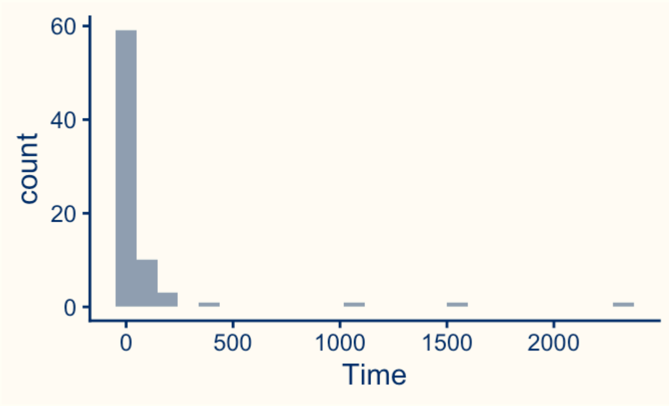
Remedial Measures: Transformations

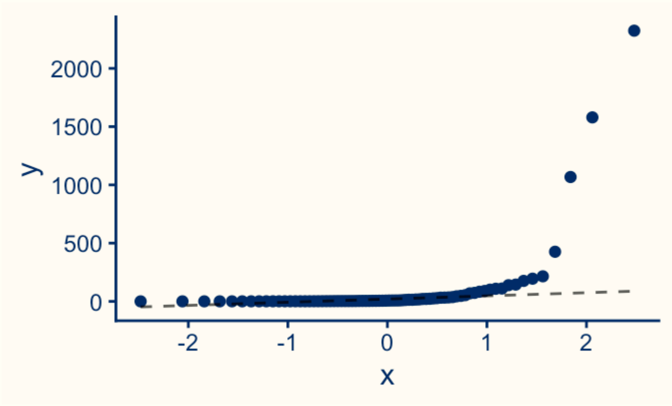
Stat 230: Applied Regression Analysis

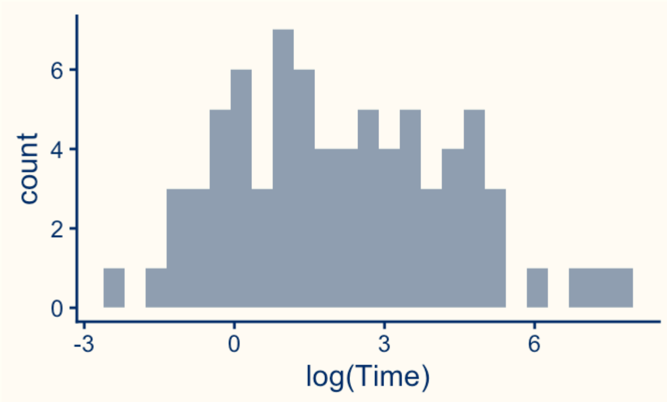
PDF version of slides

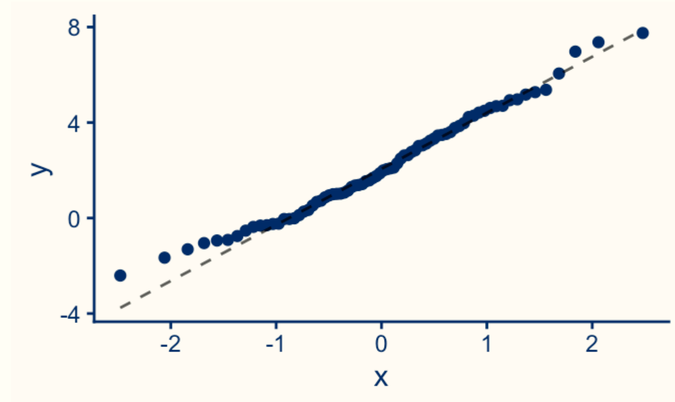
Easing skew

If a set of data values is skewed to the right, taking the (natural) log of each data value *can* result in a data set that is roughly symmetric and often roughly normal.

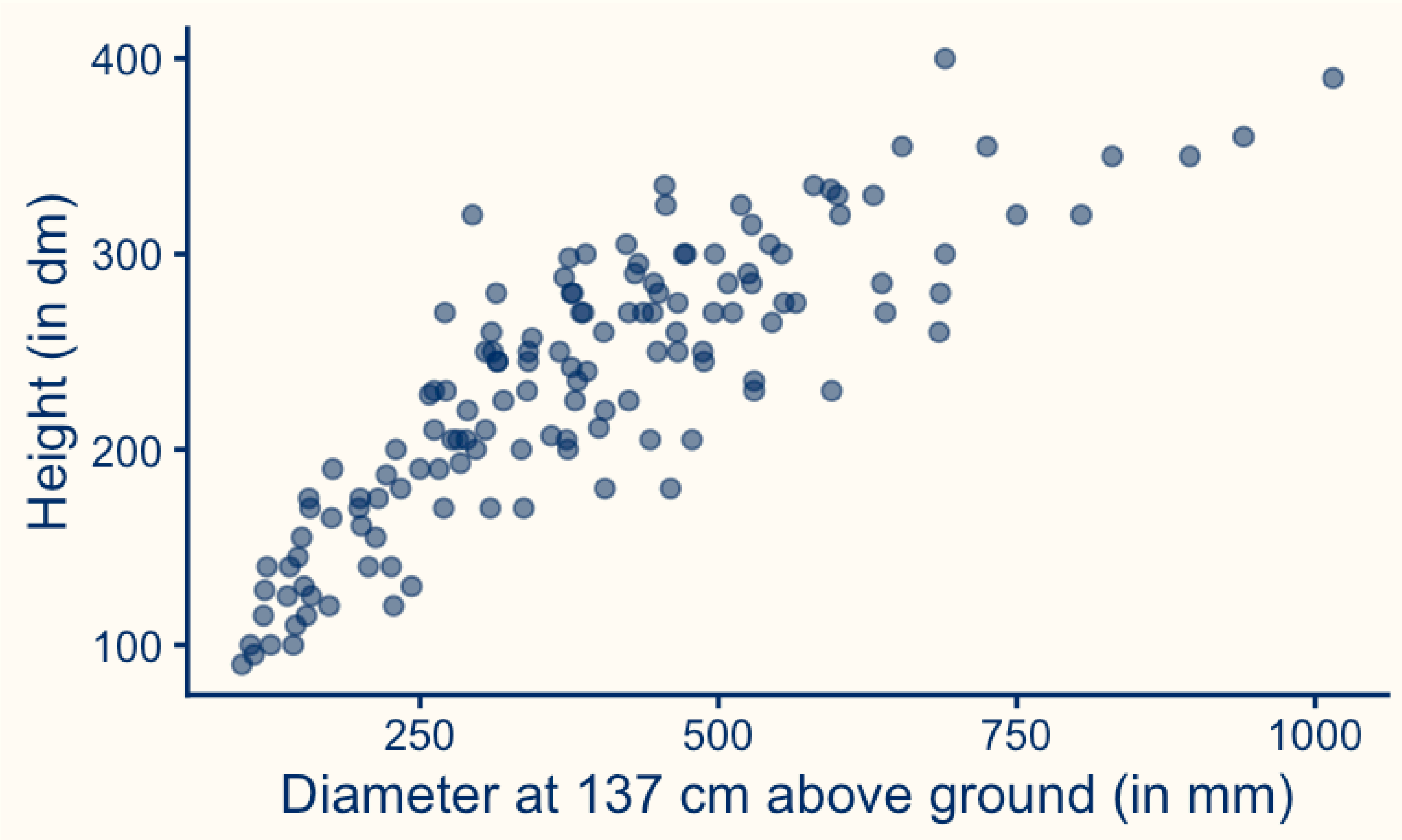




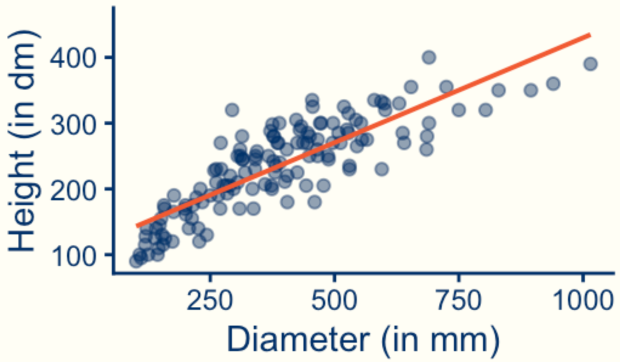

count
log(Time)
6
4
2
0
-3
0
3
6

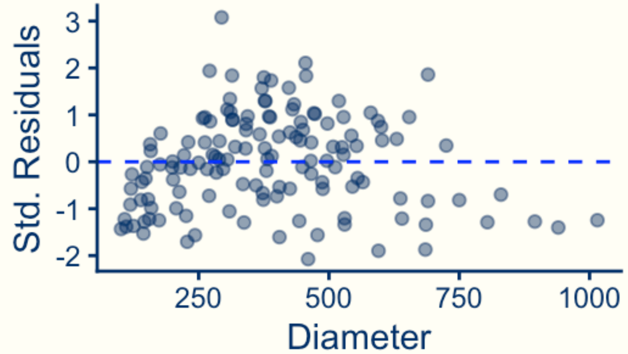

y
x
8
4
0
-4
-2
-1
0
1
2

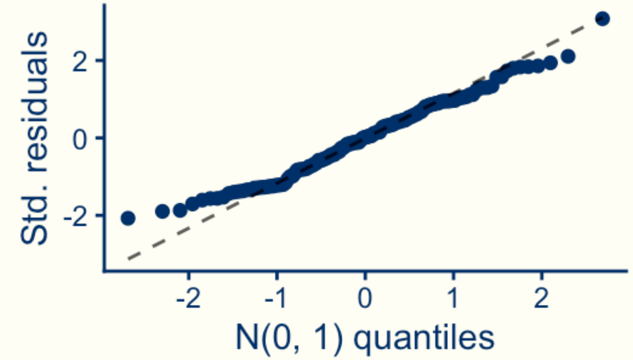
How are tree height and tree diameter related for the western red cedar?



Are the conditions violated?







a. linearity

b. constant errors

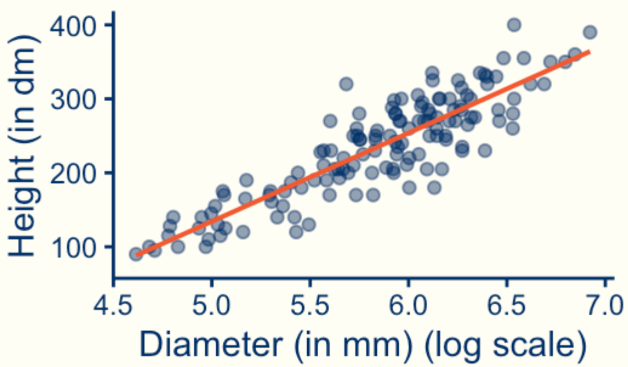
c. independent errors

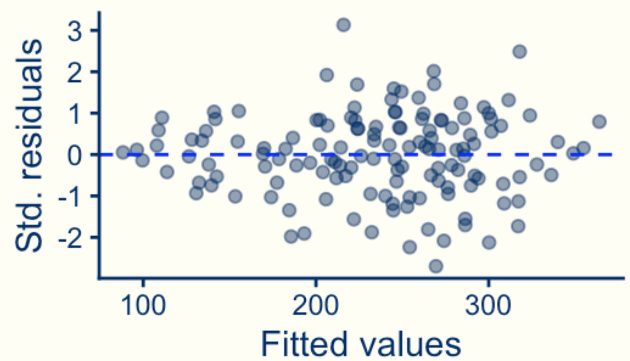
d. normal errors

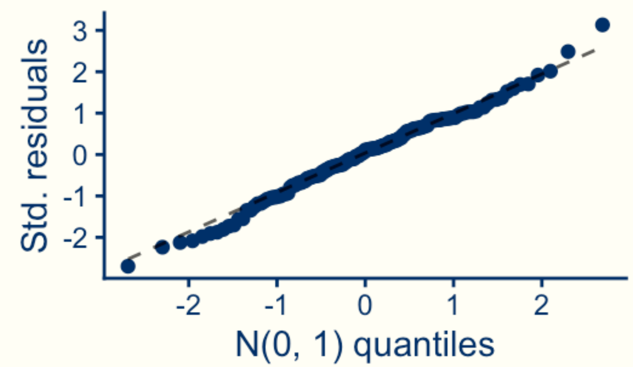
e. outliers

f. none

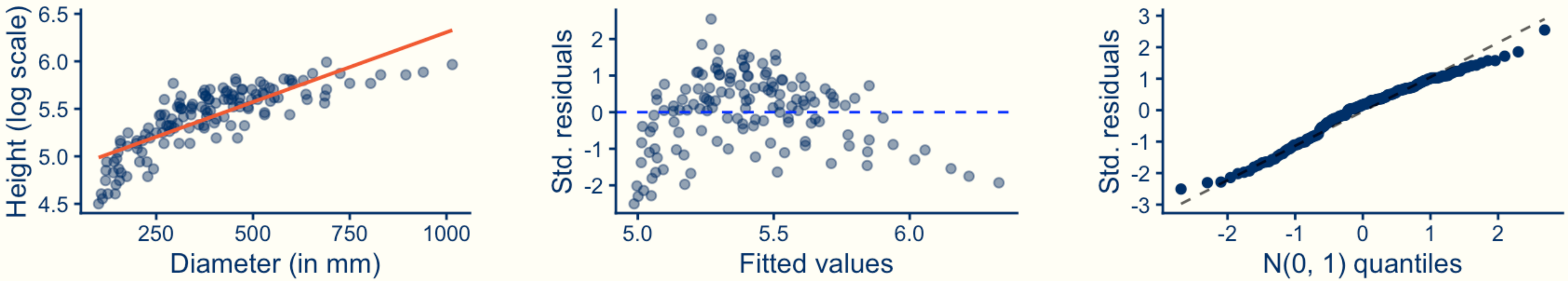
Does transforming X help?



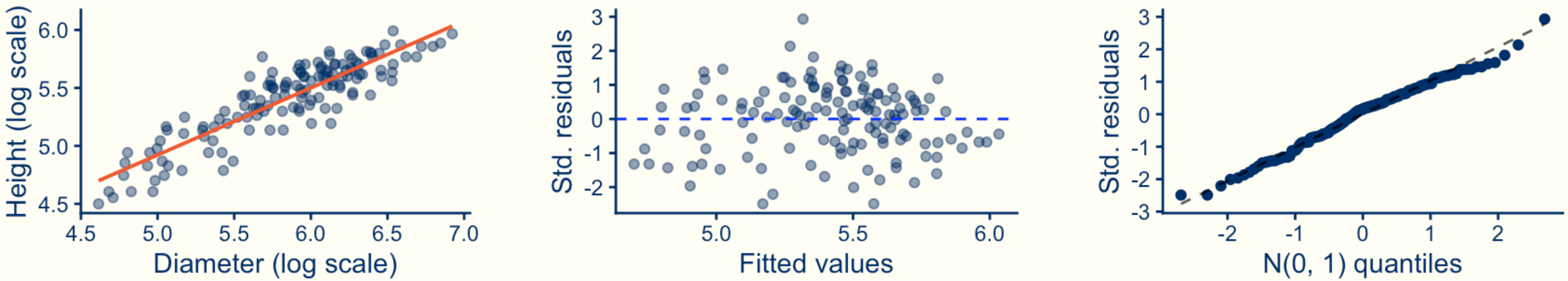




Does transforming Y help?



Transforming both X and Y?



Your turn

- Work through the first example on the handout with your neighbor(s)
- Online version with R chunks:

Back-Transforming

Converting a transformed variable back to its original scale

Back-Transforming

Log scale

mean
2.146

Original scale

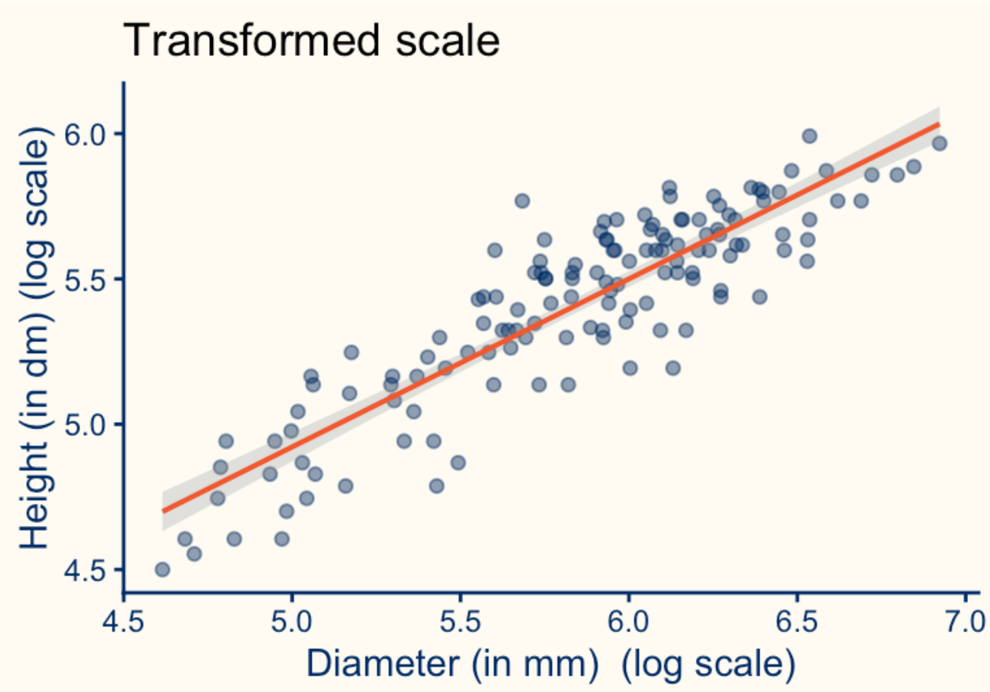
mean
98.558

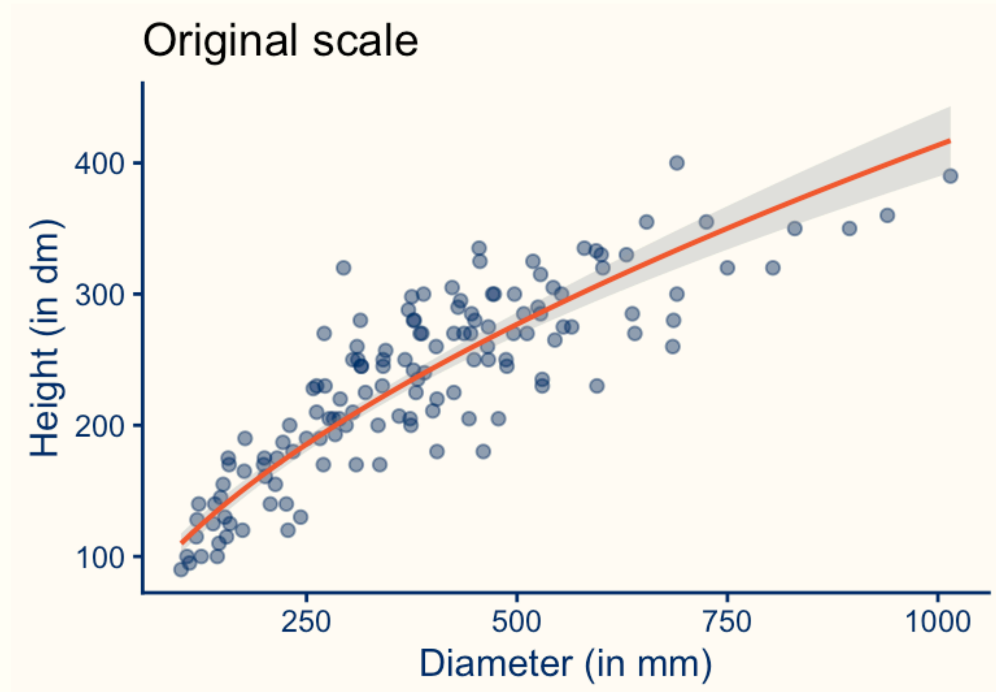
- Back-transformed mean: $e^{2.146} \approx 8.55$

R Note

- `log` is the natural log
- `exp(x)` calculated e^x

Displaying a transformed model

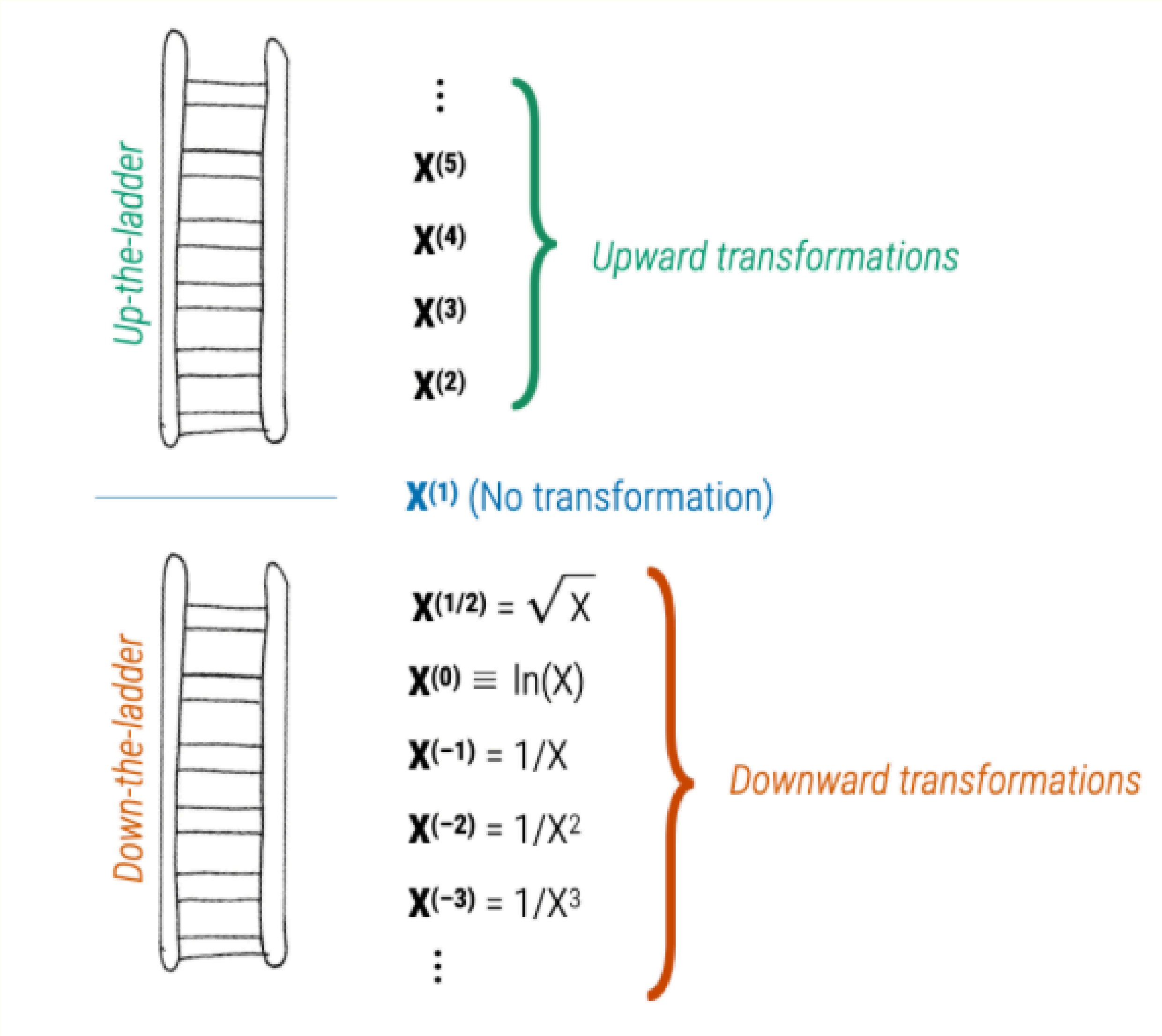




Rules of thumb

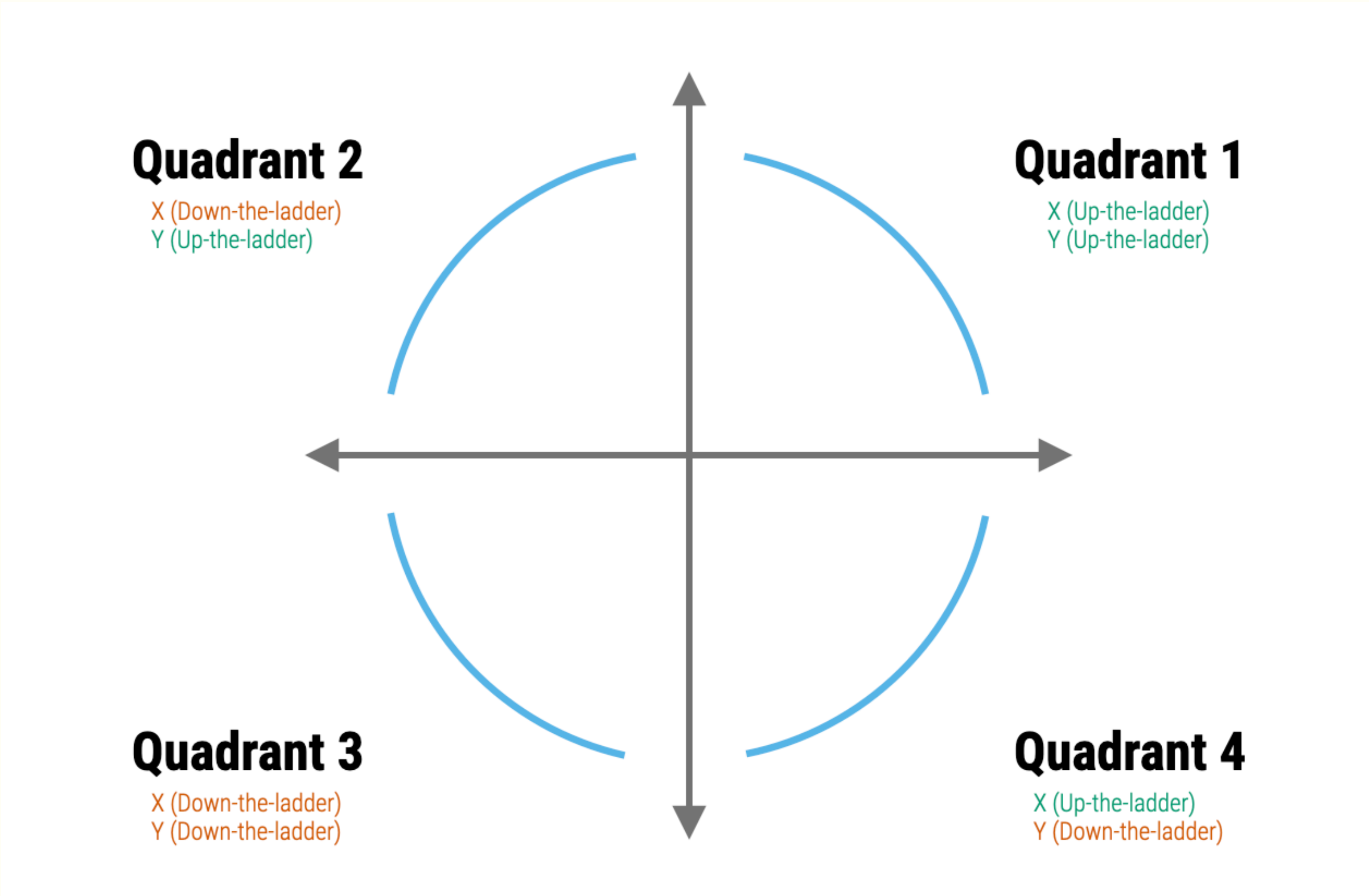
- **transform x**: if mean function is nonlinear, but is monotonic and the residual variance is constant
- **transform y**: if mean function is nonlinear and the residual variance increases as the mean increases (log, reciprocal, or square root often work)
- **log rule**: if values range over more than 1 order of magnitude and are strictly positive, then the natural log is likely helpful
- **range rule**: if the range is considerably less than 1 order of magnitude, then transformations are unlikely to help
- **square roots** are useful for count data

Ladder of transformations



Rule of the Bulge

Introduced by John Tukey and Frederick Mosteller for "straightening" data to better meet the assumption of linearity



Interpreting a log-transformed model

Back-Transforming

Log scale

mean	median
2.146	1.933

Original scale

mean	median
98.558	6.925

- Back-transformed median: $e^{1.933} \approx 6.91$
- Back-transformed mean: $e^{2.146} \approx 8.55$

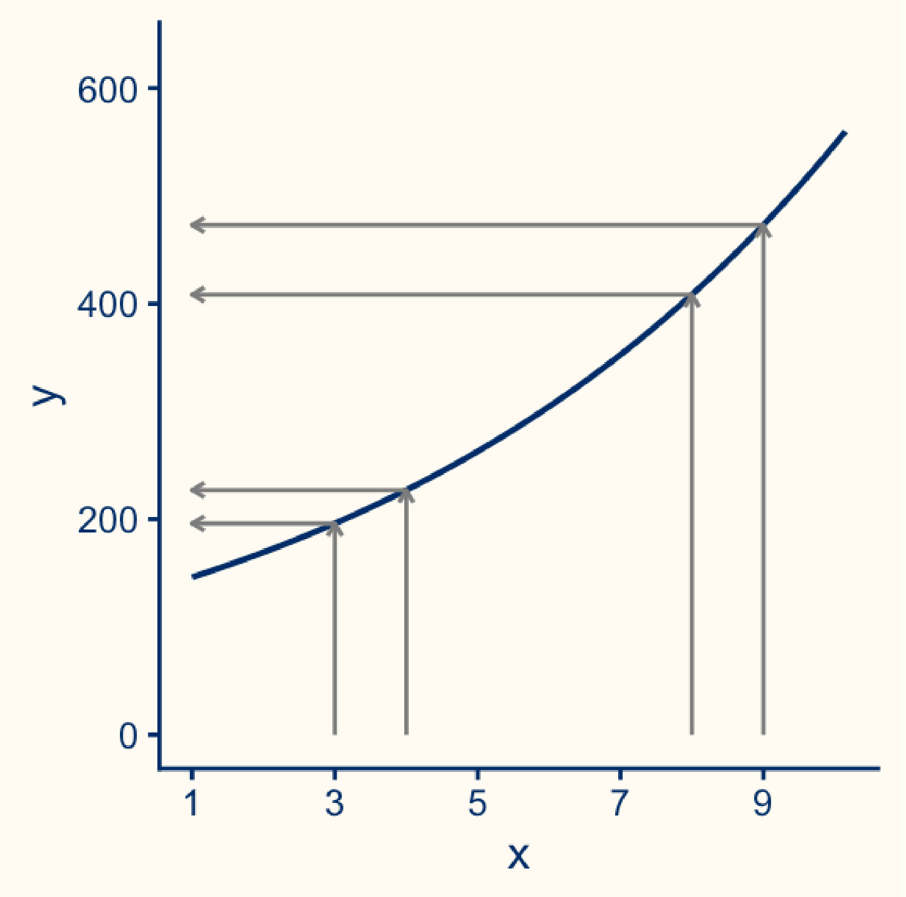
Back-transforming log transformations

- Often log-transforming a variable makes in approximately symmetric
- If symmetric, then the median ≈ mean on the log scale
-
- Inference made mean on the log scale can thought of as inference for the median on the log scale

Log-transform of Y only

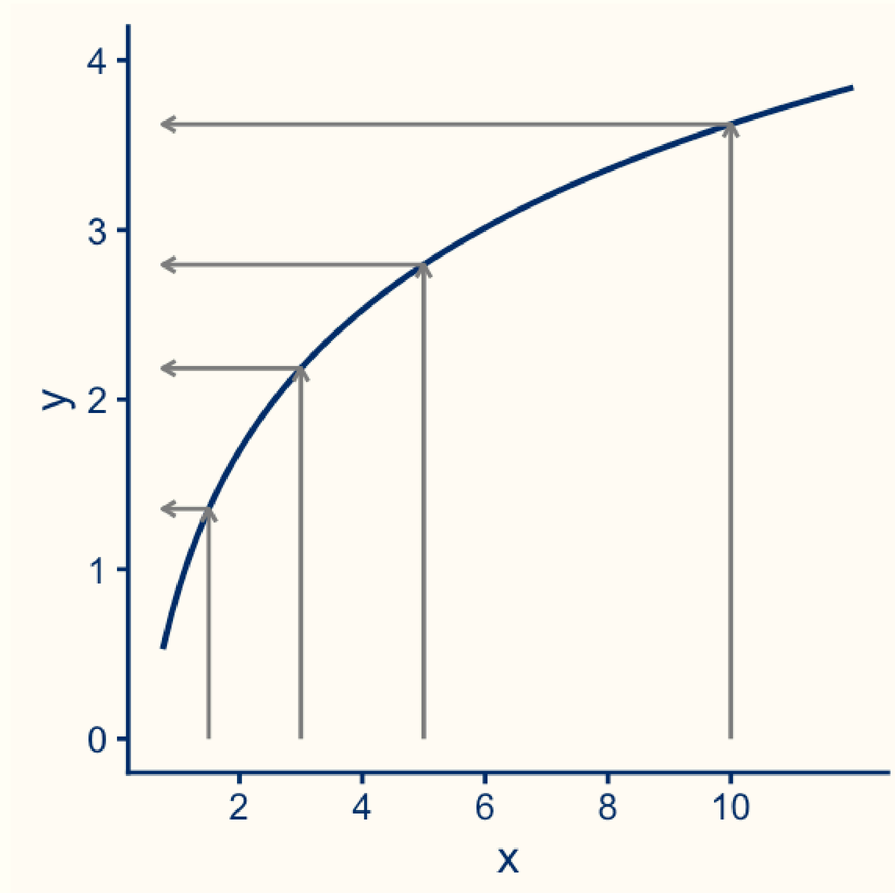
The median of at is times larger (smaller) than the median of at .

Or… increasing by 1 increases (decreases) the median of by a factor of .



Log-transform of X only

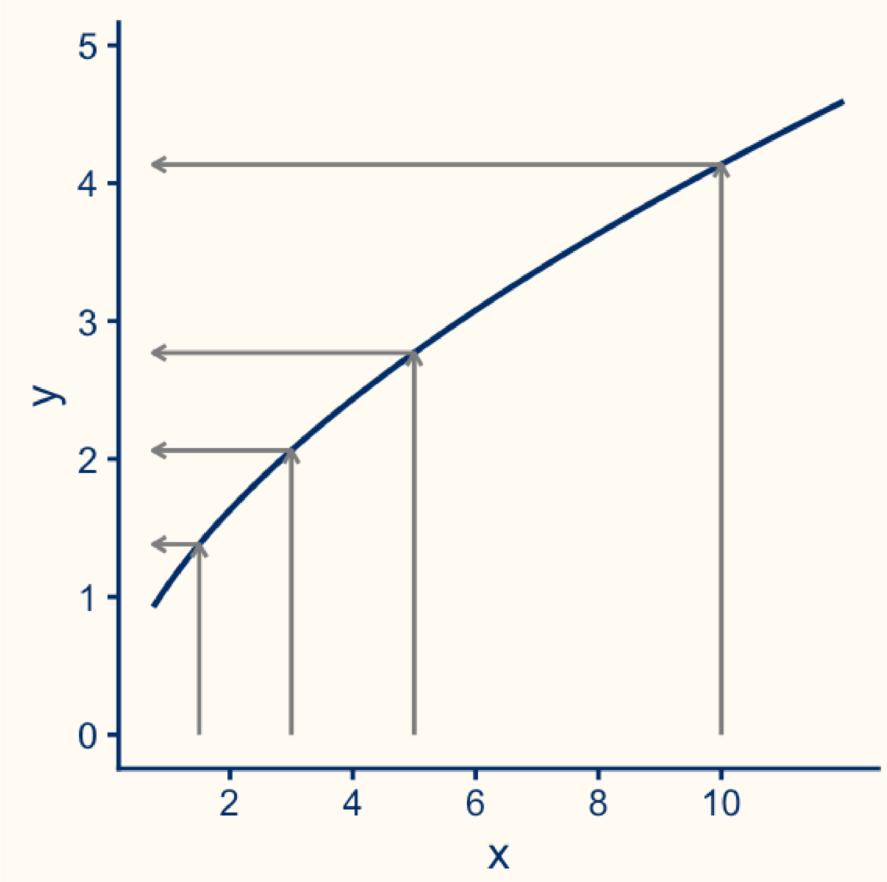
A doubling of is associated with the mean response increasing (decreasing) by units.



Log-transform both Y and X

The median of at is times greater (smaller) than the median of at .

Or… A doubling of x is associated with the median of Y increasing (decreasing) by a factor of .



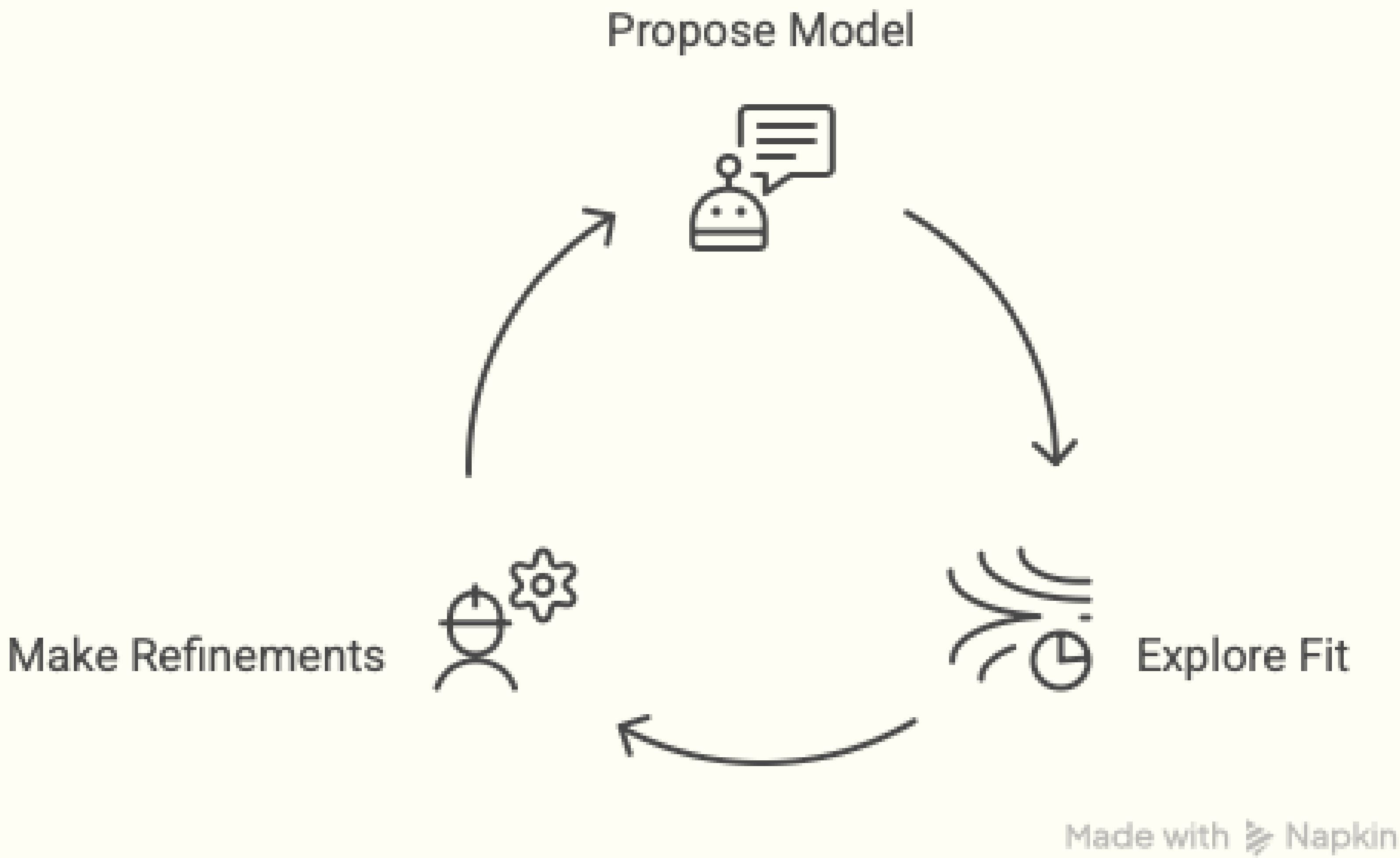
Your turn

You estimated the model

term	estimate	std.error	statistic	p.value
(Intercept)	2.190	0.176	12.439	<0.001
log(bodyweight)	0.759	0.042	18.163	<0.001

1. Interpret the slope in context
2. Interpret the intercept in context

Modeling is an iterative process



Issues with transformations

- You're often guessing — Statistics is an art AND a science!
- Changes the interpretation of the parameters — need to back-transform to provide interpretable results
- Changes SEs of the parameters
- Not always easy to keep track of all your assumptions