Homework 6

Math 315, Fall 2019

Due 8 October by 4:00 p.m.

Instructions: Complete the following problems and submit them by 4 p.m. on the due date. Please make sure that your solution is neatly written, clearly organized, and stapled (if there are multiple pages). You may complete derivations by hand, but any R work should be completed using R Markdown to render the final write up. You may hide the plotting code chunks, but please do not hide the code chunks where you derive your posteriors or simulate from distributions, since we may need to "dig into" these to point out minor errors.

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In addition, complete the following problems:

1. *Jeffreys' prior.* In class we showed that the Jeffreys' prior for the Binomial(n, θ) model is Beta(1/2, 1/2). In this problem you will explore why a Jeffreys' prior is said to be transformation invariant.

   (a) Suppose you reparameterize the binomial distribution with $\gamma = \log[\theta/(1-\theta)]$, so that

   $$f(y|\gamma) = \binom{n}{y}(e^{\gamma})^{y}(1-e^{\gamma})^{n-y}.$$

   Derive the Jeffreys' prior distribution for γ under this model.

   (b) Take the Beta(1/2, 1/2) prior distribution we derived in class and apply the change of variables formula to obtain the induced prior density on γ. Does this agree with your answer to part (a)? (If you have forgotten the change of variables formula, see equation 2.36 for a reminder.)

2. *Multiparameter model.* Suppose we are interested in learning about the sleeping habits of students at Carleton. We collect the sleeping times (in hours) for 20 randomly selected students in a statistics course. These are the observations:

   $$9.0, 8.5, 7.0, 8.5, 6.0, 12.5, 6.0, 9.0, 8.5, 7.5,$$
   $$8.0, 6.0, 9.0, 8.0, 7.0, 10.0, 9.0, 7.5, 5.0, 6.5$$

   (a) Use a Normal(μ, σ^2) sampling distribution to model the data, and assign the noninformative prior $\pi(\mu, \sigma^2) = 1/\sigma^2$. Describe how to draw a random sample from the posterior distribution for (μ, σ^2), and then draw a random sample of size 10,000 from this distribution.

   (b) Plot the marginal posterior distributions for μ and σ^2, and calculate the posterior mean and 90% credible interval.

   (c) Plot the posterior distribution for the upper 75th quantile, $p_{75} = \mu + 0.674\sigma$ and calculate its mean and a 90% credible interval.