is worth more than a dollar one or more years from now (for several reasons) because of interest and/or profit it can earn.

Hence, money has a *time value*

- Other factors affecting money time-value are inflation/deflation and currency exchange (chapter 8).

Return on Capital

- Interest and profit pay the providers of capital for forgoing its use during the time the capital is being used.
- Interest and profit are payments for the *risk* the investor takes in letting another use his or her capital.
- Any project or venture must provide a sufficient return to be financially attractive to the suppliers of money or property.

Focusing on interest effect

Simple Interest (used infrequently)

Interest earned or charged is linearly proportional to the initial amount of the loan (**P**rincipal), the ***i***nterest rate, and the **N**umber of interest periods.

Compound Interest

For any interest period, compound interest reflects both the remaining principal and any accumulated interest up to the beginning of that period.

Simple vs. Compound Interest

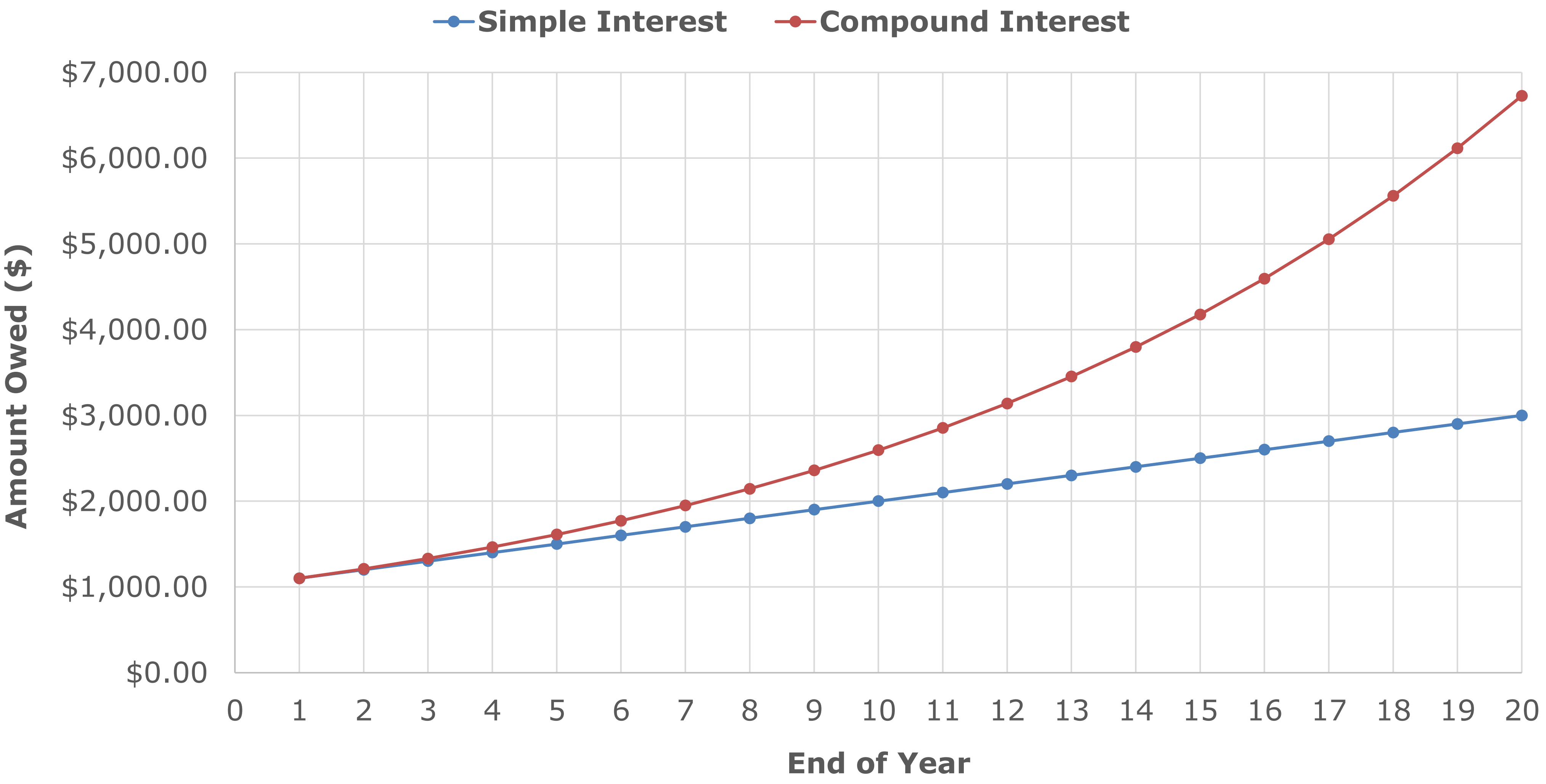
For $1,000 loaned for three years at an interest rate of 10% compounded each period, what is the total due for repayment at the end of the third year?

Period	Simple Interest			Compound Interest		
	Balance (BOY)	Interest	Balance (EOY)	Balance (BOY)	Interest	Balance (EOY)
1						
2						
3						

Simple vs. Compound Interest

Year	Simple Interest			Compound Interest		
	Balance Beg. of Yr. (BOY)	Interest Earned	Balance End of Yr. (EOY)	Balance Beg. of Yr. (BOY)	Interest Earned	Balance End of Yr. (EOY)
1	$1,000.00	$100.00	$1,100.00	$1,000.00	$100.00	$1,100.00
2	$1,100.00	$100.00	$1,200.00	$1,100.00	$110.00	$1,210.00
3	$1,200.00	$100.00	$1,300.00	$1,210.00	$121.00	$1,331.00
4	$1,300.00	$100.00	$1,400.00	$1,331.00	$133.10	$1,464.10
5	$1,400.00	$100.00	$1,500.00	$1,464.10	$146.41	$1,610.51
6	$1,500.00	$100.00	$1,600.00	$1,610.51	$161.05	$1,771.56
7	$1,600.00	$100.00	$1,700.00	$1,771.56	$177.16	$1,948.72
8	$1,700.00	$100.00	$1,800.00	$1,948.72	$194.87	$2,143.59
9	$1,800.00	$100.00	$1,900.00	$2,143.59	$214.36	$2,357.95
10	$1,900.00	$100.00	$2,000.00	$2,357.95	$235.79	$2,593.74
11	$2,000.00	$100.00	$2,100.00	$2,593.74	$259.37	$2,853.12
12	$2,100.00	$100.00	$2,200.00	$2,853.12	$285.31	$3,138.43
13	$2,200.00	$100.00	$2,300.00	$3,138.43	$313.84	$3,452.27
14	$2,300.00	$100.00	$2,400.00	$3,452.27	$345.23	$3,797.50
15	$2,400.00	$100.00	$2,500.00	$3,797.50	$379.75	$4,177.25
16	$2,500.00	$100.00	$2,600.00	$4,177.25	$417.72	$4,594.97
17	$2,600.00	$100.00	$2,700.00	$4,594.97	$459.50	$5,054.47
18	$2,700.00	$100.00	$2,800.00	$5,054.47	$505.45	$5,559.92
19	$2,800.00	$100.00	$2,900.00	$5,559.92	$555.99	$6,115.91
20	$2,900.00	$100.00	**$3,000.00**	$6,115.91	$611.59	**$6,727.50**

Simple vs. Compound Interest



Computation of Simple Interest

- The total interest, $\underline{I}$, earned or paid may be computed using the formula below.

$$\underline{I} = (P)(N)(i)$$

  - P = principal amount lent or borrowed
  - N = number of interest periods (e.g., years)
  - i = interest rate per interest period
- The total amount repaid at the end of N interest periods is $P + \underline{I}$.

Another Simple Interest Example

- If $5,000 were loaned for five years at a simple interest rate of 7% per year, the interest earned would be

$$I = \$5{,}000 \times 5 \times 0.07 = \$1{,}750$$

- So, the total amount repaid at the end of five years would be

$$P + I = \$5{,}000 + \$1{,}750 = \$6{,}750$$

In-Class Assignment 4-B

You borrow $500 from a family member and agree to pay it back in six months. Because you are part of the family, you are only being charged simple interest at the rate of 0.5% per month. How much will you owe after six months? How much is the interest?

Engineering economic analysis can play a role in many types of situations…

- Choosing the best design for a high-efficiency gas furnace
- Selecting the most suitable robot for a welding operation on an automotive assembly line
- Equipment replacement decisions
- Making a recommendation about whether jet airplanes for an overnight delivery service should be purchased or leased
- Determining the optimal staffing plan for a computer help desk

Economic Equivalence

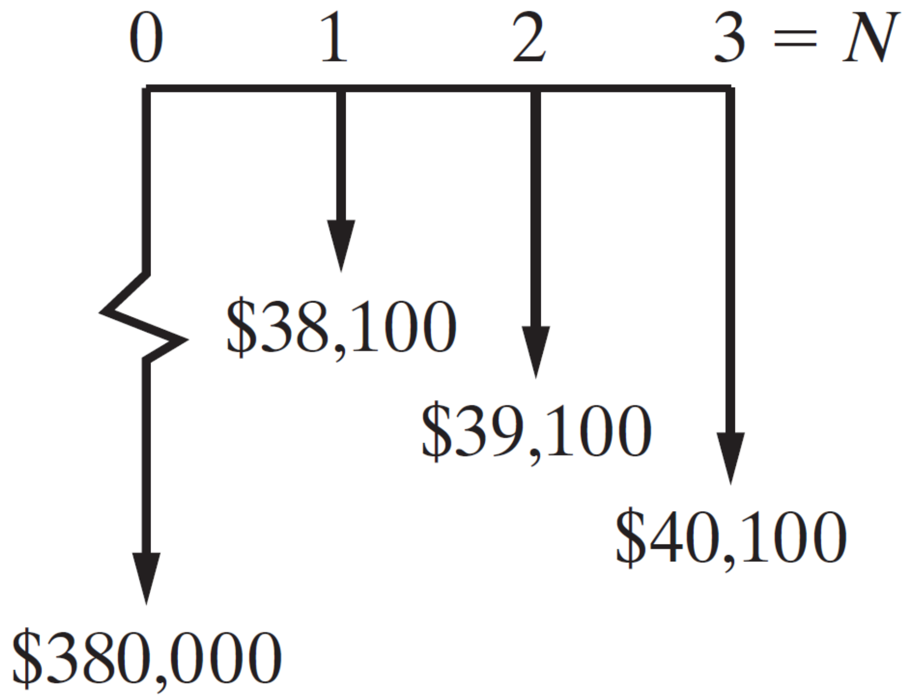
- How can alternatives for providing the same service or accomplishing the same function be compared when interest is involved over extended periods of time?

Each alternative can be reduced to an equivalent basis dependent on ***interest rate***, ***amount of money involved***, and ***timing of monetary receipts or expenses***

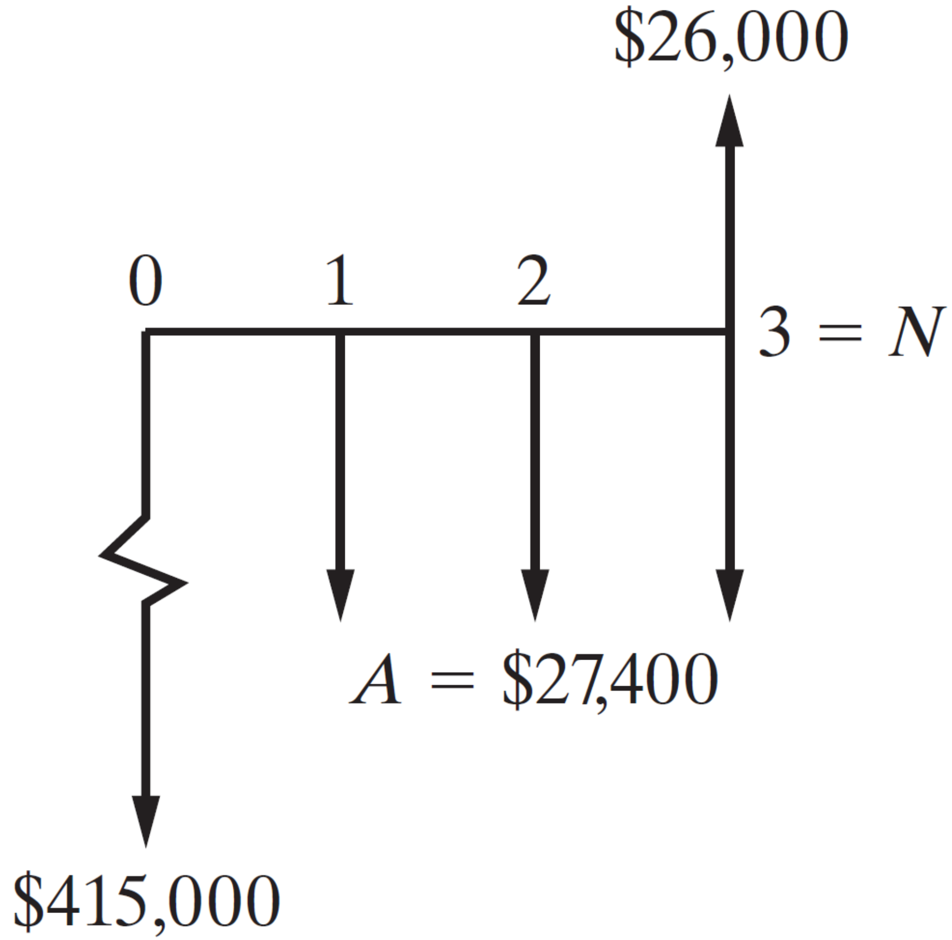
- Using these elements, we can “move” cash flows on the time-scale so that we can compare them at particular points in time

Economic Equivalence

Alternative A



Alternative B



Notation

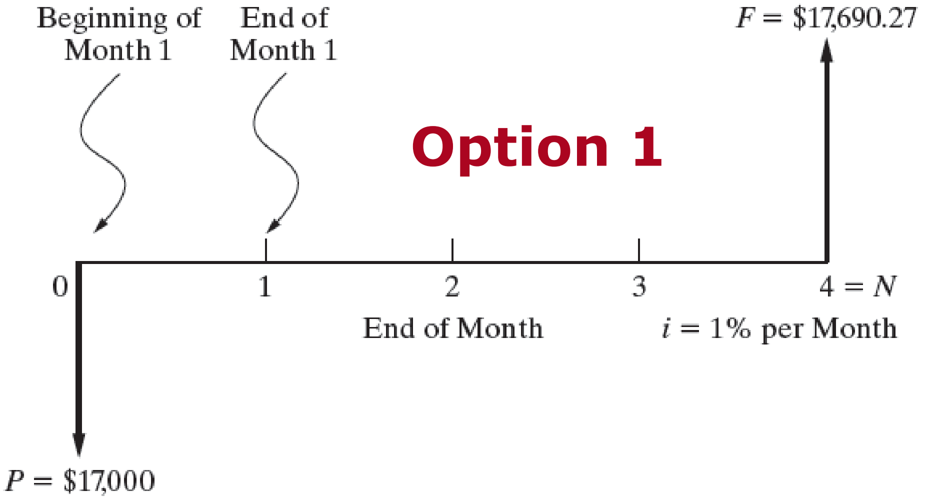
- ***i*** = effective interest rate per interest period
- ***N*** = number of compounding (interest) periods
- ***P*** = present sum of money; equivalent value of one or more cash flows at a reference point in time; the present
- ***F*** = future sum of money; equivalent value of one or more cash flows at a reference point in time; the future
- ***A*** = end-of-period cash flows in a uniform series continuing for a certain number of periods, starting at the end of the first period and continuing through the last

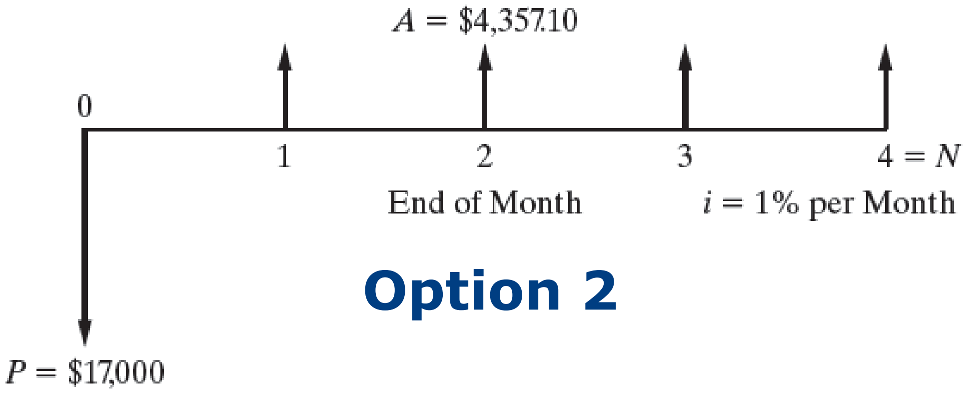
Cash Flow Diagrams – Basics

An indispensable tool for clarifying and visualizing a series of cash flows

Example

Suppose you have a $17,000 balance on your credit card, and you decide to repay the $17,000 debt in four months. An unpaid credit card balance at the beginning of a month will be charged interest at the rate of 1% by your credit card company.



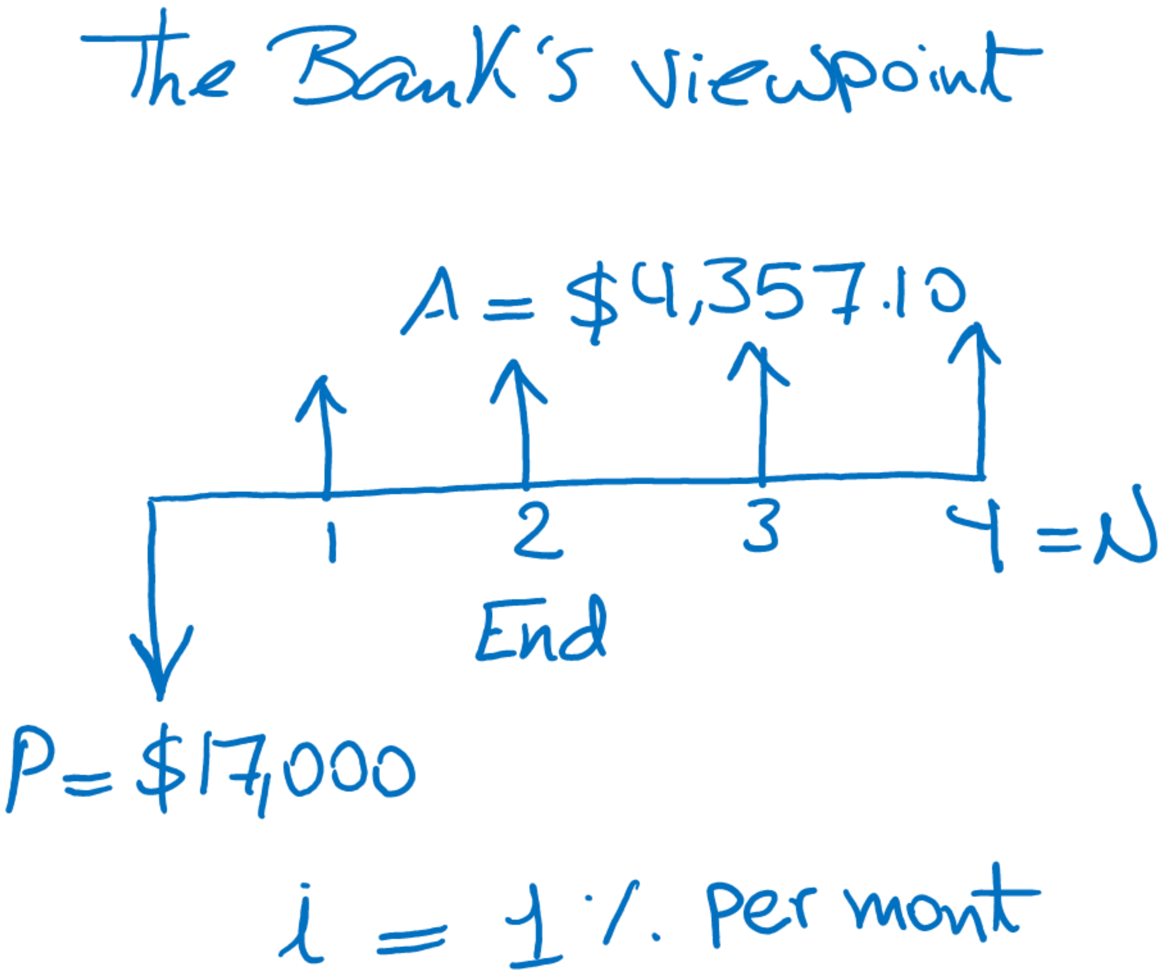


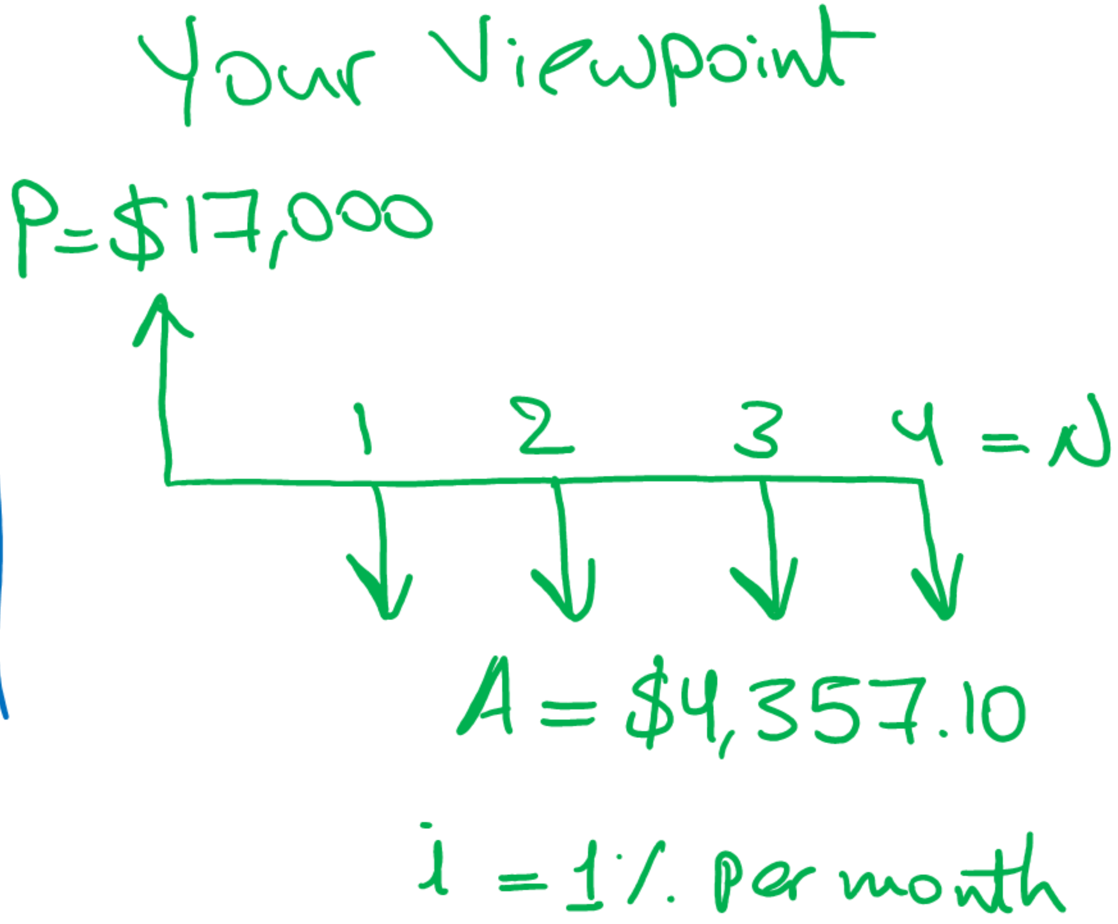
Expenses, negative cash Flows, or cash outflows

Receipts, positive cash flows, or cash inflows

Cash Flow Diagrams – Viewpoint

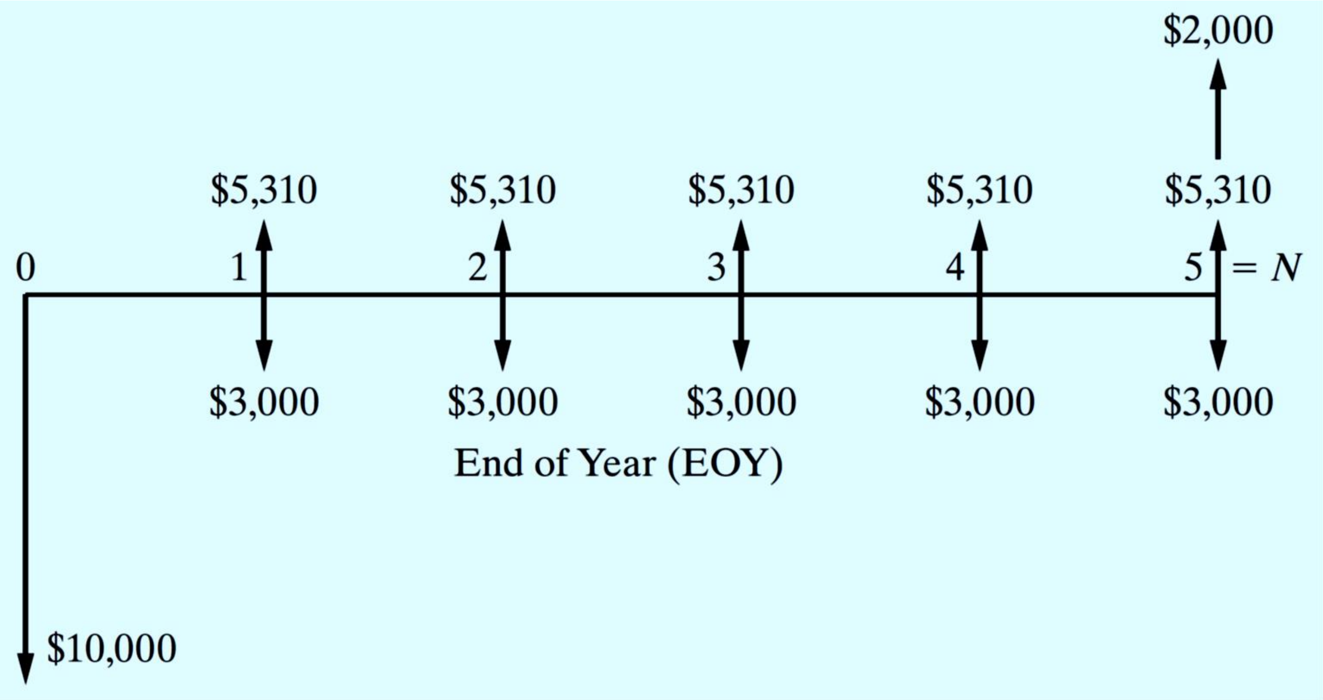
Cash-flow diagrams depends on the point of view.





Cash Flow Diagrams – Example

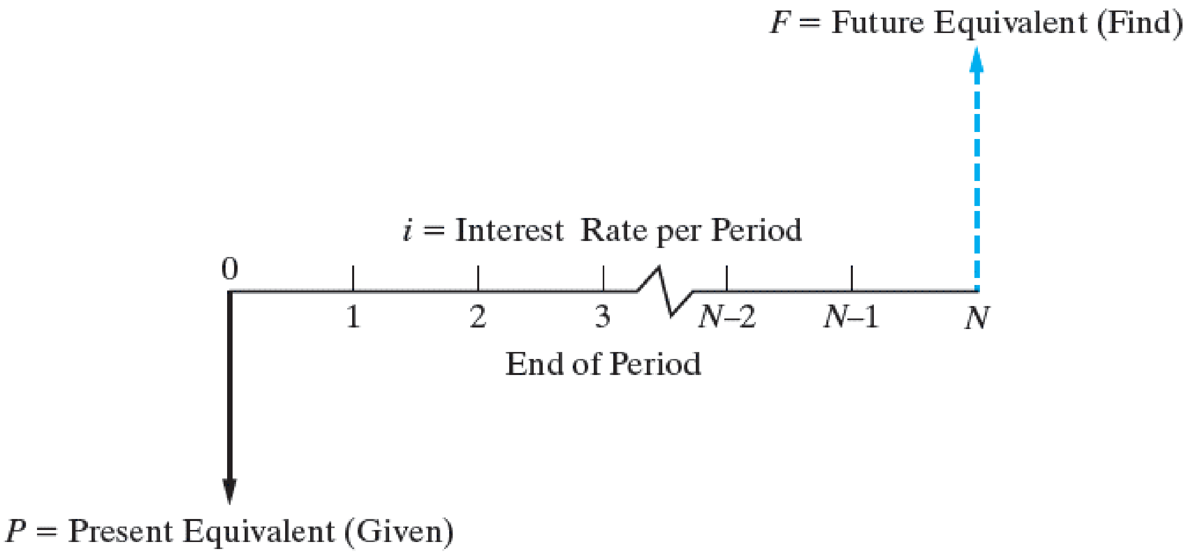
An investment of $10,000 can be made that will produce uniform annual revenue of $5,310 for five years and then have a market (recovery) value of $2,000 at the end of year (EOY) five. Annual expenses will be $3,000 at the end of each year for operating and maintaining the project. Draw a cash-flow diagram for the five-year life of the project.



Relating Present and Future Equivalent Values of Single Cash Flows

- We Can Apply Compound Interest Formulas to “Move” Cash Flows Along the Cash Flow Diagram
- Using the standard notation, we find that a present amount, *P*, can grow into a future amount, *F*, in *N* time periods at interest rate *i* according to the formula below.

$$\boxed{F = P\left(1+i\right)^{N}}$$



- In a similar way we can find *P* given *F* by

$$\boxed{P = F\left(1+i\right)^{-N}}$$

Relating Present and Future Equivalent Values of Single Cash Flows (Cont'd)

It is common to use standard notation for interest factors

$$F = P\boxed{(1+i)^N} \qquad \boxed{(1+i)^N = (F/P, i, N)}$$

(*F*/*P*, *i* , *N*) is known as the single payment compound amount factor. The term on the right is read "*F* given *P* at *i*% interest per period for N interest periods."

$$P = F\boxed{(1+i)^{-N}} \qquad \boxed{(1+i)^{-N} = (P/F, i, N)}$$

is called the single payment present worth factor.

These factors can be calculated or extracted from standard tables

Interest Table Example

TABLE C-1 Discrete Compounding; $i = 1/4\%$

	Single Payment		Uniform Series				Uniform Gradient		
	Compound Amount Factor	Present Worth Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Gradient Present Worth Factor	Gradient Uniform Series Factor	
N	To Find F Given P F/P	To Find P Given F P/F	To Find F Given A F/A	To Find P Given A P/A	To Find A Given F A/F	To Find A Given P A/P	To Find P Given G P/G	To Find A Given G A/G	N
1	1.0025	0.9975	1.0000	0.9975	1.0000	1.0025	0.000	0.0000	1
2	1.0050	0.9950	2.0025	1.9925	0.4994	0.5019	0.995	0.4994	2
3	1.0075	0.9925	3.0075	2.9851	0.3325	0.3350	2.980	0.9983	3
4	1.0100	0.9901	4.0150	3.9751	0.2491	0.2516	5.950	1.4969	4
5	1.0126	0.9876	5.0251	4.9627	0.1990	0.2015	9.901	1.9950	5
6	1.0151	0.9851	6.0376	5.9478	0.1656	0.1681	14.826	2.4927	6
7	1.0176	0.9827	7.0527	6.9305	0.1418	0.1443	20.722	2.9900	7
8	1.0202	0.9802	8.0704	7.9107	0.1239	0.1264	27.584	3.4869	8
9	1.0227	0.9778	9.0905	8.8885	0.1100	0.1125	35.406	3.9834	9
10	1.0253	0.9753	10.1133	9.8639	0.0989	0.1014	44.184	4.4794	10
11	1.0278	0.9729	11.1385	10.8368	0.0898	0.0923	53.913	4.9750	11
12	1.0304	0.9705	12.1664	11.8073	0.0822	0.0847	64.589	5.4702	12
13	1.0330	0.9681	13.1968	12.7753	0.0758	0.0783	76.205	5.9650	13
14	1.0356	0.9656	14.2298	13.7410	0.0703	0.0728	88.759	6.4594	14
15	1.0382	0.9632	15.2654	14.7042	0.0655	0.0680	102.244	6.9534	15
16	1.0408	0.9608	16.3035	15.6650	0.0613	0.0638	116.657	7.4469	16
17	1.0434	0.9584	17.3443	16.6235	0.0577	0.0602	131.992	7.9401	17
18	1.0460	0.9561	18.3876	17.5795	0.0544	0.0569	148.245	8.4328	18
19	1.0486	0.9537	19.4336	18.5332	0.0515	0.0540	165.411	8.9251	19
20	1.0512	0.9513	20.4822	19.4845	0.0488	0.0513	183.485	9.4170	20
21	1.0538	0.9489	21.5334	20.4334	0.0464	0.0489	202.463	9.9085	21
22	1.0565	0.9466	22.5872	21.3800	0.0443	0.0468	222.341	10.3995	22
23	1.0591	0.9442	23.6437	22.3241	0.0423	0.0448	243.113	10.8901	23
24	1.0618	0.9418	24.7028	23.2660	0.0405	0.0430	264.775	11.3804	24
25	1.0644	0.9395	25.7646	24.2055	0.0388	0.0413	287.323	11.8702	25
30	1.0778	0.9278	31.1133	28.8679	0.0321	0.0346	413.185	14.3130	30
36	1.0941	0.9140	37.6206	34.3865	0.0266	0.0291	592.499	17.2306	36
40	1.1050	0.9050	42.0132	38.0199	0.0238	0.0263	728.740	19.1673	40
48	1.1273	0.8871	50.9312	45.1787	0.0196	0.0221	1040.055	23.0209	48
60	1.1616	0.8609	64.6467	55.6524	0.0155	0.0180	1600.085	28.7514	60
72	1.1969	0.8355	78.7794	65.8169	0.0127	0.0152	2265.557	34.4221	72
84	1.2334	0.8108	93.3419	75.6813	0.0107	0.0132	3029.759	40.0331	84
100	1.2836	0.7790	113.4500	88.3825	0.0088	0.0113	4191.242	47.4216	100
∞				400.0000		0.0025			∞

Or we can program it in a spreadsheet (Excel)

Using Factors to Relate P and F

We can use the interest factors to find economically equivalent values at different points in time

- **Example 1:** $2,500 at time zero is equivalent to how much after six years if the interest rate is 8% per year?

- **Example 2:** $3,000 at the end of year seven is equivalent to how much today (time zero) if the interest rate is 6% per year?