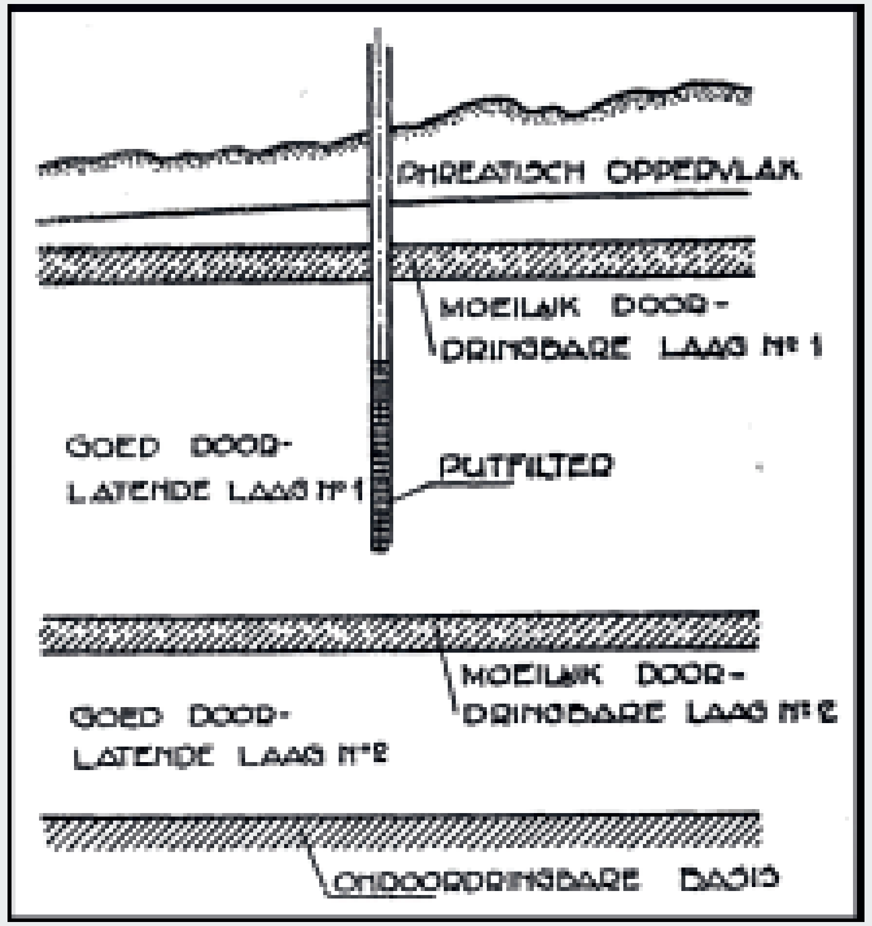
FACULTY OF SCIENCES

LABORATORY FOR APPLIED GEOLOGY AND HYDROGEOLOGY
DEPARTMENT OF GEOLOGY

Axisymmetric Flow in Multilayer Aquifer Systems: Solutions and Theoretical Considerations

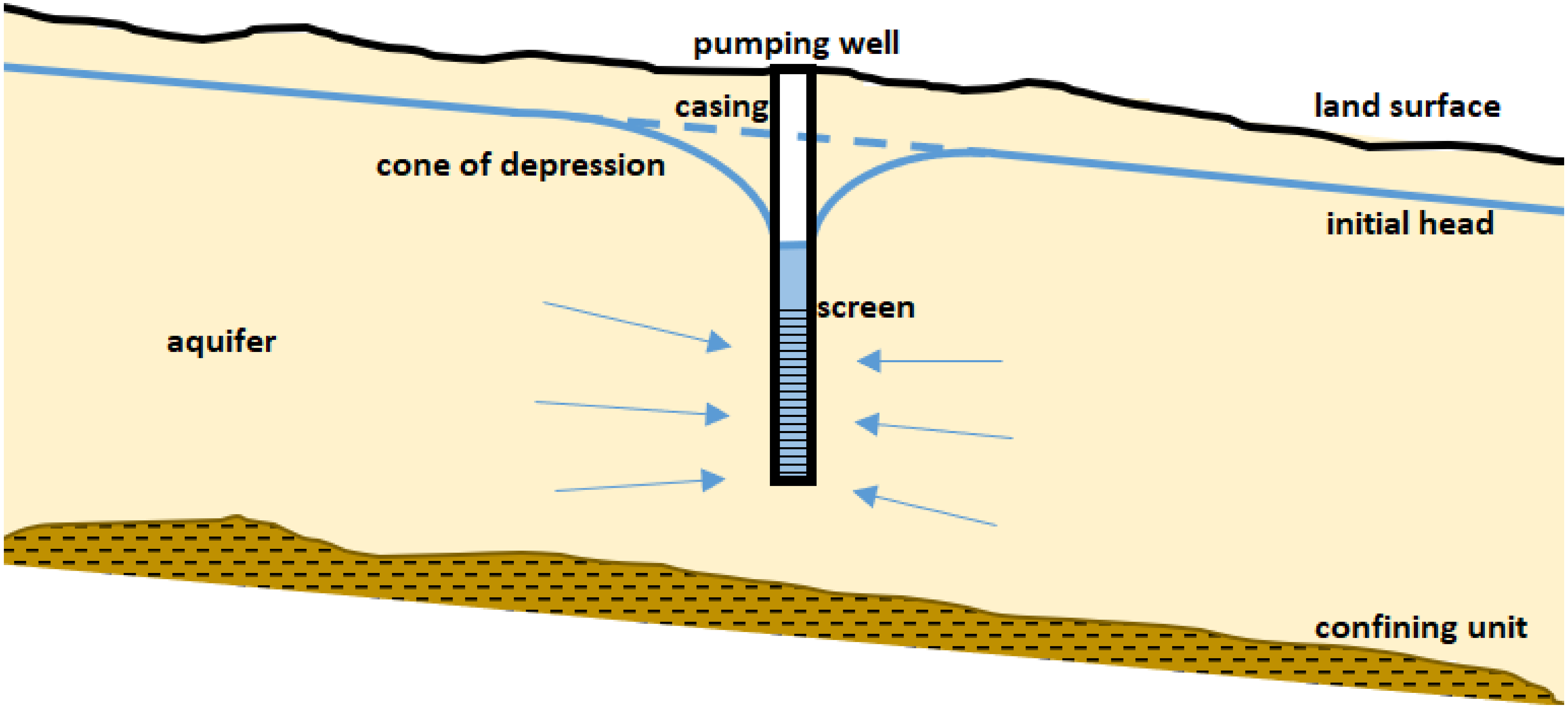


Andy Louwyck

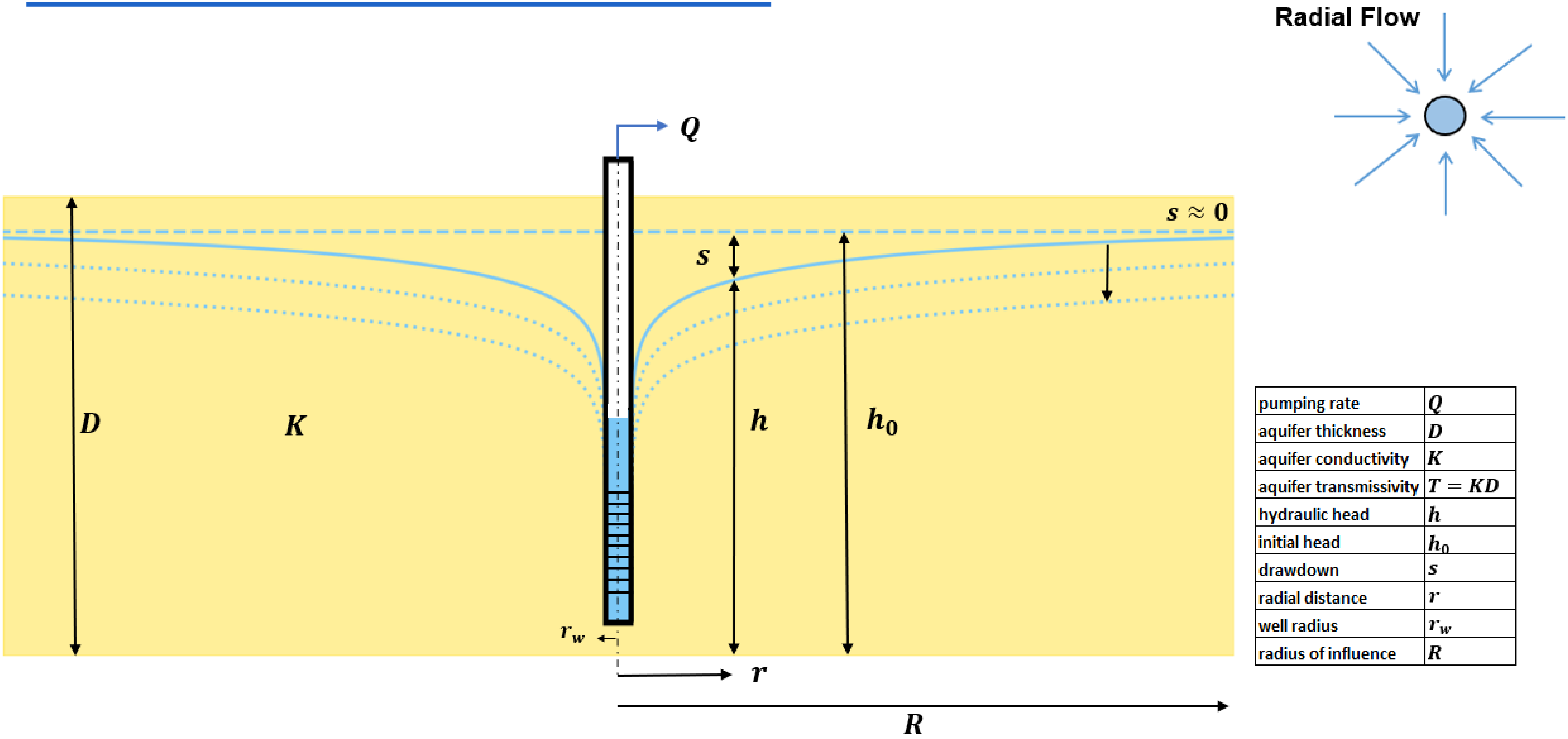
September 2023

GHENT UNIVERSITY

FLOW TO A PUMPING WELL



AXISYMMETRIC MODEL



pumping rate	Q
aquifer thickness	D
aquifer conductivity	K
aquifer transmissivity	$T = KD$
hydraulic head	h
initial head	h_0
drawdown	s
radial distance	r
well radius	r_w
radius of influence	R

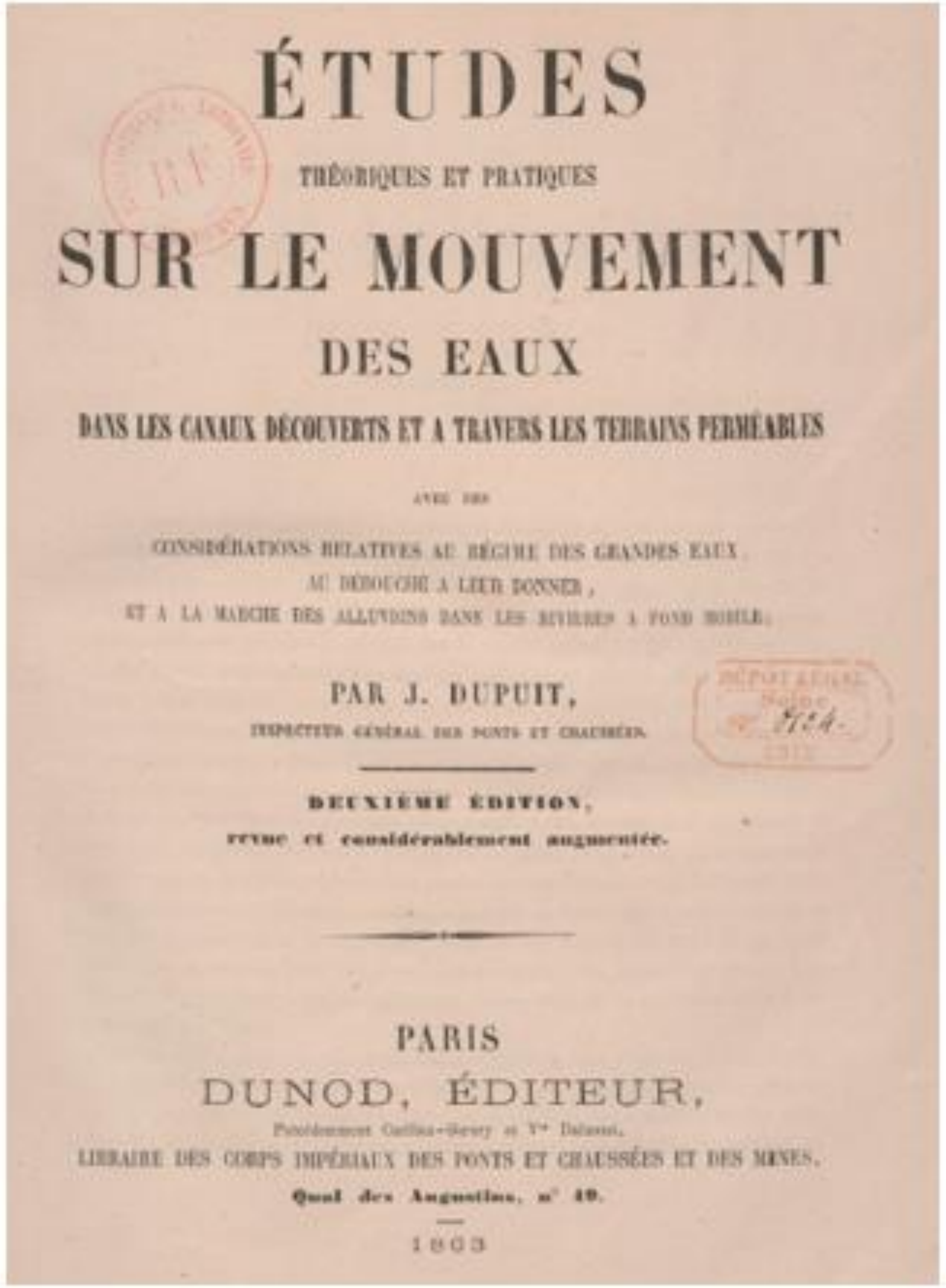
THE THIEM-DUPUIT FORMULAS

– Steady confined flow (Thiem, 1870, 1906)

$$s(r) = \frac{Q}{2\pi KD} \ln\left(\frac{R}{r}\right)$$

– Steady unconfined flow (Dupuit, 1857, 1863)

$$s(r) = h_0 - \underbrace{\sqrt{h_0^2 - \frac{Q}{\pi K} \ln\left(\frac{R}{r}\right)}}_{h}$$


ÉTUDES

THÉORIQUES ET PRATIQUES

SUR LE MOUVEMENT

DES EAUX

DANS LES CANAUX DÉCOUVERTS ET A TRAVERS LES TERRAINS PERMÉABLES

AVEC DES

CONSIDÉRATIONS RELATIVES AU RÉGIME DES GRANDES EAUX,

AU DÉBOUCHÉ A LEUR DONNER,

ET A LA MARCHE DES ALLUVIONS DANS LES RIVIÈRES A FOND MOBILE;

PAR J. DUPUIT,

INSPECTEUR GÉNÉRAL DES PONTS ET CHAUSSÉES.

DEUXIÈME ÉDITION,

revue et considérablement augmentée.

PARIS

DUNOD, ÉDITEUR,

LIBRAIRE DES CORPS IMPÉRIAUX DES PONTS ET CHAUSSÉES ET DES MINES,

Quai des Augustins, n° 49.

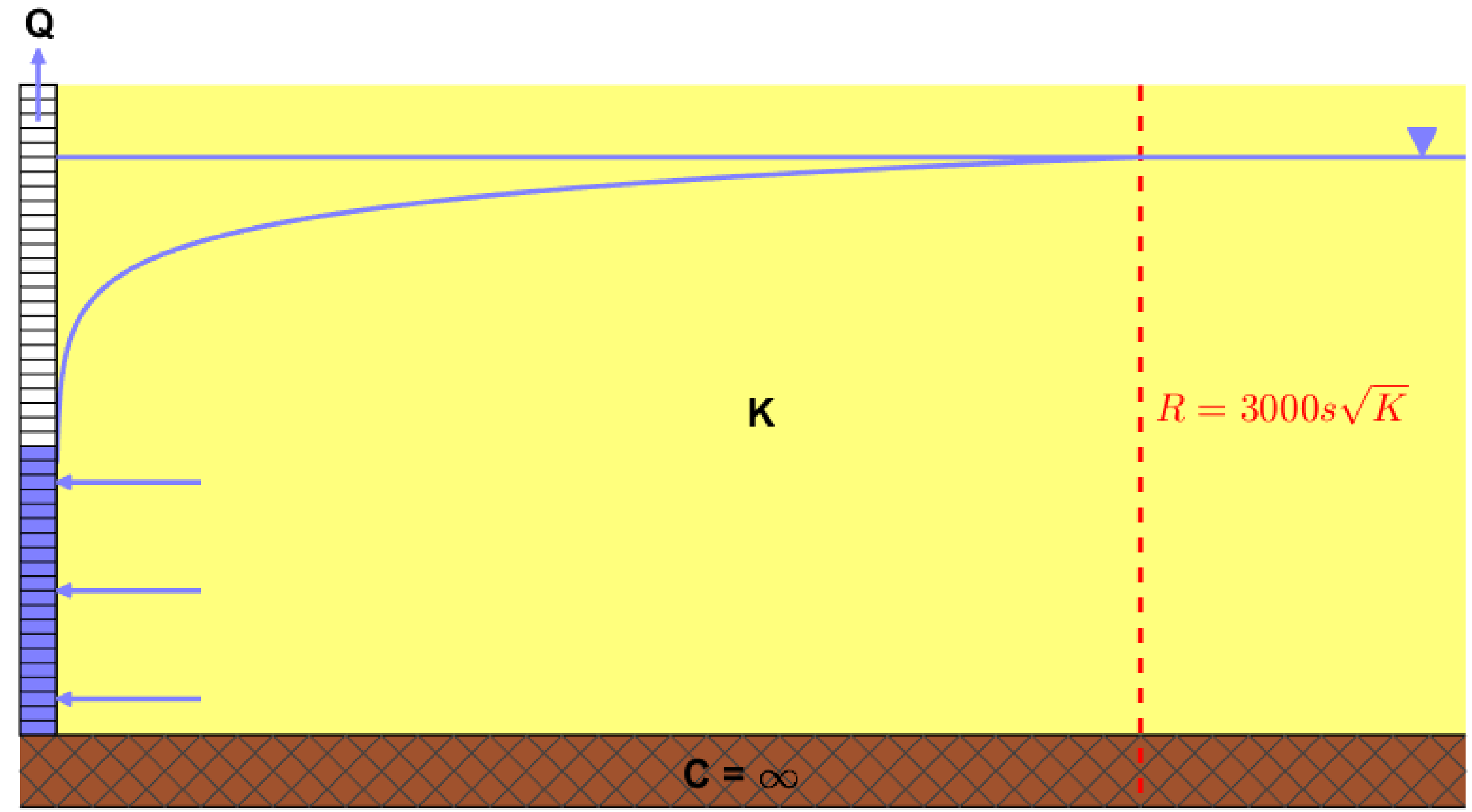
1863

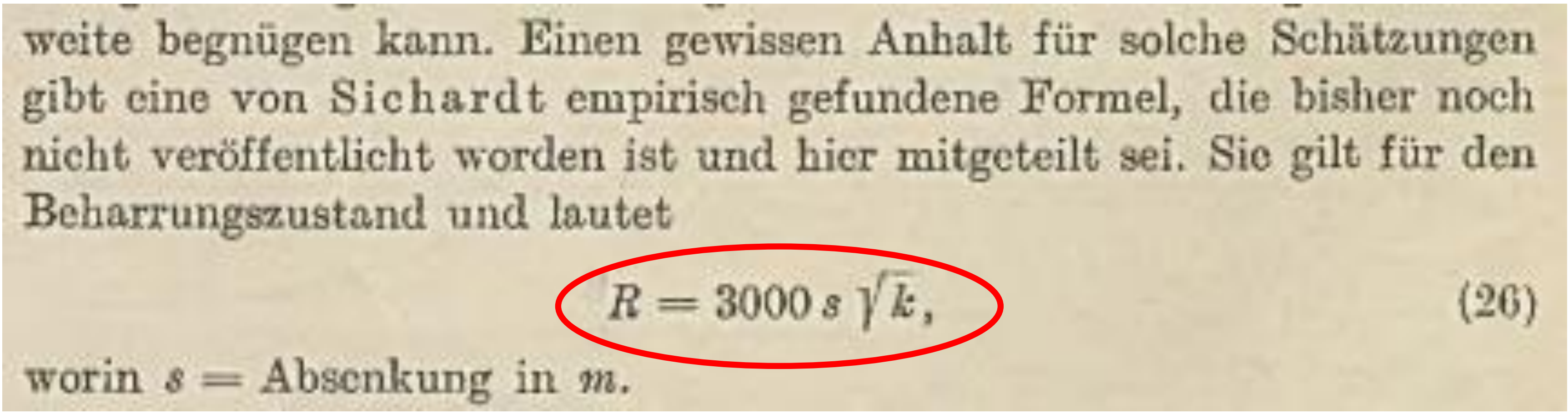

Jules Dupuit


Adolf Thiem


Günther Thiem

EMPIRICAL SICHARDT FORMULA

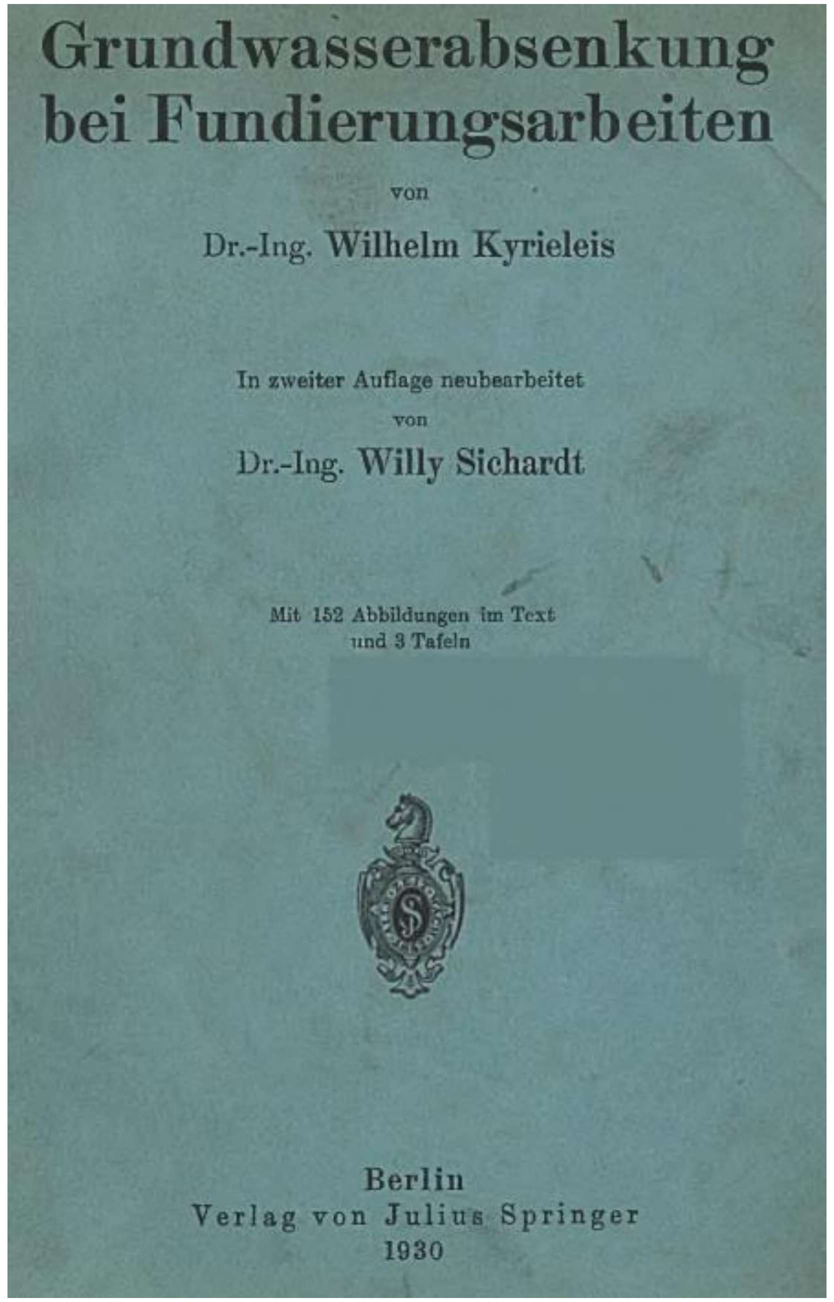




weite begnügen kann. Einen gewissen Anhalt für solche Schätzungen gibt eine von Sichardt empirisch gefundene Formel, die bisher noch nicht veröffentlicht worden ist und hier mitgeteilt sei. Sie gilt für den Beharrungszustand und lautet

$$R = 3000\, s \sqrt{k}, \qquad (26)$$

worin s = Absenkung in m.



FORWARD AND INVERSE PROBLEMS

- **forward problem**
  - simulate head *h* or drawdown *s*
  - e.g. assessing the environmental impact of extractions

$$s = \frac{Q}{2\pi KD} \ln\left(\frac{R}{r}\right)$$

- **inverse problem type I**
  - derive transmissivity *KD*
  - e.g. pumping test interpretation

$$KD = \frac{Q}{2\pi} \frac{\ln r_2 - \ln r_1}{s_1 - s_2}$$

- **inverse problem type II**
  - derive pumping rate *Q*
  - e.g. construction dewatering

$$Q = 2\pi KD \frac{s_w}{\ln R - \ln r_w}$$

THE RADIUS OF INFLUENCE MYTH

Applying the Sichardt formula:

- is inconsistent with fundamental hydrogeological principles
- may underestimate the extent of the cone of depression
- is not recommended to assess the impact of extractions

water MDPI

Article

The Radius of Influence Myth

by Andy Louwyck [1,*], Alexander Vandenbohede [2], Dirk Libbrecht [3], Marc Van Camp [1] and Kristine Walraevens [1]

[1] Laboratory for Applied Geology and Hydrogeology, Department of Geology, Ghent University, Krijgslaan 281-S8, 9000 Ghent, Belgium

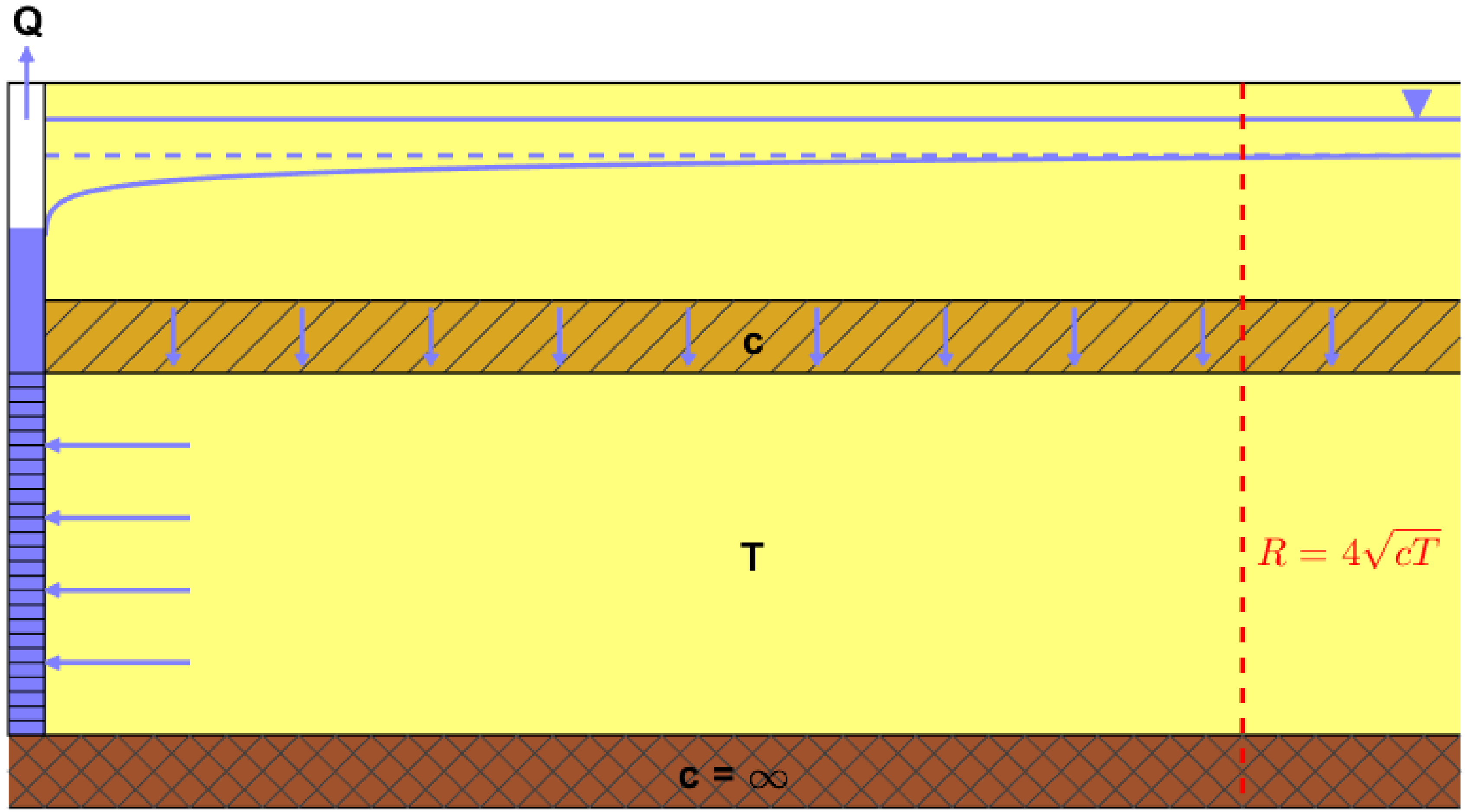
[2] De Watergroep, Water Resources and Environment, Vooruitgangstraat 189, 1030 Brussels, Belgium

[3] Arcadis Belgium nv/sa, Gaston Crommenlaan 8, Bus 101, 9050 Ghent, Belgium

[*] Author to whom correspondence should be addressed.

Water **2022**, *14*(2), 149; **https://doi.org/10.3390/w14020149**

THE DE GLEE FORMULA



OVER GRONDWATERSTROOMINGEN BIJ WATERONTTREKKING DOOR MIDDEL VAN PUTTEN.

PROEFSCHRIFT

TER VERKRIJGING VAN DEN GRAAD VAN DOCTOR IN DE TECHNISCHE WETENSCHAP AAN DE TECHNISCHE HOOGESCHOOL TE DELFT, OP GEZAG VAN DEN RECTOR MAGNIFICUS IR. F. WESTENDORP, HOOGLEERAAR IN DE AFDEELING DER WERKTUIGBOUWKUNDE EN SCHEEPSBOUWKUNDE, VOOR EENE COMMISSIE UIT DEN SENAAT TE VERDEDIGEN OP WOENSDAG 2 APRIL 1930, DES NAMIDDAGS TE 3 UUR,

DOOR

GERRIT JAN DE GLEE,

CIVIEL-INGENIEUR,

GEBOREN TE ASSEN.

GEDRUKT BIJ DE TECHNISCHE BOEKHANDEL EN DRUKKERIJ J. WALTMAN JR. DELFT. — 1930.

Johan Kooper

Charles E. Jacob

Steady leaky flow (Kooper, 1914; de Glee, 1930; Jacob, 1946)

$$s(r) = \frac{Q}{2\pi KD} \mathrm{K}_0\left(r\sqrt{\frac{1}{cKD}}\right)$$

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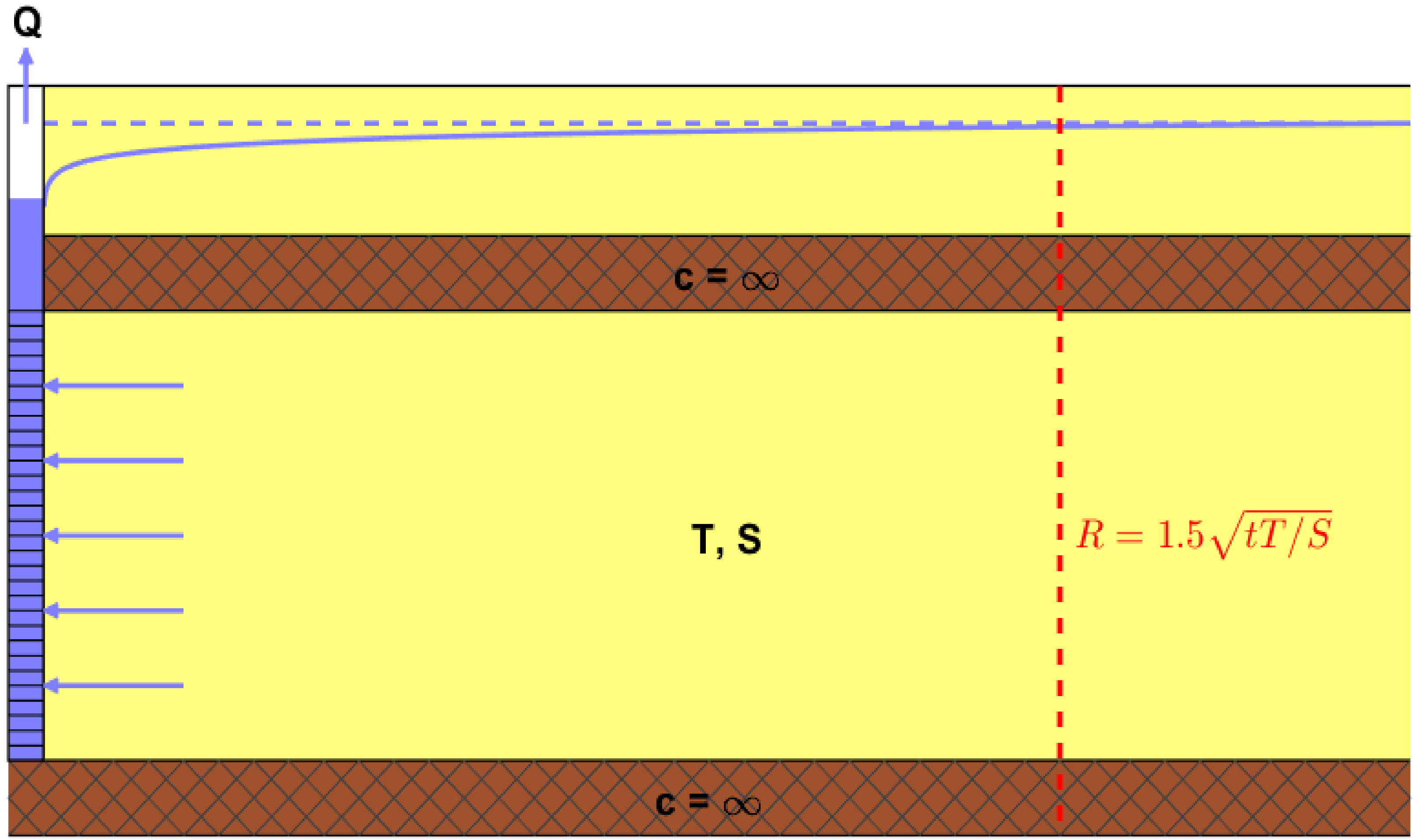
THE THEIS EQUATION



UNITED STATES
DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY
WATER RESOURCES DIVISION
GROUND WATER BRANCH
Washington 25, D. C.

GROUND WATER NOTES
HYDRAULICS

No. 5

August 1952

THE RELATION BETWEEN THE LOWERING OF THE PIEZOMETRIC SURFACE AND THE RATE AND DURATION OF DISCHARGE OF A WELL USING GROUND WATER STORAGE

By
Charles V. Theis

This paper develops some of the basic concepts on which much of our present-day theory of ground-water hydraulics is founded. Although published in the Transactions of the American Geophysical Union, part 2 (pp. 519-524), August 1935, the supply of reprints has long since been exhausted and the paper is now generally to be found only in the better-stocked reference libraries.

It is reproduced as part of the new series of Ground Water Notes to permit distribution to all ground-water field personnel for their ready reference and use. Minor changes have been made in notation only, to be consistent with current Branch usage.

When a well is pumped or otherwise discharged, water levels in its neighborhood are lowered. Unless this lowering occurs instantaneously it represents a loss of storage, either by the unwatering of a portion of the previously saturated sediments if the aquifer is nonartesian or by release of stored water by the compaction of the aquifer due to lowered pressure if the aquifer is artesian. The mathematical theory of ground-water hydraulics has been based, apparently entirely, on a postulate that equilibrium has been attained and therefore that water-levels are no longer falling. In a great number of hydrologic problems, involving a well or pumping district near or in which water-levels are falling, the

Open file

Charles V. Theis

Hilton H. Cooper, Jr.

Transient confined flow (Theis, 1935; Cooper & Jacob, 1946)

$$s(r,t) = \frac{Q}{4\pi KD} \mathrm{W}\left(\frac{r^2 S}{4tKD}\right) \approx \frac{-Q}{2\pi KD} \ln\left(r\sqrt{\frac{e^{\gamma} S}{4tKD}}\right)$$

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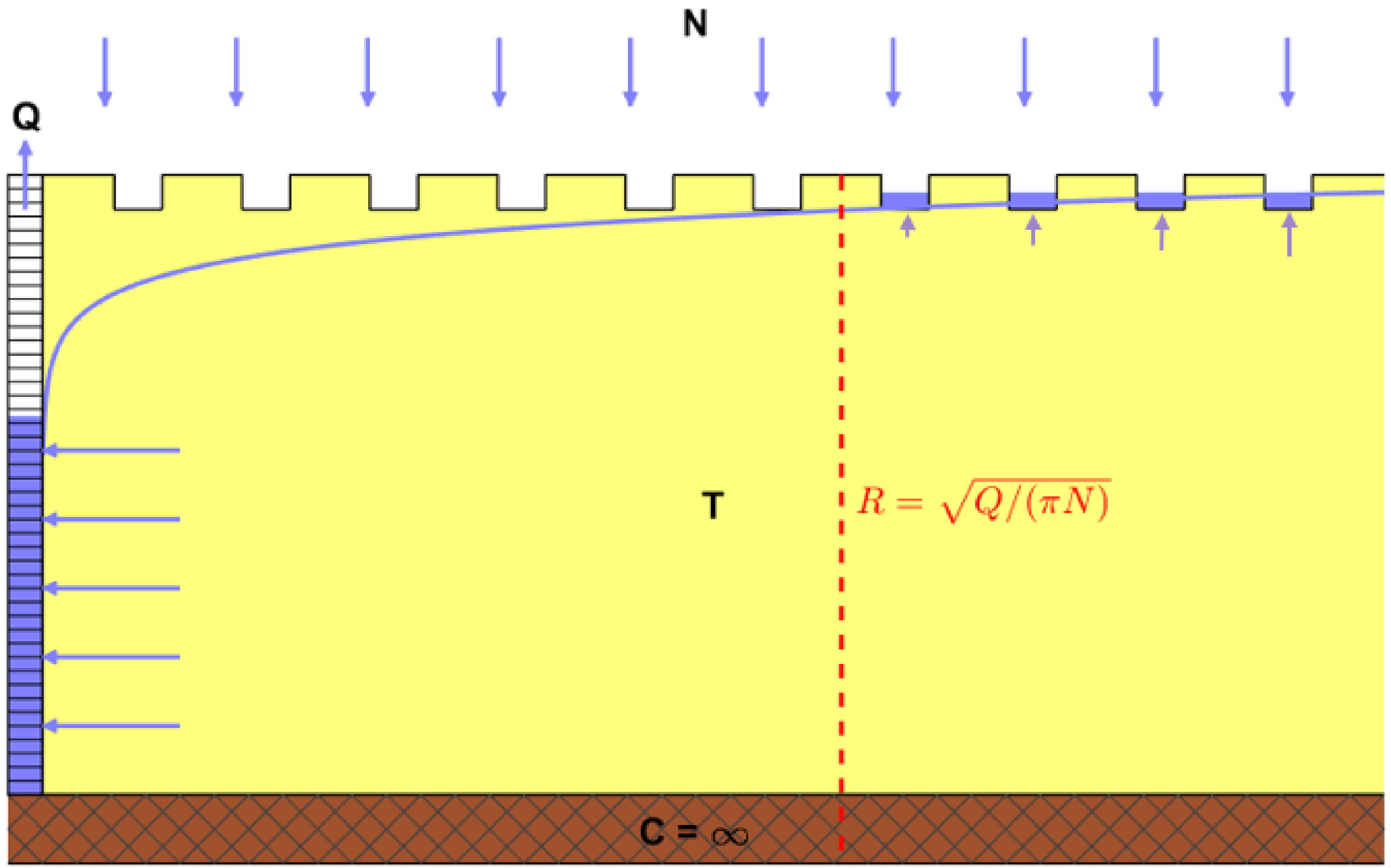
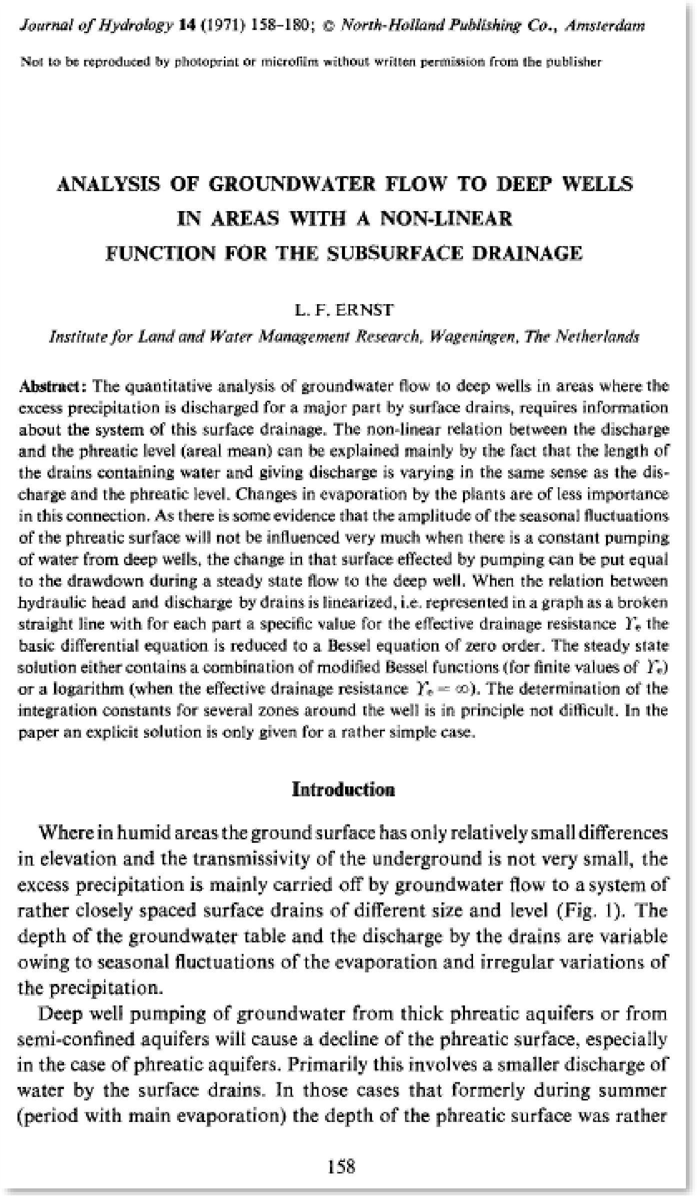
THE ERNST MODEL





Journal of Hydrology **14** (1971) 158–180; © *North-Holland Publishing Co., Amsterdam*

ANALYSIS OF GROUNDWATER FLOW TO DEEP WELLS IN AREAS WITH A NON-LINEAR FUNCTION FOR THE SUBSURFACE DRAINAGE

L. F. ERNST

Institute for Land and Water Management Research, Wageningen, The Netherlands

Abstract: The quantitative analysis of groundwater flow to deep wells in areas where the excess precipitation is discharged for a major part by surface drains, requires information about the system of this surface drainage. The non-linear relation between the discharge and the phreatic level (areal mean) can be explained mainly by the fact that the length of the drains containing water and giving discharge is varying in the same sense as the discharge and the phreatic level. Changes in evaporation by the plants are of less importance in this connection. As there is some evidence that the amplitude of the seasonal fluctuations of the phreatic surface will not be influenced very much when there is a constant pumping of water from deep wells, the change in that surface effected by pumping can be put equal to the drawdown during a steady state flow to the deep well. When the relation between hydraulic head and discharge by drains is linearized, i.e. represented in a graph as a broken straight line with for each part a specific value for the effective drainage resistance Υ_e the basic differential equation is reduced to a Bessel equation of zero order. The steady state solution either contains a combination of modified Bessel functions (for finite values of Υ_e) or a logarithm (when the effective drainage resistance $\Upsilon_e = \infty$). The determination of the integration constants for several zones around the well is in principle not difficult. In the paper an explicit solution is only given for a rather simple case.

Introduction

Where in humid areas the ground surface has only relatively small differences in elevation and the transmissivity of the underground is not very small, the excess precipitation is mainly carried off by groundwater flow to a system of rather closely spaced surface drains of different size and level (Fig. 1). The depth of the groundwater table and the discharge by the drains are variable owing to seasonal fluctuations of the evaporation and irregular variations of the precipitation.

Deep well pumping of groundwater from thick phreatic aquifers or from semi-confined aquifers will cause a decline of the phreatic surface, especially in the case of phreatic aquifers. Primarily this involves a smaller discharge of water by the surface drains. In those cases that formerly during summer (period with main evaporation) the depth of the phreatic surface was rather

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Steady flow with recharge and drainage (Ernst, 1971)

$$s(r) = \frac{Q}{2\pi KD} \ln\left(\frac{R}{r}\right) - \frac{N}{4KD}\left(R^2 - r^2\right) \quad \text{if } c_{drain} \to 0$$

THE WATER BUDGET MYTH

– **Safe yield:** $\cancel{Q = R_0}$

– **Capture equation:** $Q = {\color{red}\Delta R} - {\color{green}\Delta D} - {\color{blue}\frac{dV}{dt}}$

(Theis, 1940; Bredehoeft el al., 1982; Bredehoeft, 2002)

Michael E. Campana (Read this paper before the one by Bredehoeft et al.)

The Source of Water Derived from Wells

Essential Factors Controlling the Response of an Aquifer to Development

From a Paper Presented Before the Arizona Section

By Charles V. Theis

Bredehoeft, J.D., S.S. Papadopulos and H.H. Cooper. 1982. Groundwater: the Water-Budget Myth. In *Scientific Basis of Water-Resource Management*, Studies in Geophysics, Washington, DC: National Academy Press, pp. 51-57.

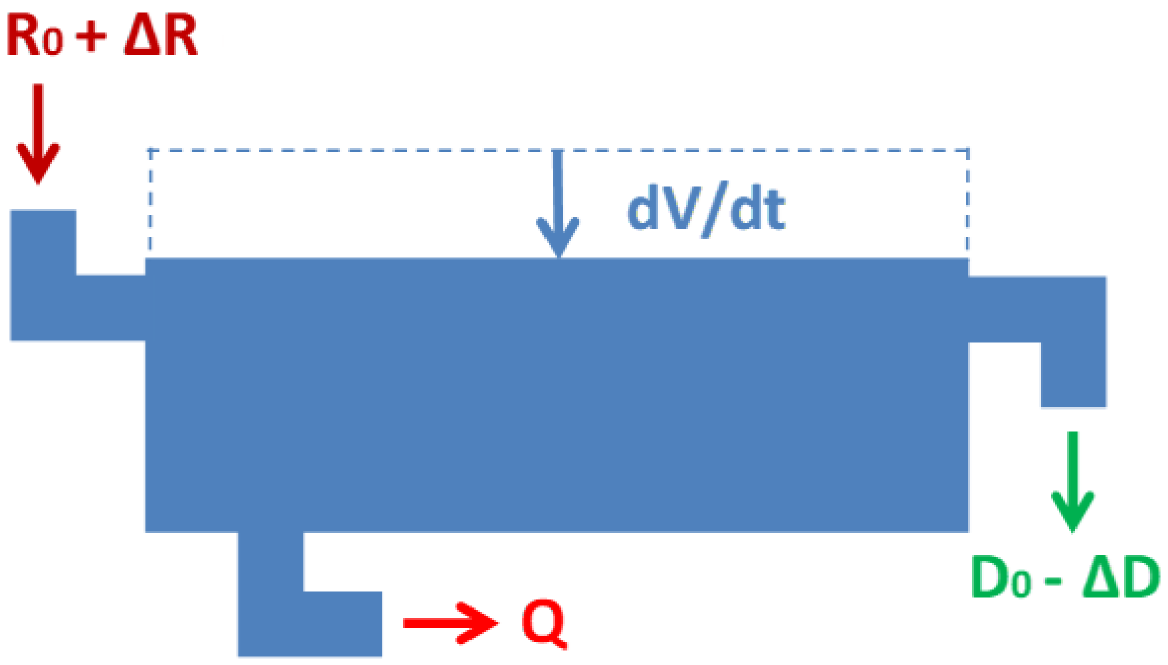
Groundwater:
The Water-Budget Myth

JOHN D. BREDEHOEFT
U.S. Geological Survey

STEPHEN S. PAPADOPULOS
S. S. Papadopulos and Associates, Inc.

H. H. COOPER, JR.
U.S. Geological Survey

John D. Bredehoeft



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LINEAR VS NONLINEAR MODELS

- Linear model: $\boldsymbol{Q = \Delta R - \Delta D - \frac{dV}{dt}}$
  - superposition: recharge is canceled out
  - implicit assumption of infinite sources of water

- Nonlinear model: $\boldsymbol{Q = [R_t - R_0] - [D_t - D_0] - \frac{dV}{dt}}$
  - initial conditions are relevant
  - so is recharge!

  - Ernst model: $\boldsymbol{Q = [D_t - D_0] = \pi R^2 N}$

groundwater NGWA The Groundwater Association

Research Paper/

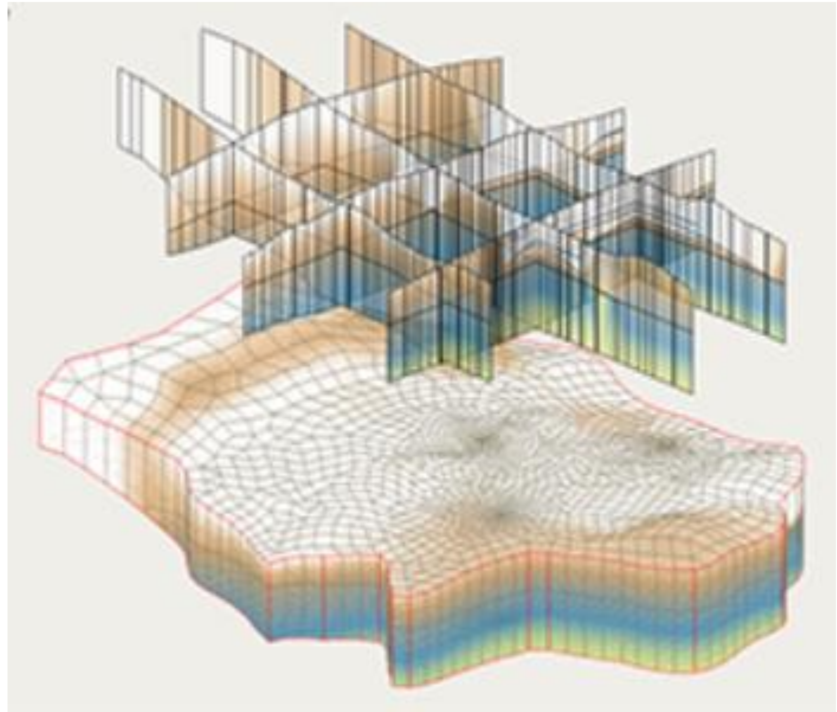
The Water Budget Myth and Its Recharge Controversy: Linear vs. Nonlinear Models

Andy Louwyck, Alexander Vandenbohede, Griet Heuvelmans, Marc Van Camp, Kristine Walraevens

First published: 15 August 2022 | https://doi.org/10.1111/gwat.13245 | Citations: 1

CONCLUSIONS

- assessing sustainability and impact of extractions requires **advanced numerical modeling**



- **analytical models** may be useful:
  - time and budget constraints
  - lack of data
  - *they offer insight!*

groundwater NGWA The Groundwater Association

The Role of Hand Calculations in Ground Water Flow Modeling

Henk Haitjema

First published: 08 March 2006 | https://doi.org/10.1111/j.1745-6584.2006.00189.x | Citations: 66

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EVOLUTION OF AXISYMMETRIC MODELS

- **1 layer**
- incompressible aquitards
- well:
  - fully penetrating (mostly)
  - infinitesimal radius (mostly)

1856	Darcy
1857 & 1863	Dupuit
1870	A. Thiem
1906	G. Thiem
1914	Kooper
1930	de Glee
1935	Theis
1946	Jacob
1955	Hantush & Jacob

EVOLUTION OF AXISYMMETRIC MODELS

- **1, 2 or 3 layers**
- compressible aquitards
- anisotropy
- well:
  - partially penetrating
  - multi-aquifer
  - finite diameter (wellbore storage)
  - instantaneous head change (slug test)
  - finite-thickness skin
- water table conditions:
  - delayed yield
  - infiltration and drainage
  - confined-unconfined flow

1951	Huisman & Kemperman
1954 & 1963	Boulton
1964 & 1967	Hantush
1966	Papadopulos
1967	Papadopulos & Cooper
1967	Cooper et al.
1969	Neuman & Witherspoon
1971	Ernst
1972	Moench & Prickett
1972	Bruggeman
1972 & 1974	Neuman
1983	Javandel & Witherspoon
1984	Moench
1984	Wikramaratna
1988	Butler
1994	Hyder et al.
1995 & 1996	Moench
2012	Mishra et al.
2022	Louwyck et al.

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EVOLUTION OF AXISYMMETRIC MODELS

- **N layers**
- compressible aquitards
- anisotropy
- well:
  - partially penetrating
  - multi-aquifer
  - finite diameter (wellbore storage)
  - instantaneous head change (slug test)
  - finite-thickness skin
- water table conditions:
  - delayed yield
  - infiltration and drainage
  - confined-unconfined flow

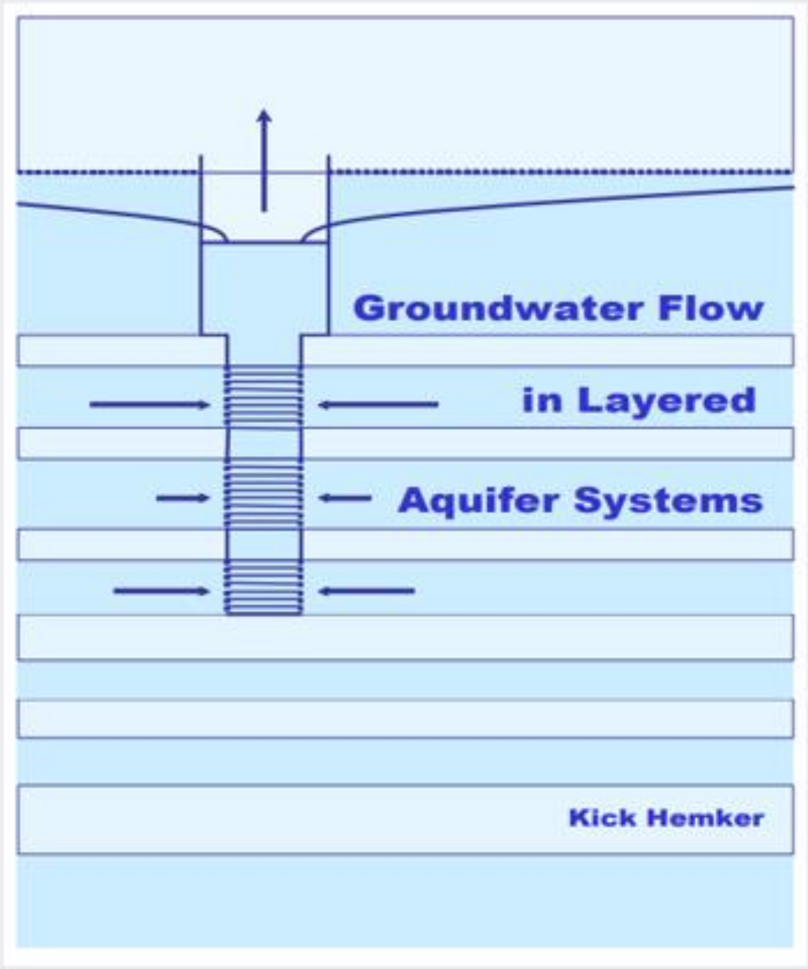
1984 & 1985	Hemker
1985 & 1986	Hunt
1986 & 1987	Maas
1987	Hemker & Maas
1987	Yu
1993	Cheng & Morohunfola
1999	Hemker
2001	Bakker
2002 & 2004	Bakker & Hemker
2003	Bakker & Strack
2004	Meesters et al.
2006	Bakker & Hemker
2007	Hunt & Scott
2009	Veling & Maas
2023	Louwyck

GENERALIZED SEMI-ANALYTICAL SOLUTION

- Python code
  - axisymmetric or parallel flow
  - steady or transient state
  - specified discharge or head
  - laterally bounded or unbounded
  - confined or leaky + recharge
  - superposition in space and time

```
model = Transient(T=[100, 200, 50],      # transmissivities (m²/d)
                  S=[0.1, 0.05, 0.01],   # storativities (-)
                  Q=[-100, -250, 0],     # pumping rates (m³/d)
                  c=[500, 1000],         # resistances (d)
                  c_top=100)             # top resistance (d)
t = np.logspace(-5, 5, 100)  # simulation times (d)
r = 0.1  # well-radius (m)
s = model.h(r, t)  # drawdown s (m)
```

- based on earlier work
  - Hemker (1984, 1985, 1999, 2000)
  - Bakker & Strack (2003)




Kick Hemker


Mark Bakker

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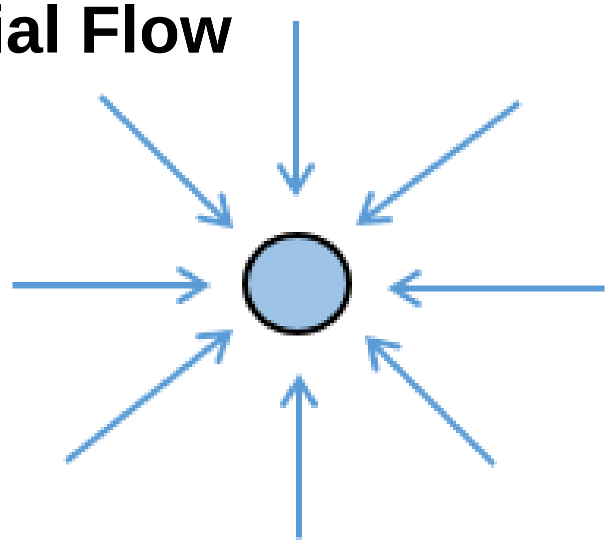
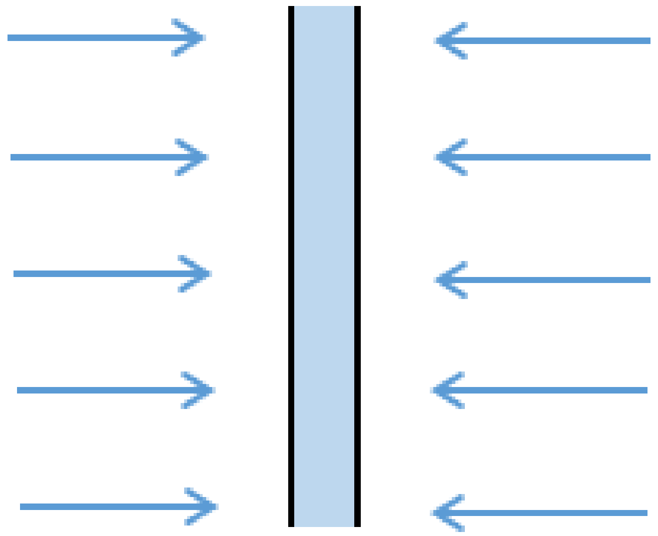
2D MULTILAYER FLOW

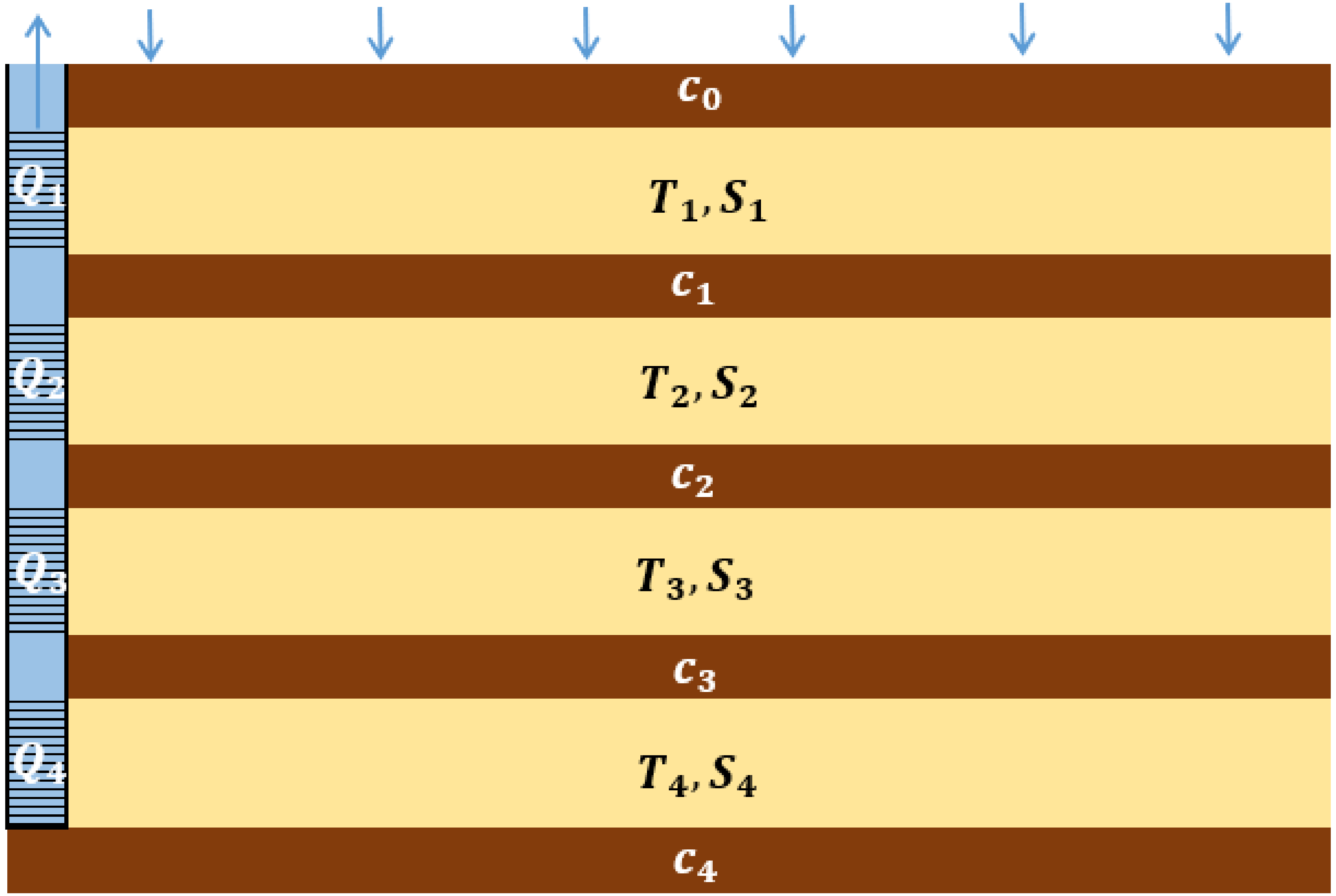
Radial Flow



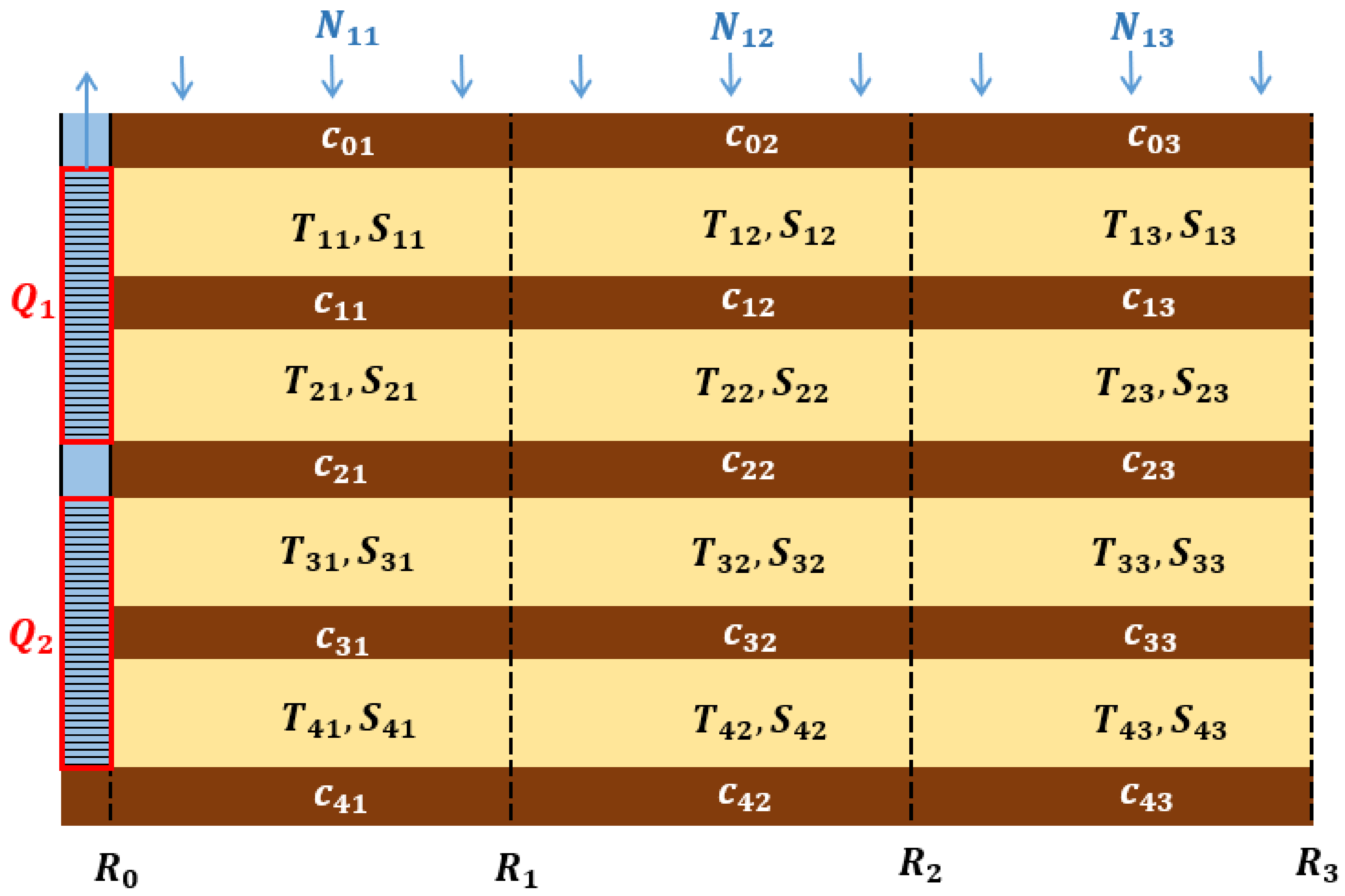
Parallel Flow

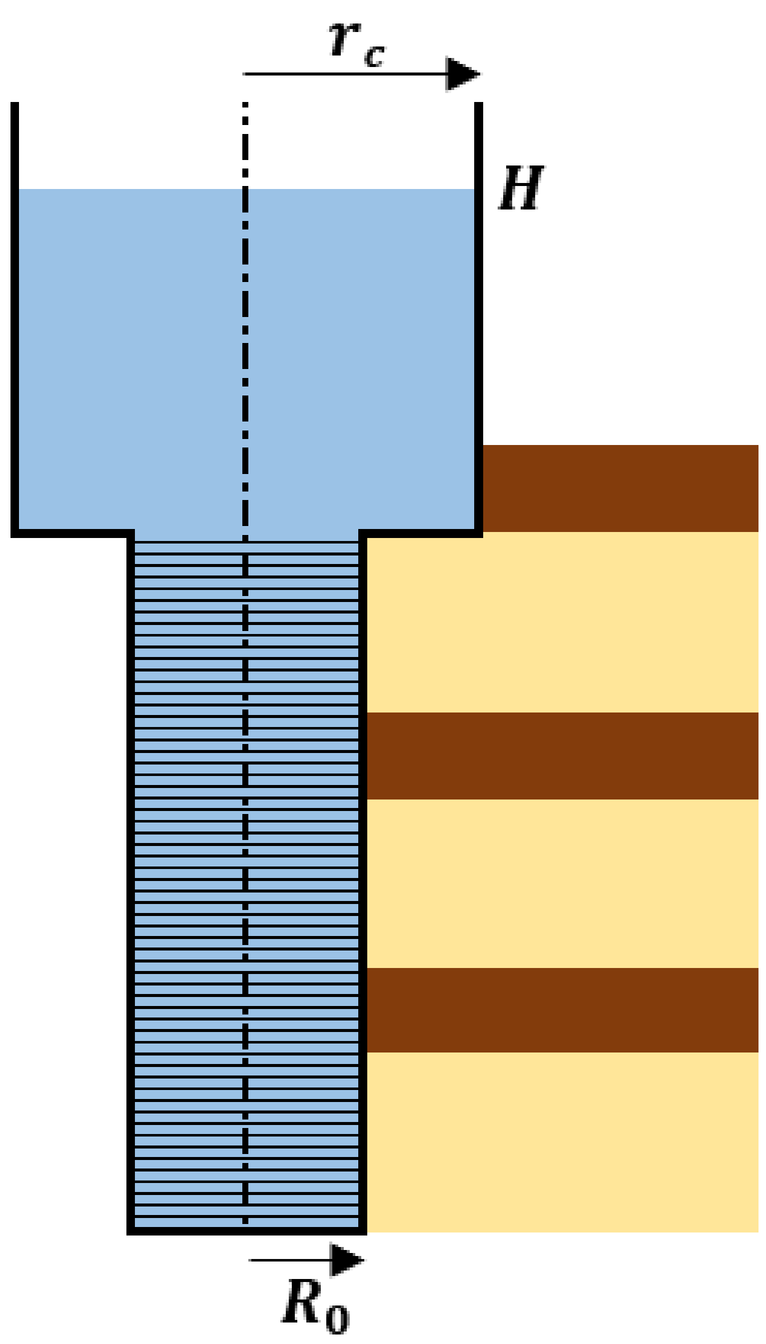


Multilayer Aquifer System



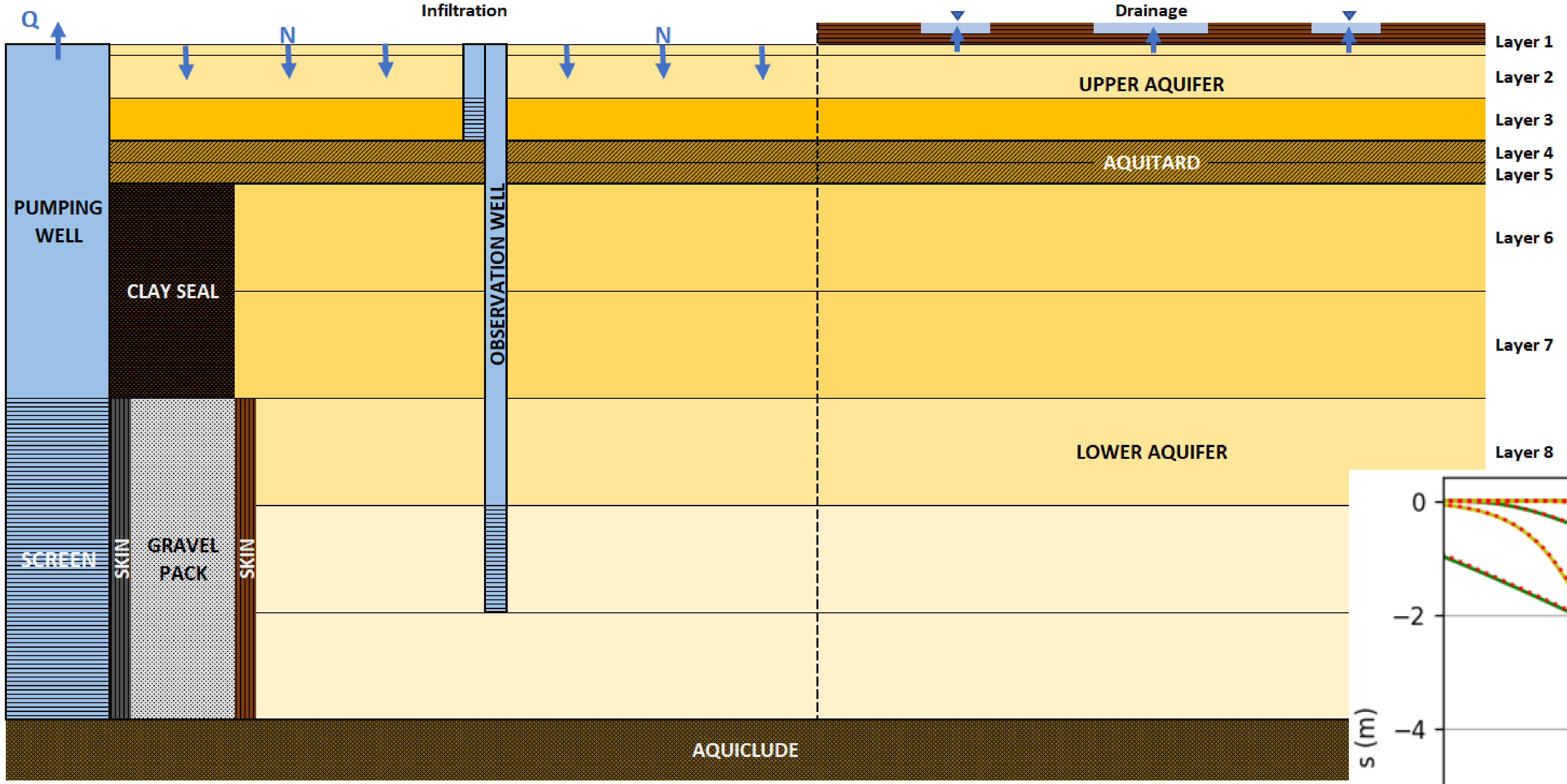
MULTILAYER-MULTIZONE FLOW

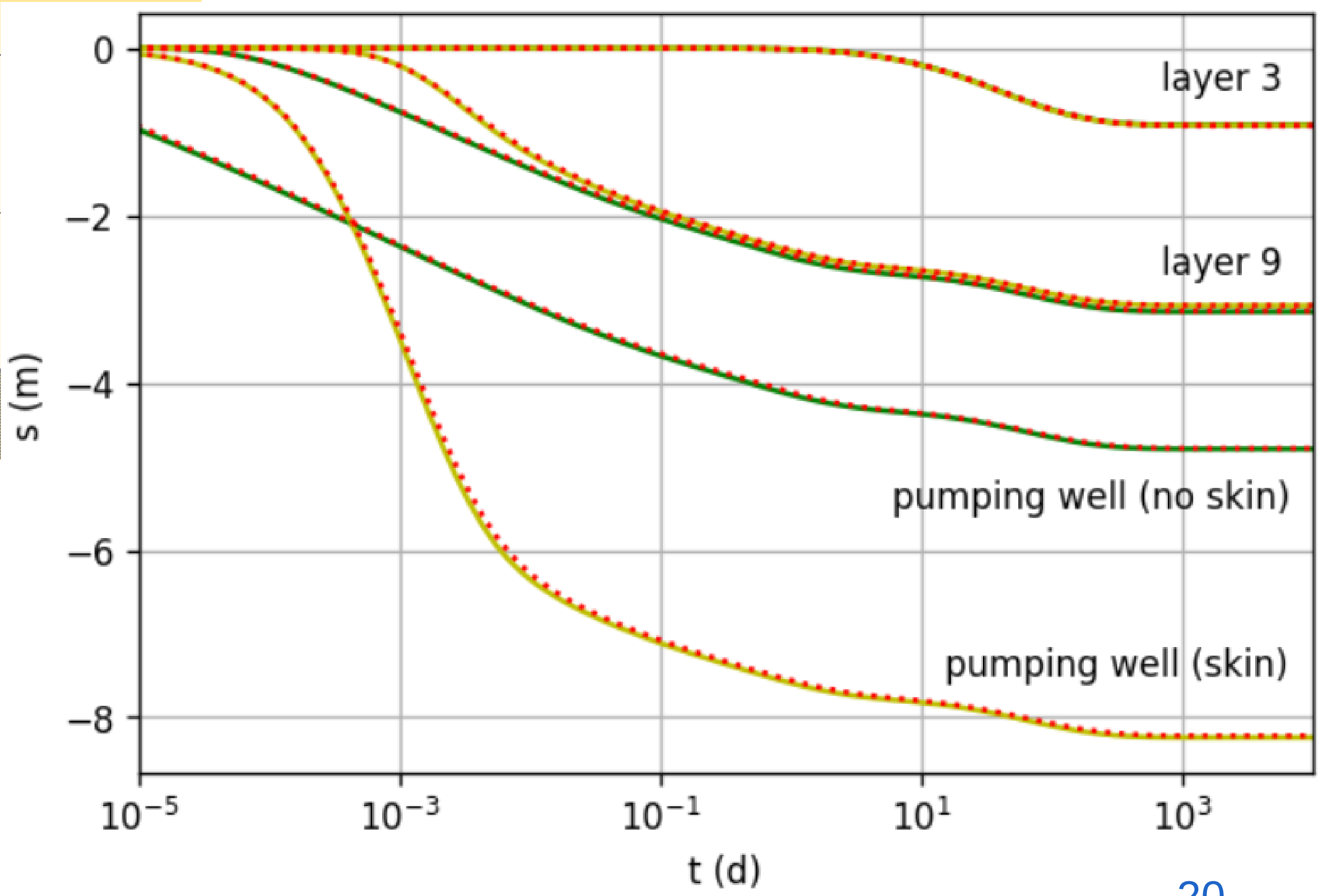




EXAMPLE

alouwyck / PhD





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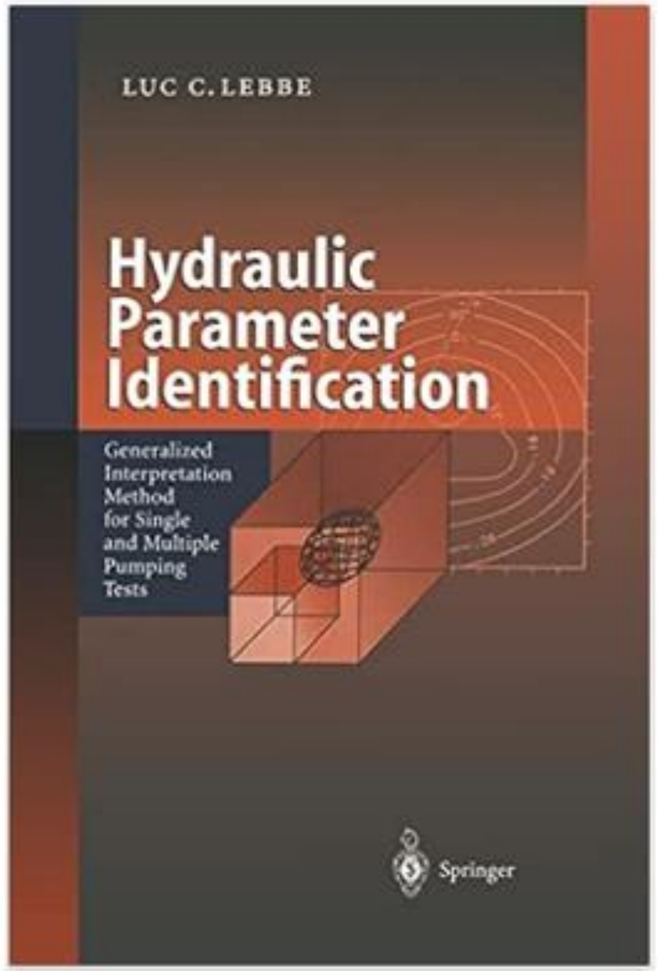
FINITE-DIFFERENCE APPROACH

alouwyck / MAxSym

- software implementations
  - AS2D Matlab wrapper
  - OGMA-RF (Louwyck et al., 2007, 2010; Vandenbohede et al., 2008, 2009)
  - MAxSym (Louwyck, 2011, 2015; Louwyck et al., 2012)
  - MODFLOW procedure (Louwyck et al., 2012, 2014)
  - Python version

- based on earlier work
  - AS2D (Lebbe, 1983, 1988, 1999)
  - MODFLOW (1984, 1988, 1996, 2000, 2005)
  - MODFLOW procedure (Langevin, 2008)



Luc Lebbe

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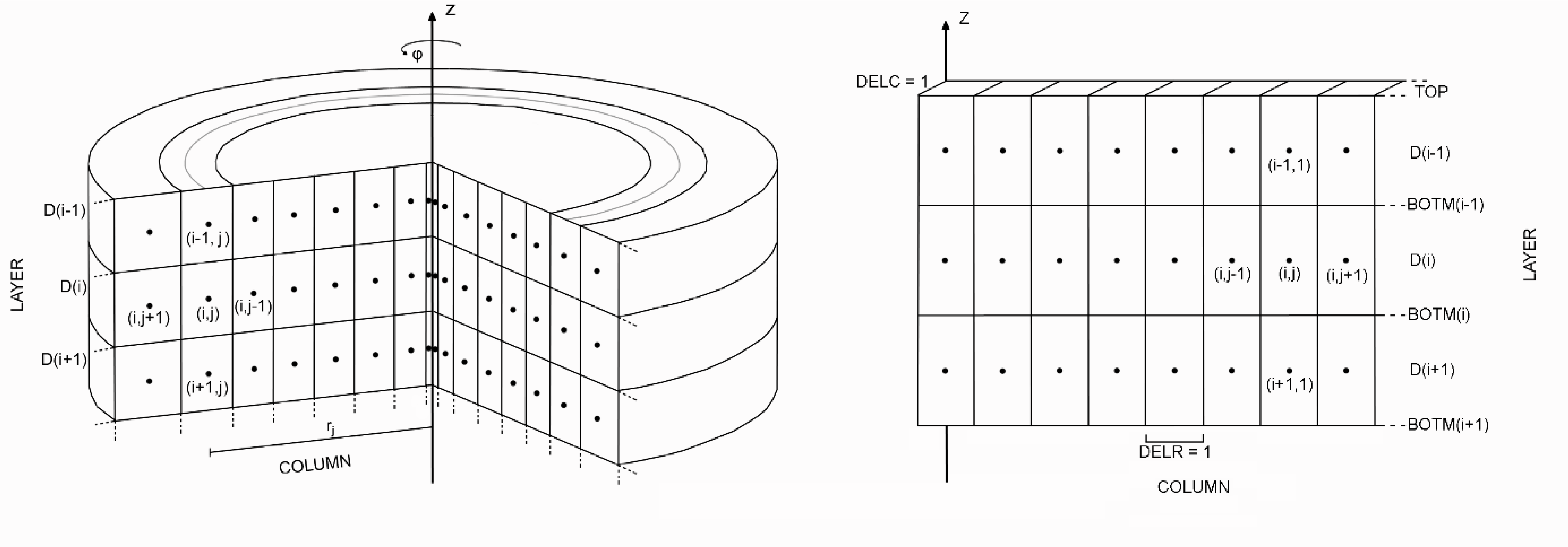
MODFLOW PROCEDURE



Hydrogeology Journal (2014) 22: 1217–1226
DOI 10.1007/s10040-014-1150-0

MODFLOW procedure to simulate axisymmetric flow in radially heterogeneous and layered aquifer systems

Andy Louwyck · Alexander Vandenbohede ·
Mark Bakker · Luc Lebbe

GHENT UNIVERSITY

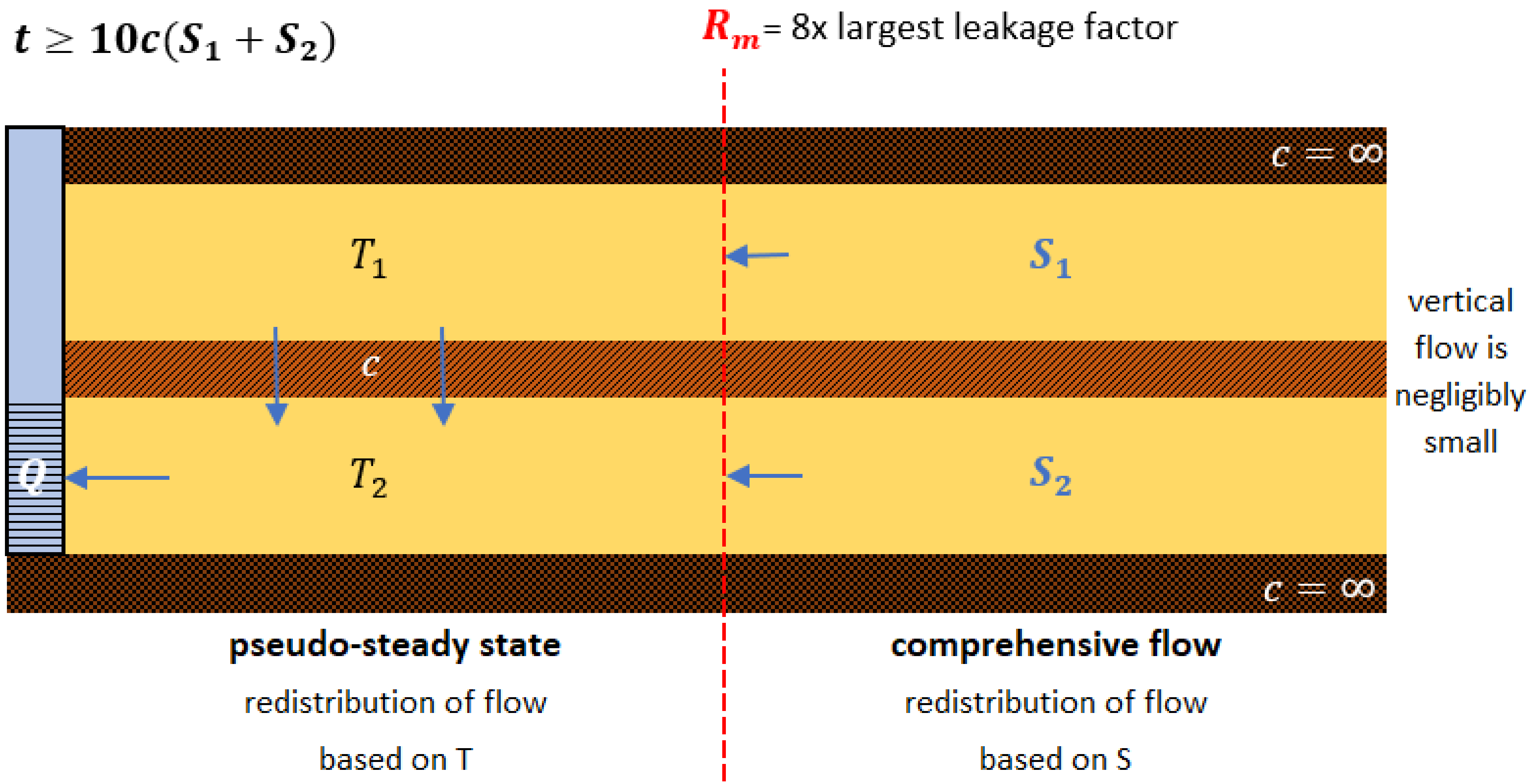
CONCLUSIONS

Semi-Analytical (SA) vs Finite-Difference (FD):

- both very accurate and fast
- FD easier to implement in case of
  - heterogeneities
  - nonlinearities
- SA offers insight!

PSEUDO-STEADY STATE

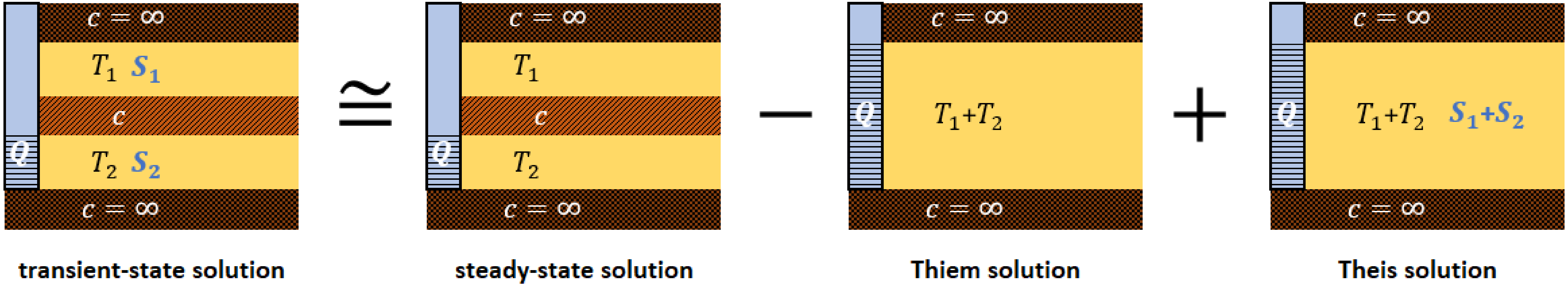
also called *steady shape* (Bohling et al., 2002)



LARGE TIME APPROXIMATION

for confined multilayer well-flow:

$$s(i,r,t) \sim s_{steady}(i,r) - s_{thiem}(r) + s_{theis}(r,t) \qquad (t \to \infty)$$



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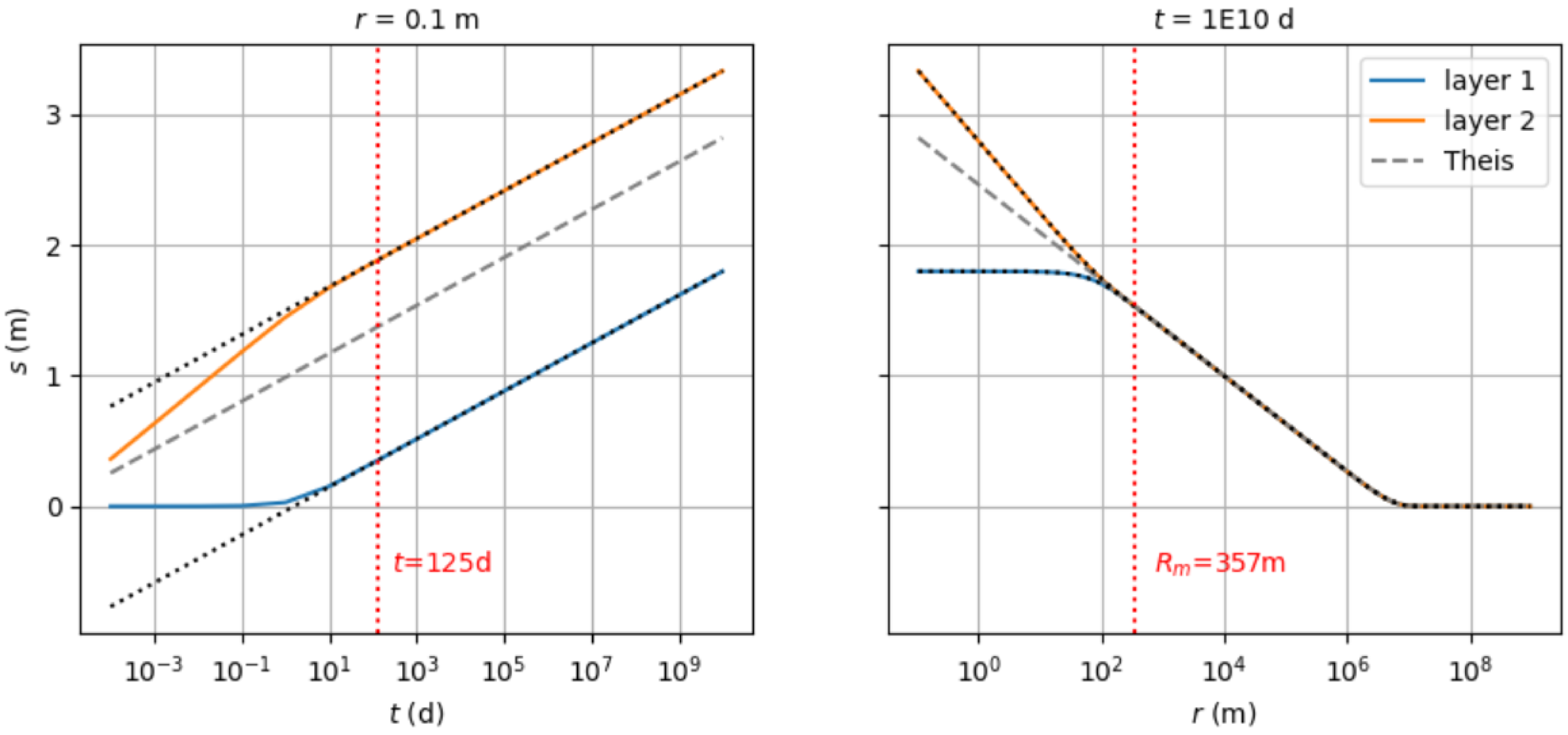
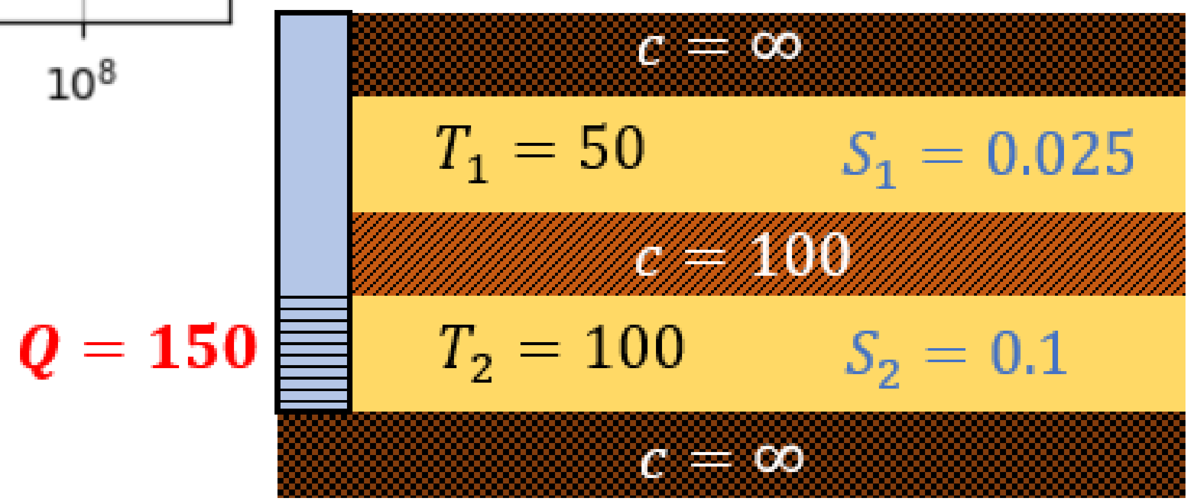
EXAMPLE





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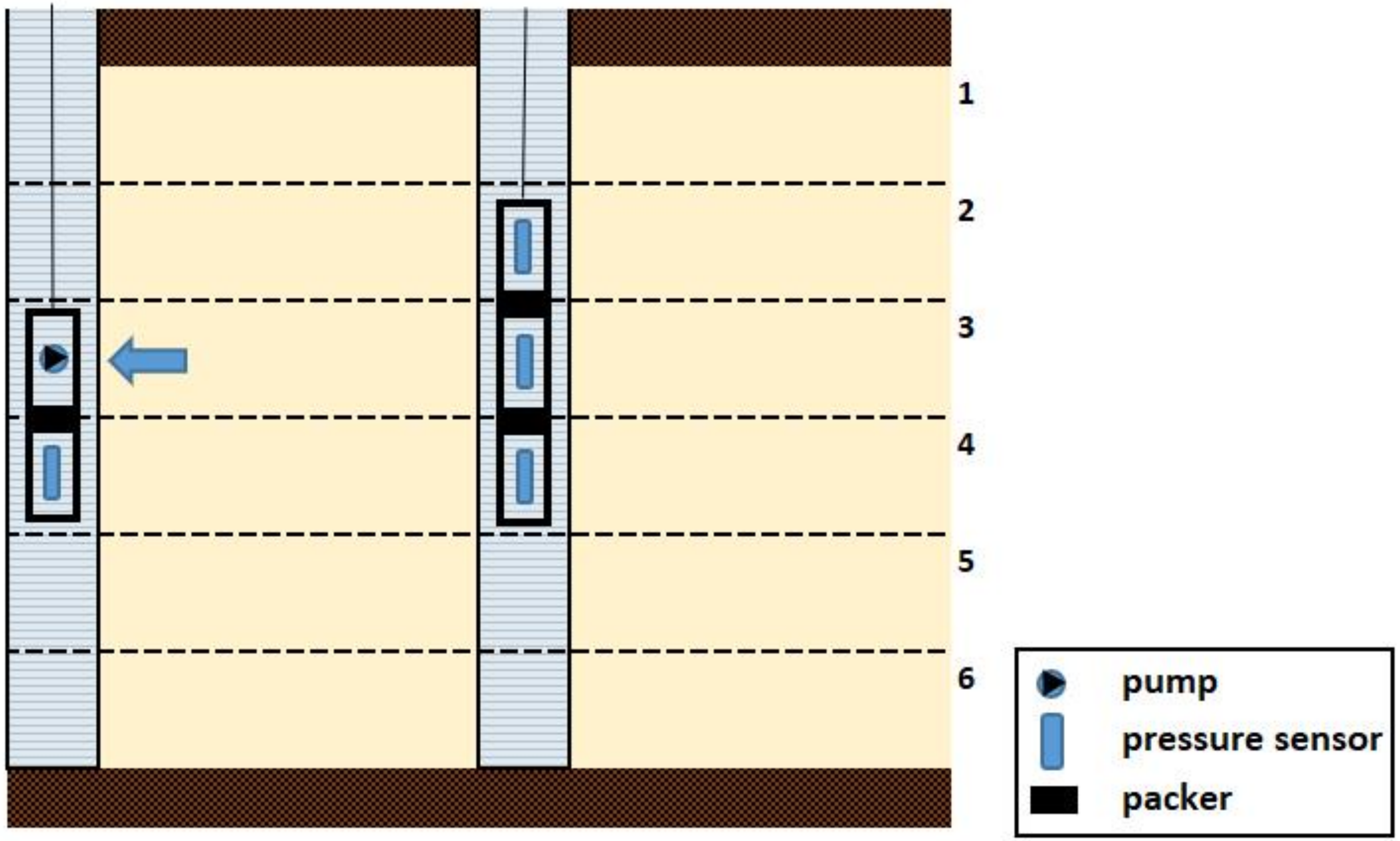
HYDRAULIC TOMOGRAPHY



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CONCLUSIONS

- Theis curves (again) at large values of time
- inversion of flow in the distal zone possible
- spatial averaging in drawdown measurements
- multilevel pumping tests have inherent limitations

groundwater NGWA The Groundwater Association

Inherent Limitations of Hydraulic Tomography

Geoffrey C. Bohling, James J. Butler Jr.

First published: 22 September 2010 | https://doi.org/10.1111/j.1745-6584.2010.00757.x |

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- **Members of the Jury:**
  - Prof. Dr. Stephen Louwye (Chairman, UGent)
  - Prof. Dr. Thomas Hermans (Secretary, UGent)
  - Prof. Dr. Herman Peiffer (UGent)
  - Prof. Dr. Marijke Huysmans (VUB)
  - Prof. Dr. Mark Bakker (TU Delft)
- **Coauthors**:
  - Dr. Griet Heuvelmans (VMM)
  - MSc. Marc Van Camp (UGent)
  - MSc. Dirk Libbrecht (Arcadis)

FACULTY
OF SCIENCES

Andy Louwyck

Hydrogeologist[1] & Lecturer in AI[2]

[1] Laboratory for Applied Geology and Hydrogeology
Department of Geology - Ghent University
[1] Flemish Environment Agency (VMM)
[2] VIVES University of Applied Sciences

andy.louwyck@gmail.com
Andy Louwyck
github.com/alouwyck/PhD

Universiteit Gent
@ugent
@ugent
Ghent University

www.ugent.be

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