Tuning beta prior distributions

Stat 340: Bayesian Statistics

Informative prior distributions

1. Data augmentation

2. Using domain expertise

3. Sensitivity to priors

(Problem topics 1 & 5)

Your turn 1

Work through the Bechdel Test prior elicitation example.

Discuss ideas with your neighbors

Explore the use of a few functions in the `{bayesrule}` R package

- `plot_beta`
- `plot_beta_binomial`
- `summarize_beta_binomial`

Run `install.packages("bayesrules")` first if working on your own computer.

10:00

Informative priors

1. Data augmentation priors

Assumption — we have n independent and identically distributed (iid) success/failure trials

Likelihood — $Y|p \sim \mathrm{Binomial}(n, p)$

Prior — $p \in [0, 1]$, so Beta(a, b) is a good choice

Posterior — $p|Y \sim \mathrm{Beta}(a + Y, b + n - Y)$

$a =$ prior successes; $b =$ prior failures

2. Domain expertise

Choose the functional form of your prior (e.g., it's Beta)

Interview a domain expert to determine key characteristics of the distribution, then solve the system of equations

EXAMPLE

Through prior elicitation you determine the following information

- The researchers believe the probability of success, p, is equally likely to be above or below 2/3
- The researchers believe there is only a 5% chance that $p > 0.9$

Specifying quantiles

- The researchers believe the probability of success, p, is equally likely to be above or below 2/3

- The researchers believe there is only a 5% chance that $p > 0.9$

```
library(ProbBayes)
beta.select(
  # x = value of quantile, p = what quantile
  list(x = 2/3, p = 0.5),
  list(x = 0.9, p = 0.95)
)
```

```
## [1] 4.63 2.48
```

Specifying other characteristics

Set up a system of equations to solve with any two characteristics (not just quantiles)

Mean $$E(X) = \frac{a}{a+b}$$

Mode $$\text{mode}(X) = \frac{a-1}{(a-1)+(b-1)}$$ if $a, b > 1$

Variance $$\text{Var}(X) = \frac{ab}{(a+b+1)(a+b)^2}$$

Another parameter solver

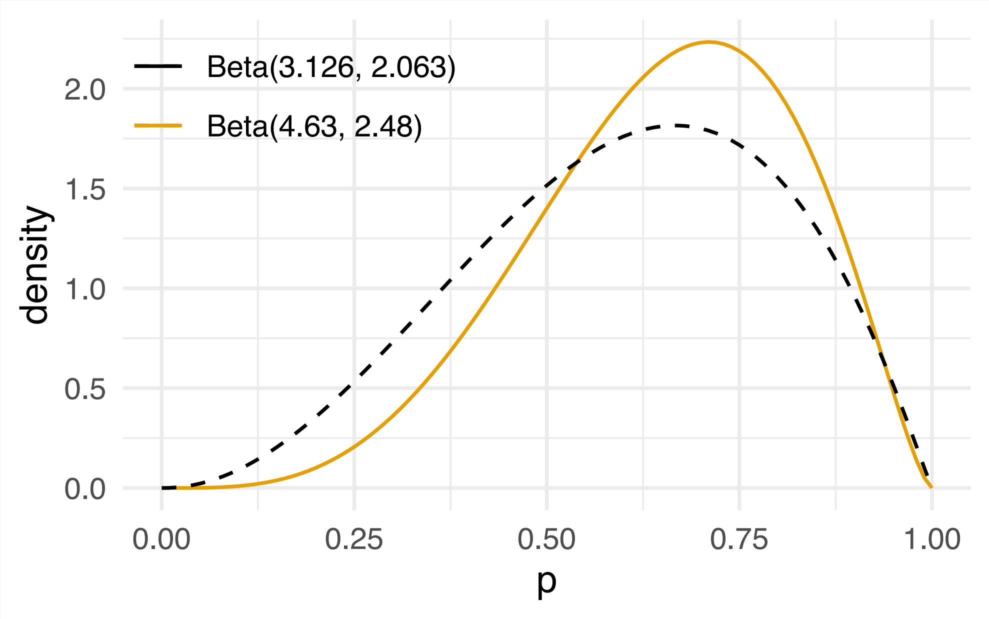
- The researchers believe the probability of success, p, is **most likely** to be 2/3
- The researchers believe there is only a 5% chance that $p > 0.9$

```
devtools::source_gist("https://gist.github.com/aloy/e1fcefb4e04c349777f030762dfdb301")
```

```
beta_solver(guess = c(2, 2), .mode = 2/3, .quantile = c(value = 0.9, p = 0.95))
```

```
## $par
## [1] 3.126138 2.063069
##
## $residual
## [1] 2.938904e-08
##
## $fn.reduction
```

Always plot your priors!



Compare what the plot shows to your prior belief

Reflect on what the expert said

- Is p most likely 2/3 or is it the median?
- The researchers believe there is only a 5% chance that $p > 0.9$

Your turn 2

Set up the equations you would use to tune a Beta(a, b) model that accurately reflects the given prior information. If you have time, use R to find the parameters.

Often, there's no single "right" answer, but rather multiple "reasonable" answers.

1. A scientist has created a new test for a rare disease. They expect that the test is accurate 80% of the time with a variance of 0.05.

2. Your friend tells you "I think that I have a 80% chance of getting a full night of sleep tonight, and I am pretty certain." When pressed further, they put their chances between 70% and 90%.

05 : 00

☢ Be aware of prior sample sizes ☢

Posterior: $p|Y \sim \text{Beta}(a + Y, b + n - Y)$

Posterior mean: $\hat{p} = w\dfrac{Y}{n} + (1 - w)\dfrac{a}{a + b}$, where $w = \dfrac{n}{n + a + b}$

What happens to the posterior mean if n is much larger than $a + b$?

What happens to the posterior mean if $a + b$ is much larger than n?

What if the experts disagree?

Expert 1:

- $E(p) = 0.6$
- $\mathrm{Var}(p) = .05$

$\Rightarrow p \sim \mathrm{Beta}(a = 2.28, b = 1.52)$

Expert 2

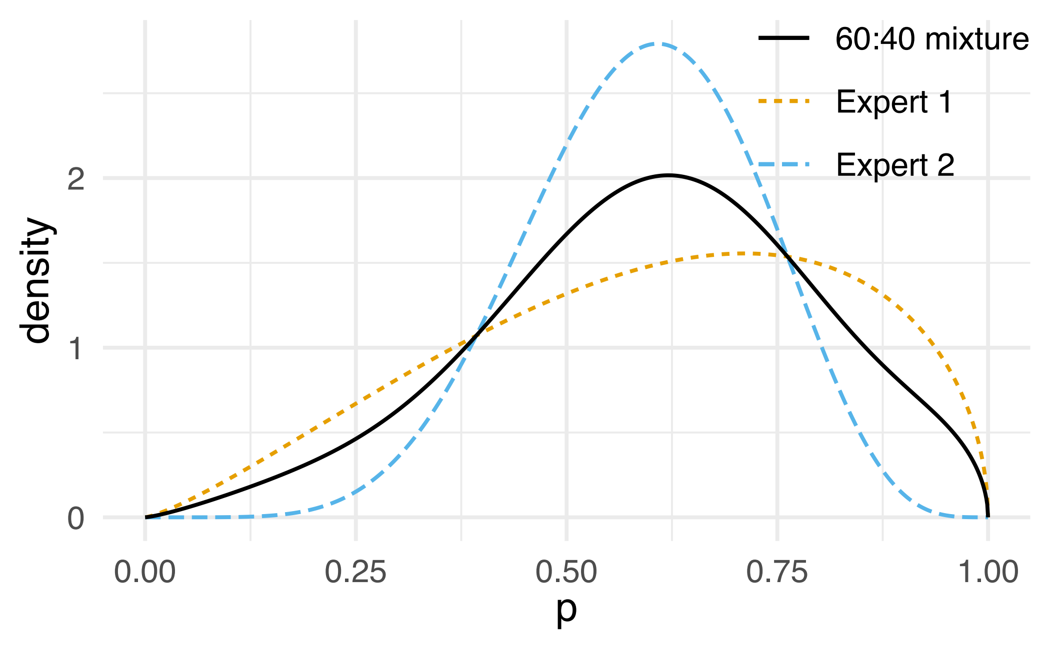
- $\mathrm{mode}(p) = 0.8$
- 0.9 quantile of p is 0.8

$\Rightarrow p \sim \mathrm{Beta}(a = 7.2, b = 5)$

- Suppose we give expert 1 weight $w_1 = 0.6$ and expert 2 weight $w_2 = 0.4$
- We can create a **mixture prior**

Mixture prior

```r
p_grid <- seq(0, 1, by = 0.001)
mix_prior <-
  0.6 * dbeta(p_grid, 2.28, 1.52) + 0.4 * dbeta(p_grid, 7.2, 5)
```

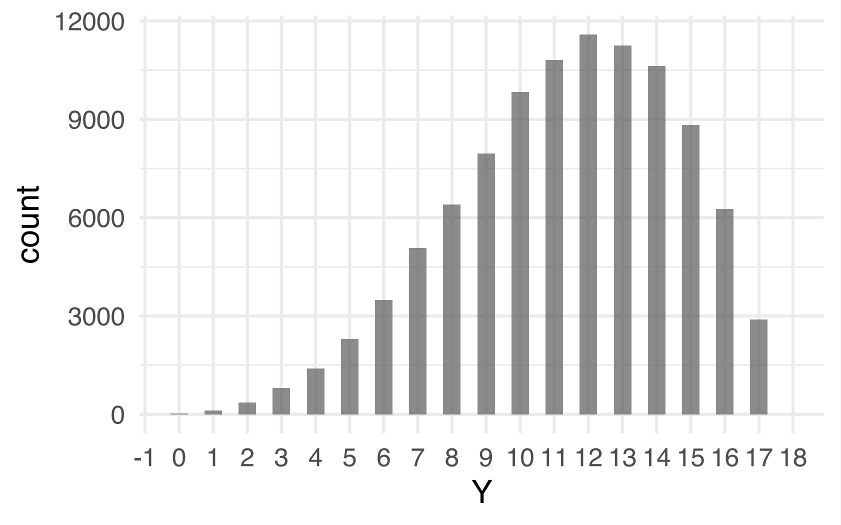


Sanity check: Prior predictive distributions

Simulate data implied by your prior specification to see if it seems reasonable

Example: Blindsight continued

```r
prior_p_sims <- rbeta(1e5, shape1 = 5.268, shape2 = 2.634)  # Draw 1e5 ps from prior
prior_ys     <- rbinom(1e5, size = 17, prob = prior_p_sims) # Draw one y for each p
```



Sensitivity analysis

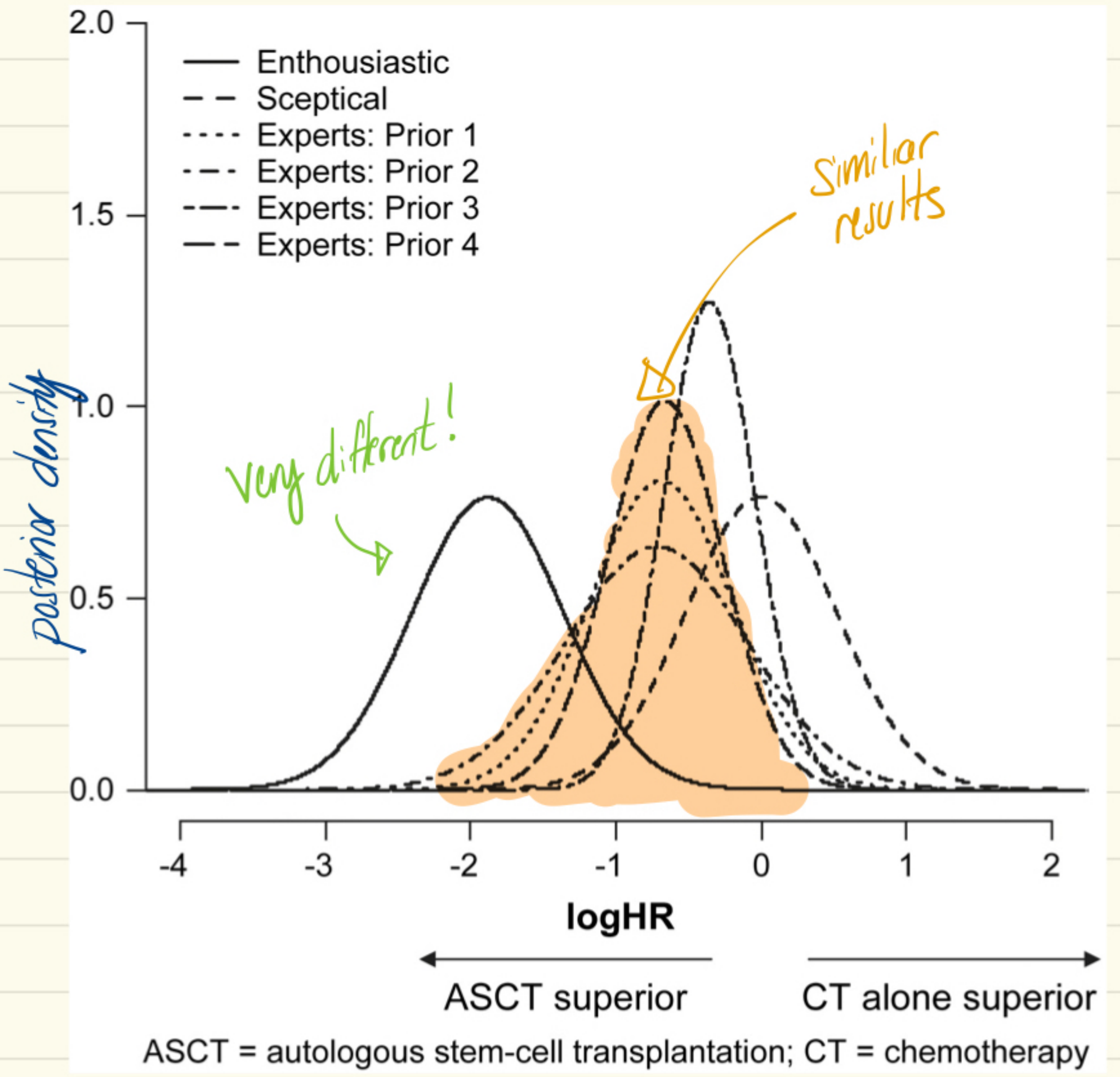
Sensitivity analysis

Any time informative priors are used you should conduct a sensitivity analysis

- Compare the posterior for several priors
- Describe how the posterior changes
- Reflect on the impact

It's about transparency

- many justifiable analyses are tried, and all of them are described.



Example

- Compared 6 different prior distributions
- Posterior distributions similar for 3-4 priors
- Posterior strikingly different for 1 prior

Hiance A, Chevret S, Lévy V. A practical approach for eliciting expert prior beliefs about cancer survival in phase III randomized trial. *J Clin Epidemiol.* 2009 Apr;62(4):431-437.e2.