More on Permutation tests

Stat 250

Click here for PDF version

Questions?

What questions do you have from last class, including the extra examples?

Implementation

Data overview

Data frame with 47 rows and two columns

```
dplyr::glimpse(motivation)
```

```
Rows: 47
Columns: 2
$ Score     <dbl> 5.0, 5.4, 6.1, 10.9, 11.8, 12.0, 12.3,
14.8, 15.0, 16.8, 17.…
$ Treatment <fct> Extrinsic, Extrinsic, Extrinsic,
Extrinsic, Extrinsic, Extri…
```

1. Calculate the observed difference in means

```r
group_means <- mosaic::mean(~Score | Treatment, data = motivation)
group_means
```

```
Extrinsic Intrinsic
 15.73913  19.88333
```

```r
observed <- group_means["Intrinsic"] - group_means["Extrinsic"]
observed
```

```
Intrinsic
 4.144203
```

2. Use a for loop to run the permutations

```r
y <- motivation$Score # vector with response variable
n <- nrow(motivation) # total sample size
ngrp1 <- 24            # sample size for group 1
N <- 10^4 - 1          # number of resamples
result <- numeric(N)   # place to store results

for(i in 1:N) {
  index <- sample(n, size = ngrp1, replace = FALSE)
  result[i] <- mean(y[index]) - mean(y[-index])
}
```

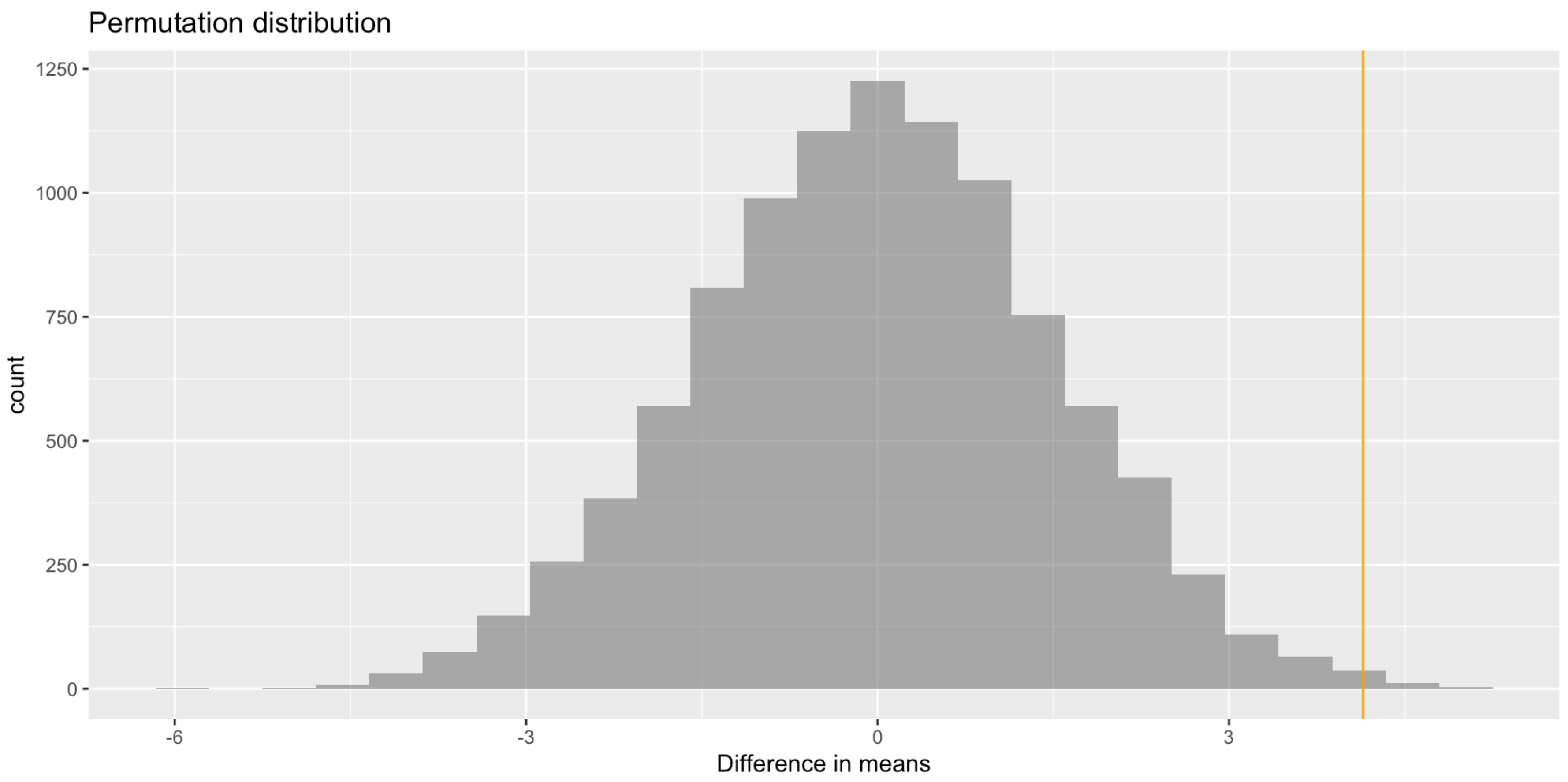
`result` is a vector

```r
head(result)
```

```
[1] -2.2331522 -3.5614130 -2.7780797 -1.6626812
0.1849638 -2.6503623
```

3. Plot the permutation (null) distribution

```
gf_histogram(~result, title = "Permutation distribution"
    xlab = "Difference in means") |>
  gf_vline(xintercept = ~observed, color = "orange")
```



4. Calculate the p-value

⚠ Be sure to **look at the alternative hypothesis** to select the inequality (tail)

```
(sum(result >= observed) + 1) / (N + 1)
```

```
[1] 0.0025
```

To get a two-sided p-value, multiple by 2

```
2 * (sum(result >= observed) + 1) / (N + 1)
```

```
[1] 0.005
```

Permutation test for difference in proportions

Example: Hearing loss in U.S. teens, 1988-1994 vs. 2005-06

```
glimpse(hearing_loss)
```

```
Rows: 4,699
Columns: 2
$ year  <chr> "2005-2006", "2005-2006", "2005-2006",
"2005-2006", "2005-2006",…
$ hloss <chr> "yes", "yes", "yes", "yes", "yes", "yes",
"yes", "yes", "yes", "…
```

1. Calculate the observed difference in means

```
group_props <- mosaic::prop(~hloss | year, data = hearing_loss, success = "yes")
group_props
```

```
prop_yes.1988-1994 prop_yes.2005-2006
         0.1639344          0.1880294
```

```
observed <- group_props["prop_yes.1988-1994"] - group_props["prop_yes.2005-2006"
observed
```

```
prop_yes.1988-1994
       -0.02409494
```

2. Use a for loop to run the permutations

```r
y <- hearing_loss$hloss      # vector with response variable
n <- nrow(hearing_loss)      # total sample size
ngrp1 <- 2928                # sample size for group 1
N <- 10^4 - 1                # number of resamples
result <- numeric(N)         # place to store results

for(i in 1:N) {
  index <- sample(n, size = ngrp1, replace = FALSE)
  result[i] <- mean(y[index] == "yes") - mean(y[-index] ==
}
```

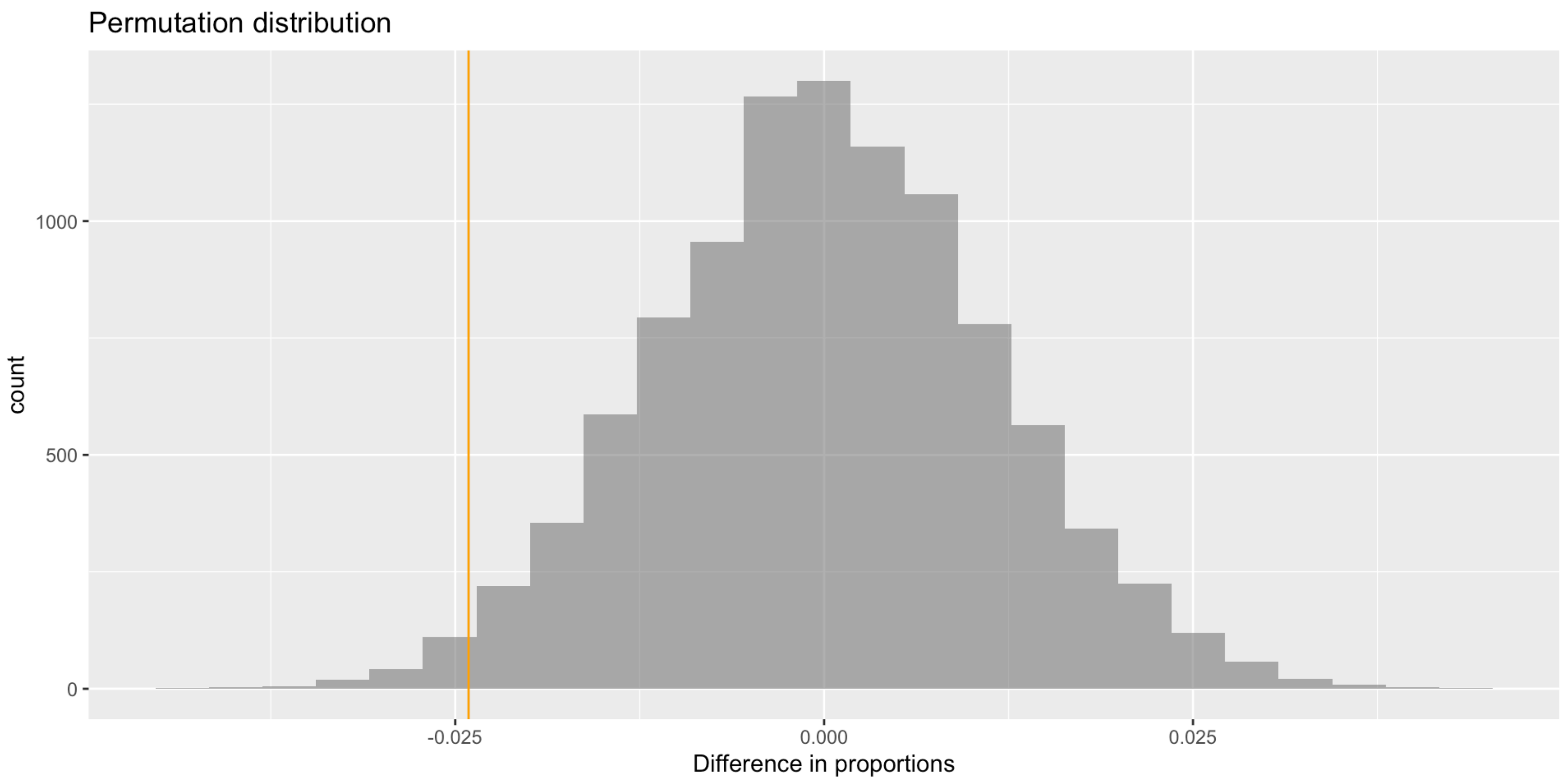
result is a vector

```r
head(result)
```

```
[1]  0.004902914 -0.013220742 -0.023188753  0.007621462
0.013964742
[6] -0.025001119
```

3. Plot the permutation (null) distribution

```
gf_histogram(~result, title = "Permutation distribution"
     xlab = "Difference in proportions") |>
  gf_vline(xintercept = ~observed, color = "orange")
```



4. Calculate the p-value

⚠ Be sure to **look at the alternative hypothesis** to select the inequality (tail)

```
(sum(result <= observed) + 1) / (N + 1)
```

```
[1] 0.0184
```

To get a two-sided p-value, multiple by 2

```
2 * (sum(result <= observed) + 1) / (N + 1)
```

```
[1] 0.0368
```

Reproducibility

Every time you run sampling code you will received a different random sample **unless** you set the seed (random number generator state).

Add `set.seed()` to the code chunk where you sample (or at the top of an .Rmd file)

```
set.seed(1234) # Choose some large integer
```

Your turn

Work through the R examples.

Matched pairs

Data are **paired** if the groups being compared are clearly *linked*

- Two measurements on each case
- Twin studies
- Each case is matched with a similar case, and one case in each pair is given each treatment
- Other situations where the data are naturally paired

Your turn

Which of the following scenarios should be analyzed as paired data?

1. Students take an MCAT prep course. Their before and after scores are compared.
2. 20 first-year and 20 second-year students in class take a midterm. We compare their scores.
3. A group of freshman are asked about the quality of food on campus. A year later, the same students are asked this question again. Do student’s opinions change over time?

Is it safe to look at social media while driving?

- Previous research on smart phone use while driving has primarily focused on phone calls and texting.
- Study looked at the effects of different smart phone tasks on car-following performance in a driving simulator.
- Drivers performed driving only baseline simulation
- Drivers performed other phone tasks: texting, reading Facebook posts, exchanging photos on Snapchat, viewing updates on Instagram
- Brake reaction times (in seconds) recorded

Your turn

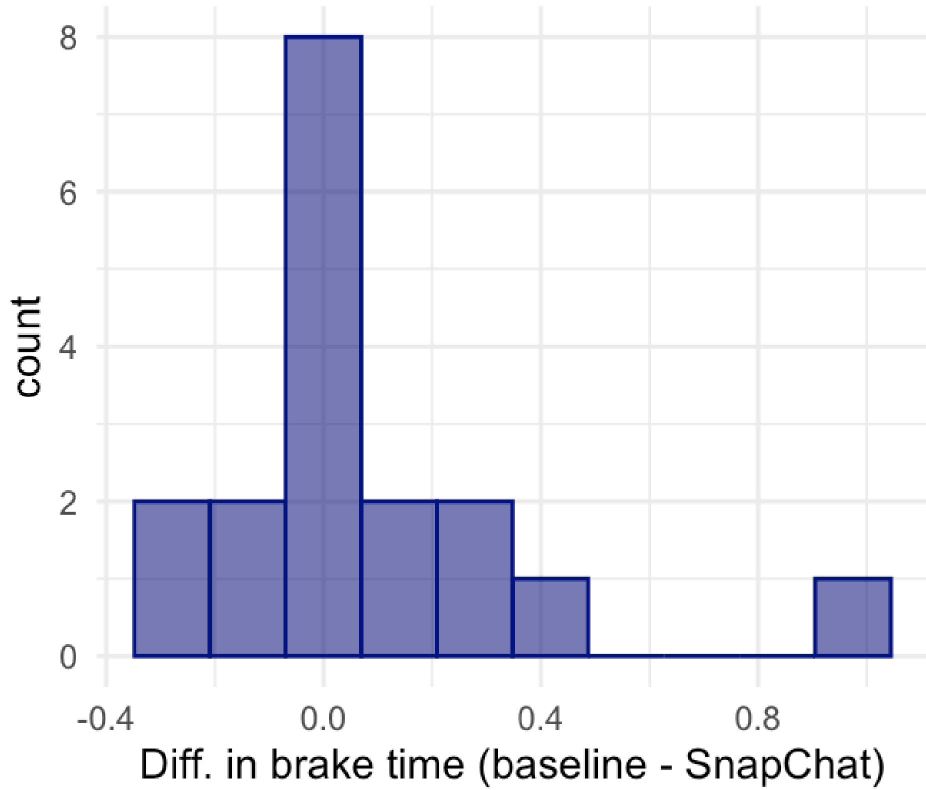
- Is it safe to look at social media while driving?
- What are logical null and alternative hypotheses for this research question?

Test statistic

Subject	Baseline	SnapChat	Diff
1	0.863	0.865	**0.002**
2	0.847	0.783	**-0.063**
3	0.836	0.808	**-0.028**
4	0.655	1.010	**0.354**
5	0.900	0.837	**-0.063**
6	0.957	1.175	**0.218**
7	0.780	0.817	**0.037**
8	0.954	0.861	**-0.094**
9	0.970	0.717	**-0.253**
10	1.102	1.141	**0.039**
11	0.925	0.583	**-0.342**
12	0.833	0.883	**0.050**
13	0.833	0.995	**0.161**
14	0.773	0.837	**0.064**
15	0.914	1.008	**0.095**
16	0.858	1.137	**0.278**
17	0.822	1.733	**0.911**
18	0.963	0.883	**-0.079**

- For a matched pairs experiment, look at the differences between responses for each unit (pair)
- Compute a new variable for differences
- We use the mean difference as our test statistic

EDA



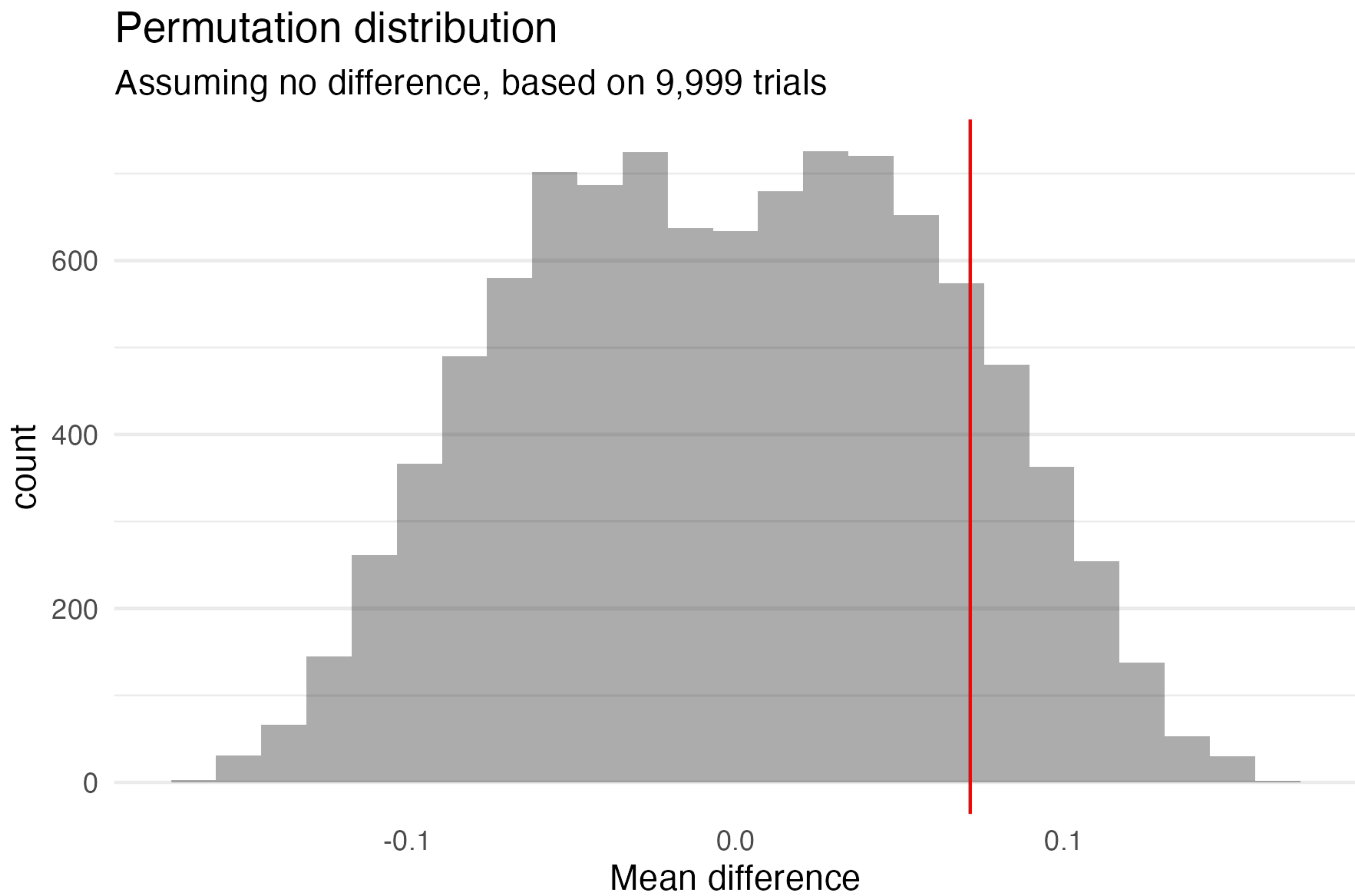
mean	sd	n
0.072	0.27	18

min	Q1	median
-0.342	-0.063	0.038

Generating the null distribution

Subject	Baseline	SnapChat	Diff
1	0.863	0.865	**0.002**
2	0.847	0.783	**-0.063**
3	0.836	0.808	**-0.028**
4	0.655	1.010	**0.354**
5	0.900	0.837	**-0.063**
6	0.957	1.175	**0.218**
7	0.780	0.817	**0.037**
8	0.954	0.861	**-0.094**
9	0.970	0.717	**-0.253**
10	1.102	1.141	**0.039**
11	0.925	0.583	**-0.342**
12	0.833	0.883	**0.050**
13	0.833	0.995	**0.161**
14	0.773	0.837	**0.064**
15	0.914	1.008	**0.095**
16	0.858	1.137	**0.278**
17	0.822	1.733	**0.911**
18	0.963	0.883	**-0.079**

- Can’t permute one of the columns and recalculate the difference
- Instead, **randomly select a sign (+/-) for each difference**
- Use the mean difference as the test statistic


Permutation distribution
Assuming no difference, based on 9,999 trials
count
600
400
200
0
-0.1
0.0
0.1
Mean difference

Strength of evidence

- 1481 simulated test statistics exceed the observed
- 9999 total statistics are in the null distribution
- What's the *p*-value?
- Is there a statistical discernible difference in braking time?

Implementation

Data frame with 18 rows and 6 columns

```
glimpse(brake)
```

```
Rows: 18
Columns: 6
$ Subject   <dbl> 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13,
14, 15, 16, 17, 18
$ Baseline  <dbl> 0.863358, 0.846684, 0.836120, 0.655340,
0.900022, 0.956534, …
$ Facebook  <dbl> 1.254010, 1.100017, 1.020859, 0.864210,
0.793354, 1.252108, …
$ Texting   <dbl> 1.011476, 0.900027, 1.063912, 0.973921,
0.856002, 1.178000, …
$ Instagram <dbl> 0.962927, 0.600018, 0.946600, 0.725775,
0.816683, 1.134345, …
$ SnapChat  <dbl> 0.865000, 0.783353, 0.808357, 1.009547,
0.836783, 1.175025, …
```

Add a `Diff` column

```
brake <- mutate(brake, Diff = Baseline - SnapChat)
```

Implementation

```r
Diff <- brake$Diff             # vector of differences
observed <- mean(Diff)         # observed test stat
n <- length(Diff)              # sample size
N <- 10^4 - 1                  # no. of permutation resamples

set.seed(120)                  # set seed for reproducibility
result <- numeric(N)           # place to store the results

for (i in 1:N){
   swap <- sample(c(-1,1), n, replace = TRUE)
   result[i] <- mean(swap * Diff)
}
```