Homework 9 Solution

Math 315, Fall 2019

BSM Chapter 3 exercise 11

Below, I use a Normal candidate distributions with standard deviation 0.2 and 0.06 for α and β, respectively. These standard deviations were selected by trial and error to give an acceptance rate around 0.4 for each parameter. Both chains have clearly converged quickly.

```
# Data
Y       <- c(64,13,33,18,30,20)
t       <- 1:6

# Initialization
S        <- 25000                         # No. MCMC draws
beta     <- c(2, 0)                       # Initial guess for alpha, beta
can.sd   <- c(0.2, 0.06)                  # SD for candidate dsn
draws    <- matrix(NA, nrow = S, ncol = 2) # Storage for MCMC draws
colnames(draws) <- c("alpha", "beta")     # Adding column names

post    <- function(Y, t, beta, prior.sd = 10){
  mn     <- exp(beta[1] + t * beta[2])
  llike  <- sum(dpois(Y, mn, log = TRUE))
  lprior <- sum(dnorm(beta, 0, prior.sd, log = TRUE))
  return(exp(llike + lprior))
}

# Tracking acceptance rates
acc <- c(0, 0)

# Metropolis sampling

for(i in 1:S){
  for(j in 1:2){
    can    <- beta
    can[j] <- rnorm(1, beta[j], can.sd[j])
    R      <- post(Y, t, can) / post(Y, t, beta)
    if(runif(1) < R){
      beta <- can
      acc[j] <- acc[j] + 1
    }
  }
  draws[i,] <- beta
}

# Calc. acceptance rates
acc / nrow(draws)
```

```
## [1] 0.41296 0.39708
```

```
# Trace plots
par(mfrow = c(1, 2))
plot(draws[,1], type = "l", xlab = "Iteration", ylab = expression(alpha))
```

```r
plot(draws[,2], type = "l", xlab = "Iteration", ylab = expression(beta))
```