Other Confidence Intervals

Stat 250

Click here for PDF version

Your turn

Let $X \sim \text{Gamma}(2, \lambda)$. It is a fact that $2\lambda X$ has a chi-square distribution with 4 degrees of freedom. Use this fact to find a 95% confidence interval for λ via the pivotal method.

One-sided confidence intervals

- So far, all of our CIs were two-sided intervals of the form: $P(L \leq \theta \leq U) = 1 - \alpha$
- When we only care about the lower or upper bound, then a one-sided interval is required

Lower confidence bound

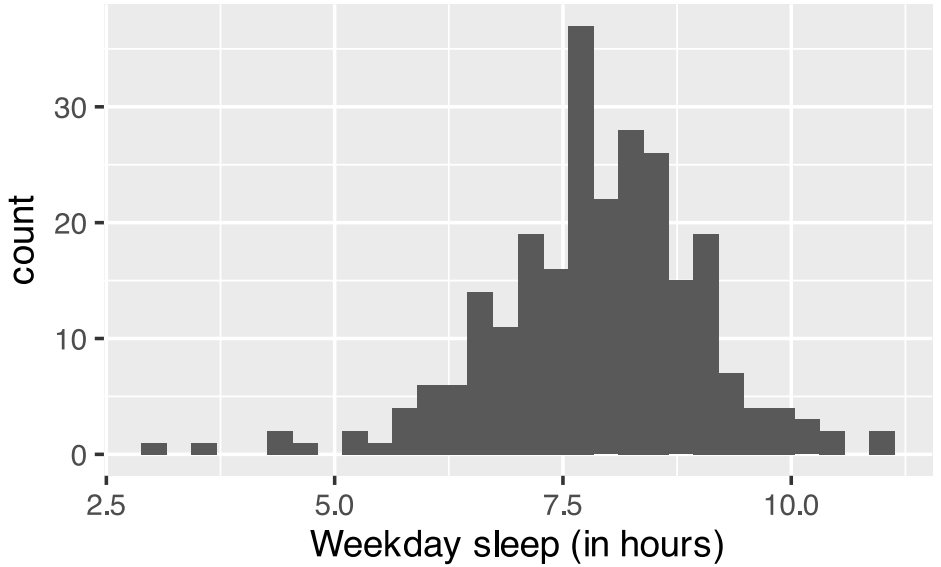
Found via $P(L \leq \theta) = 1 - \alpha$

Upper confidence bound

Found via $P(\theta \leq U) = 1 - \alpha$

Example

- Do college students sleep < 8 hours per night?
- Average hours of sleep on weekdays collected for random sample of students from one college
- Find a 95% lower confidence bound for the average hours of sleep on weekdays for students at this college



min	Q1	median	Q3	max	mean	sd	n
3.00	7.20	7.95	8.60	10.97	7.87	1.17	253

Score intervals for proportions

A Gallup poll surveyed 3,731 randomly sampled US in April 2021, asking how they felt about requiring proof of COVID-19 vaccination for travel by airplane. The poll found that 57% said they would favor it.

Let X = number of respondents in favor of proof of vaccine

Assume that $X \sim \text{Binom}(n, p)$

$\implies \hat{p} = X/n$ is an unbiased estimator of p

$\implies$ For large n, $\hat{p} \dot{\sim} N\left(p, \frac{p(1-p)}{n}\right)$

Score intervals for proportions

For large n, $Z = (\hat{p} - p)/\sqrt{\frac{p(1-p)}{n}} \dot{\sim} N(0,1)$

Using the pivotal method:

$$P\left(-q \leq \frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}} \leq q\right) = 1 - \alpha,$$

$q = 1 - \alpha/2$ quantile from $N(0,1)$

Now solve for p (you'll need to remember your quadratic formula!)

Score intervals for proportions

$$L = \frac{\hat{p} + q^2/(2n) - q\sqrt{\hat{p}(1-\hat{p})/n + q^2/(4n^2)}}{1 + q^2/n}$$

$$U = \frac{\hat{p} + q^2/(2n) + q\sqrt{\hat{p}(1-\hat{p})/n + q^2/(4n^2)}}{1 + q^2/n}$$

Example

- A Gallup poll surveyed 3,731 randomly sampled US in April 2021, asking how they felt about requiring proof of COVID-19 vaccination for travel by airplane.
- The poll found that 57% (2,127 respondents) said they would favor it.
- Construct a 90% confidence interval for p

```
prop.test(x = 2127, n = 3731, conf.level = 0.9)$conf
```

```
[1] 0.5565766 0.5834977
attr(,"conf.level")
[1] 0.9
```