Repositioning in Bike-sharing Systems

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Abstract

Bike-sharing systems (BSSs) are quickly gaining popularity worldwide, as they help to reduce traffic congestion and vehicle CO_2 emissions. Throughout the day, BSSs stations are often unbalanced, which leads to demand unsatisfaction. As a remedy, the operators employ trucks to rebalance bikes among stations. However, due to the uncertain rental and return demand, as well as the complexity of the dynamic planning problem, manual planning tends to be sub-optimal. To provide decision support, mixed integer linear programming (MILP) models are proposed to obtain the repositioning strategies under certain modeling assumptions ranging from the planning objective to the actual decisions and practical constraints.

1. Basic MILP Model

Bike-sharing rebalancing problems can be divided into static bicycle repositioning problems (SBRP) and dynamic bicycle repositioning problems (DBRP) (Raviv et al. 2013). SBRP typically rebalance stations overnight, while the operations during the day are not considered. In contrast, DBRP focus on intraday rebalancing, where customer trips carried out during the day heavily affect the availability of bikes and docks (Espegren et al. 2016). We here focus on dynamic rebalancing planning, which has a higher impact on practice.

We consider a multi-period dynamic model for routing and repositioning of bikes in the bike-sharing system with minimal assumptions. To remain computationally feasible, the planning horizon (e.g., 6 a.m. - 1 p.m.) is typically divided into a set of time-periods (e.g., 30 minutes).

The input parameters are summarized in Table 1. Namely, S denotes the set of stations, while V denotes the set of available vehicles. Each station $s \in S$ has a total of C_s docks, referred to as its capacity. Each vehicle $v \in V$ has a bike capacity $\hat{C}_v$. Parameters $D_{i,j}$ and $R^t_{s,s'}$ denote respectively the distance and transit time between stations i and j at time-period t. The expected rental/return ($f^{+,t}_s/f^{-,t}_s$) can be learned from historical data. We consider a planning horizon with

$|T|$ time-periods, where each time-period $t \in T$ has a duration of L_t minutes.

Table 1: Input parameters of the optimization model

Input	Definition
S	The set of stations.
V	The set of vehicles.
T	The set of discretized time-periods.
$D_{i,j}$	The distance between station $i \in S$ and $j \in S$.
C_s	The capacity of station $s \in S$.
$\hat{C}_v$	The capacity of vehicle $v \in V$.
L_t	The length (in minutes) of time-period $t \in T$.
d_s^1	The initial number of bikes in station $s \in S$.
$\hat{d}_v^1$	The initial number of bikes in vehicle $v \in V$.
$z_{s,v}^1$	1, if vehicle $v \in V$ is at station $s \in S$ at the beginning of planning; 0, otherwise.
$f_s^{+,t}$	The expected rental demand at station $s \in S$ in period $t \in T$.
$f_s^{-,t}$	The expected return demand at station $s \in S$ in period $t \in T$.
$R_{s,s'}^t$	Transit time for vehicles from station $s \in S$ to station $s' \in S$ in period $t \in T$.
$F_{s,s'}^{t,t'}$	The number of trips from station $s \in S$ at period $t \in T$ to $s' \in S$ at period $t' \in T$.

The decision variables are summarized in Table 2. Variables $r_{s,v}^{+,t}/r_{s,v}^{-,t}$ represent the number of bikes picked up/dropped off at station s by vehicle v during period t. Variable $z_{s,v}^t$ takes value 1 if station s visited by vehicle v at period t; 0 otherwise. For each time-period, intermediate variables are used, such as the number of bikes available at stations/in vehicles, successful trips, and the routes of the vehicles. We employ rental and return demand as variables $x_s^{+,t}$ and $x_s^{-,t}$.

Table 2: Decision variables of the optimization model

Variable	Definition
$\boldsymbol{r_{s,v}^{+,t}}$	The number of bikes picked up at station s by vehicle v in period t
$\boldsymbol{r_{s,v}^{-,t}}$	The number of bikes dropped off at station s by vehicle v in period t
$\boldsymbol{z_{s,v}^{t}}$	1, if vehicle $v \in V$ is at station $s \in S$ at period $t \in T$; 0, otherwise.
d_s^t	The number of bikes available in station $s \in S$ at the beginning of period t
$\hat{d}_v^t$	The number of bikes in vehicle $v \in V$ at the beginning of period t
$p_{s,s',v}^t$	1, if vehicle v is at station s in period t and at station s' in period $t+1$; 0, otherwise
$x_s^{+,t}$	The number of successful bike trips starting from station s in period t
$x_s^{-,t}$	The number of successful bike trips ending at station s in period t

The following MILP model (1)-(9) is an example of dynamic repositioning that can be used to obtain rebalancing strategies for BBSs. Note that for vehicle routing, we assume that one vehicle can visit at most T stations during one planning epoch and one vehicle can only visit one station in each period.

$$\min \quad \sum_{s\in S}\sum_{t\in T}(f_s^{+,t} - x_s^{+,t}) + \sum_{s\in S}\sum_{t\in T}(f_s^{-,t} - x_s^{-,t}) \tag{1}$$

$$\text{s.t.} \quad \hat{d}_v^{t+1} = \hat{d}_v^t + \sum_{s\in S}(r_{s,v}^{+,t} - r_{s,v}^{-,t}) \qquad \forall\, v \in V,\ t \in T \tag{2}$$

$$d_s^{t+1} = d_s^t - \sum_{v\in V}(r_{s,v}^{+,t} - r_{s,v}^{-,t}) - x_s^{+,t} + x_s^{-,t} \qquad \forall\, s \in S,\ t \in T \tag{3}$$

$$\sum_{s\in S} z_{s,v}^t = 1 \qquad \forall\, v \in V,\ t \in T \tag{4}$$

$$r_{s,v}^{+,t} + r_{s,v}^{-,t} \le \hat{C}_v z_{s,v}^t \qquad \forall\, s \in S,\ v \in V,\ t \in T \tag{5}$$

$$0 \le \hat{d}_v^t \le \hat{C}_v, 0 \le d_s^t \le C_s \qquad \forall\, s \in S,\ v \in V,\ t \in T \tag{6}$$

$$0 \le x_s^{+,t} \le f_s^{+,t}, 0 \le x_s^{-,t} \le f_s^{-,t} \qquad \forall\, s \in S,\ t \in T \tag{7}$$

$$0 \le r_{s,v}^{+,t}, r_{s,v}^{-,t} \le \hat{C}_v \qquad \forall\, s \in S,\ v \in V,\ t \in T \tag{8}$$

$$z_{s,v}^t \in \{0, 1\} \qquad \forall\, s \in S,\ v \in V,\ t \in T \tag{9}$$

Objective function (1) minimizes the total lost rental and return demand in the planning horizon over all stations and time-periods. If required, it can be modified according to the preferences of the BSS operators. The constraints are as below:

- **The flow of bikes for each vehicle.** Constraint (2) ensures that the number of bikes in each vehicle in the next period equals to the sum of the number of bikes in each vehicle in the current period and the number of bikes picked up.

- **The flow of bikes for each station.** Constraints (3) manage the station inventory along time, considering the rebalancing operations and successful customer trips (i.e., rentals and returns).

- **Vehicle visiting.** Constraints (4) ensure that each vehicle is at exactly one station at each time-period, which forms the flow of vehicles sequentially.

- **Picked up/dropped off bikes by vehicles.** Constraints (5) ensure that a vehicle only operates at the station where it is currently located.

- **Inventory of stations and vehicles.** Constraints (6) enforce that the number of bikes in each vehicle is limited by its capacity and the number of bikes at each station is within the station's capacity.

- **Expected demand and successful trips.** Constraints (7) impose that the number of successful trips is bounded by the expected demand for rentals and returns.

- **The capacity of vehicles and rebalancing operations** Constraints (8) force the number of picked-up/dropped-off bikes to respect the vehicle capacity.

2. Alternatives in the model

In the model proposed, some alternative constraints are always applicable according to decision-makers. We give two examples of time constraints and initial settings.

2.1. Time Constraints

The basic model assumes that vehicles can relocate to any other stations and carry out the rebalancing operations within the duration of a time-period. When the duration of the time-period is short, the resulting planning solution may become infeasible in practice. Since vehicles may not have sufficient time to relocate and rebalance bikes, time constraints may be added to restrict the vehicle relocation between stations and rebalancing operations to the time available. We formulate time constraints as follows. First, for each pair of stations s and s', vehicle v, and time-period t, Constraints (10) enforce variable $p^t_{s,s',v}$ to take value 1 if both variables $z^t_{s,v}$ and $z^{t+1}_{s',v}$ are have value 1. Then, time constraints (11) guarantee that the transit time between stations and the operation time for picking up/dropping off bikes for each period will not surpass the available time L_t.

$$z^t_{s,v} + z^{t+1}_{s',v} - 1 \le p^t_{s,s',v} \qquad \forall\ s, s' \in S,\ v \in V,\ t \in T \tag{10}$$

$$\sum_{s\in S}\sum_{s'\in S} p^t_{s,s',v} R^t_{s,s'} + op \sum_{s\in S}(r^{+,t}_{s,v} + r^{-,t}_{s,v}) \le L_t, \qquad \forall\ v \in V,\ t \in T \tag{11}$$

$$p^t_{s,s',v} \in \{0,1\} \qquad \forall\ s, s' \in S,\ v \in V,\ t \in T, \tag{12}$$

where *op* is the average operational time to pick up/drop off a single bike.

2.2. Initial Settings

In our basic model, we assume that the initial location and inventory of each truck are known. For example, each truck can be set to the same amount of bikes, i.e., 20 bikes. However, the operator may have the possibility and desire to specify an initial location and inventory for the trucks. To this end, Constraints (13) and (14) are created. Constraint (13) implies that the number of vehicles located at specific stations at the first time-period is equal to the number of vehicles *num_v* in the system, which, along with the Constraints (4), means that the trucks can be assigned to any station at the beginning of rebalancing. Constraint (14) indicates that the total number of bikes in all vehicles equals the total number of bikes *av_bike* initially available in vehicles.

$$\sum_{s,v} z^1_{s,v} = num\_v \tag{13}$$

$$\sum_{v} \hat{d}^1_v = av\_bike \tag{14}$$

Note that there are still many other constraints can be adapted into mathematical model according to decision-makers. For example, initial inventory of stations, trip distribution, and etc.

3. Dataset

We here provide a dataset of a bike-sharing system with 30 stations. The data is generated based on the trip pattern obtained from the real-world dataset. The dataset contains the station network, trip information (expected rental and return), and rebalancing fleet.

The 30 stations are indexed as 0-29. The first five stations (stations 0-4) are located inside the city center, each of which has a larger capacity of 40 docks. The regular stations (stations 5-29) are distributed outside the city center with a capacity of 20 docks. An initial inventory of each station at the beginning of the planning horizon can be defined according to decision-makers, as long as the total number of bikes in all the stations is 304. An example of initial inventory at 6 a.m. is given by the file *Initial_Inven.json* based on a static rebalancing model. Another file named *Dis.json* implies the distance of each pair of stations. It is a 30×30 upper triangular matrix calculated with longitude and latitude. Assuming that the routes between two stations i and j (i to j and j to i) are symmetric, only one element in the matrix $d_{i,j}(i < j)$ is used to indicate the distance between station i and j. The transit time between two stations can be presented by $\alpha \times d_{i,j}$, where $\alpha = 2$.

Trip information for 500 days is provided within 500 *.json* files, namely from *simu0_0.json* to *simu0_499.json*, each of which depicts trip details of one entire day. Each file *simu0_*.json* is a set of N^* individual trips $\{[t_d(n), s_d(n), t_a(n), s_a(n)], n \in N^*\}$, including rental and return demands with $t_d(n)$ ascending. For a trip n, $t_d(n)$ is the time of departure, $s_d(n)$ is the departure station, $t_a(n)$ is the arriving time, and $s_a(n)$ is the arrival station. Each day (24 hours) is well-grained into 1440 one-minute segments (indexed from 0 to 1439). $t_d(n)$ and $t_a(n)$ present the exact minute when the rental and return are expected to happen.

For a system with 30 stations, two trucks are suggested to be hired for rebalancing. Each truck has a capacity of 40, which is the maximum number of bikes that a truck can hold. At the beginning of the planning horizon, the available bikes for trucks are totally 40. The average operation time for picking up or dropping off one bike *op* can be set as 1 minute.

References

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