Useful distributions

Math 315, Fall 2019

Bernoulli

$$f(x|\theta) = \theta^x(1-\theta)^{1-x};\ x = 0, 1;\ 0 \leq \theta \leq 1$$
$$E(X) = \theta,\ Var(X) = \theta(1-\theta)$$

Beta

$$f(x|a,b) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}x^{a-1}(1-x)^{b-1};\ 0 \leq x \leq 1,\ a > 0,\ b > 0$$
$$E(X) = \frac{a}{a+b},\ Var(X) = \frac{ab}{(a+b)^2(a+b+1)}$$

Binomial

$$f(x|n,\theta) = \binom{n}{x}\theta^x(1-\theta)^{n-x};\ x = 0, 1, \ldots, n,\ 0 \leq \theta \leq 1$$
$$E(X) = n\theta,\ Var(X) = n\theta(1-\theta)$$

Exponential

$$f(x|\lambda) = \lambda e^{-\lambda x};\ x \geq 0,\ \lambda \geq 0$$
$$E(X) = \frac{1}{\lambda},\ Var(X) = \frac{1}{\lambda^2}$$

Gamma

$$f(x|a,b) = \frac{b^a}{\Gamma(a)}e^{-bx}x^{a-1};\ x \geq 0,\ a > 0 \text{ (shape)},\ b > 0 \text{ (scale)}$$
$$E(X) = \frac{a}{b},\ Var(X) = \frac{a}{b^2}$$

Inverse gamma

$$f(x|a,b) = \frac{b^a}{\Gamma(a)} e^{-b/x} x^{-a-1};\ x \geq 0,\ a > 0 \text{ (shape)},\ b > 0 \text{ (scale)}$$
$$E(X) = \frac{b}{a-1} \text{ (if } a > 1),\ Var(X) = \frac{b^2}{(a-1)^2(a-2)} \text{ (if } a > 2)$$

Note: $1/X \sim \text{Gamma}(a,b)$

Multinomial

$$f(\mathbf{x}|\boldsymbol{\theta}) = \frac{n!}{\prod_{j=1}^{p} x_j!} \prod_{j=1}^{p} \theta_j^{x_j},\ x_j = 0,1,\ldots,n,\ 0 \leq \theta_j \leq 1, \sum_{j=1}^{p} \theta_j = 1$$
$$E(X_j) = n\theta_j,\ Var(X_j) = n\theta_j(1-\theta_j)$$

Negative Binomial

$$f(x|\theta,m) = \binom{x+m-1}{x} \theta^m (1-\theta)^x;\ x = 0,1,\ldots,\ m = 1,2,\ldots,\ 0 \leq \theta \leq 1$$
$$E(X) = m(1-\theta)/\theta,\ Var(X) = m(1-\theta)/\theta^2$$

Normal

$$f(x|\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/\left(2\sigma^2\right)};\ -\infty < x < \infty,\ -\infty < \mu < \infty,\ \sigma^2 > 0$$
$$E(X) = \mu,\ Var(X) = \sigma^2$$

Poisson

$$f(x|\lambda) = \frac{e^{-\lambda}\lambda^x}{x!};\ x = 0,1,2,\ldots,\lambda \geq 0$$
$$E(X) = \lambda,\ Var(X) = \lambda$$