Programming Project of INF 421:

Two Centers in a Tree

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1 Problem Description

Let $T = (V, E)$ be a tree where V is the set of vertices and E is the set of edges. For every vertex $v \in V$, it has a weight $w(v)$, which is a positive integer. Denote $d(u, v)$ as the distance between u and v in the tree, i.e., the number of edges on the unique path connecting u and v. If $u = v$, then $d(u, v) = 0$.

Your task is to find two vertices x and y in the tree, such that the following expression $S(x, y)$ is minimized:

$$S(x, y) = \sum_{v \in V} \big(w(v) \cdot \min\{d(v, x), d(v, y)\}\big).$$

Such vertices x and y are the *two centers in the tree*.

Notice that the naive algorithm for finding the two centers in the tree would be very slow when the tree is huge. What is the fastest algorithm that you can find? And what is the complexity of your algorithm?

You are asked to implement your algorithm in either C++ or Java.

2 Tests

There are 10 independent tests. The input files `centers.1.in`,...,`centers.10.in` can be downloaded by clicking here.

Input Format

Your program reads from an input file `centers.k.in` (where $1 \leq k \leq 10$), which contains the following:

- Line 1: an integer N (where $1 < N \leq 50000$), which is the number of vertices in the tree. The vertices are labeled from 1 to N.
- Lines $2, \ldots, N$: each line consists of two integers u and v (where $1 \leq u, v \leq N$), representing an edge between the vertex u and the vertex v.
- Lines $N + 1, \ldots, 2N$: each line consists of a positive integer, such that the integer in the $(N + i)$-th line corresponds to the weight $w(i)$ of the vertex i.

Output Format

Your program writes into an output file `centers.k.out` (where $1 \leq k \leq 10$), which contains only one integer, the minimum value of $S(x, y)$ achievable for all pairs of vertices x and y. We guarantee that the solution value is at most 10^9.

3 An Example

Sample Input

```
5
1 2
1 3
3 4
3 5
5
7
6
5
4
```

Sample Output

```
14
```

Explanations

The above value is achieved when the x and y are the vertices 2 and 3, respectively.

4 Your Report

Please describe in your report your algorithm in details, together with the analysis on the complexity of your algorithm.

You should also include in your report the output of your program as well as its running time on each of the 10 tests.