Working with continuour prior distributions

Stat 340, Fall 2021

Your turn 1: Undertanding kernels

The kernel of the Beta distribution is $x^{a-1}(1-x)^{b-1}$. To help you develop your understanding of the kernel and how it relates to the PDF, we will compare plots of the beta kernel to the full PDF.

1. Pick a value for the `a` and `b` parameters.

2. Using these parameters, run the below code chunk. (This code chunk is posted on teh course webpage, so just copy and paste it to save time!)

```
a <- # insert your a value
b <- # insert your b value
x <- seq(0, 1, by = .001)
kernel <- x^{a - 1} * (1 - x)^{b - 1}
pdf <- gamma(a + b)/(gamma(a) * gamma(b)) * kernel
par(mfrow=c(1,2))
plot(x, pdf, type="l", main = "PDF")
plot(x, kernel, type = "l", main = "Kernel")
```

3. What are the similarities between the pdf and the kernel? What are the differences?

4. If we know the kernel for a distribution, why can we find the PDF?

Your turn 2: Geometric-Beta model

Let $X_1, X_2, \ldots, X_n$ be a random sample from the geometric distribution with PMF

$$f(x|p) = p(1-p)^{x-1};\ x = 1, 2, \ldots, \quad 0 < p < 1$$

1. Write down the likelihood function, $f(x_1, \ldots, x_n|\lambda)$. (Remember joint distributions!)

2. Suppose that you decide to use a Beta(a,b) prior distribution for p with PDF

$$\pi(p) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} p^{a-1}(1-p)^{b-1}$$

   Find the posterior density of p.

3. Is the beta prior a conjugate family to the geometric likelihood?