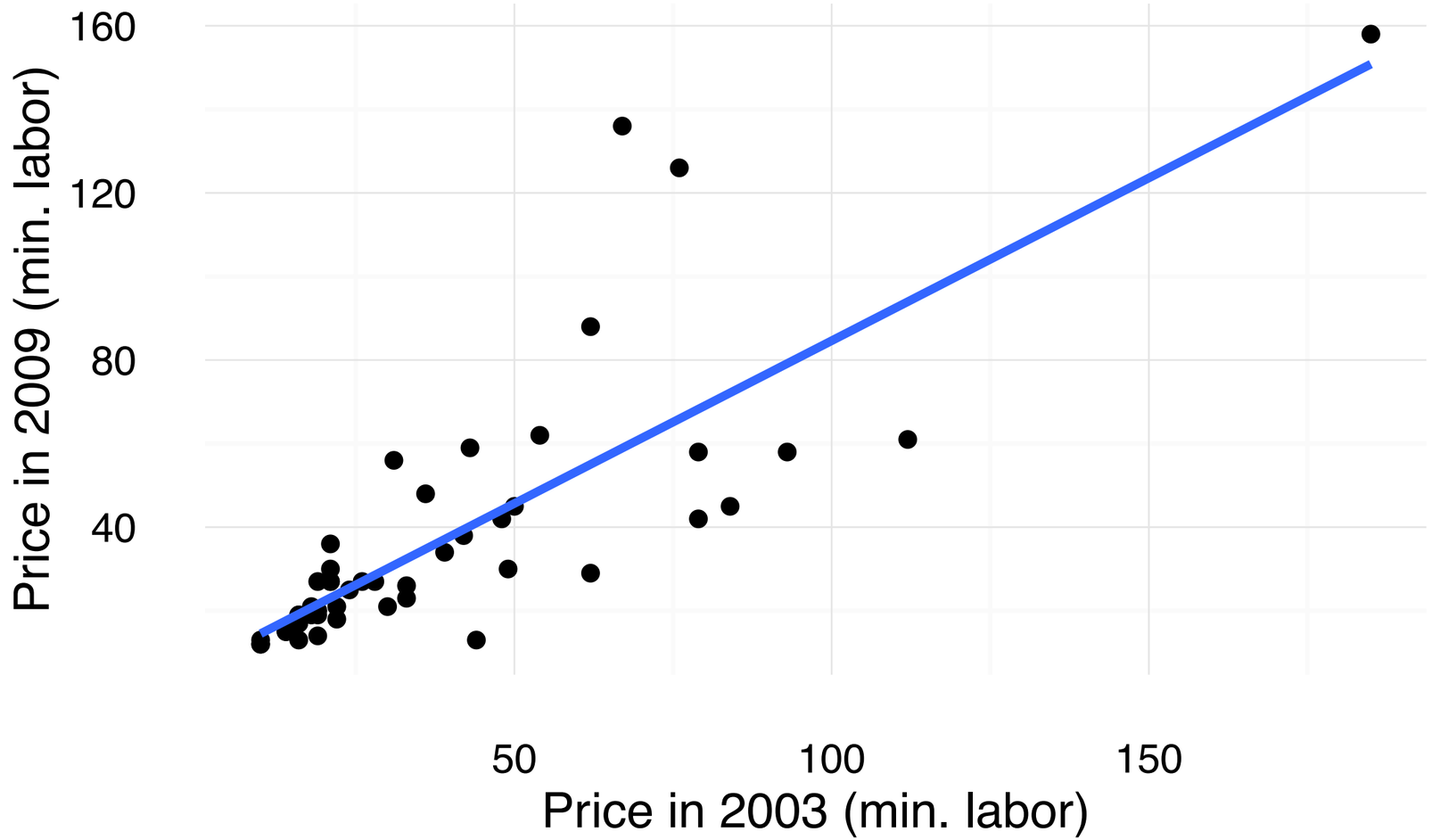
Assessing the Model

Union Bank of Switzerland (UBS) reports

Produces regular reports on prices & earnings in major cities throughout the world, in minutes of labor required for "typical" worker to purchase the commodity:

- **prices of basic commodities (1 kg rice, 1 kg bread)**
- **price of a Big Mac at McDonald's**

Data from 2003 (before recession) and 2009 (after recession) reports




```
cor(bigmac2009 ~ bigmac2003, data = UBSprices)
[1] 0.804481
```

```
bigmac.lm <- lm(bigmac2009 ~ bigmac2003, data = UBSprices)
summary(bigmac.lm)

Call:
lm(formula = bigmac2009 ~ bigmac2003, data = UBSprices)

Residuals:
    Min      1Q  Median      3Q     Max 
-32.968  -5.258  -2.159   0.187  77.081 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  6.73612    3.84985   1.750   0.0861 .  
bigmac2003   0.77886    0.07975   9.767 2.33e-13 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 18.35 on 52 degrees of freedom
Multiple R-squared:  0.6472,    Adjusted R-squared:  0.6404 
F-statistic: 95.39 on 1 and 52 DF,  p-value: 2.334e-13
```

Bootstrapping regression models

1. Number cases in data set from 1 to n.
2. Take a random sample *with replacement* of size n from these numbers.
3. Create a new data set by pulling the rows (cases) from the original data set that were selected in the random sample.
4. Fit the regression model to this new data set and save the values of the estimated coefficients or other summary statistics.
5. Repeat steps 2-4 R times.

Bootstrap confidence intervals

95% plug-in intervals

Percentile intervals

Plug-in intervals in R

```
bigmac.boot <- Boot(bigmac.lm, R = 999)

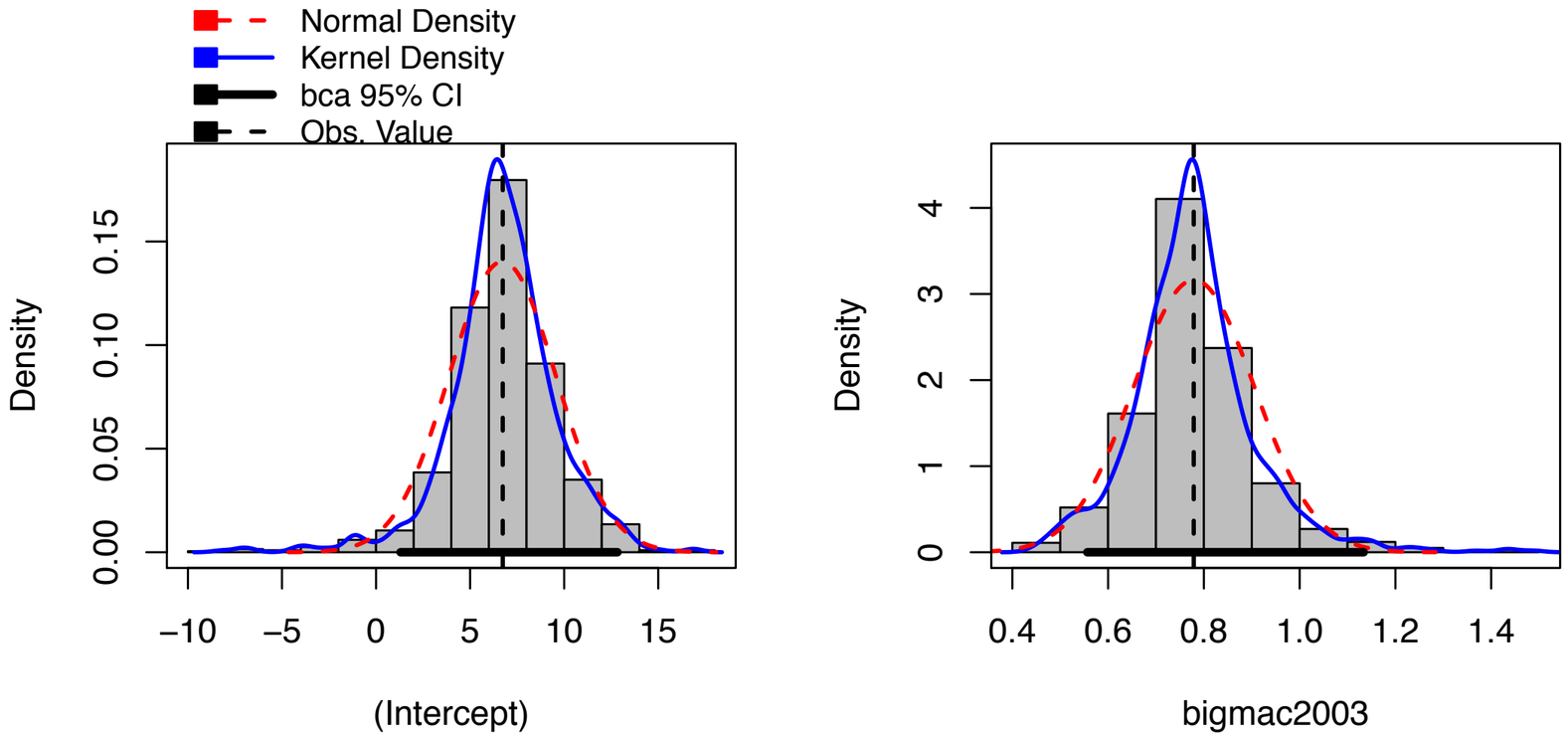
summary(bigmac.boot)

                 R original    bootBias  bootSE bootMed
(Intercept) 999  6.73612 -0.01700515 2.84323 6.69652
bigmac2003  999  0.77886  0.00049195 0.12613 0.77432
```

Interpreting confidence intervals for the slope

Are the bootstrap distributions bell-shaped?

```
hist(bigmac.boot, col = "gray")
```



Percentile intervals in R

```
confint(bigmac.boot, type = "perc", level = .95)

Bootstrap quantiles, type =  percent

                  2.5 %    97.5 %
(Intercept) 0.2406487 12.243993
bigmac2003  0.5309925  1.058041
```

Is the model any good?

Coefficient of determination: R^2

Interpretation: Proportion of the variability in y explained by the linear model.

Calculation: For single variable regression, $R^2 = r^2$

Intuition: A better model explains more of the variability in y

Pitfall: R^2 does not talk about predictive ability of the model

```
summary(bigmac.lm)

Call:
lm(formula = bigmac2009 ~ bigmac2003, data = UBSprices)

Residuals:
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Model assumptions

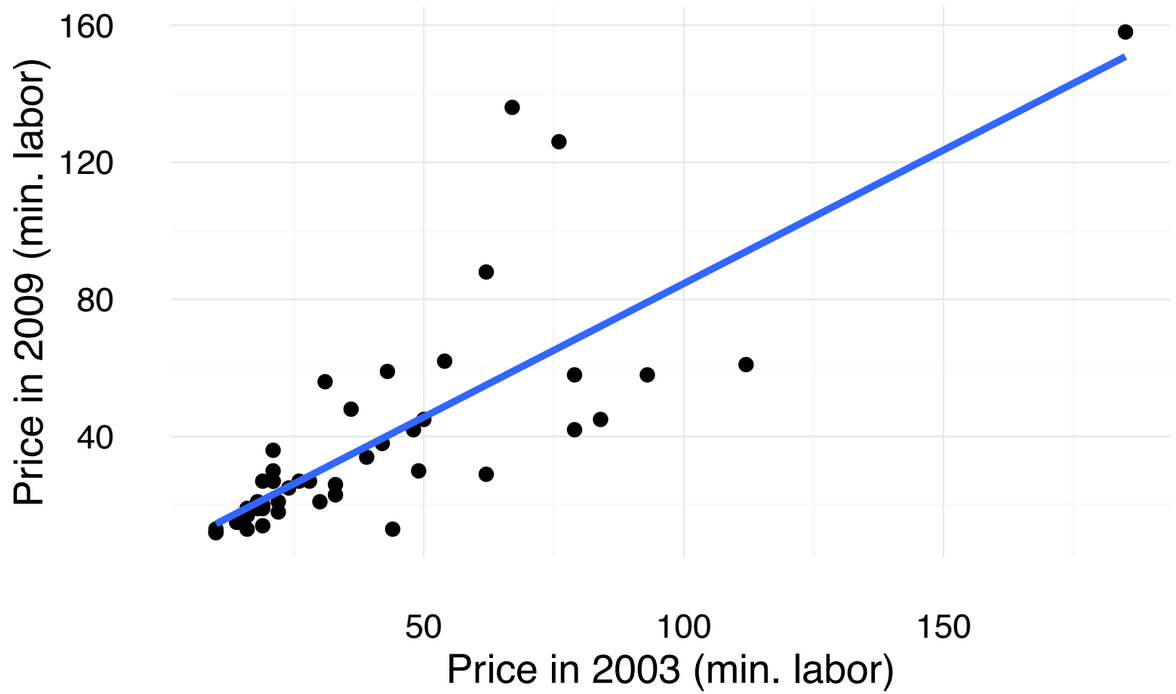
Linearity

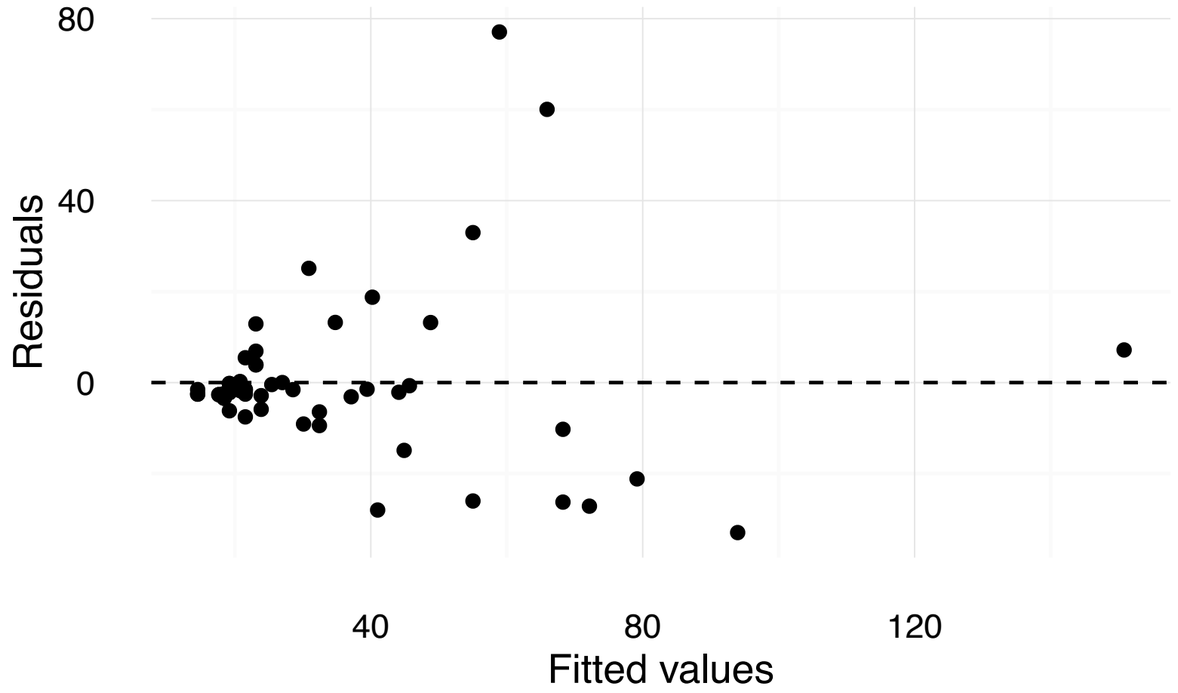
Constant variability around the line

Bell-shaped residuals

Residuals

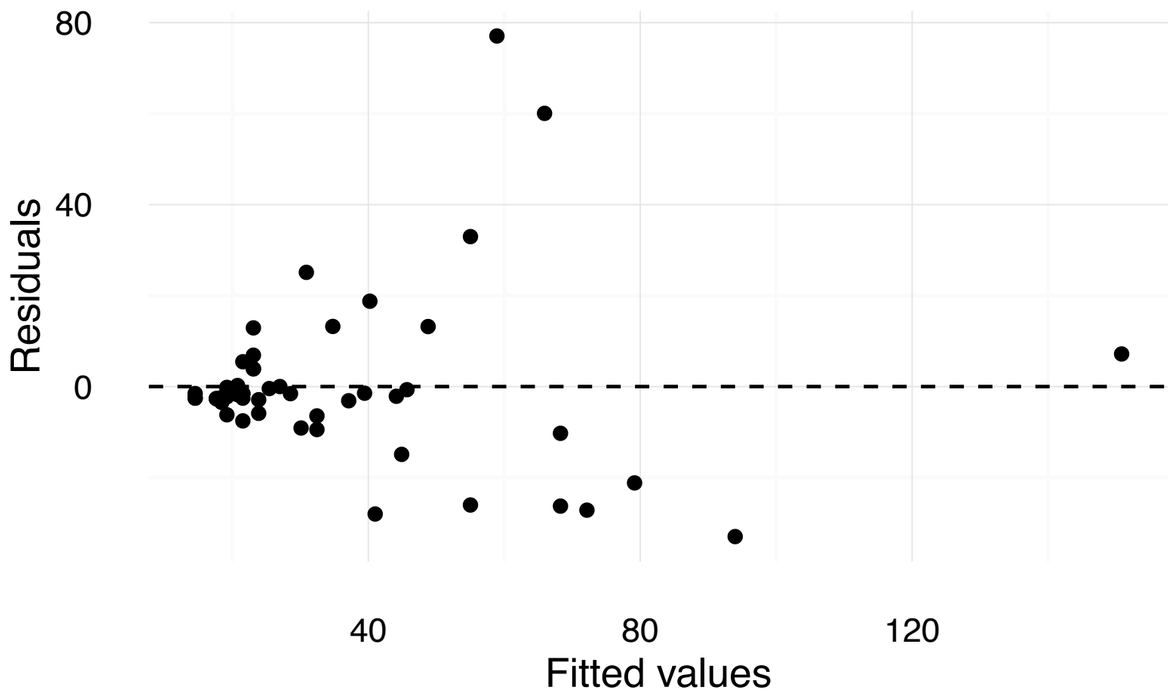
$$e_i = y_i - \hat{y}_i$$

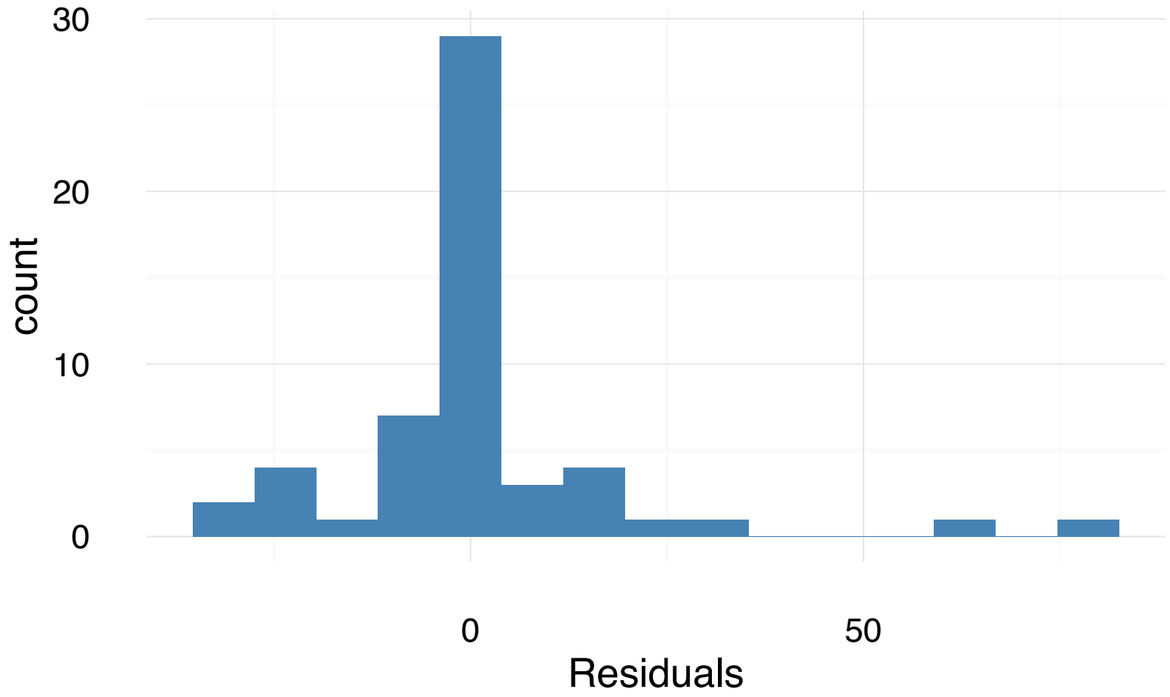




Residuals

$$e_i = y_i - \hat{y}_i$$



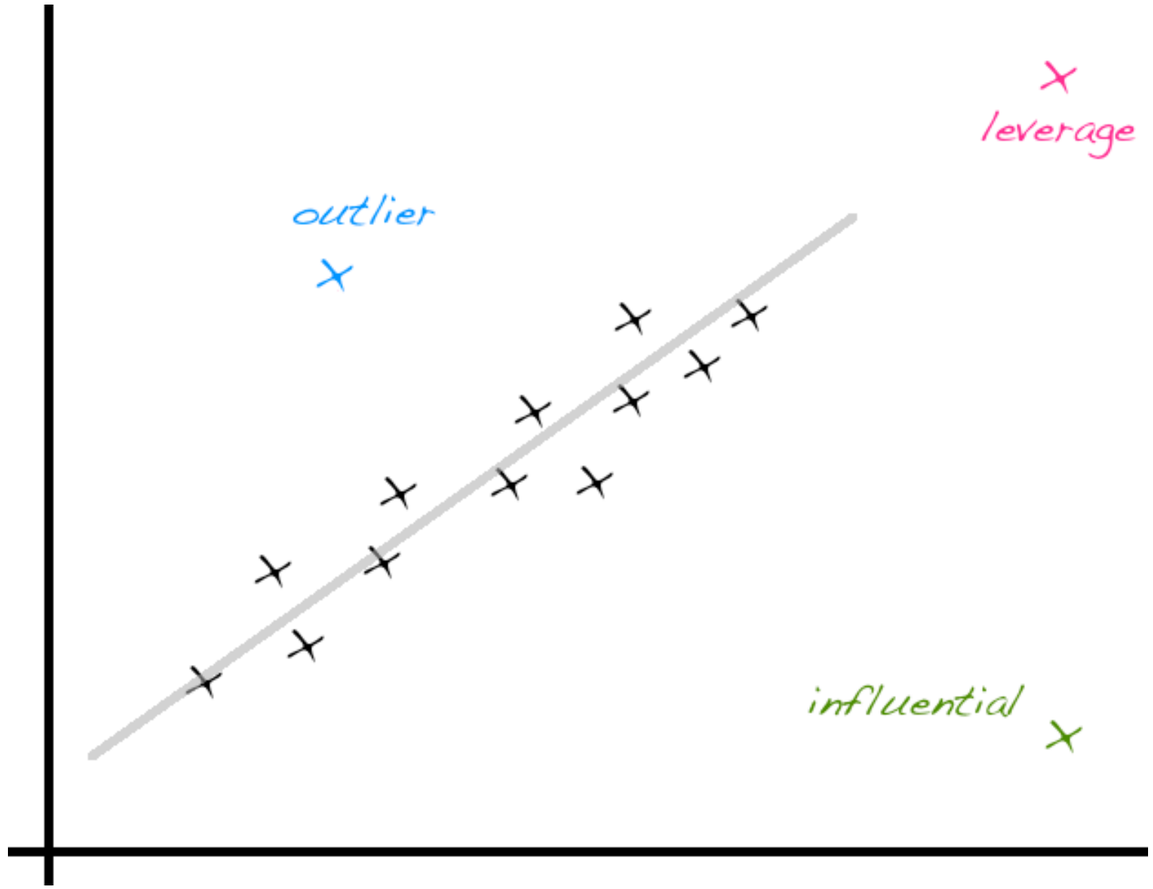


Calibrate your intuition

https://gallery.shinyapps.io/slr_diag/

Outliers

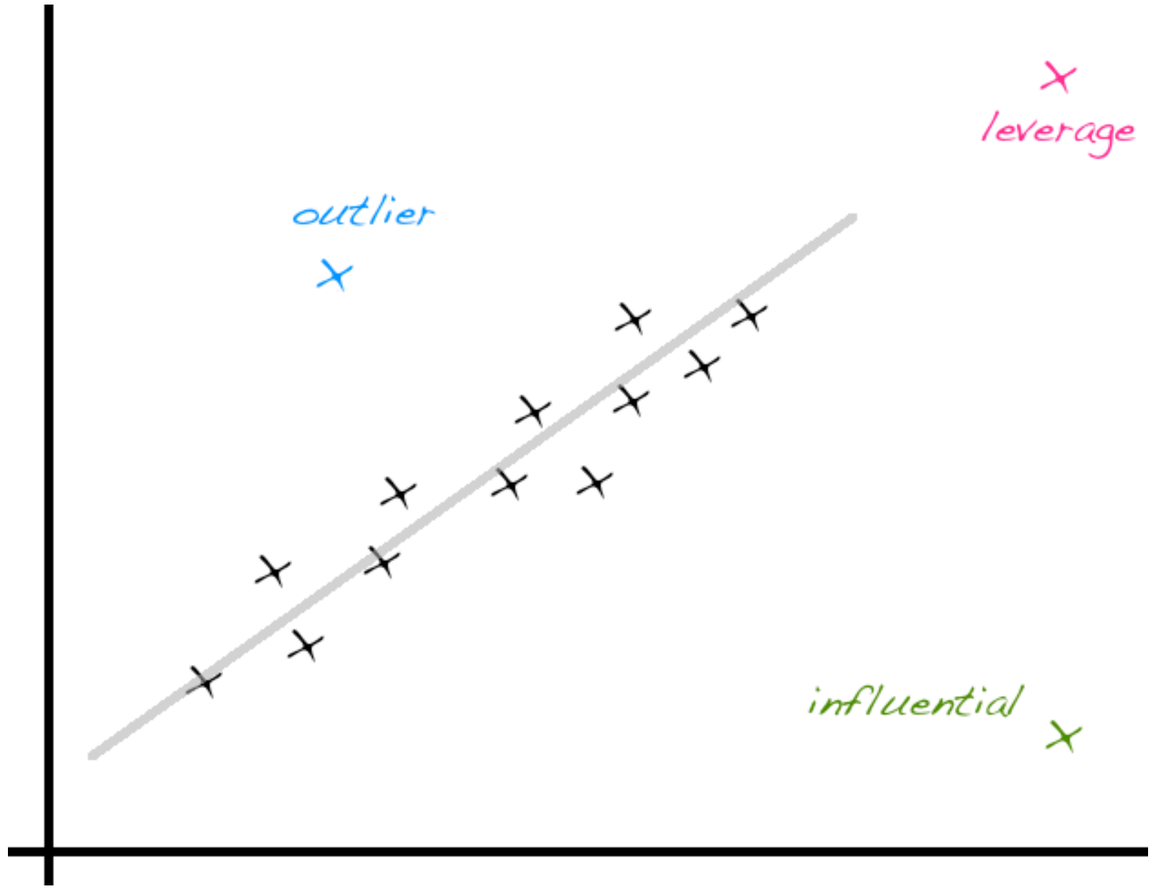
Leverage point



Away from the cloud of points horizontally

Does not necessarily change the slope

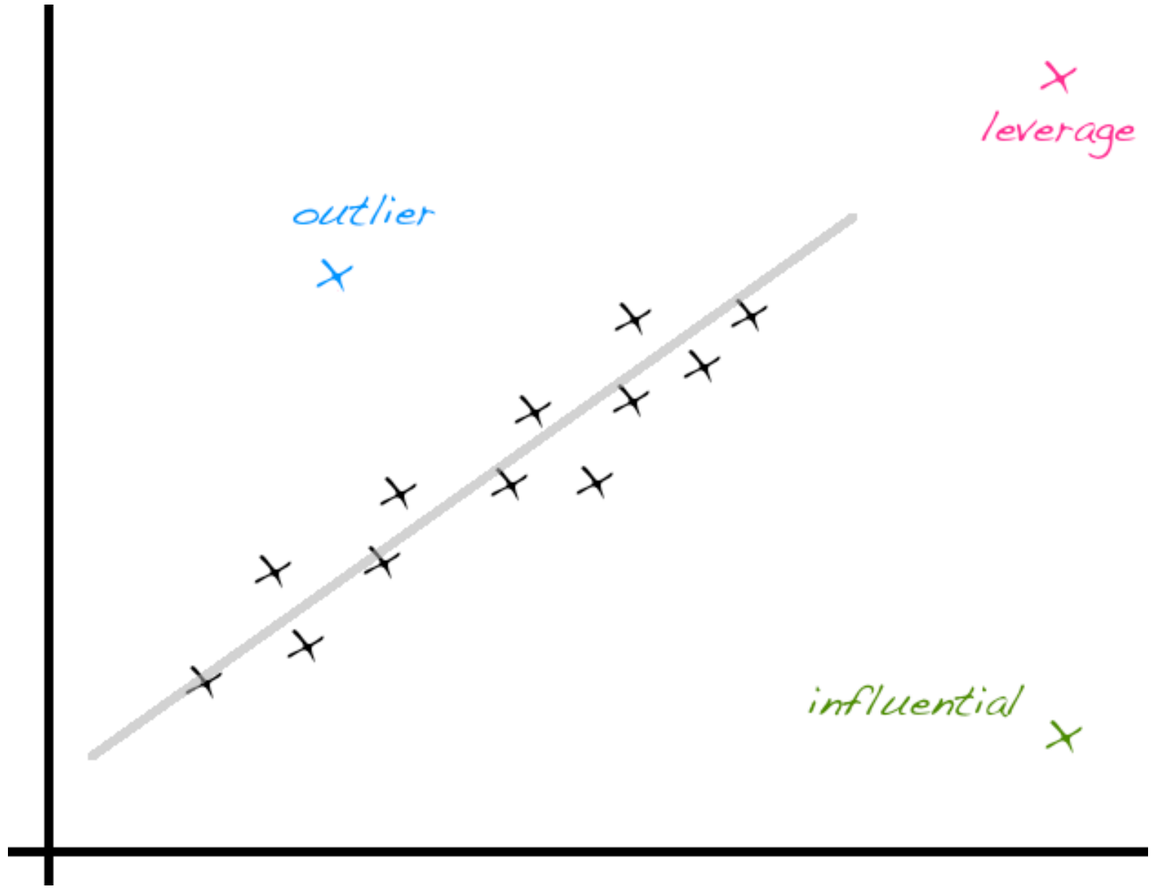
Influential point



Changes the slope (most likely also has high leverage)

Run the regression with and without that point to determine

Outlier



An unusual point without these special characteristics (this one likely affects the intercept only)

If clusters (groups of points) are apparent in the data, it might be worthwhile to model the groups separately