Posterior derivations

Stat 340, Fall 2021

Your turn 1

Let $Y_1, Y_2, \ldots, Y_n$ be a random sample (i.i.d.) from the Normal distribution with PDF

$$f(y) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y-\mu)^2}{2\sigma^2}\right), \quad -\infty < y < \infty, \quad -\infty < \mu < \infty, \quad \sigma > 0$$

Derive the likelihood, $f(y_1, \ldots, y_n|\mu, \sigma)$.

Your turn 2

To simplify the derivation and make this a univariate problem, assume that $\sigma = 8$. If we use specify $\mathcal{N}(\mu_0, \sigma_0)$ (where μ_0 and σ_0 are known constants) as our prior for μ, then what is the posterior of μ?

Example: Poisson-Gamma model

Let $X_1, X_2, \ldots, X_n$ be a random sample from the Poisson distribution with PMF

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!},\ x = 0, 1, 2, \ldots$$

1. Write down the likelihood function, $f(x_1, \ldots, x_n|\lambda)$.

2. Suppose that you decide to use a Gamma(a,b) prior distribution for λ with PDF

   $$\pi(\lambda) = \frac{b^a}{\Gamma(a)}\lambda^{a-1}e^{-b\lambda},\ \lambda > 0.$$

   Find the posterior density of λ.

3. Is the gamma prior a conjugate family to the Poisson likelihood?