

QBank Scaffold v6: Full Simpson–Hetter iterative calibration

```
# - Offline simulator + iterative SH estimator that targets per-item exposure cap  $\tau$ 
# - Admin endpoint to trigger calibration (with dry-run + parameters)
# - Updates selection service to count "proposals" (instrumentation hook)
#
# Output: /mnt/data/qbank_scaffold_v6.zip
```

```
import os, zipfile, pathlib, textwrap, json
```

```
ROOT = "/mnt/data/qbank_scaffold_v6"
os.makedirs(ROOT, exist_ok=True)
```

```
def write(path, content):
    os.makedirs(os.path.dirname(path), exist_ok=True)
    with open(path, "w", encoding="utf-8") as f:
        f.write(textwrap.dedent(content))
```

```
# ----- README -----
```

```
write(f"{ROOT}/README.md", """
```

```
# QBank Scaffold (v6) — Full Simpson–Hetter (S–H) iterative calibration
```

This drop wires a **true S–H iterative estimator** that simulates CAT sessions and updates

per-item acceptance probabilities `sh_p` so the **probability an examinee sees an item**

(i.e., exposure rate) does not exceed τ .

What's included:

- `analytics/calibration/sh_iterative.py`: Iterative S–H with simulation → updates `item_exposure_control`.
- Admin API: `POST /v1/admin/exposure/calibrate_sh` to run (or dry-run) calibration on demand.
- Selector instrumentation: a hook to count **proposals** (times an item is proposed before acceptance).

Quick run (dev-safe defaults):

```
```bash
```

```
python analytics/calibration/sh_iterative.py \
```

```
--dsn postgresql://qbank:qbank@localhost:5432/qbank \
```

```
--exam DEMO-EXAM --tau 0.2 --n 500 --len 30 --iters 6 --alpha 0.6
```

API trigger (synchronous; use low n/iters in prod or run offline via cron/job):

```
bash
```

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```
curl -X POST http://localhost:8000/v1/admin/exposure/calibrate_sh \
-H "Authorization: Bearer <ADMIN_JWT>" -H "Content-Type: application/json" \
-d '{"exam_code":"DEMO-EXAM","tau":0.2,"n":400,"test_len":25,"iters":5,"alpha":0.6,"dry_run":false}'
```

Notes:

We estimate exposure as the share of examinees who received the item at least once.

Update rule (damped):  $k_i \leftarrow \text{clamp}(\text{floor}, \text{ceil}, k_i * (\tau / \max(r_i, \epsilon))^\alpha)$ , where  $r_i$  is observed exposure rate under the current  $k$ ;  $\alpha \in (0, 1]$  damps oscillations.

If an item is chronically under-exposed ( $r_i \ll \tau$ ),  $k_i$  will drift up toward 1; if over-exposed,

$k_i$  reduces multiplicatively.

""")

----- Simulator & Iterative SH -----

```
write(f"{ROOT}/analytics/calibration/sh_iterative.py", """
import argparse, math, random, statistics
import psycpg2, psycpg2.extras
```

D = 1.7

```
def logistic(x: float) -> float: return 1.0 / (1.0 + math.exp(-x))
def prob_3pl(theta: float, a: float, b: float, c: float) -> float:
 return c + (1.0 - c) * logistic(D * a * (theta - b))
def fisher_info_3pl(theta: float, a: float, b: float, c: float) -> float:
 P = prob_3pl(theta, a, b, c); Q = 1.0 - P
 if P <= 0 or Q <= 0 or (1.0 - c) <= 0: return 0.0
 return (D**2) * (a**2) * (Q / P) * ((P - c) / (1.0 - c))**2
```

```
def sample_theta(dist: str = "normal0,1"):
 # extensible: normal0,1 | uniform-1,1 | fixed0
 if dist.startswith("normal"):
 mu, sd = 0.0, 1.0
 return random.gauss(mu, sd)
 if dist.startswith("uniform"):
 return random.uniform(-1.0, 1.0)
 return 0.0
```

```

def load_pool(conn, exam_code: str):
 sql = '''
 SELECT qv.question_id, qv.version, qv.topic_id,
 COALESCE(ic.a, 1.0) AS a, COALESCE(ic.b, 0.0) AS b, COALESCE(ic.c, 0.2) AS c,
 COALESCE(iec.sh_p, 1.0) AS sh_p
 FROM question_publications qp
 JOIN question_versions qv ON qv.question_id=qp.question_id AND
 qv.version=qp.live_version
 LEFT JOIN item_calibration ic ON ic.question_id=qv.question_id AND
 ic.version=qv.version AND ic.model='3pl'
 LEFT JOIN item_exposure_control iec ON iec.question_id=qv.question_id AND
 iec.version=qv.version
 WHERE qp.exam_code=%s AND qv.state='published'
 '''

 with conn.cursor(cursor_factory=psycopg2.extras.DictCursor) as cur:
 cur.execute(sql, (exam_code,))
 rows = cur.fetchall()
 items = []
 for r in rows:
 items.append({
 "qid": int(r["question_id"]), "ver": int(r["version"]),
 "topic": r["topic_id"], "a": float(r["a"]), "b": float(r["b"]), "c": float(r["c"]),
 "k": float(r["sh_p"]) if r["sh_p"] is not None else 1.0
 })
 return items

def simulate_once(pool, n: int, test_len: int, theta_dist: str, use_k: bool = True,
seed: int | None = None):
 if seed is not None: random.seed(seed)
 # Tallies:
 # seen[i_key] = number of examinees who saw item at least once (exposure rate
 numerator)
 # proposed[i_key] = number of times item was proposed (instrumentation; unused
 by update but exposed for analysis)
 seen = { (it["qid"], it["ver"]): 0 for it in pool }
 proposed = { (it["qid"], it["ver"]): 0 for it in pool }

python
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for _ in range(n):
 theta = sample_theta(theta_dist)

```

```

administered_keys = set()
for _pos in range(test_len):
 # rank by Fisher information for this theta among items not yet seen in this
test
 candidates = [it for it in pool if (it["qid"], it["ver"]) not in administered_keys]
 if not candidates: break
 scored = [(fisher_info_3pl(theta, it["a"], it["b"], it["c"]), it) for it in
candidates]
 scored_xsort(key=lambda x: x[0], reverse=True)

 chosen = None
 for _, it in scored:
 key = (it["qid"], it["ver"])
 proposed[key] += 1
 if use_k:
 if random.random() <= max(0.0, min(1.0, it["k"])):
 chosen = it; break
 else:
 continue
 else:
 chosen = it; break
 if chosen is None: # if everything rejected, force top
 chosen = scored[0][1]
 key = (chosen["qid"], chosen["ver"])
 if key not in administered_keys:
 administered_keys.add(key)

update seen tallies (each administered item counts once per examinee)
for key in administered_keys:
 seen[key] += 1

return seen, proposed
def iterative_sh(pool, tau: float, n: int, test_len: int, iters: int, alpha: float,
theta_dist: str, floor: float, ceil: float, seed: int | None = None):
 # copy ks
 k = { (it["qid"], it["ver"]): float(it["k"]) for it in pool }
 history = []
 for t in range(iters):
 # write ks back into pool
 for it in pool:
 it["k"] = k[(it["qid"], it["ver"])]
 seen, proposed = simulate_once(pool, n=n, test_len=test_len,
theta_dist=theta_dist, use_k=True, seed=(None if seed is None else seed + t))
 # exposure rates: proportion of examinees who saw the item at least once

```

```

r = { key: seen[key] / max(1, n) for key in seen }
update rule (damped multiplicative adjustment)
newk = {}
diffs = []
for it in pool:
 key = (it["qid"], it["ver"])
 ri = rxget(key, 0.0)
 ki = k[key]
 if ri <= 0.0:
 newk[key] = min(1.0, max(floor, ki * 1.1)) # nudge up gently if never seen
 else:
 ratio = tau / ri
 adj = ratio ** alpha
 cand = ki * adj
 newk[key] = min(1.0, max(floor, min(ceil, cand)))
 diffs.append(abs(ri - min(tau, ri))) # track magnitude (ri itself relative to cap)
k = newk
avg_exp = statistics.mean(rxvalues()) if r else 0.0
max_over = max((ri - tau for ri in r.values()), default=0.0)
history.append({"iter": t+1, "avg_exp": avg_exp, "max_over": max_over})
return final k, last seen/proposed, history
return k, seen, proposed, history

```

```

def upsert_k(conn, kmap):
 with conn.cursor() as cur:
 cur.execute("SET search_path TO public")
 for (qid, ver), kval in kmap.items():
 cur.execute(
 '''INSERT INTO item_exposure_control(question_id, version, sh_p)
VALUES (%s,%s,%s)
ON CONFLICT (question_id,version) DO UPDATE SET sh_p=EXCLUDED.sh_p,
updated_at=now()''',
 (qid, ver, float(kval))
)
 conn.commit()

```

```

def main():
 ap = argparse.ArgumentParser()
 ap.add_argument("--dsn", required=True)
 ap.add_argument("--exam", "--exam_code", dest="exam_code", required=True)
 ap.add_argument("--tau", type=float, default=0.2, help="Max exposure per item (probability)")
 ap.add_argument("--n", type=int, default=500, help="Number of examinees to simulate per iteration")

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ap.add_argument("--len", "--test_len", dest="test_len", type=int, default=30,
help="Test length (items)")
ap.add_argument("--iters", type=int, default=6, help="Number of S-H iterations")
ap.add_argument("--alpha", type=float, default=0.6, help="Damping exponent in
(0,1]")
ap.add_argument("--theta", "--theta_dist", dest="theta_dist",
default="normal0,1")
ap.add_argument("--floor", type=float, default=0.02)
ap.add_argument("--ceil", type=float, default=1.0)
ap.add_argument("--seed", type=int, default=None)
ap.add_argument("--dry_run", action="store_true")
args = ap.parse_args()

```

lua

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```

conn = psycopg2.connect(args.dsn)
pool = load_pool(conn, args.exam_code)
if not pool:
 print("No items found for exam", args.exam_code); return

```

```

kmap, seen, proposed, hist = iterative_sh(
 pool, tau=args.tau, n=args.n, test_len=args.test_len, iters=args.iters,
 alpha=args.alpha, theta_dist=args.theta_dist, floor=args.floor, ceil=args.ceil,
 seed=args.seed
)

```

```

print("Iterations summary:", hist)
top = sorted([(kmap[(it["qid"], it["ver"])], it["qid"], it["ver"]) for it in pool],
reverse=True)[:5]
print("Top k after calibration (first 5):", top[:5])

```

```

if not args.dry_run:
 upsert_k(conn, kmap)
 print(f"Upserted {len(kmap)} k-values into item_exposure_control")
conn.close()
if name == "main":
 main()
"""

```

```

----- Admin API: trigger calibration -----
write(f"{ROOT}/qbank-backend/app/api/admin.py", """
from fastapi import APIRouter, Depends, HTTPException

```

```
from pydantic import BaseModel, Field
from typing import List, Optional
from sqlalchemy.orm import Session
from sqlalchemy import text
import os, subprocess, sys
from app.core.database import get_db
from app.core.auth import require_roles
from app.models.orm import ItemExposureControl
```

```
router = APIRouter()
```

```
class ItemRow(BaseModel):
 question_id: int
 version: int
 topic_id: Optional[int] = None
 sh_p: float
 recent_attempts: int | None = 0
```

```
@router.get("/exposure/items", response_model=List[ItemRow],
dependencies=[Depends(require_roles("admin"))])
def list_items(limit: int = 100, db: Session = Depends(get_db)):
 rows = db.execute(text("""
SELECT qv.question_id, qv.version, qv.topic_id,
COALESCE(iec.sh_p, 1.0) as sh_p,
(SELECT count(*) FROM user_responses ur WHERE ur.question_id=qv.question_id
AND ur.version=qv.version AND ur.created_at>now()-interval '7 days') as
recent_attempts
FROM question_versions qv
LEFT JOIN item_exposure_control iec ON iec.question_id=qv.question_id AND
iec.version=qv.version
WHERE qv.state='published'
ORDER BY recent_attempts DESC NULLS LAST
LIMIT :lim
"""), {"lim": limit}).all()
 return [ItemRow(question_id=r[0], version=r[1], topic_id=r[2], sh_p=float(r[3]),
recent_attempts=r[4] or 0) for r in rows]
```

```
class SetSh(BaseModel):
 question_id: int
 version: int
 sh_p: float
```

```
@router.post("/exposure/set", dependencies=[Depends(require_roles("admin"))])
def set_sh(payload: SetSh, db: Session = Depends(get_db)):
```

```

if payload.sh_p < 0 or payload.sh_p > 1: raise HTTPException(400, "sh_p must be
in [0,1]")
row = db.get(ItemExposureControl, {"question_id": payload.question_id, "version":
payload.version})
if row: row.sh_p = payload.sh_p
else: db.add(ItemExposureControl(question_id=payload.question_id,
version=payload.version, sh_p=payload.sh_p))
db.commit()
return {"ok": True}

```

```

class CalibrateRequest(BaseModel):
 exam_code: str = Field(...)
 tau: float = 0.2
 n: int = 400
 test_len: int = 25
 iters: int = 5
 alpha: float = 0.6
 theta_dist: str = "normal0,1"
 floor: float = 0.02
 ceil: float = 1.0
 dry_run: bool = False

```

```

@router.post("/exposure/calibrate_sh",
dependencies=[Depends(require_roles("admin"))])
def calibrate_sh(payload: CalibrateRequest):
 # This is a simple synchronous wrapper around the CLI; in production, prefer a job
 runner.
 dsn = os.getenv("DATABASE_URL", "postgresql+psycopg2://
qbank:qbank@localhost:5432/qbank")
 cmd = [
 sys.executable, "analytics/calibration/sh_iterative.py",
 "--dsn", dsn, "--exam", payload.exam_code,
 "--tau", str(payload.tau), "--n", str(payload.n), "--len", str(payload.test_len),
 "--iters", str(payload.iters), "--alpha", str(payload.alpha),
 "--theta", payload.theta_dist, "--floor", str(payload.floor), "--ceil",
 str(payload.ceil)
] + (["--dry_run"] if payload.dry_run else [])
 try:
 proc = subprocess.run(cmd, capture_output=True, text=True, check=True)
 return {"ok": True, "stdout": proc.stdout}
 except subprocess.CalledProcessError as e:
 raise HTTPException(status_code=500, detail=e.stdout + "\n" + e.stderr)
 """)

```



```
----- Selection service: proposal hook (documentation stub) -----
write(f"{ROOT}/qbank-backend/app/services/proposals.py", """
```

Placeholder for live instrumentation.

If you want to count proposals in production (item was proposed prior to S-H acceptance),

wire this function from your selector path.

```
from app.core.cache import redis_client
```

```
def bump_proposed(question_id: int, version: int) -> None:
```

```
Daily counter; useful for diagnostics
```

```
import datetime as dt
```

```
day = dt.datetime.utcnow().strftime("%Y%m%d")
```

```
redis_client.incr(f"proposed:{day}:{question_id}:{version}", 1)
"""
```

```
----- Keep minimal main to include admin router -----
```

```
write(f"{ROOT}/qbank-backend/app/main.py", """
```

```
from fastapi import FastAPI
```

```
from fastapi.middleware.cors import CORSMiddleware
```

```
from app.api.admin import router as admin_router
```

```
app = FastAPI(title="QBank API v6 (admin demo)", version="6.0.0")
```

```
app.add_middleware(CORSMiddleware, allow_origins=[""],
```

```
allow_credentials=True, allow_methods=[""], allow_headers=["*"])
```

```
app.include_router(admin_router, prefix="/v1/admin", tags=["admin"])
```

```
@app.get("/health")
```

```
def health(): return {"status": "ok"}
```

```
"""
```

```
----- Zip -----
```

```
zip_path = "/mnt/data/qbank_scaffold_v6.zip"
```

```
with zipfile.ZipFile(zip_path, "w", zipfile.ZIP_DEFLATED) as z:
```

```
for p in pathlib.Path(ROOT).rglob("*"):
 z.write(p, p.relative_to(ROOT))
```

```
zip_path
```

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Result

'/mnt/data/qbank\_scaffold\_v6.zip'

Done — full Sympson–Hetter iterative calibration is wired up.

Download v6 (full SH calibration)

What's in v6:

Offline simulator + iterative SH estimator

analytics/calibration/sh\_iterative.py simulates CAT sessions and iteratively updates sh\_p so each item's exposure (probability an examinee sees the item at least once) stays  $\leq \tau$ . It uses a damped multiplicative update to avoid oscillation and writes results to item\_exposure\_control.

Admin calibration endpoint

POST /v1/admin/exposure/calibrate\_sh (sync) wraps the CLI; supports dry\_run, tau, n, test\_len, iters, alpha, theta\_dist, and bounds.

Proposal instrumentation hook

app/services/proposals.py gives you a one-liner to increment daily "proposed" counters if you want live diagnostics.

How to run it quickly:

Simulate and update sh\_p offline

bash

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```
python analytics/calibration/sh_iterative.py \
 --dsn postgresql://qbank:qbank@localhost:5432/qbank \
 --exam DEMO-EXAM --tau 0.2 --n 500 --len 30 --iters 6 --alpha 0.6
```

Or via API (synchronous; use smaller n/iters in dev):

bash

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```
POST /v1/admin/exposure/calibrate_sh
{
 "exam_code": "DEMO-EXAM",
 "tau": 0.2,
 "n": 400,
 "test_len": 25,
 "iters": 5,
 "alpha": 0.6,
```

```
"dry_run": false
}
```

(Optional) Wire live proposal counting

Call `bump_proposed(qid, ver)` from your selector right before SH acceptance sampling.

Defaults & tuning tips:

`tau`: typical caps are 0.15–0.25 for large pools; tighten for high-stakes pools.

`alpha`: 0.5–0.7 damps volatility; raise toward 1.0 if you want faster convergence.

`n` & `test_len`: increase for smoother estimates; they scale runtime linearly.

Bounds: floor 0.02 guards against items becoming “dead”; keep ceil at 1.0.