Homework 3

Math 315, Fall 2019

Due 27 September by 4:00 p.m.

Instructions: Complete the following problems and submit them by 4 p.m. on the due date. Please make sure that your solution is neatly written, clearly organized, and stapled (if there are multiple pages). For this homework, I expect you to use R Markdown. You may hide the plotting code chunks, but please do not hide the code chunks where you derive your posteriors, since we may need to "dig into" these to point out minor errors.

1. Exercise 1.14 from *BSM*

   In your answer, display either a grid of plots (either 3 x 2 or 2 x 3) or a single plot with the different posteriors clearly labeled by color, and shape/linetype. Be sure to resize your plots so that they are legible, and give them clear labels/legends so that you can tell what is happening. You should write your observations clearly.

2. Suppose $Y_1, \ldots, Y_5$ are independent samples from a Cauchy distribution with unknown center θ and known scale 1. The PDF for this Cauchy distribution is given below:

$$p(y_i|\theta) \propto \frac{1}{1 + (y_i - \theta)^2}$$

   Assume, for simplicity, that the prior distribution for θ is Unif(0, 100). Suppose that you observed the the data points 43, 44, 45, 46.5, and 47.5.

(a) Compute the unnormalized posterior density function on a grid of points for $\theta = 0, \frac{1}{m}, 2m, \ldots, 100$, for some large integer m. Using the grid approximation, compute and plot the normalized posterior density function as a function of θ.

(b) Sample 1000 draws of θ from the posterior density and plot a histogram of the draws. (Be sure to use a reasonable number of bins in your histogram, the default isn't always reasonable.)

(c) Use the 1000 samples of θ to obtain 1000 samples from the predictive distribution of a future observation, Y_6, and plot a histogram of the predictive draws.