Modeling count data

Stat 340, Fall 2021

Your turn 1: Poisson-Gamma model

Let $X_1, X_2, \ldots, X_n$ be a random sample from the Poisson distribution with PMF

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \ x = 0, 1, 2, \ldots$$

1. Write down the likelihood function, $f(x_1, \ldots, x_n | \lambda)$.

2. Suppose that you decide to use a Gamma(a,b) prior distribution for λ with PDF

$$\pi(\lambda) = \frac{b^a}{\Gamma(a)} \lambda^{a-1} e^{-b\lambda}, \ \lambda > 0.$$

   Find the posterior density of λ.

3. Is the gamma prior a conjugate family to the Poisson likelihood?

Your turn 2: Tuning a gamma prior

After thinking about how many scam phone calls I receive per day (before collecting data), my best guess is that this rate...

- is expected to be around 5 calls per day, and
- could reasonably range from 2 to 7 calls per day.

Translate my prior belief into a gamma prior distribution for λ. (You can use the `bayesrules::plot_gamma()` function to quickly plot a proposed distribution to help check you prior.)

Your turn 3: Inference for count data

Over the first two weeks of the term, I recorded the number of scam calls I received and stored them in a vector:

```
calls <- c(2, 1, 5, 5, 1, 3, 3, 1, 3, 3, 0, 6, 1, 1)
```

Give statistically sound estimates using the posterior distribution we discussed (though more work may be required in some cases).

1. How many scam calls should I expect on average in a typical day?

2. How many scam calls should I expect tomorrow?

3. How many scam calls should I expect in the next week (7 days)?