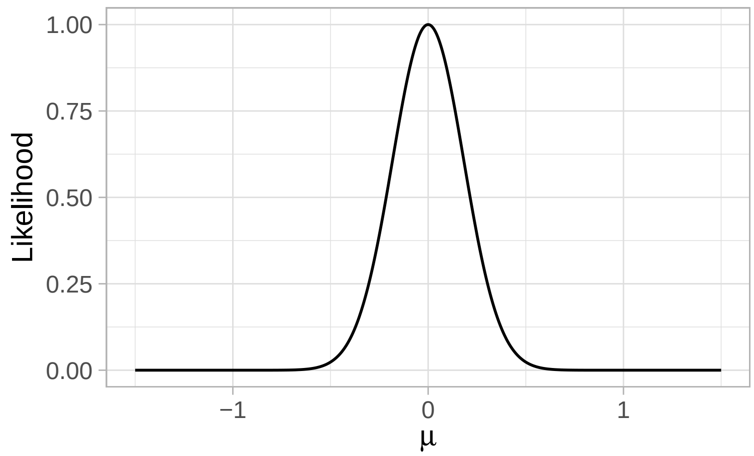
Exam 1 Practice Problems

Math 315, Fall 2019

1. **Priors.** The plot below is the likelihood function for the mean of a $\mathcal{N}(\mu, 1)$ distribution evaluated using 30 observations.



   (a) Sketch (and clearly label) a *uninformative/diffuse* prior on the plot above. Write a short definition of a uninformative/diffuse prior.

   (b) Sketch (and clearly label) an *informative* prior on the plot above. Write a short definition of an informative prior.

2. **Bayesian updating and prediction.** Suppose that $Y_1, \ldots, Y_n | \theta \sim \text{Gamma}(1, \theta)$ and that $\theta \sim \text{InvGamma}(a, b)$.

   (a) Find the posterior distribution of θ. If it is a member of a named family of distributions, be sure to specify this, along with its parameter values.

   (b) Is the inverse-gamma prior a conjugate family to the gamma likelihood?

   (c) Write down two integrals (but do not evaluate them) that could be solved to find the 97% percentile interval for θ.

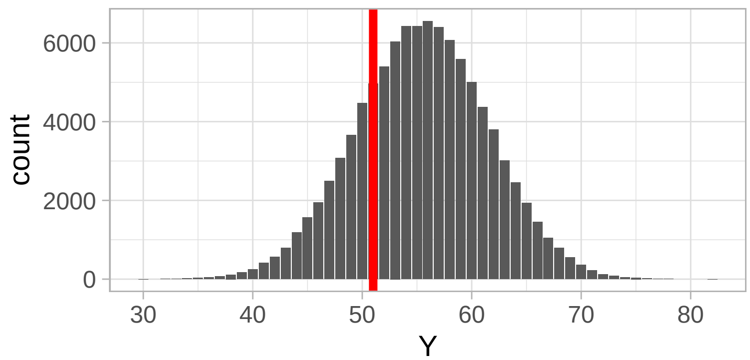
   (d) Outline (describe) the steps necessary to draw a sample from the posterior predictive distribution of $\widetilde{Y}$, assuming that you have a sample $\theta^{(1)}, \theta^{(2)}, \ldots, \theta^{(m)}$ from the posterior distribution of θ.

3. **Sampling.**

   (a) Suppose that you are interested in a posterior distribution, $p(\theta | x_1, \ldots, x_n)$, with support on $(0, 1)$. Describe how you would use grid approximation to draw a random sample from $p(\theta | x_1, \ldots, x_n)$.

   (b) Suppose that we have a sample of size m drawn from $p(\theta | x_1, \ldots, x_n)$—that is, we have $\theta^{(1)}, \theta^{(2)}, \ldots, \theta^{(m)}$ in hand. How could we use this sample to evaluate the posterior probability that $\theta > 0.8$?

4. **Model checking.** Below is the posterior predictive distribution for the following model: $Y_i|p \sim \text{Binom}(n, p)$, where $p \sim \text{Beta}(0.5, 0.5)$. The observed value of Y is diplayed as a vertical line.



What does this plot reveal about the model's fit?

5. **Conditional posteriors.** Suppose $y_1, \ldots, y_n$ form a random sample from $\mathcal{N}(\mu, \sigma^2)$. The joint posterior distribution that results from the reference prior is

$$p(\mu, \sigma^2 | y_1, \ldots, y_n) \propto \left(\sigma^2\right)^{-n/2-1} \exp\left\{ \sum_{i=1}^{n} -\frac{1}{2\sigma^2}(y_i - \mu)^2 \right\}$$

Find $p(\sigma^2|\mu, y_1, \ldots, y_n)$, the conditional posterior of σ^2 given μ and the data. If it is a member of a named family of distributions, be sure to specify this, along with its parameter values.