Exam 2 Study Guide

Math 315, Fall 2019

Logistics

- In-class exam: 13 November
- You may bring one standard (8.5 × 11 inch) sheet of paper with notes written on both sides.
- I will provide a sheet with PDFs/PMFs for any distribution that you need on the exam.
- The exam covers material from chapters 3.2-3.4, 4.1-4.2, 5.1, 5.5-5.6 of *BSM*. I will update this list if we don't quite get to section 5.6.

Topics

Disclaimer: This list may not be exhaustive, but I hope that you find it useful in preparing for the exam.

Topics since exam 1

1. Markov chains. Understand what a Markov chain is and why we are interested in a stationary distribution.
2. MM. Understand why we need MCMC and what the goal of MCMC is. What should a traceplot of a "good" chain look like (after enough draws)? Once you have draws via MCMC, how do you conduct inference? Know how to use all of our tools to diagnose convergence.
    - Metropolis and Metropolis-Hastings algorithms. How does it work (generally)? Be able to sketch an outline of the algorithm. Understand the acceptance probability of a proposed draw. What types of proposal (candidate) densities are permitted?
    - Gibbs sampler. How does it work (generally)? Be able to sketch an outline of the algorithm. Be able to derive full-conditional posteriors for a reasonable number of parameters.
    - Improving convergence. Be able to discuss strategies to improve the convergence of your chains (there are 7 discussed in the book).
3. Understand the relative advantages/disadvantages of the varios computational methods we have learned to approximate the posterior distribution.
4. JAGS. Understand the basic syntax of JAGS so that you will be able to read it if given to you on an exam problem.
5. Interpret plots for models (not just regression models).
6. Formulate the model for one- and two-sample analyses for normal means.
7. Formulate the model for regression problems. What's the reference prior? How do you approximate it in JAGS?
8. How do you create either the prior or posterior predictive distribution for a regression problem?
9. Given posterior estimates from a regression model, interpret the model coefficients and predictions.
10. Model selection. Understand the considerations involved in selecting a model for both prediction and description. Understand how the below issues relate to model selection.
    - Underfitting and overfitting. Understand each concept and why we want to avoid these issues.

11. Regularization. What problem does a regularizing prior help us avoid? How can you tune a regularizing prior? What are the two ways to use a fully Bayesian analysis to tune the penalty term?
12. Cross validation. Understand what it is and how we can use it to compare models based on their predictive performance.
13. Deviance. Understand the definition of deviance and how we use it to assess models. Understand the properties of deviance, notably that it improves as models become more complex and that it scales with sample size. Understand how deviance relates to the information criteria discussed.
14. Information criteria. Understand what DIC and WAIC are telling you about a model's performance and (at least roughly) how they are calculated. What are they attempting to approximate? Which criterion is the most general? What assumptions are required for each? What do the effective number of parameters measure? How do we use these criteria to compare models?

Review topics

We still make use of many of the ideas from the first exam, so it is important to understand the topics outlined for exam 1 even if I don't ask as explicit questions on those topics. Some topics (e.g. deriving posteriors, drawing samples from posteriors for inference) are very central to our tasks. Don't expect me to ignore central ideas!

1. Bayes' rule. Be able to apply Bayes' rule and the Bayesian thought process to conditional probability problems.
2. "The posterior is proportional to prior times likelihood." Given a prior and a likelihood, write down the posterior up to the normalizing constant.
3. Given a posterior distribution, calculate the posterior mean, median, MAP estimate, variance, standard deviation, equal-tailed credible interval (i.e. percentile interval), and $P(\theta > k)$.
4. Define informative prior, weak/diffuse/flat prior, (natural) conjugate prior, improper prior, Jeffreys' prior
5. Define elicitation and describe what it means to do elicitation well.
6. Given two quantiles, or the mean and standard deviation, describe how to determine the hyperparameters of your prior distribution in the one-parameter setting.
7. Given a sampling distribution (likelihood), derive the Jeffreys' prior.
8. Given a sampling distribution (likelihood), show that a prior is conjugate.
9. Given a sampling distribution (likelihood), derive the natural conjugate prior.
10. Given a sampling distribution and a posterior distribution, write down the integral expression for the posterior predictive distribution. Describe how you would generate samples from the predictive distribution.
11. Given a sampling distribution and a prior distribution, write down the integral expression for the prior predictive distribution. Describe how you would generate samples from the predictive distribution.
12. Describe the algorithm for grid approximation. Describe how you would generate samples from the resulting posterior distributions.
13. Describe how to use the Bayesian CLT to approximate the posterior distribution, and how you would generate samples from the approximate posterior.
14. Describe how to get a sample from the posterior distribution of a function of a parameter, given that you have a sample from its posterior distribution.

15. Given a two-dimensional posterior distribution, write the integral expression for the marginal distribution of one of the parameters.
16. Given a two-dimensional posterior distribution, write an expression for the conditional distribution of one of the parameters, given the other.
17. Describe how to get a sample from the marginal posterior for a parameter, given that you have a sample from the joint posterior distribution.
18. Describe how to get a sample from the posterior distribution of a function of parameters, given that you have a sample from their joint posterior distribution.
19. Describe how to "calculate" the posterior mean, median, variance, standard deviation, credible interval (i.e. percentile interval), highest posterior density interval, and $P(\theta > k)$ given a sample from the posterior distribution.