Groundwater

The Water Budget Myth and Its Recharge Controversy: Linear vs. Nonlinear Models

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Abstract

The water budget myth, which is the idea that safe pumping must not exceed the initial recharge, gave rise to a controversy about the role of recharge in assessing the sustainability of groundwater development. To refute the concept of safe yield, a simplified water budget equation is used, which equals the total pumping rate to the sum of capture and storage change. Since initial recharge and discharge are canceled out from this equation, it is concluded that sustainable pumping has nothing to do with recharge. Investigating the assumptions underlying this equation, it is seen that it expresses the superposition principle, which implicitly assumes the groundwater reservoir can be depleted indefinitely and boundary conditions are an infinite source of water. To evaluate sustainability, however, the limits of the aquifer system must be examined accurately. Theoretically, this can only be accomplished applying nonlinear models, in which case setting up the simplified water budget equation is impossible without knowing the initial conditions. Hence, excluding recharge when assessing sustainable pumping may not be done inconsiderately, which is illustrated by two examples. An analytical solution, developed by Ernst in 1971 to simulate flow to a well in a polder area with a nonlinear function for drainage, even shows that it is not necessarily a misconception to assume the cone of depression stops expanding when the pumping rate is balanced by the infiltration rate.

Introduction

This paper addresses the controversy about the role of recharge in assessing whether a groundwater extraction is sustainable or not. The controversy started with an editorial written by Sophocleous (1997) explaining why the concept of safe yield, which limits groundwater pumping to the natural recharge, leads to continued groundwater depletion, stream dewatering, and loss of wetland and riparian ecosystems. Sophocleous (1997) refers to Theis (1940), who concisely stated the underlying hydrologic principles, which were revisited by Konikow and Leake (2014). Under natural conditions, aquifer systems are in a dynamic state of equilibrium, hence, recharge is balanced by discharge. If water is pumped from an aquifer system, the extracted volume is balanced by storage change, an increase in recharge, and/or a decrease in discharge. When pumping starts, the water almost exclusively comes from storage, whereas it is the capture, that is the increase in recharge plus the decrease in discharge (Lohman 1972a), which balances the extracted amount of water when the system is brought into a new state of equilibrium. According to Sophocleous (1997), the timing of this transition is a key factor in developing sustainable water-use policies, but it is exceedingly difficult to distinguish between natural and induced recharge. These ideas are further developed and translated into practice by Sophocleous (2000, 2002, 2005).

In a subsequent editorial, Bredehoeft (1997) called the editorial written by Sophocleous (1997) an especially important one, as it explains why the idea of safe yield is fallacious. Bredehoeft (1997) also refers to Theis (1940), and to Bredehoeft et al. (1982), who term this idea the groundwater budget myth. Bredehoeft (1997) elaborates on the dynamic nature of the capture that depends on both the geometry of the aquifer system and the hydraulic properties such as permeability and specific storage. In his experience, the change in recharge due to pumping is difficult or even impossible to quantify. Recharge usually is fixed by rainfall and does not increase by development; hence, only reduced discharge may bring

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the aquifer system into a new state of equilibrium. Bredehoeft (1997) concludes sustainable groundwater development has almost nothing to do with recharge, and therefore, the focus of research should not be on induced recharge, as suggested by Sophocleous (1997).

Bredehoeft (2002) revisits the water budget myth, because the Bredehoeft et al. (1982) paper appeared in an obscure publication. Again, Bredehoeft (2002) refers to Theis (1940), and additionally, to Brown (1963), who also debunks the water budget myth. He also mentions more recent studies (e.g., Alley et al. 1999; Sophocleous 2000), but remarks that the fundamental principles stated by Theis (1940) are not well understood by many groundwater professionals. Bredehoeft (2002) points to the use of superposition to calculate the cone of depression and concludes that the initial rate of recharge is not important at all in determining the size of a sustainable extraction. Maddock and Vionnet (1998) show that even seasonal variations of recharge and discharge do not determine capture.

Sophocleous and Devlin (2004) agree with the arguments given by Bredehoeft (2002), but they believe he condemns the association between recharge and sustainability so forcefully, groundwater practitioners may get the impression that recharge is not worth considering at all. According to Sophocleous and Devlin (2004), recharge is important because sustainability encompasses more than sustainable pumping; other factors such as water quality, ecology and socioeconomic considerations also play an important role. This is in agreement with Alley et al. (1999), who define groundwater sustainability as the development and use of groundwater in a manner that can be maintained for an indefinite time without causing unacceptable environmental, economic, or social consequences. Alley and Leake (2004) even argue that sustainability is not a purely scientific concept, but an evolving perspective that challenges hydrologists to translate complex and vague socioeconomic questions into quantifiable problems. This requires “integrated groundwater management,” which is an approach of “thinking beyond the aquifer” to get to a more encompassing view of steps needed to have sustainable groundwater resources (Jakeman et al. 2016). Since sustainability cannot be considered from solely a groundwater perspective, Devlin and Sophocleous (2005) suggest to separate the concepts of sustainable pumping and sustainability, and they conclude that recharge rates are irrelevant to the first, whereas they are not to the latter.

Loáiciga (2006) interprets the issue as a maximization problem, and the solution to this problem shows that the sustainable pumping rate cannot exceed the initial recharge. Loáiciga (2006) concludes that the water budget myth is an innocuous, if not laudable concept, as it is a meaningful and correctly derived pumping rate under the definition of sustainable pumping proposed by Loáiciga (2003). Loáiciga (2006) also solves the transient groundwater budget equation which yields a sustainable pumping rate as a function of time. This adaptive sustainable pumping requires knowledge of the recharge, which is also a function of time, as precipitation varies with time (Loáiciga 2006).

Devlin and Sophocleous (2006) argue that Loáiciga’s (2006) comment illustrates there is confusion in the hydrogeological community about sustainable pumping and sustainability, which is partially semantic. According to Devlin and Sophocleous (2006), Loáiciga’s (2006) definition of sustainable pumping is consistent with use of the term sustainability by Devlin and Sophocleous (2005), and introducing the transient case redefines the scope of the discussion. Consequently, Devlin and Sophocleous (2006) maintain that recharge estimates are not necessary for the calculation of sustainable pumping rates, whereas they are required for the assessment of sustainability.

Kalf and Woolley (2005) agree with Bredehoeft (2002) to the extent that a determination of natural recharge alone is an oversimplification for determining sustainability, but conclude that the water budget myth is not necessarily a myth, from a basin groundwater sustainability perspective, and that natural recharge is not irrelevant. It is required to determine the predevelopment conditions, and the interception of natural discharge must always be equal to or less than the rate of natural recharge, given the basin initially was in equilibrium. They also discuss the hypothetical Basin and Range example given by Bredehoeft et al. (1982), Bredehoeft (2002), and Bredehoeft and Durbin (2009), and consider the same basin in a more humid environment with a large permanent river meandering across it. In this case, the well-field performance would be controlled largely by induced recharge provided that this river flow is comprised of runoff. By contrast, Bredehoeft et al. (1982) assume recharge is independent of the pumping in the basin, which is a typical condition in the arid west of the United States.

As already mentioned above, Bredehoeft (1997, 2002) generalizes this assumption to most groundwater situations, arguing recharge is a function of external conditions such as rainfall and vegetation. This implies there is no change in recharge and the extracted water is balanced by a decrease in initial discharge. In an editorial titled “It is the discharge,” Bredehoeft (2007) repeats his point of view, and advocates to examine the discharge of a groundwater system, as it is more fruitful than focusing on the recharge; the latter is difficult to quantify, and human activities that impact a groundwater system ultimately impact the former. Bredehoeft (2007) also states that many aquifers can be analyzed mathematically as if they are linear systems, even water table aquifers where the change in head does not change the saturated thickness greatly. Leake (2011) emphasizes that rates and directions of groundwater flow do not matter when determining capture in a reasonably linear system. If the system is linear, it is justified to apply the principle of superposition, which is discussed in more detail by Bredehoeft (2002). The nonlinear case is discussed by Leake et al. (2010), who conclude that in general, the capture is overestimated when nonlinearities are linearized.

Zhou (2009) agrees that the traditional idea of safe yield is a misconception, but states that both the natural recharge and the dynamic development of the capture determine the sustainability of a groundwater development. Zhou (2009) concludes that the aquifer storage will be depleted if the pumping rate is larger than the total recharge, that is, initial plus induced recharge. Zhou (2009) also believes that it is difficult to determine the capture, whereas it is more convenient to determine total recharge and total discharge under development conditions, an argument that is also given by Van der Gun and Lipponen (2010).

Both Zhou (2009) and Van der Gun and Lipponen (2010) mention that the natural recharge and the sustainable pumping rate, which is balanced by the maximum capture, are correlated. Van der Gun and Lipponen (2010) indicate this correlation is strong when capture mainly consists of reduced discharge, and they argue that in case of a closed arid groundwater basin, such as the Basin and Range example discussed by Bredehoeft (2002), a model is not required to see the maximum capture is virtually equal to the initial recharge. The situation is different when the groundwater system interacts with a permanent surface water body, in which case the induced recharge from the latter plays an important role. In many other cases, however, the maximum rate of sustainable pumping may be expressed as a fraction of the initial recharge (Van der Gun and Lipponen 2010).

Seward et al. (2015) question Zhou's (2009) statement that aquifer sustainability depends on both natural recharge and capture, as the first is missing in Bredehoeft's (2002) capture equation. Seward et al. (2015) also believe a large part of the hydrogeological community see no harm in "pumping the recharge" approaches to sustainability, referring to Balleau (2013), although the latter suggests to abandon the traditional concept of safe yield from the administrative functions, as the concept was dismissed by groundwater experts long time ago. Balleau (2013) even disapproves the use of a ratio of natural recharge to well-field withdrawals as indicator of sustainability by Wada et al. (2010). Ferguson (2021) also criticizes Wada (2016) and Bierkens and Wada (2019) for applying recharge based approaches, and substantiates his comment referring to Bredehoeft (2002).

The cited literature proves that there is still controversy about the role of recharge in studying groundwater sustainability. The objective of this paper is to show that, within the scope of sustainable pumping, making the distinction between linear and nonlinear models is of utmost importance. This is proven by unraveling the assumptions underlying the simplified water budget equation presented by Bredehoeft et al. (1982) and Bredehoeft (2002). It is also argued that the mathematical assumption of a linear system comes down to the assumption that the extraction is sustainable. This underlying circular reasoning makes the use of linear models theoretically questionable if applied to evaluate sustainable pumping and sustainability in general. These ideas are illustrated by discussing two simple well-flow problems that can be solved applying analytical models.

First, the Bredehoeft et al. (1982) and Bredehoeft (2002) example of a circular island is used to demonstrate that the nonlinear solution with head-dependent aquifer transmissivity requires knowledge of the initial conditions. In this case, the initial recharge also determines whether the pumping is sustainable or not, and superposition is applicable only if specific conditions are met. Second, Ernst's solution (1971) is discussed in detail by adopting the circular island example and reframing it to humid climatic conditions. As the Ernst (1971) model includes a nonlinear drainage boundary condition, it is examined again under what conditions superposition is applicable. In this case, the recharge appears to be one of the key parameters in determining whether the system can be analyzed as a linear system or not. It is also shown that balancing pumping and infiltration rate to determine the extent of the cone of depression corresponds to the asymptotic solution with zero drainage resistance.

The Water Budget Equation Revisited

The basic idea that the amount of extracted groundwater is balanced by a change in storage, a change in recharge, and a change in discharge, is stated first by Theis (1940), and expressed using water balance equations by Lohman (1972b), who mentions Cooper formulating these equations back in 1967. The same equations are presented by Bredehoeft et al. (1982) and Bredehoeft (2002) to debunk the water budget myth, and therefore, they are an integral part of most papers discussing the myth.

In general, the groundwater balance at time t (T) is

$$R_t - D_t - \frac{\mathrm{d}V_t}{\mathrm{d}t} = 0 \tag{1}$$

with R_t is the total recharge (L^3/T), D_t is the total discharge (L^3/T), and V_t is the groundwater storage (L^3). Note that R and D in Equation 1 are always positive, whereas storage change $\mathrm{d}V/\mathrm{d}t$ is positive when water is removed from the aquifer. If $\mathrm{d}V/\mathrm{d}t = 0$, then the aquifer system is in a steady state. In natural circumstances, recharge may result from rainfall percolating through the soil or from surface water infiltrating into the aquifer system (Theis 1940). If Equation 1 expresses the total water balance of a multiaquifer system, it includes all layers in the system, in which case the vertical flow between layers is not considered as recharge.

To study the hydrological impact of a groundwater extraction with pumping rate Q_t (L^3/T), the water budget Equation 1 is reformulated as:

$$(R_0 + \Delta R_t) - (D_0 + \Delta D_t) - Q_t - \frac{\mathrm{d}V_t}{\mathrm{d}t} = 0 \tag{2}$$

with R_0 and D_0 are the initial recharge and discharge, respectively, at time $t = 0$ when the extraction starts; $\Delta R_t = R_t - R_0$ is the change in initial recharge at time t, and $\Delta D_t = D_t - D_0$ is the change in initial discharge at time t. Note that in Equation 2 total pumping rate Q_t is separated from the other sinks that discharge water from the aquifer system. When discussing the water budget myth, a constant pumping rate is assumed, hence $Q_t = Q$, which is positive if water is extracted.

From Equation 1 it follows that $R_0 = D_0$ if it is assumed the groundwater system is in a state of dynamic equilibrium at $t = 0$, as storage change is zero by definition under this assumption. This reduces Equation 2 to:

$$Q = \Delta R_t - \Delta D_t - \frac{\mathrm{d}V_t}{\mathrm{d}t} \tag{3}$$

with $\Delta R_t - \Delta D_t$ the capture at time t, also termed "depletion" according to Leake (2011). Equation 3 expresses the basic principle that the extracted water is balanced by the capture and the storage change. The initial recharge R_0 is absent from Equation 3; hence, the concept of safe yield is a myth, as it states that pumping is safe if $Q \leq R_0$ (Bredehoeft et al. 1982; Bredehoeft 2002). Another water budget myth is the idea that the volume V of groundwater in storage is by itself meaningful in the analyses of water availability (Alley 2007). Besides the fact that not all groundwater in storage is recoverable with pumping wells (Alley 2007), Equation 3 shows indeed that aquifer storage is not the only quantity that must be taken into account.

If the system is brought into a new state of equilibrium, then the pumping rate is only balanced by the capture:

$$Q = \lim_{t \to \infty} (\Delta R_t - \Delta D_t) \tag{4}$$

Mathematically, Equation 4 is obtained after an infinitely large period of pumping. In reality, the time to full capture is finite, ranging from a few seconds to centuries or possibly millions of years (Bredehoeft and Durbin 2009; Sophocleous 2012). It is also possible a new steady state may never be reached if the pumping rate is too large to be balanced by the capture. In this case, the groundwater reservoir continues to be depleted until the extraction runs dry. Therefore, sustainable pumping means that the aquifer system can be brought into a new state of dynamic equilibrium, which implies Equation 4 has a real solution.

Recall that Equation 3 is only valid under the assumption of a constant pumping rate Q and an initial steady state. In many real world cases, however, pumping rates are time dependent, and new extractions may be planned in exploited aquifer systems that are not in a state of dynamic equilibrium. The transient case was discussed by Loáiciga (2006), who showed that in this case, recharge as a function of time may be considered. If the second assumption is not met, the initial recharge does not equal the initial discharge, which means that both are required to simulate the initial conditions before pumping.

A third assumption mentioned by Bredehoeft (2002, 2007) must also be taken into account: the superposition property, which is explained in many hydrogeology textbooks (e.g., Verruijt 1970; Kruseman and de Ridder 1990; Haitjema 1995; Bruggeman 1999). Applied to pumping wells, the principle of superposition or linearity principle states that drawdowns due to individual constant-discharge wells can be summed to obtain the total drawdown caused by these extractions. However, this principle is only valid if the governing differential equation is homogeneous and linear, and if all individual solutions satisfy the boundary and initial conditions (Bruggeman 1999). The differential equation describing horizontal flow in an unconfined aquifer is the canonical example of a nonlinear problem for which the superposition principle does not hold, because the saturated aquifer thickness is a function of the hydraulic head. Superposition is also not allowed if the aquifer is recharged by a constant infiltration flux, as in this case, the governing differential equation is nonhomogeneous. A typical example of a nonlinear boundary condition is the MODFLOW drain (Harbaugh 2005), because its conductance is head-dependent.

Equation 1 only expresses the law of conservation of matter (Devlin and Sophocleous 2005), whereas flow within the system is determined by the hydraulic heads according to Darcy's law. Hence, R, D, and V are functions of hydraulic head h (L), and therefore, the change in recharge ΔR_t and the change in discharge ΔD_t should be written as, respectively:

$$\Delta R_t = R(h_t) - R(h_0) \tag{5}$$

$$\Delta D_t = D(h_t) - D(h_0) \tag{6}$$

In essence, the discussion about the relevance of the initial recharge (and discharge) comes down to the question if ΔR_t (and ΔD_t) can be calculated directly without knowing R_0 (and D_0). Since this question is not answered yet, Equation 3 needs to be rewritten using Equations 5 and 6:

$$Q = [R(h_t) - R(h_0)] - [D(h_t) - D(h_0)] - \frac{\partial V(h_t)}{\partial t} \tag{7}$$

Drawdown s (L) expresses the effect of pumping on the hydraulic head, as it is defined as the change in head due to pumping:

$$s_t = h_t(h_0) - h_0 \tag{8}$$

Note that in general, hydraulic head h_t at time t is a function of initial head h_0 at $t = 0$. However, if h_0 is constant, then there is no flow in the predeveloped aquifer system, hence, initial recharge and discharge are zero, and hydraulic head h_t is independent of initial head h_0. Under this assumption, Equation 7 simplifies to:

$$Q = R(s_t) - D(s_t) - \frac{\partial V(s_t)}{\partial t} \tag{9}$$

Since Equation 9 is equivalent to Equation 3, quantity $R(s_t) - D(s_t)$ is the capture.

Assuming a horizontal piezometric surface before pumping is common in aquifer test interpretation (Kruseman and de Ridder 1990), although Equation 9 is also valid under a less strict assumption. Indeed, if the differential equation and the boundary conditions describing the groundwater flow problem are linear, then the superposition principle holds:

$$L(ah_t + bh_0) = aL(h_t) + bL(h_0) \quad (10)$$

with a and b arbitrary constants, and L a response function, in this case R, D, and V. If Equation 10 holds and $\mathrm{d}V_0/\mathrm{d}t = 0$ implying h_0 is independent of time, Equation 7 can be written as:

$$Q = R(h_t - h_0) - D(h_t - h_0) - \frac{\partial V(h_t - h_0)}{\partial t} \quad (11)$$

Applying Equation 8 to Equation 11 indeed results in Equation 9, which is the water budget equation for models simulating drawdown s_t directly applying superposition. In these models, hydraulic head h_t is independent of initial head h_0, which follows from Equation 10. Recall that the initial head must be steady, although uniformity is not required, which means constant groundwater flow is allowed in the initial aquifer system.

Models that simulate drawdown directly, under the assumption of linearity, only consider changes in recharge and discharge, and they do not distinguish between both. That is why "rates and directions of groundwater flow don't matter", to quote Leake (2011). If it is assumed recharge through percolation is not affected by the pumping, then it is not included in these models. And that is exactly the point made by Bredehoeft (1997, 2002, 2007). To evaluate the change in flow in the aquifer system or the alterations in the interactions with sources and sinks, the drawdowns are superimposed on the initial heads afterwards.

If some of the equations involved in the mathematical problem statement are not linear, then superposition is not allowed, and Equation 11 cannot be applied. This means that hydraulic head h_t during pumping depends on initial head h_0, and the capture on initial recharge R_0 and initial discharge D_0. In other words, Equation 7 cannot be simplified to Equation 11 if the problem is not linear. Therefore, the question whether initial recharge is relevant or not in evaluating sustainable pumping comes down to the question if the assumption of linearity is justified when assessing the sustainability of an extraction.

As already mentioned, it is common practice to apply superposition when interpreting aquifer tests. If these tests are temporary and have limited impact on the hydraulic head, the assumption of linearity is justified indeed. Using superposition to evaluate the impact of permanent groundwater extractions, however, is questionable or at least subject to caution, since linear systems implicitly assume unlimited resources of water. In a linear model the saturated thickness of aquifers and aquitards are constant, as are the boundary conditions implementing the interaction with surface water bodies. As a consequence, the system may continue to be depleted if a new state of equilibrium is not reached, because wells cannot go dry and sources cannot dry up.

Sheets et al. (2014) investigate the approximation error produced by the simulation of water-table aquifers using a constant saturated thickness, and they report a maximum error of about 20% when maximum drawdowns are about 35% of the initial saturated thickness. Therefore, they recommend the use of the specified-thickness approximation in early phases of model development and in applications that require many model runs to save execution time and improve solution stability. Leake et al. (2010) argue that nonlinear responses are more likely to occur when pumping rates are large, but weaken the importance of nonlinearities by stating that their general effect is to overestimate the capture when they are removed from the model. However, water budget Equation 3 clearly indicates that overestimating the capture leads to an underestimation of the storage change, and hence, the cone of depression. This confirms that applying superposition should be done carefully, not only to assess sustainability in the broadest sense of the word, but also to evaluate whether an extraction is sustainable or not.

Bredehoeft's Island

Consider the circular island situated in a freshwater lake shown in Figure 1 and discussed by Bredehoeft et al. (1982) and Bredehoeft (2002). The phreatic aquifer on the island is recharged by rainfall and discharges into the lake. It is bounded below by an impermeable aquiclude, and a well at the center of the island extracts water from it at constant pumping rate. After a period of pumping, the system is brought into a new equilibrium, and the steady head in the unconfined aquifer is calculated using the following nonlinear equation (Verruijt 1970):

$$h(r) = \sqrt{h_c^2 + \frac{N}{2K}\left(r_c^2 - r^2\right) + \frac{Q}{\pi K}\ln\frac{r}{r_c}} \quad (12)$$

where h is the hydraulic head (L), r is the radial distance (L) to the well, h_c is the water level in the lake, which defines the constant head (L) at the outer boundary of the island at distance r_c, which is the radius (L) of the island, K is the hydraulic conductivity (L/T) of the aquifer, N is the infiltration flux (L/T), and Q is the pumping rate (L^3/T) of the well. In case of recharge, N is positive. Recall that Q is positive in case of extraction. The initial steady state head h_0, before the extraction starts, is also given by Equation 12 for $Q = 0$. If there is no infiltration, that is, $N = 0$, then $h_0 = h_c$, which means the aquifer's initial saturated thickness is equal to the water level in the lake. If $N = 0$, then Equation 12 simplifies to the well-known Dupuit (1863) equation for radial flow in an unconfined aquifer.

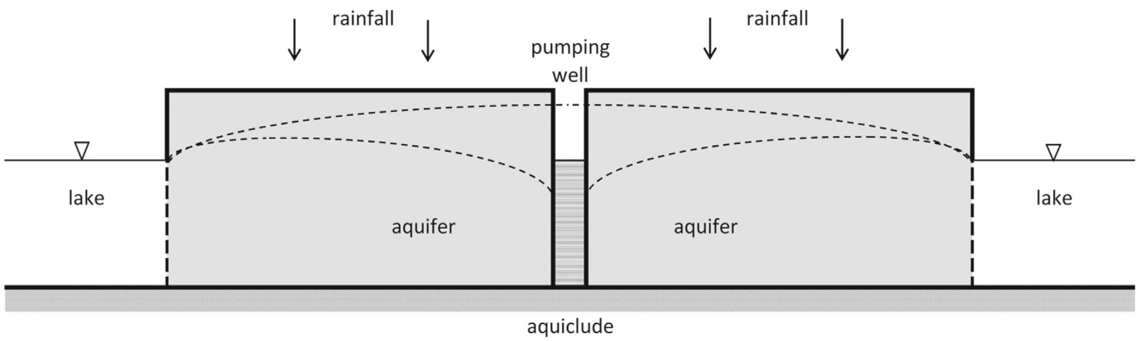

Figure 1. Sketch of the circular island aquifer system discussed by Bredehoeft et al. (1982) and Bredehoeft (2002).

Drawdown s (L) defined by Equation 8 is found by taking the difference between the head h during pumping and the initial head h_0:

$$s(r) = \sqrt{h_c^2 + \frac{N}{2K}\left(r_c^2 - r^2\right) + \frac{Q}{\pi K}\ln\frac{r}{r_c}} - \sqrt{h_c^2 + \frac{N}{2K}\left(r_c^2 - r^2\right)} \quad (13)$$

Because Equation 13 is not linear, it cannot be simplified to obtain an expression independent of initial head h_0 and recharge N. Rearranging and rewriting Equation 13 as a function of initial head h_0 gives:

$$s(r) = h_0\left[\sqrt{1 + \frac{Q}{\pi K h_0^2}\ln\frac{r}{r_c}} - 1\right] \quad (14)$$

Equation 14 is valid only if the argument of the square root is positive. If this is not the case at distance r_w, the radius (L) of the well, then the latter goes dry, and the extraction is not sustainable. Therefore, the condition to have a sustainable development is

$$\frac{Q}{\pi K h_0^2}\ln\frac{r_c}{r_w} \leq 1 \quad (15)$$

Usually $r_w^2 \ll r_c^2$, in which case $h_0^2 \approx h_c^2 + \frac{N r_c^2}{2K}$; hence, condition (15) may be rearranged into:

$$N\pi r_c^2 \geq Q\ln\frac{r_c^2}{r_w^2} - 2\pi K h_c^2 \quad (16)$$

Before the extraction begins, the system is in a steady state, and both initial recharge R_0 and initial discharge D_0 equal $N\pi r_c^2$, which is the left-hand side of Equation 16. The groundwater budget myth erroneously states that the pumping rate should not exceed the initial recharge for the extraction to be safe. That would imply the right-hand side of Equation 16 should be equal to Q, which is clearly not the case. Sustainable pumping in this example also depends on the relative distance r_w/r_c between well and lake, on the constant water level h_c in the lake, and on the aquifer's conductivity K: if the lake is too far away from the well, and/or the aquifer transmissivity is too low, the well goes dry before the cone of depression reaches the lake. This confirms Bredehoeft's (1997, 2002, 2007) point of view. On the other hand, condition (16) clearly shows that whether the well goes dry or not, also depends on the initial recharge R_0.

Applying series expansion $\sqrt{1-x} \rightarrow \left(1 - \frac{x}{2}\right)$ if $x \rightarrow 0$, Equation 14 may be approximated as:

$$s(r) \approx \frac{Q}{2\pi K h_0}\ln\frac{r}{r_c} \quad (17)$$

To approximate Equation 14 by Equation 17, $Q/\left(\pi K h_0^2\right)\ln\left(r_c/r\right)$ must be much smaller than 1. Rearranging this condition after substituting h_0^2 by $h_c^2 + \left(N r_c^2/2K\right)$ gives:

$$\frac{Q}{2\pi K h_c}\ln\frac{r_c}{r} \ll \frac{h_c}{2} + \frac{N\left(r_c^2 - r^2\right)}{4K h_c} \quad (18)$$

Recall that if there is no infiltration, then $h_0 = h_c$, and Equation 17 is the well-known Thiem (1870) equation with constant transmissivity $K h_c$ (L^2/T). This means the left-hand side of Equation 18 is the absolute value of the drawdown according to the Thiem (1870) equation. The latter may be used if drawdown is smaller than 10% of the initial saturated aquifer thickness (Louwyck et al. 2022), and if the infiltration flux is negligibly small. Only if these conditions are met, the aquifer transmissivity may be assumed constant, in which case applying superposition is justified. However, if superposition is applied inaccurately in this case, then the approximated drawdown will underestimate the exact drawdown, unless it is corrected (Jacob 1944).

Devlin and Sophocleous (2005) also discuss the island example, and they show that the aquifer continues to discharge into the lake in case of gentle pumping, that is, $Q < R_0$. In this case, the capture only consists of reduced discharge ΔD, and there is no induced recharge ΔR as infiltration flux N is constant. If $Q > R_0$, then water flows from the lake into the aquifer, and there is also induced recharge. Note that the constant head boundary conceptualizes the lake as an infinite source of water which never dries up, and therefore, the amount of

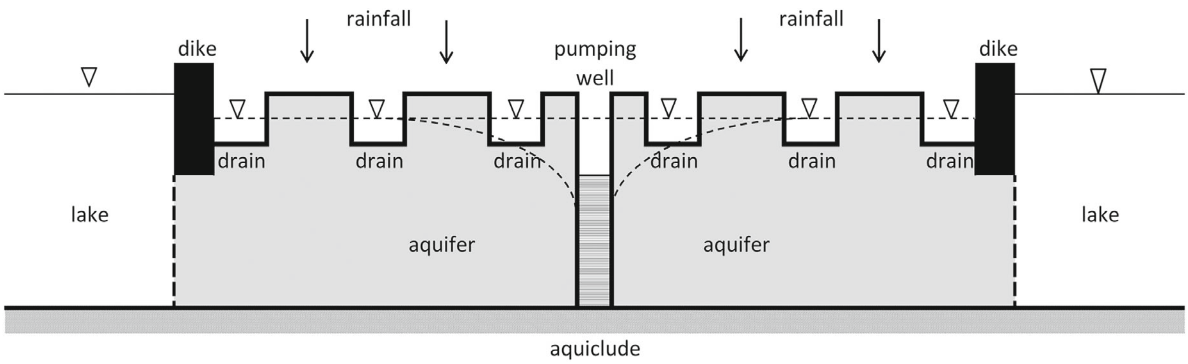

Figure 2. Sketch of the polder island aquifer system.

water in the lake is not a limiting factor to the extraction in this model.

Polder Island

The previous example illustrates that the superposition principle may be applied if recharge is negligibly small, and if the aquifer's saturated thickness is relatively large, a typical condition in the arid west of the United States, according to Bredehoeft et al. (1982). Suppose now the circular island is reclaimed from the lake in an area where the climate is humid and infiltration cannot be ignored (Figure 2). This is a typical condition in deltaic or low lying polder areas (Ernst 1971). On the island, a dense network of canals and ditches is present which drains the excess of water from rainfall.

The interaction between the aquifer and the draining elements may be conceptualized in a similar way a river boundary condition is defined in MODFLOW (Harbaugh 2005):

$$Q_d = 2\pi \int_0^{r_c} \frac{h_d(r) - h(r)}{c} r\mathrm{d}r \tag{19}$$

with Q_d (L^3/T) is the total amount of water per unit of time drained by the network of ditches, h_d (L) is the drainage level, and c (T) is the drainage resistance. Note that Q_d is negative if water is removed from the aquifer.

If it is assumed drainage resistance c and aquifer transmissivity T (L^2/T) are constant, the problem is linear. In this case, superposition may be applied by simulating hydraulic head h relatively to drainage level h_d. Hence, it is allowed to set h_d to zero in Equation 19, and superimpose the calculated head on drainage level h_d afterwards. If the island's radius r_c is large enough so that the outer boundary condition has no impact on the pumping induced drawdown, it is also justified to assume the aquifer is unbounded, that is, $r_c \to \infty$. Under these assumptions, the problem of flow to a well in the middle of the polder island is equivalent to the problem of well-flow in a leaky aquifer of infinite extent (Ernst 1971; Hemker 1984). If the aquifer's outer boundary is at large distance, the initial steady state head h_0 is uniform (Ernst 1971):

$$h_0 = Nc \tag{20}$$

In contrast to Bredehoeft's island case, the infiltrated water is not discharged by outflow into the lake, but by drainage. Substituting Equation 20 into Equation 19, and taking into account $h_d = 0$, the drained amount of water per unit of time at $t = 0$ is

$$D_0 = N\pi r_c^2 = R_0 \tag{21}$$

When the system is brought into a new dynamic equilibrium after a period of pumping, the head is given by the following equation (Bruggeman 1999):

$$h(r) = h_0 - \frac{Q}{2\pi T}\left(\frac{K_0\left(\frac{r}{\lambda}\right)}{\frac{r_w}{\lambda}K_1\left(\frac{r_w}{\lambda}\right)}\right) \tag{22}$$

with $\lambda = \sqrt{Tc}$, the leakage factor (L); K_0 and K_1 are the modified Bessel functions of the second kind with order zero and one, respectively. If $r_w \to 0$, then $(r_w/\lambda)\,K_1\,(r_w/\lambda) \to 1$, and Equation 22 simplifies to:

$$h(r) = h_0 - \frac{Q}{2\pi T}K_0\left(\frac{r}{\lambda}\right) \tag{23}$$

Substituting Equation 23 into Equation 19, and making use of $\int_0^\infty rK_0(r)\mathrm{d}r = 1$, the amount of water per unit of time Q_d exchanged between the aquifer and the network of ditches during pumping is obtained:

$$Q_d = Q - N\pi r_c^2 \tag{24}$$

Using Equations 4 and 21, it follows from Equation 24 that:

$$Q_d = -D_0 - \Delta D + \Delta R \tag{25}$$

Recharge through infiltration N is unchanged, and therefore, the extracted water is balanced by reduced drainage, and possibly induced infiltration from the ditches. Indeed, if head h drops below the drainage level h_d, the ditches will start to irrigate. As a consequence, two zones around the well can be distinguished: a proximal zone where

$h < h_d$ in which irrigation takes place, and a distal zone where $h > h_d$ in which groundwater is still being drained. Taking into account $h_d = 0$, it follows from Equations 20 and 23 that the boundary r_d (L) between those two zones is found by solving the following equation:

$$\frac{2\pi N\lambda^2}{Q} = K_0\left(\frac{r_d}{\lambda}\right) \tag{26}$$

If well radius r_w is greater than r_d, there will be no induced recharge. It can be proven mathematically that this is the case when the pumping rate is smaller than the initial recharge given by Equation 21. Otherwise, there is always induced recharge.

Since the system is linear, it is possible to calculate drawdown s directly according to the principle of superposition, and Equation 23 becomes the well-known formula to calculate drawdown due to pumping in a leaky aquifer (De Glee 1930; Jacob 1946):

$$h(r) - h_0(r) = s(r) = \frac{-Q}{2\pi T}K_0\left(\frac{r}{\lambda}\right) \tag{27}$$

In case of a leaky aquifer, the leakage is usually interpreted as an increase of vertical flow through the overlying aquitard (Jacob 1946). If it is assumed there is no flow before the extraction, this interpretation of induced recharge is correct. However, the polder island example clearly demonstrates the no-flow assumption is too strict, and the leakage may be reduced discharge.

No prior knowledge about the infiltration is required in order to assess the impact of the extraction on the water table applying Equation 27. The leakage, which is equal to the capture $\Delta R - \Delta D$, can also be calculated directly using Equation 27 without knowing the initial recharge or discharge:

$$\Delta Q_d = 2\pi \int_0^{\infty} \frac{-s(r)}{c} r\,\mathrm{d}r = Q \tag{28}$$

Note that Equation 28 gives a change in flow ΔQ_d (L^3/T), as it uses the change in head s. In fact, Equation 28 is equivalent to Equation 9, with $(\partial V(s_t)/\partial t) = 0$. As a consequence, no distinction can be made between induced recharge and reduced discharge. To distinguish between those two components, the initial head $h_0 = Nc$ is needed. Therefore, knowing the infiltration flux N is required to evaluate the impact of the extraction on the ditches and canals as they may switch from draining to irrigating.

This is even more pronounced if the ditches are restricted to draining. The steady state solution for this system is developed by Ernst (1971), whereas the transient state solution is presented by Louwyck et al. (2022). In this case, boundary condition (19) expressing the interaction between groundwater and surface water, needs to be redefined as:

$$Q_d = 2\pi \int_0^{r_c} \frac{h_d(r) - h(r)}{f(h)} r\,\mathrm{d}r$$

with

$$f(h) = \begin{cases} c & \text{if } h > h_d \\ \infty & \text{if } h \le h_d \end{cases} \tag{29}$$

Equation 29 states that there is drainage in the distal zone ($r > r_d$), and no interaction in the proximal zone ($r \le r_d$). If boundary condition (29) is applied instead of Equation 19, the system is nonlinear, as drainage resistance c depends on head h in the aquifer, which is expressed by function f. In the proximal zone, $Q_d = 0$, because of the infinitely large resistance c, whereas $Q_d < 0$ in the distal zone. Note that boundary condition (29) can be seen as a MODFLOW drain (Harbaugh 2005) defined over the whole area of the island.

As resistance c is infinitely large in the proximal zone around the well where drainage is inactive, the steady state head h_1 in this zone must be calculated as (Haitjema 1995):

$$h_1(r) = \left[\frac{Nr_w^2}{2T} + \frac{Q}{2\pi T}\right]\ln\left(\frac{r}{r_d}\right) + \frac{N}{4T}\left(r_d^2 - r^2\right) \quad (r \le r_d) \tag{30}$$

The constant head at boundary r_d equals the drainage level h_d which is set to zero. The head h_2 in the distal zone, where the drainage is still active, is given by Equation 22. To ensure continuity of flow at boundary r_d, pumping rate Q in Equation 22 must be replaced by:

$$-2\pi T r_d \frac{\mathrm{d}h_1(r_d)}{\mathrm{d}r} = N\pi r_d^2 - Q \tag{31}$$

Inserting the right-hand side of Equation 31 into Equation 22 and replacing r_w by r_d gives:

$$h_2(r) = h_0 + \frac{\left(N\pi r_d^2 - Q\right)}{2\pi T}\left(\frac{K_0\left(\frac{r}{\lambda}\right)}{\frac{r_d}{\lambda}K_1\left(\frac{r_d}{\lambda}\right)}\right) \quad (r > r_d) \tag{32}$$

Recall that initial head $h_0 = Nc$. As the system is nonlinear, this initial head cannot be removed from Equation 32. Drawdown in the proximal and distal zone is found by subtracting h_0 from Equations 30 and 32, respectively.

Using Equation 32, distance r_d is found by solving $h_2(r_d) = h_d = 0$. Rearranging this equation gives:

$$\left[2\frac{K_1\left(\frac{r_d}{\lambda}\right)}{K_0\left(\frac{r_d}{\lambda}\right)} + \frac{r_d}{\lambda}\right]\frac{r_d}{\lambda} - \frac{Q}{N\pi\lambda^2} = 0 \tag{33}$$

Finding the root of the left-hand side of Equation 33 can be done numerically applying a nonlinear solver. Equation 33 clearly indicates that for a given leakage factor λ, ratio Q/N determines the distance r_d up to where the draining elements are depleted. Louwyck et al. (2022) show that the largest possible value for r_d^2 is $Q/(\pi N)$, and that this maximum radius is attained if $\left(Q/\pi N\lambda^2\right) > 100$. Mathematically, the no-drainage zone is maximal if the drainage is perfect or $c \to 0$. In this case, there is no flow in the distal zone, as head $h_2 = 0$. This implies the left-hand side of Equation 31 is zero, from which the following well-known formula can be derived:

$$Q = N\pi r_d^2 \tag{34}$$

Equation 34 is used to calculate the radius of the capture zone (Haitjema 1995), also called the area contributing recharge to the well (Reilly and Pollock 1996). Note that this "capture" may not be confused with the water budget capture (Seward et al. 2015; Barlow et al. 2018). Since h_2 is zero in this case, by definition, r_d is also the radius of influence. As a consequence, the radius of the cone of depression and the radius of the capture zone coincide in this special case, an idea Brown (1963) called fallacious. This example shows, however, it is justified to estimate the radius of influence using Equation 34 for wells in densely drained areas with flat water table.

Taking a closer look at the total groundwater budget, it is clear that Equations 21 and 24 still hold if the ditches can only drain water, but in this case, it is certain that the capture only contains reduced discharge as there is no induced recharge, given the outer boundary is at a sufficiently large distance from the well. Therefore, Equation 25 is reduced to:

$$Q_d = -D_0 - \Delta D \tag{35}$$

From Equations 24 and 35, it follows that $\Delta D = -Q$, which confirms the extracted amount of water is balanced by reduced drainage only. According to Ernst (1971), deep well pumping of groundwater in this case primarily involves a smaller discharge of water by the drains, whereas changes in evaporation by plants are of less importance, which justifies the assumption of a constant infiltration flux N. This is in complete agreement with the statements made by Bredehoeft (1997, 2002, 2007).

If the proximal zone without drainage is negligibly small, then this zone may be neglected. Indeed, if $r_d \to 0$, then $\frac{r_d}{\lambda}K_1\left(\frac{r_d}{\lambda}\right) \to 1$, and Equation 32 reduces to Equation 23, the solution for the linear system. Louwyck et al. (2022) show that this approximation is justified if $\left(Q/\left(\pi N\lambda^2\right)\right) < 1$. If this constraint is satisfied, no ditches are depleted, and the interaction between the groundwater system and the draining surface water is given by boundary condition (19), which is a linear function of hydraulic head. Recall that in this case, the superposition principle is valid and drawdown may be calculated directly using the De Glee (1930) Equation 27. However, if this equation is used to approximate drawdown when superposition is not allowed, the resulting cone of depression will be underestimated.

In this example, whether superposition is allowed or not, thus depends on ratio Q/N and leakage factor λ: if there is relatively much infiltration, and/or if the aquifer transmissivity and/or the resistance to drainage are large, then the problem is linear, which implies drawdown can be calculated without knowledge of the initial steady state conditions. Under the assumption of linearity, prior knowledge about the infiltration rate is not required, although infiltration flux N is one of the parameters that implicitly determine whether this assumption is valid or not. Recall that a similar conclusion was drawn from the previous island example without drainage.

Summary and Conclusions

The groundwater budget myth, contested by Bredehoeft et al. (1982) and Bredehoeft (2002), gave rise to a controversy about the role of recharge in assessing the sustainability of groundwater development. Bredehoeft (2002) explains how traditional analytical models apply the principle of superposition to assess the impact of wells on groundwater systems. These models calculate the cone of depression which is superimposed on the initial head. The recharge flux is required only to solve the initial boundary value problem. To simulate the cone of depression, only the aquifer diffusivity, the boundary conditions, and the pumping rate are needed; initial recharge and discharge are not required.

The irrelevance of the initial conditions is based on a simplified water budget equation, which equals the total pumping rate to the sum of capture and storage change (Bredehoeft et al. 1982; Bredehoeft 2002). After examining the assumptions underlying this equation, it was proven that it indeed expresses the superposition principle, which assumes a linear system. Under this assumption, drawdown is independent of initial conditions, including initial recharge.

However, like any other model assumption, this is a simplification of reality, as it implicitly assumes the groundwater reservoir can be depleted indefinitely and boundary conditions are an infinite source of water. In reality, capture and storage are limited, and this requires modeling nonlinear responses, in which case the superposition principle is not applicable, and simulating the initial conditions is inevitable. In fact, the initial conditions determine the limits of the system, and as a consequence, they paradoxically determine whether the assumption of linearity is valid or not. Therefore, it is concluded that applying linear models to assess sustainability should be subject to caution, as they tend to underestimate the cone of depression.

Two analytical examples were given to demonstrate nonlinear models are dependent on the initial conditions indeed. Bredehoeft's island case (Bredehoeft et al. 1982; Bredehoeft 2002) was discussed first. In this model, recharge implicitly determines the aquifer's saturated thickness, and hence, the sustainability of the extraction. It was also shown that in this case the superposition principle is valid only under specific conditions: recharge must be negligibly small and drawdown must be much smaller than the saturated aquifer thickness. Otherwise a nonlinear model is required in which the initial conditions, including recharge, are of importance.

The second example discussed the analytical model developed by Ernst (1971) to simulate flow to a well in a polder area with a nonlinear function for drainage. The ratio of the recharge to the pumping rate together with the leakage factor determine whether superposition can be applied or not. If drainage resistance is negligibly small, the cone of depression even coincides with the capture zone. Although this asymptotic solution is an exceptional case, it proves that the idea of balancing pumping and infiltration rate is not fallacious, but in agreement with the fundamental principles stated by Theis (1940), as in this case, the pumping rate is balanced by reduced drainage.

Although both one-dimensional models are oversimplifications of reality, they clearly demonstrate that the initial recharge cannot always be ignored if the model is not linear. In fact, the question if recharge is important in assessing the sustainability of groundwater development comes down to the question if superposition can be applied in the analysis of the groundwater system. In many cases, applying this traditional method of analysis is still justified indeed, but in many other real world cases, defining nonlinear and time-dependent stresses on the groundwater system is inevitable, making recharge a relevant parameter. And this, to quote Bredehoeft (2002), is "why hydrogeologists model."

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