Multiple Linear Regression

Stat 230: Applied Regression Analysis

PDF version of slides

Multiple Linear Regression (MLR)

$$Y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_p x_{ip} + \varepsilon_i, \quad \varepsilon_i \overset{iid}{\sim} N(0, \sigma^2)$$

This model requires that…

1. $\mu\{Y | x_1, x_2, \ldots, x_p\}$ is a linear function
2. The error terms are independent and are from a single population distribution
3. The error terms follow a normal distribution with mean 0 and variance σ^2

Example

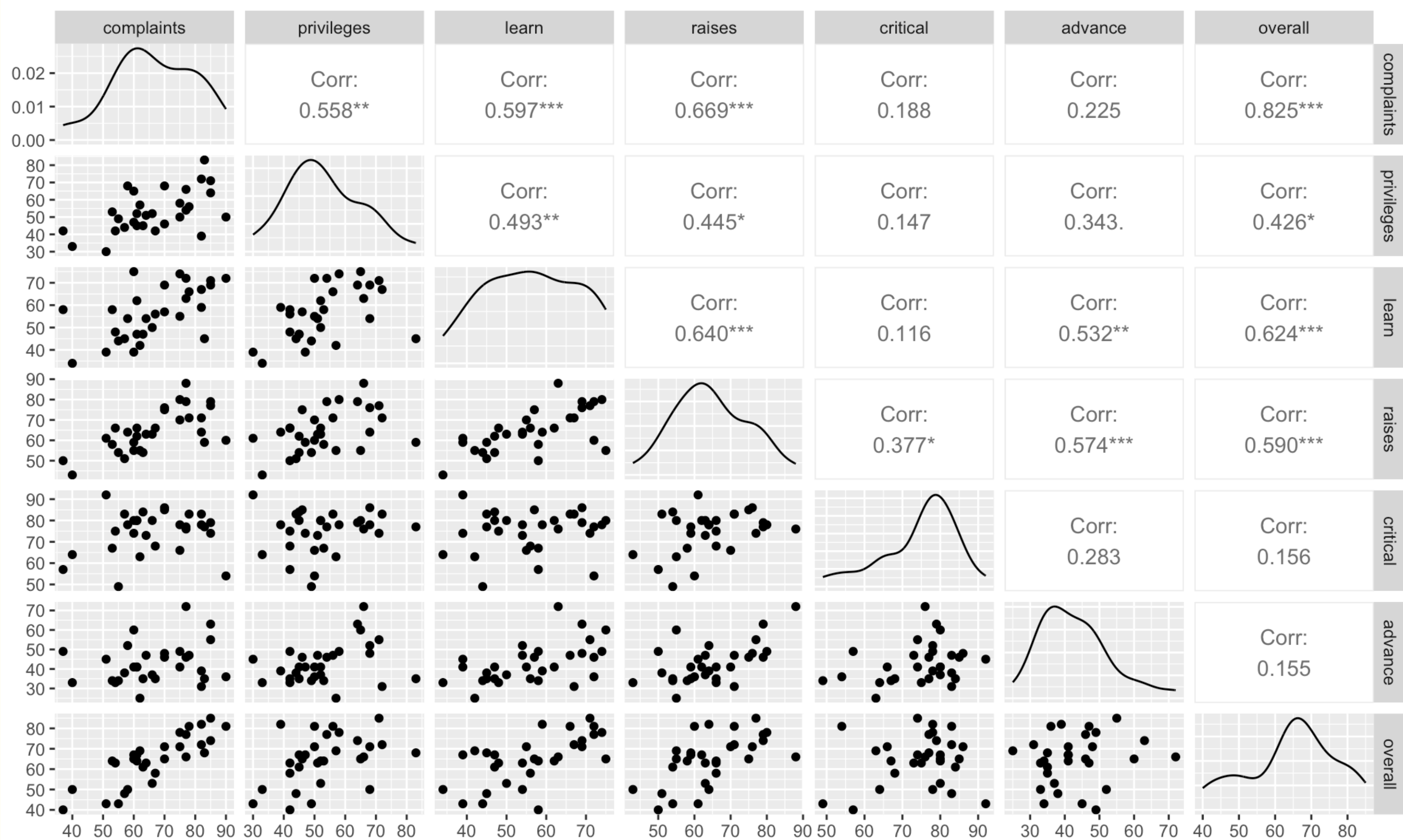
- In an effort to understand the important aspects of a satisfactory supervisor, clerical employees at a large financial organization were asked to rate their immediate supervisor.
- The survey questions were designed to measure overall performance of the supervisor, as well as six additional characteristics.
- Employees were asked to rate the following statements on a scale from 0 to 100

  (0 = "completely disagree", 100 = "completely agree")

The data

Variable	Description
`rating`	Overall rating of supervisor performance
`complaints`	Score for "Your supervisor handles employee complaints appropriately."
`privileges`	Score for "Your supervisor allows special privileges."
`learn`	Score for "Your supervisor provides opportunities to learn new things."
`raises`	Score for "Your supervisor bases raises on performance."
`critical`	Score for "Your supervisor is too critical of poor performance."
`advance`	Score for "I am not satisfied with the rate I am advancing in the company?"

EDA: Scatterplot matrix



EDA: Correlation matrix

```r
cor(supervisor)
```

```
           overall complaints privileges  learn raises critical advance
overall     1.0000     0.8254     0.4261 0.6237 0.5901   0.1564  0.1551
complaints  0.8254     1.0000     0.5583 0.5967 0.6692   0.1877  0.2246
privileges  0.4261     0.5583     1.0000 0.4933 0.4455   0.1472  0.3433
learn       0.6237     0.5967     0.4933 1.0000 0.6403   0.1160  0.5316
raises      0.5901     0.6692     0.4455 0.6403 1.0000   0.3769  0.5742
critical    0.1564     0.1877     0.1472 0.1160 0.3769   1.0000  0.2833
advance     0.1551     0.2246     0.3433 0.5316 0.5742   0.2833  1.0000
```

Fitting the model

Target:

- Fitted regression equation:
  $\widehat{Y}_i = \widehat{\beta}_0 + \widehat{\beta}_1 x_{i1} + \widehat{\beta}_2 x_{i2} + \cdots + \widehat{\beta}_p x_{ip}$
- Standard error for MLR model: $\widehat{\sigma}$

Fitting the model

Procedure:

Least squares estimation: choose the coefficients to minimize

$$\mathrm{SSE} = \sum_{i=1}^{n} \left(Y_i - \widehat{Y}_i \right)^2$$

Then use $\widehat{\sigma} = \sqrt{\frac{\mathrm{SSE}}{n-(p+1)}}$

Interpreting $\widehat{\beta}_0$

```
(Intercept)  complaints  privileges       learn      raises    critical 
     10.787       0.613      -0.073       0.320       0.082       0.038 
    advance 
     -0.217 
```

- The expected value of the response variable when all explanatory variables are 0
- The expected overall supervisor score is about 10.8 when all individual scores are 0

Interpreting $\hat{\beta}_j$

```
(Intercept)  complaints  privileges       learn      raises    critical 
     10.787       0.613      -0.073       0.320       0.082       0.038 
    advance 
     -0.217 
```

- The expected change in the response for a 1-unit increase in x_j, when all other variables are held constant.
- We expect a 0.613 point increase in a supervisor's overall score for a 1-point increase on the complaints scale, holding all other variables constant

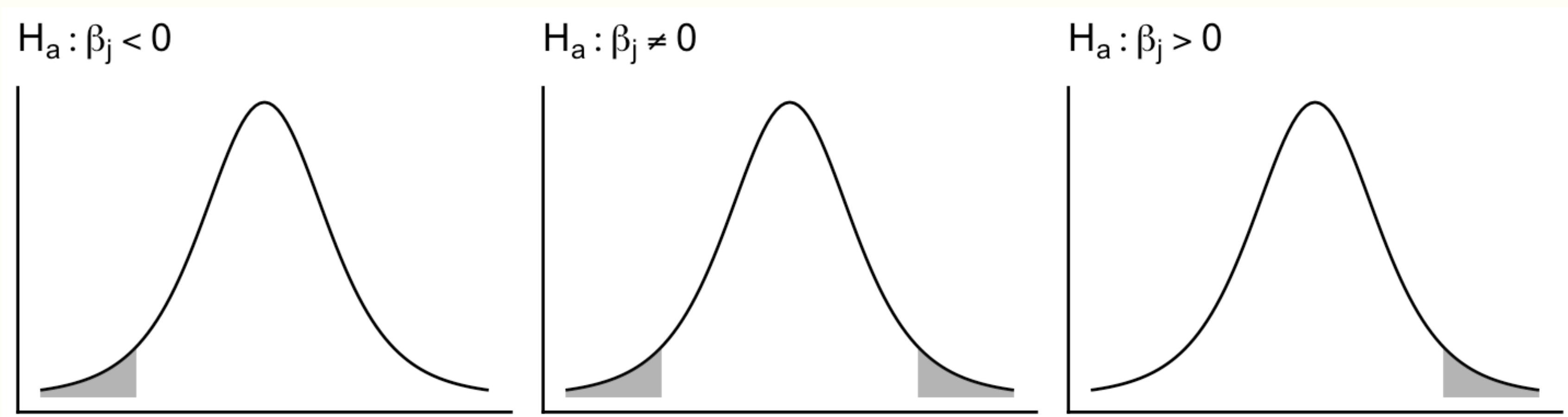
Testing a single coefficient

Hypotheses: $H_0 \; : \; \beta_j = \#$ vs. $H_a \; : \; \beta_j \; \substack{< \\ \neq \\ >} \; \#$

Test statistic: $t = \dfrac{\hat{\beta}_j - \#}{SE(\beta_j)}$

Reference distribution: t distribution with d.f. $= n - (p + 1)$

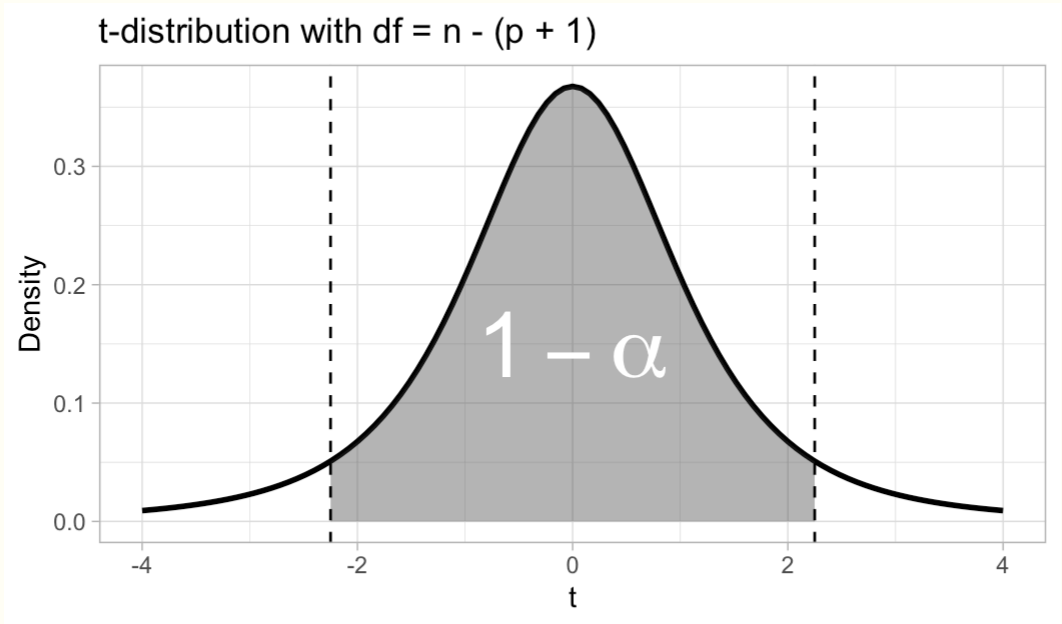
p-value: Area in the tail(s) specified by H_a



Confidence intervals

$$\hat{\beta}_j \pm t^*_{n-(p+1)} \cdot SE(\hat{\beta}_j)$$

$t^*_{n-(p+1)}$ is the $1 - \alpha/2$ quantile from the t-distribution with $d.f. = n - (p+1)$



Example

term	estimate	std.error	statistic	p.value	conf.low	conf.high
(Intercept)	10.7871	11.5893	0.9308	0.3616	-13.1871	34.7613
complaints	0.6132	0.1610	3.8090	0.0009	0.2802	0.9462
privileges	-0.0731	0.1357	-0.5382	0.5956	-0.3538	0.2077
learn	0.3203	0.1685	1.9009	0.0699	-0.0283	0.6689
raises	0.0817	0.2215	0.3690	0.7155	-0.3764	0.5399
critical	0.0384	0.1470	0.2611	0.7963	-0.2657	0.3425
advance	-0.2171	0.1782	-1.2180	0.2356	-0.5857	0.1516