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ADNAN

Part a)

| k | n |
|---|---|
| 0 | 0 |
| 1 | 2 |
| 2 | 4 |
| 3 | 8 |
| 4 | 16 |

$\theta(2)^1, 2(2)^2, 2(2)^3$

$i = 2^{2k}$

$2^k = \log_2 n$

Runtime; $\theta = \log_2 n(\log_2(n))$

Part b)

$k < i^3$

| i | $k < i^3$ |
|---|---|
| 1 | 1 |
| 2 | 8 |
| 3 | 27 |

$\sum_{i=1}^{n} (\theta(1) + \sum_{k=1}^{\sqrt{n}} i^3)$

$\sum_{i=1}^{m} \theta(1) + \sum_{i=1}^{n} \sum_{k=1}^{\sqrt{n}} (k\sqrt{n})^3$

$(\sqrt{n})^3 \sum_{k=1}^{\sqrt{n}} k^3 = \left[\frac{\sqrt{n}(\sqrt{n}+1)}{2}\right]^2 (\sqrt{n}^5 + \theta(n))$

$= \underbrace{n^2 + 2\sqrt{n} + n}_{4} \cdot n\sqrt{n} + n$

| i | n=1 | n=9 |
|---|---|---|
| 1 | yes | no |
| 2 | yes | no |
| 3 | yes | no |
| 4 | yes | no |
| 5 | yes | no |

*highest significance* $= \frac{n^3\sqrt{n} + 2n^2 + n^2\sqrt{n}}{4} + n$

$\boxed{\theta(n^{3/2})}$

Part c) for $O(n)$

k A(k) For $O(n)$

A(k) == 1 if $O(1)$

m=1 m++ For $O(n)$

m=2 m=4

$$T(n) = \sum_{i=1}^{n} \sum_{k=1}^{n} (\Theta(1) + \Theta(\sum_{m=1}^{k} \Theta(1))$$

worstcase is when A[k]=1 for all elements of k, so needs to occur n times

$$T(n) = \sum_{i=1}^{n} \sum_{k=1}^{n} \Theta(1) + n \sum_{m=1}^{n} \Theta(1)$$

$$T(n) = \Theta(n^2) + \Theta(n \cdot \log(n)) \quad m = 4^x$$

Runtime: $\Theta(n^2)$

Part d) for(int i=0, i<n, i++) runs n times

$$\sum_{i=1}^{n} (\Theta(1) + \alpha \sum_{j=1}^{10 \times \frac{3}{2} s} \Theta(1)) \qquad i = 10 \times \frac{3}{2} s$$

$$n \sim 10 \times \frac{3}{2} s$$

$$s = \log_{\frac{3}{2}} n/10$$

$$= \sum_{i=0}^{n} \Theta(1) + \sum_{j=1}^{10 \times \frac{3}{2} s} \Theta(1)$$

$$= \Theta(n) + \log_{\frac{3}{2}} n/10 \sum_{k=0} \Theta(10 \times \frac{3}{2} k)$$

$$= \Theta(n) + 10(\tfrac{3}{2}) \log_{\frac{3}{2}} n/10$$

$$= \Theta(n) + \Theta(10(n/10))$$

$$= \Theta(2n)$$

Runtime: $\Theta(n)$