Learning about a Binomial Probability

Stat 340: Bayesian Statistics

1. Comparing paradigms

2. Discrete prior

3. Posterior analysis

(Problem topics 1-3)

A Bayesian "personality quiz"

Adapted from *Bayes Rules! An Introduction to Bayesian Modeling with R* by Johnson, Ott & Dogucu

Question 1

When flipping a fair coin, we say that "the probability of flipping Heads is 0.5."
How do you interpret this probability?

1. If I flip this coin over and over, roughly 50% will be Heads.

2. Heads and Tails are equally plausible.

3. Both a and b make sense.

Question 2

An election is coming up and a pollster claims that "candidate A has a 0.9 probability of winning." How do you interpret this probability?

1. If we observe the election over and over, candidate A will win roughly 90% of the time.

2. Candidate A is much more likely to win than to lose.

3. The pollster's calculation is wrong. Candidate A will either win or lose, thus their probability of winning can only be 0 or 1.

Question 3

Consider two claims.

- Zuofu claims that he can predict the outcome of a coin flip. To test his claim, you flip a fair coin 10 times and he correctly predicts all 10.
- Kavya claims that she can distinguish natural and artificial sweeteners. To test her claim, you give her 10 sweetener samples and she correctly identifies each.

In light of these experiments, what do you conclude?

1. You're more confident in Kavya's claim than Zuofu's claim.

2. The evidence supporting Zuofu's claim is just as strong as the evidence supporting Kavya's claim.

Question 4

Suppose that during a recent doctor's visit, you tested positive for a very rare disease. If you only get to ask the doctor one question, which would it be?

1. What's the chance that I actually have the disease?
2. If in fact I don't have the disease, what's the chance that I would've gotten this positive test result?

Tally your points

Question 1:

- 1 = 1 points
- 2 = 3 points
- 3 = 2 points

Question 2:

- 1 = 1 points
- 2 = 3 points
- 3 = 1 points

Question 3:

- 1 = 3 points
- 2 = 1 points

Question 4:

- 1 = 3 points
- 2 = 1 points

What does your score mean?

- 4-5 $\rightarrow$ you're more of a frequentist thinker
- 6-8 $\rightarrow$ you see the merit in both (a pragmatist?)
- 9-12 $\rightarrow$ you're more of a Bayesian thinker

Question 1: Interpreting probability

When flipping a fair coin, we say that "the probability of flipping Heads is 0.5." How do you interpret this probability?

1. (Frequentist) If I flip this coin over and over, roughly 50% will be Heads.
2. (Bayesian) Heads and Tails are equally plausible.
3. Both a and b make sense.

Question 2: Interpreting probability

An election is coming up and a pollster claims that "candidate A has a 0.9 probability of winning." How do you interpret this probability?

1. (Frequentist) If we observe the election over and over, candidate A will win roughly 90% of the time.

2. (Bayesian) Candidate A is much more likely to win than to lose.

3. (Rabid frequentist) The pollster's calculation is wrong. Candidate A will either win or lose, thus their probability of winning can only be 0 or 1.

Question 3: Balancing prior info and observed data

Consider two claims.

- Zuofu claims that he can predict the outcome of a coin flip. To test his claim, you flip a fair coin 10 times and he correctly predicts all 10.
- Kavya claims that she can distinguish natural and artificial sweeteners. To test her claim, you give her 10 sweetener samples and she correctly identifies each.

In light of these experiments, what do you conclude?

1. (Bayesian) You're more confident in Kavya's claim than Zuofu's claim.

2. (Frequentist) The evidence supporting Zuofu's claim is just as strong as the evidence supporting Kavya's claim.

Question 4: Asking questions

Suppose that during a recent doctor's visit, you tested positive for a very rare disease. If you only get to ask the doctor one question, which would it be?

1. (Bayesian) What's the chance that I actually have the disease?

2. (Frequentist) If in fact I don't have the disease, what's the chance that I would've gotten this positive test result?

frequentist procedure

quantifies uncertainty in terms of repeating the process that generated the data many times

Would a frequentist ever claim that...

- $\mathrm{P}(Y > 14) = 0.75$?
- $\mathrm{P}(p > 0.5) = 0.75$?
- $p \sim \mathrm{Unif}(0.25, 0.5)$?
- the probability that the true proportion of correct guesses is in the interval (0.64, 1) is 0.95?
- the probability that the null hypothesis, $H_0 : \; p = 0.5$, is true is 0.0002?

Bayesian procedure

quantifies uncertainty about the parameters that remain after accounting for prior knowledge and the information in the observed data

Would a Bayesian ever claim that...

- $\mathrm{P}(Y > 14) = 0.75$?
- $\mathrm{P}(p > 0.5) = 0.75$?
- $p \sim \mathrm{Unif}(0.25, 0.5)$?
- the probability that the true proportion of correct guesses is in the interval (0.64, 1) is 0.95?
- the probability that the null hypothesis, $H_0 : \; p = 0.5$, is true is 0.0002?

Updating a discrete prior

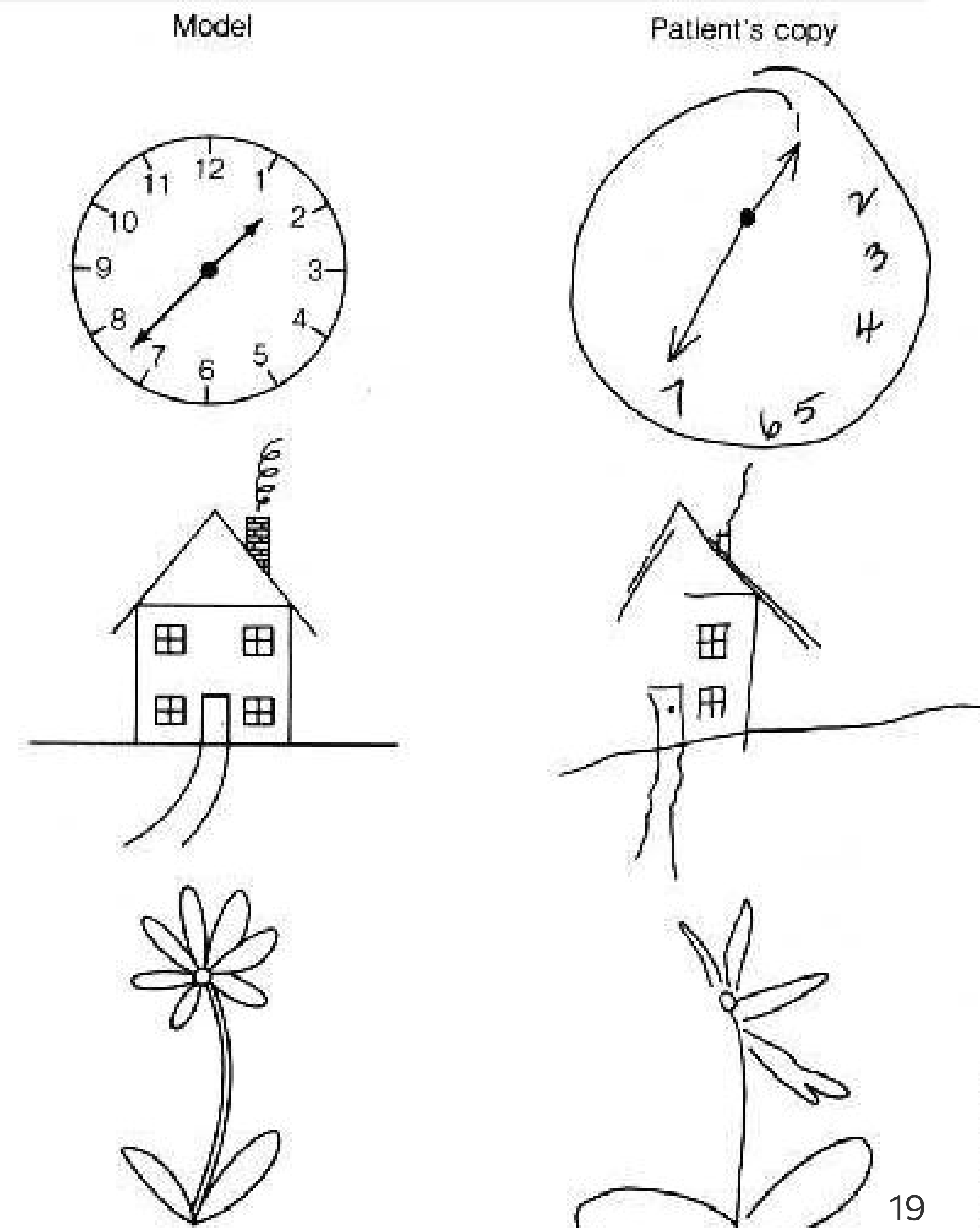
Neuroscience example

HEMISPATIAL NEGLECT

Reduced capacity to process visual info from one side of their visual space

BLINDSIGHT HYPOTHESIS

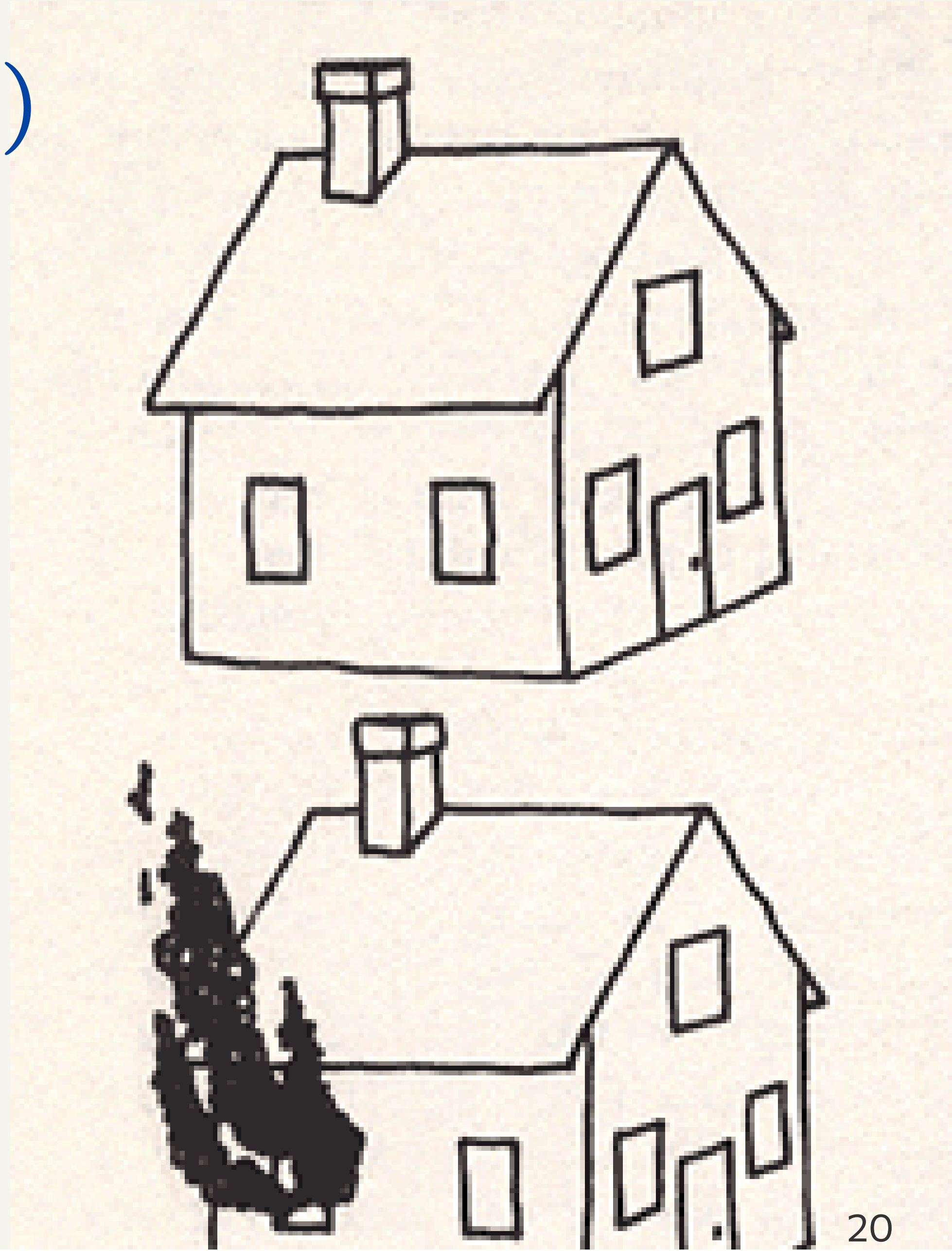
People with hemispatial neglect may be aware of stimuli that cannot be consciously recollected or identified



Marshall & Halligan (1988)

Simplified version of the study:

- Patient (P.S.) presented with two cards in random order
- Asked to identify which house she would rather live in
- $Y = \#$ times P.S. chose non-burning house
- $p =$ probability of choosing the non-burning house



Design

Data: N N N N **B B** N N N **B** N N N N N N N (14 Ns; **3 Bs**)

Data model (likelihood):

Some true proportion of guesses, p

Toss a coin with probability of heads, p

Prior belief about p:

Uniform over {0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9}

Condition

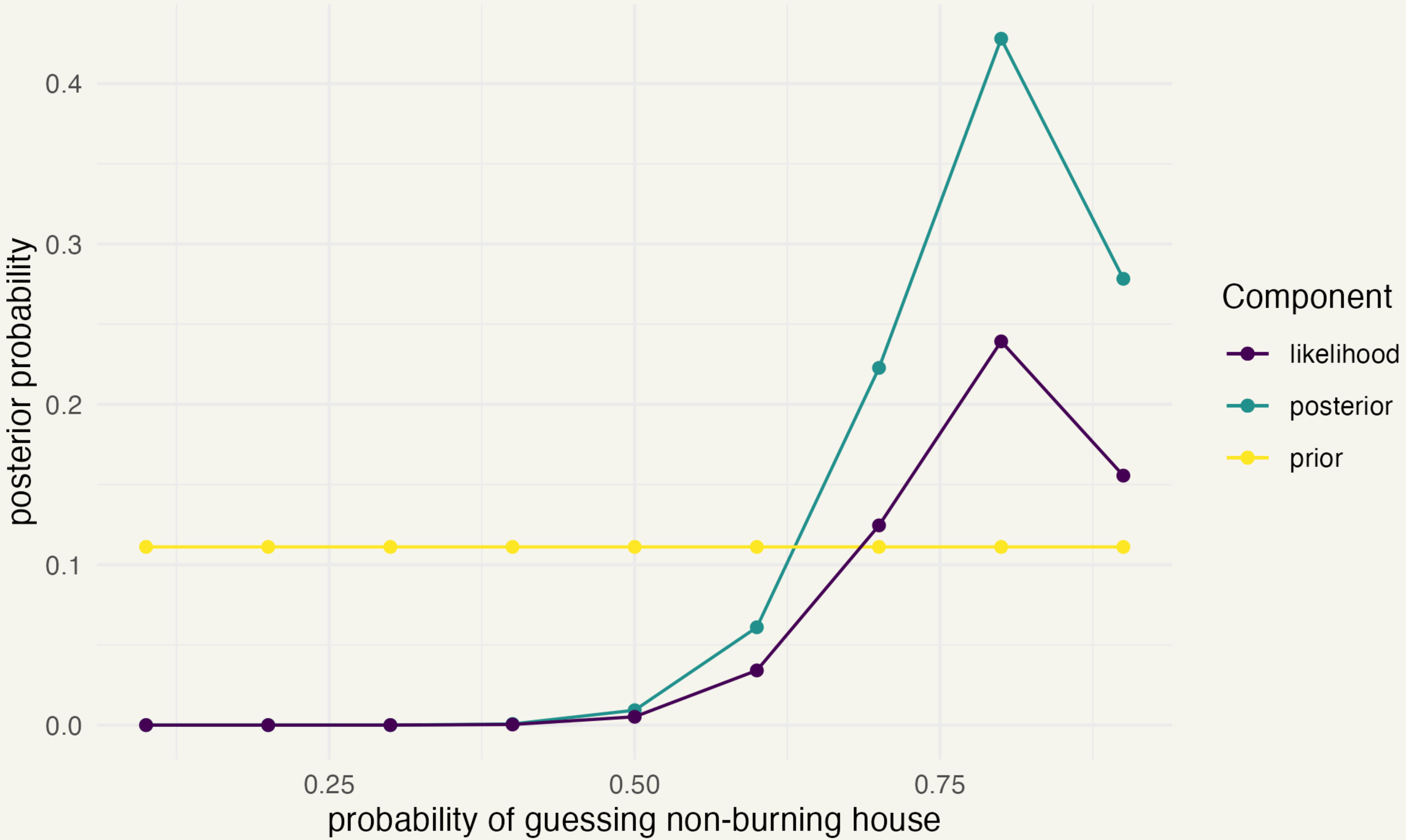
POSTERIOR DISTRIBUTION

The distribution of p that incorporates both the prior information and the data.

the **posterior** is the **prior** *conditioned* on **evidence**

$$\pi(p|y) = \mathrm{P}(p = k|y)$$

p	prior probability	likelihood	posterior plausibility (prior x likelihood)	posterior probability
0.1	0.1111111	0.0000000	0.0000000	0.0000000
0.2	0.1111111	0.0000001	0.0000000	0.0000001
0.3	0.1111111	0.0000112	0.0000012	0.0000200
0.4	0.1111111	0.0003943	0.0000438	0.0007053
0.5	0.1111111	0.0051880	0.0005764	0.0092803
0.6	0.1111111	0.0341041	0.0037893	0.0610052
0.7	0.1111111	0.1245218	0.0138358	0.2227440
0.8	0.1111111	0.2392537	0.0265837	0.4279761
0.9	0.1111111	0.1555622	0.0172847	0.2782690


0.4
0.3
0.2
0.1
0.0
posterior probability
0.25
0.50
0.75
probability of guessing non-burning house
Component
likelihood
posterior
prior

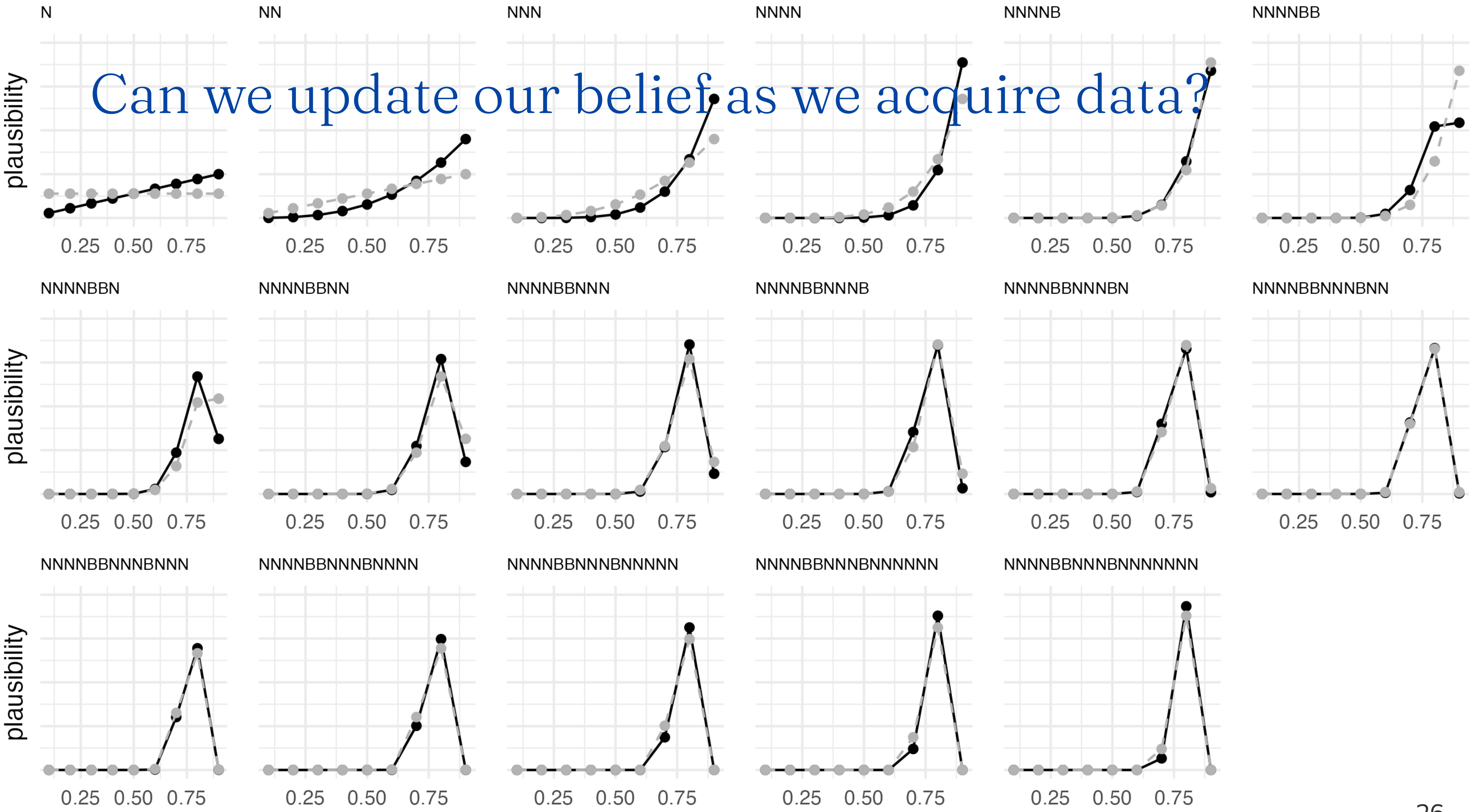
Inference

$P(p = 0.5|Y = 14)$?

$P(p \geq 0.5|Y = 14)$?

p	Prior	Likelihood	Product	Posterior
0.1	0.1111111	0.0000000	0.0000000	0.0000000
0.2	0.1111111	0.0000001	0.0000000	0.0000001
0.3	0.1111111	0.0000112	0.0000012	0.0000200
0.4	0.1111111	0.0003943	0.0000438	0.0007053
0.5	0.1111111	0.0051880	0.0005764	0.0092803
0.6	0.1111111	0.0341041	0.0037893	0.0610052
0.7	0.1111111	0.1245218	0.0138358	0.2227440
0.8	0.1111111	0.2392537	0.0265837	0.4279761
0.9	0.1111111	0.1555622	0.0172847	0.2782690

Can we update our belief as we acquire data?



☢ Bayesian inference must be supervised

- Did the model malfunction?
- Does the model's answer make sense?
- Does the question make sense?
- Check sensitivity of the answer to changes in the assumptions