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Who will win the Superbowl LX?

The Slido app must be installed on every computer you're presenting from

slido

“Calculus 3”

Multi-Variable Calculus

Instructor: Álvaro Lozano-Robledo

Day 3

Any Reminders? Any Questions?

- Class ends at 3:15.
- Slides are being posted on GitHub!
  https://github.com/alozanoroble/MATH-2110Q-Spring-2026
- Videos will be posted on YouTube… but they may lag!
- All requests for make-up quizzes need to go to your TA
- Second quiz (Friday) will be on previous week's material

Today – Lines and Planes!

- Lines
  - Parametric equations of a line
  - Symmetric equation
  - Line segments
- Planes
  - Vector equation
  - Scalar equation
  - Distance to a plane

Questions?

ALVARO: Start the recording!

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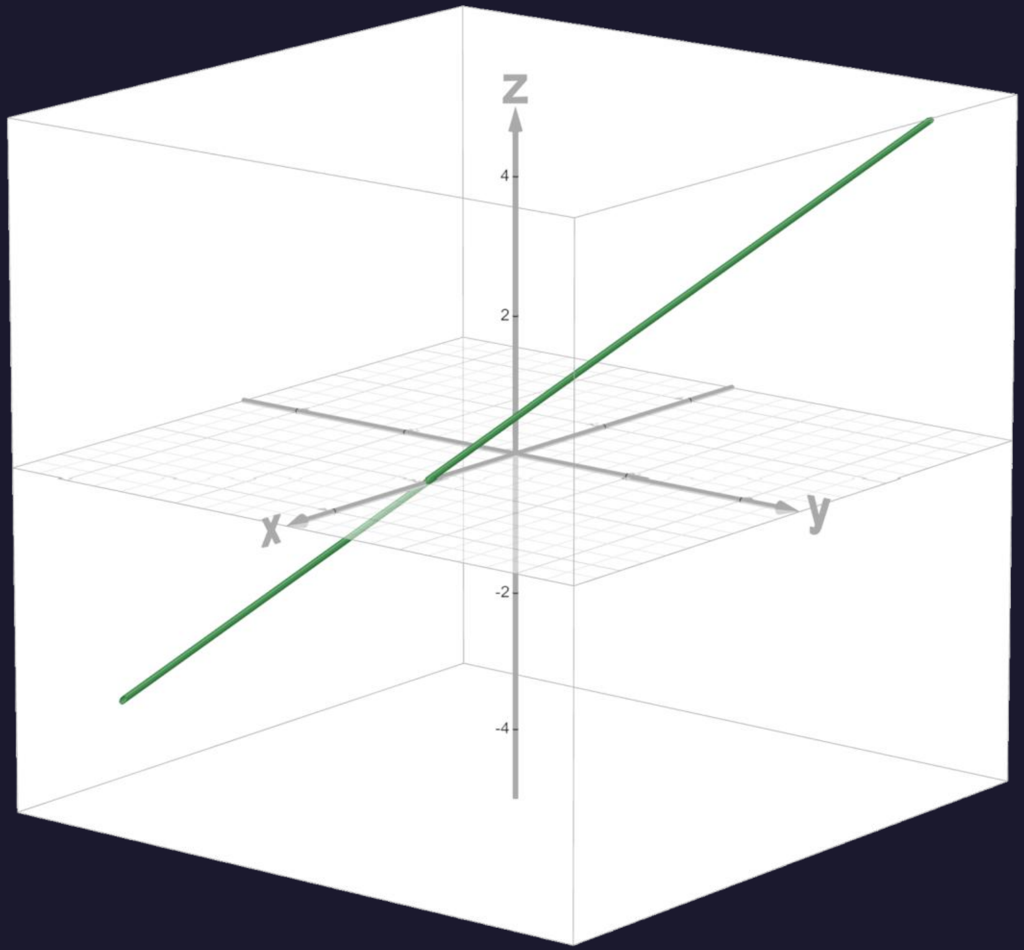
Equations of Lines

Today – Lines!

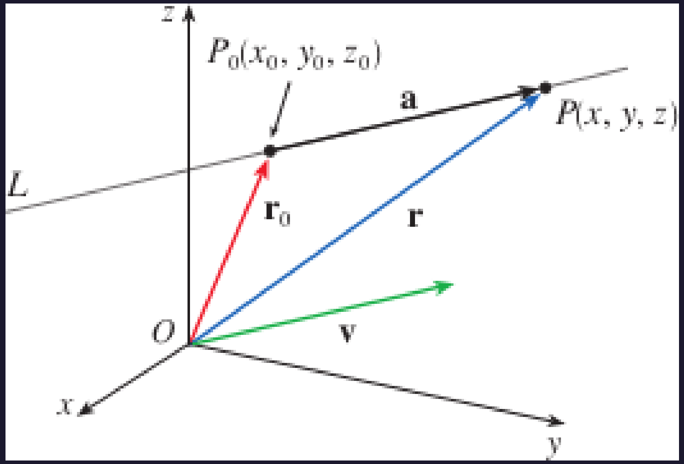
- Lines
  - Parametric equations of a line
  - Symmetric equation
  - Line segments



How to describe lines in 3D space?

Vector and Parametric Equation of a Line

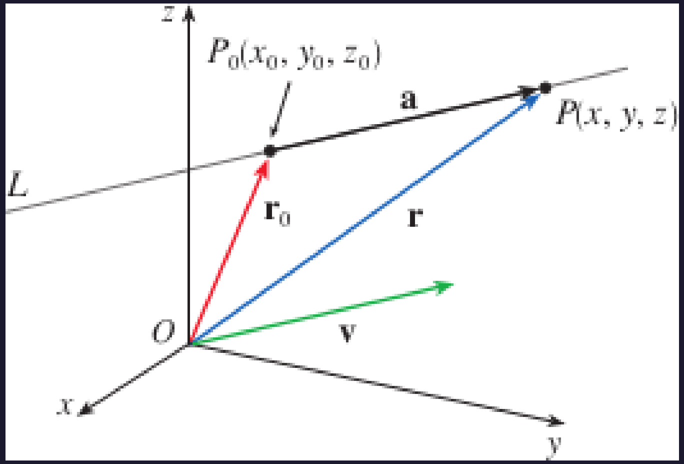
The equation of a line L that passes through $P_0 = (x_0, y_0, z_0)$ and in the direction of the vector $v = (a, b, c)$.



Example: Find the parametric equation of a line that passes through $P = (1, 2, 3)$ in the direction of $v = (-1, 0, 2)$.

From Parametric Equations to Symmetric

The equation of a line that passes through $P_0 = (x_0, y_0, z_0)$ and in the direction of the vector $v = (a, b, c)$.



Example: Find the symmetric equation of a line that passes through $P = (1, 2, 3)$ in the direction of $v = (-1, 0, 2)$.

Example: Find the parametric and symmetric equation of a line that passes through $P = (2, 4, -3)$ and $Q = (3, -1, 1)$.

Relative Position of Two Lines in Space

Parallel, Intersecting, and Skew Lines

Example: Show that the lines L_1 and L_2 intersect, and find the point of intersection. [Desmos]

$$L_1 : x = -2+t,\ y = 2-2t,\ z = -1+3t \quad \text{and} \quad L_2 : x = -2-s,\ y = 1+s,\ z = -2s.$$

Example: Show that the lines L_1 and L_2 given below are skew lines, that is, they are not parallel and they do not intersect. [Desmos]

$$L_1 : x = 1+t, y = -2+3t, z = 4-t \quad \text{and} \quad L_2 : x = 2s, y = 3+s, z = -3+4s.$$

Do not edit
How to change the design

The lines (1+t, -2t, -1+3t) and (2+t, -2-2t, 2+3t) are...

Example: Investigate the relative position of the lines

L_1 : $(1+t, -2t, -1+3t)$ and L_2 : $(2+t, -2-2t, 2+3t)$.

A Line Segment from P to Q

The equation of a line segment from $P=(x_0, y_0, z_0)$ to $Q=(x_1, y_1, z_1)$.

$L : (1-t)\cdot P + t\cdot Q$ with $0 \leq t \leq 1$.

Example: Find the parametric equation for the segment that goes from $P = (1, 0, 0)$ to $Q = (0, 1, 1)$. [Desmos]

Questions?

ALVARO: Start the recording!

“Calculus 3”

Multi-Variable Calculus

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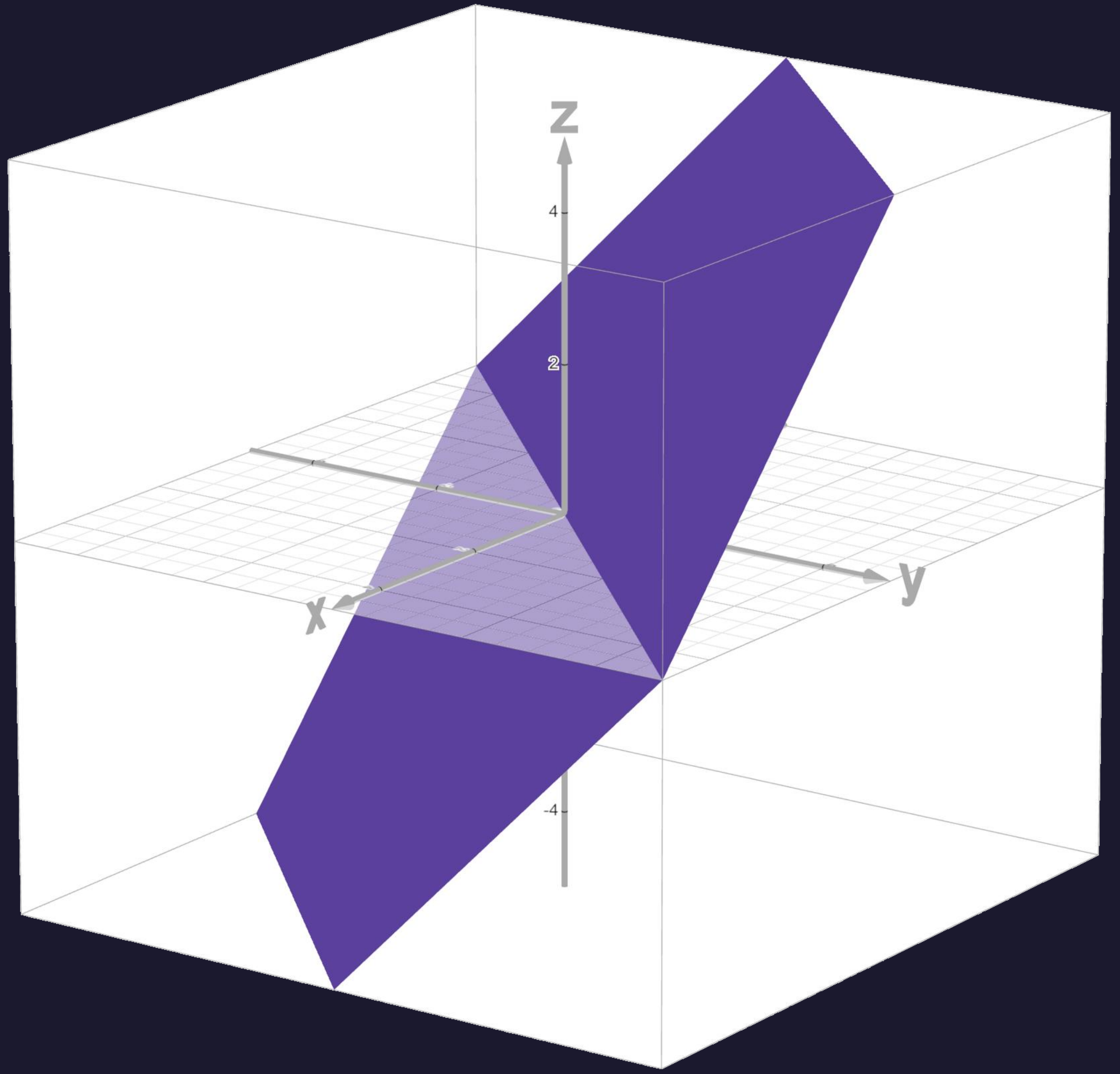
Equations of Planes

Today –Planes!

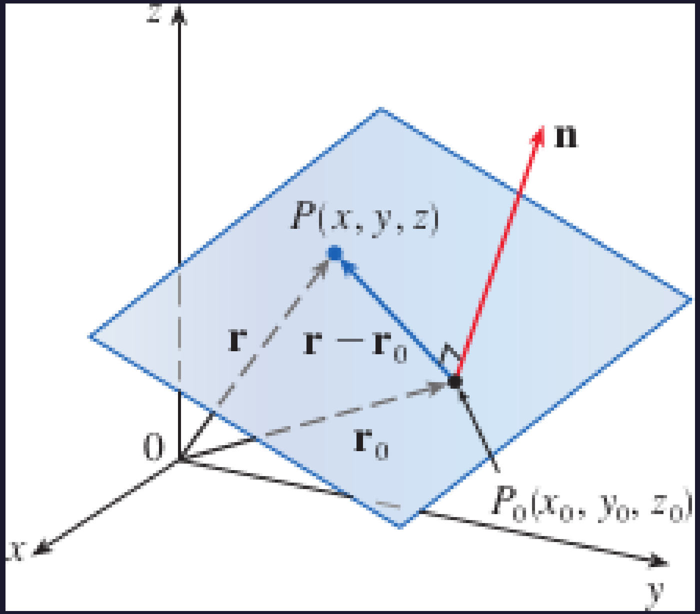
- Planes
  - Vector equation
  - Scalar equation
  - Distance to a plane



How to describe planes in 3D space?

Vector and Scalar Equation of a Plane

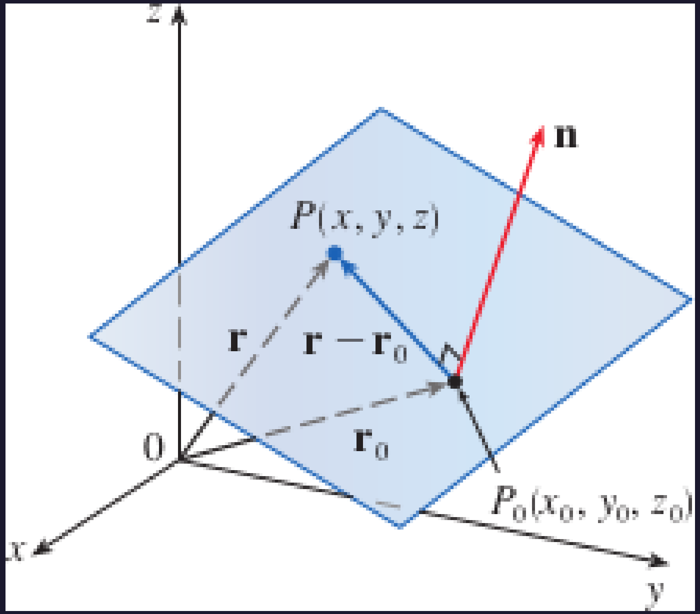
The equation of a plane M that passes through $P_0 = (x_0, y_0, z_0)$ and is perpendicular to the direction of the vector n = (a, b, c).



Example: Find the scalar equation of a plane that goes through $P = (1, 0, 0)$ and it is perpendicular to $n = (0, 1, -1)$. [Desmos]

Vector, Scalar, and Linear Equation of a Plane

The equation of a plane M that passes through $P_0 = (x_0, y_0, z_0)$ and is perpendicular to the direction of the vector $n = (a, b, c)$.



Example: Find the linear equation of a plane that goes through $P = (2, 4, -1)$ and it is perpendicular to $n = (2, 3, 4)$. [Desmos]

Example: Find the equation of the plane that passes through the points $(1,0,0)$, $(0,1,0)$, and $(0,0,1)$.

Extra space just in case

Example: Find the equation of the plane that passes through the points (2,0,1), (0,-1,1), and (0,0,-1).

Extra space just in case

Relative position of two planes

Parallel planes

The planes x-z = 2 and 2x-2z = 5 are...

The planes $x-z = 2$ and $2x-2y = 5$ are...

Example: Investigate the relative positions of the planes

$$x - z = 2,\ 2x - 2z = 5, \text{ and } 2x - 2y = 5.$$

Angle between two planes

Example: Find the angle between the planes

$$x + y + z = 1 \qquad \text{and} \qquad x - z = 3.$$

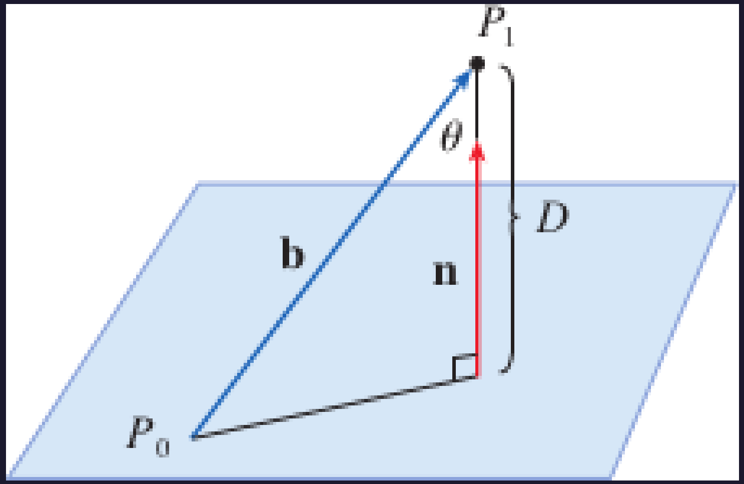
[Desmos]

Example: Find the line of intersection between the planes

$$x + y + z = 1 \qquad \text{and} \qquad x - z = 3.$$

Example: Find the angle and the line of intersection between the planes $x + z = 2$ and $x - y = 3$.

Distance from a point to a plane



The distance D from the point $P_1(x_1, y_1, z_1)$ to the plane $ax + by + cz + d = 0$ is

$$D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

Example: Find the distance from the point $P = (1, 2, 3)$ to the plane $x + z = 2$.

$$D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

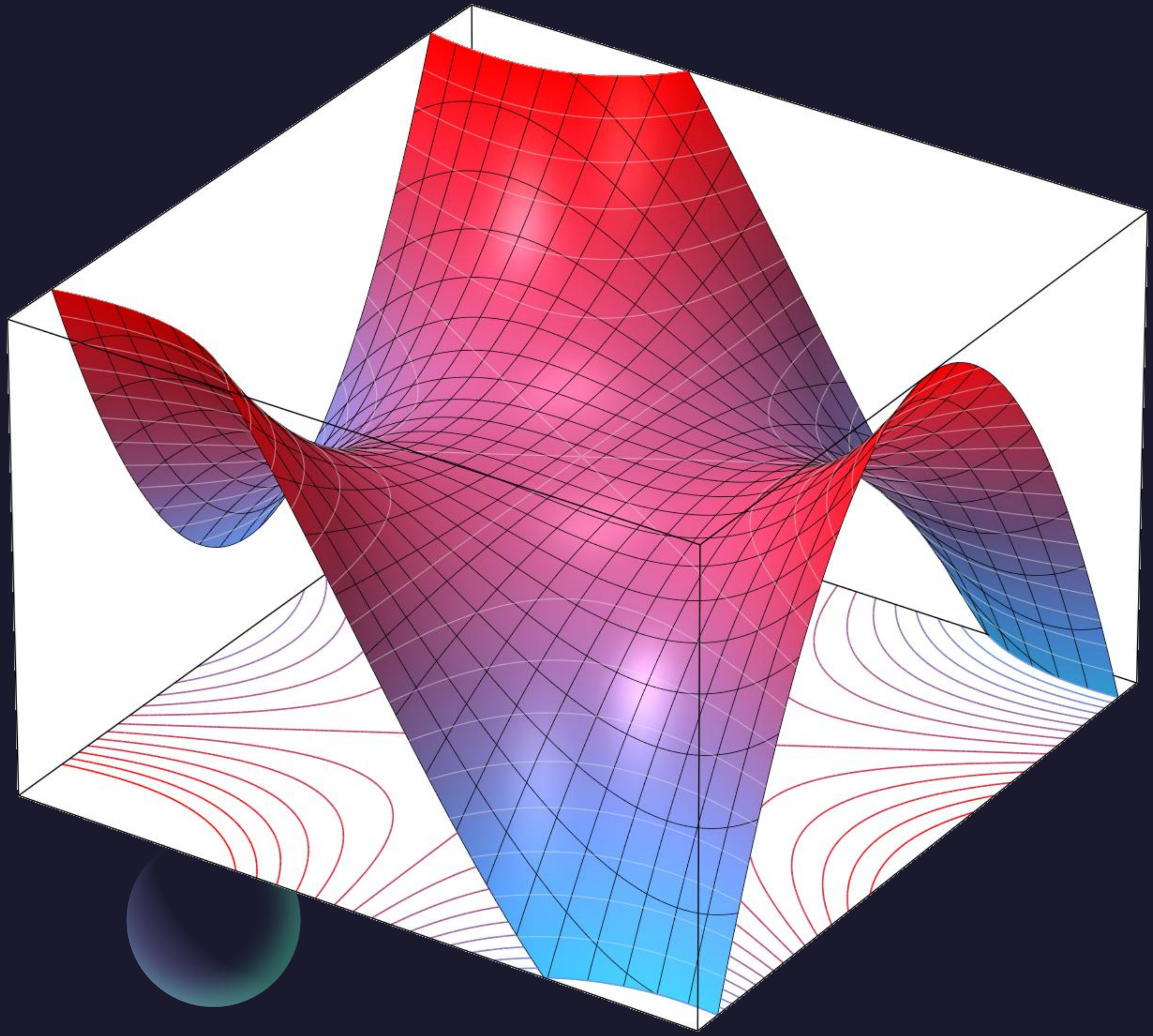
Example: Find the distance between the parallel planes $x + z = 2$ and $2x + 2y = 5$.

$$D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

Questions?

Thank you

Until next time.



“Calculus 3”

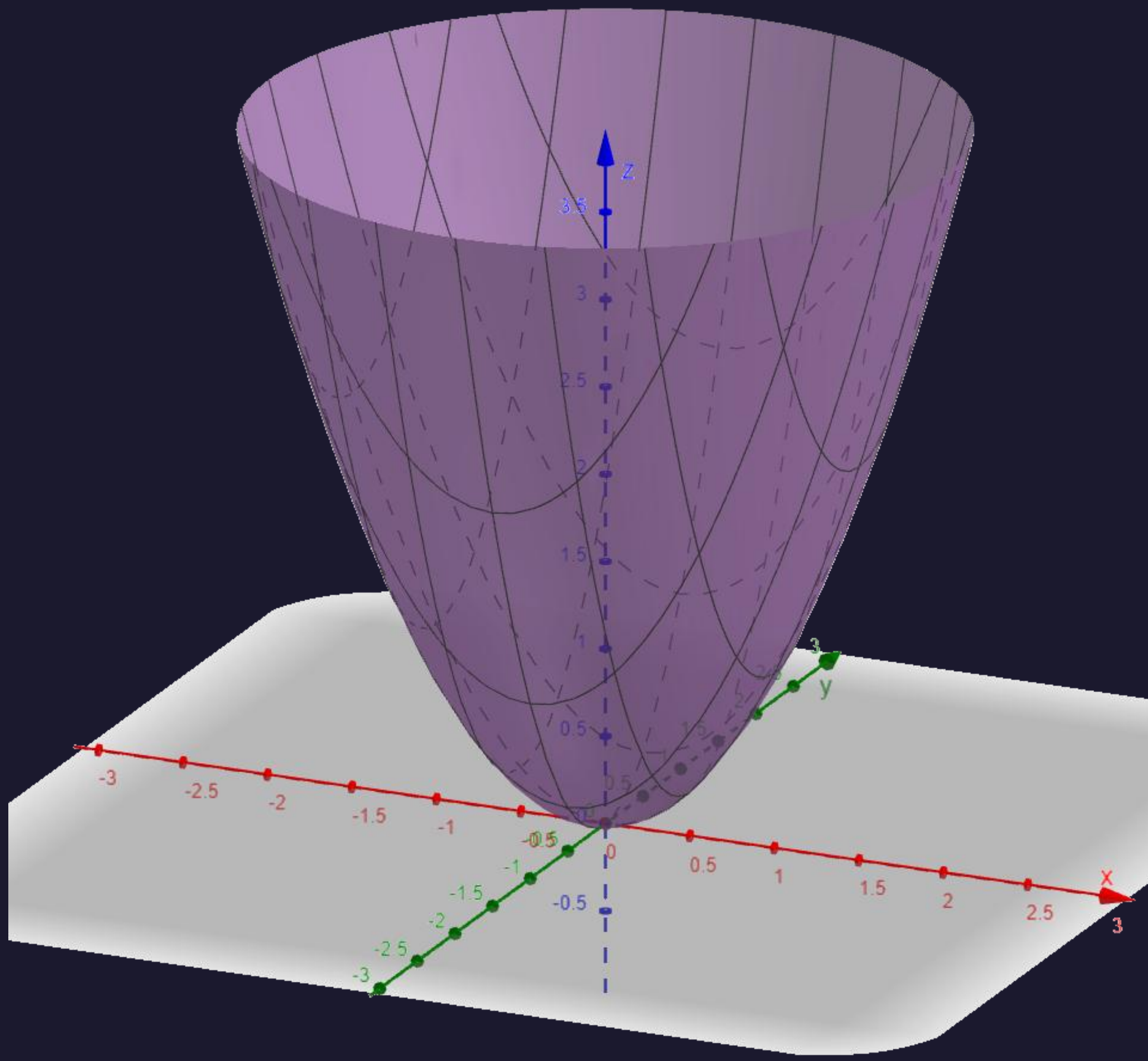
Multi-Variable Calculus

Instructor: Álvaro Lozano-Robledo

Cylinders and Quadrics

Today – Quadrics!

- Cylinders
- Quadric Surfaces
- Ellipsoids, Paraboloids, Hyperboloids.
- Sketching a Quadric Surface



“Cylinders”

A “cylinder” is a surface that consists of all lines (called “rulings”) that are parallel to a given line and pass through a given plane curve.

Recall: Circles, Ellipses, Parabolas and Hyperbolas (Conic Sections)

Example: Sketch the surfaces $x^2 + y^2 = 1$ and $x^2 + z^2 = 1$.

Example: Sketch the surfaces $y + x^2 = 1$ and $z - y^2 = -1$.

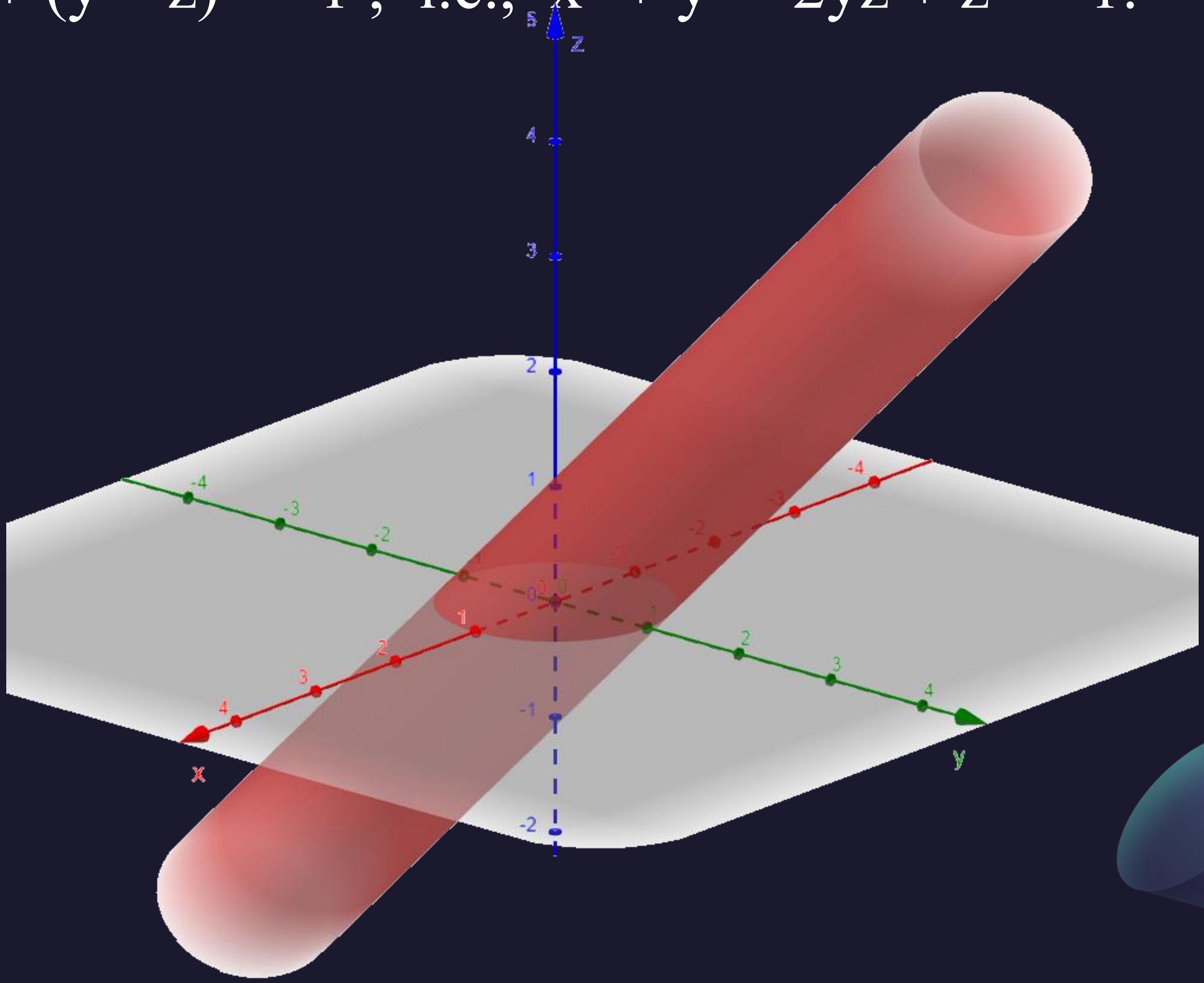
Example: Sketch the surfaces $x^2 - y^2 = 1$.

Example: Sketch the surface

$$x^2 + (y - z)^2 = 1 \text{ , i.e., } x^2 + y^2 - 2yz + z^2 = 1.$$

Example: Sketch the surface

$$x^2 + (y - z)^2 = 1 \ , \quad \text{i.e.,} \quad x^2 + y^2 - 2yz + z^2 = 1.$$



Quadric Surfaces

A **quadric surface** is the graph of a second-degree equation in three variables x, y, and z. The most general such equation is

$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + Gx + Hy + Iz + J = 0$$

where $A, B, C, \ldots, J$ are constants, but by translation and rotation it can be brought into one of the two *standard forms*

$$Ax^2 + By^2 + Cz^2 + J = 0$$

or

$$Ax^2 + By^2 + Iz = 0$$

How to sketch a quadric surface?

Traces or Cross Sections of a Surface

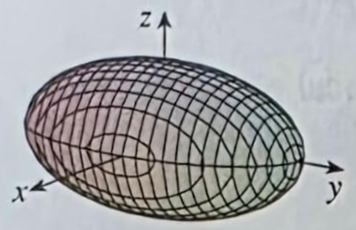
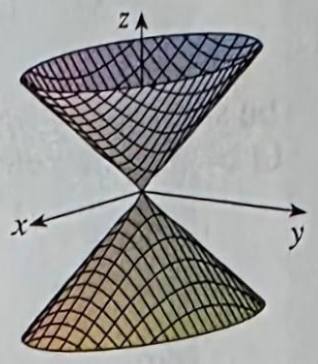
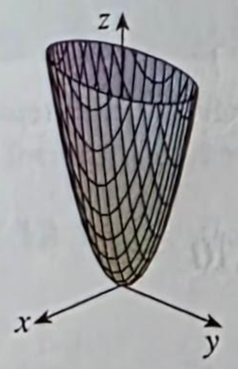
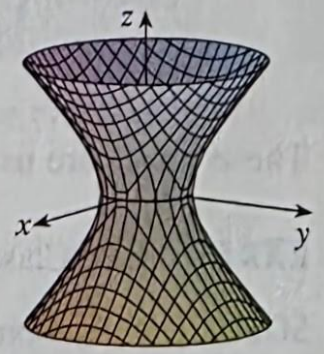
Example: Sketch the surface $x^2 + 2y^2 + 3z^2 = 1.$

Example: Sketch the surface $z = 4x^2 + y^2$.

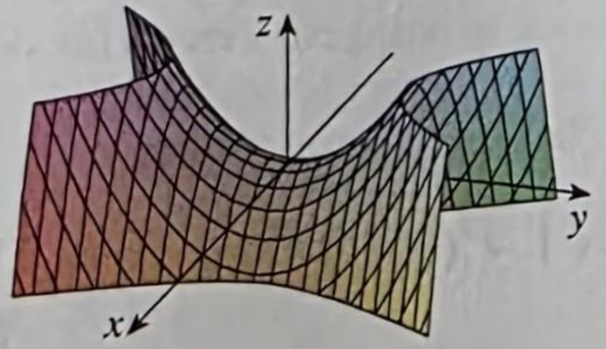
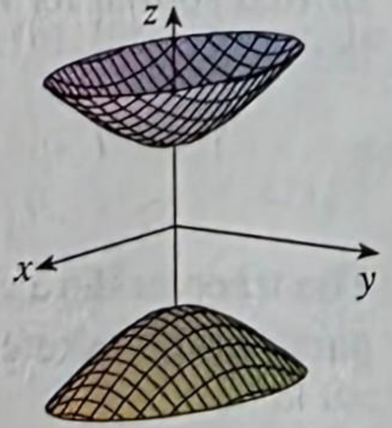
Example: Sketch the surface $z = x^2 - y^2$.

Example: Sketch the surface $x^2 + y^2 - z^2 = 1$.

Types of Quadrics (1)

Surface	Equation	Surface	Equation
Ellipsoid 	$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ All traces are ellipses. If $a = b = c$, the ellipsoid is a sphere.	**Cone** 	$\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ Horizontal traces are ellipses. Vertical traces in the planes $x = k$ and $y = k$ are hyperbolas if $k \neq 0$ but are pairs of lines if $k = 0$.
Elliptic Paraboloid 	$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ Horizontal traces are ellipses. Vertical traces are parabolas. The variable raised to the first power indicates the axis of the paraboloid.	**Hyperboloid of One Sheet** 	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ Horizontal traces are ellipses. Vertical traces are hyperbolas. The axis of symmetry corresponds to the variable whose coefficient is negative.

Types of Quadrics (2)

Hyperbolic Paraboloid 	$\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$ Horizontal traces are hyperbolas. Vertical traces are parabolas. The case where $c < 0$ is illustrated.	**Hyperboloid of Two Sheets** 	$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ Horizontal traces in $z = k$ are ellipses if $k > c$ or $k < -c$. Vertical traces are hyperbolas. The two minus signs indicate two sheets.

Example: Sketch the surface $x^2 + 2z^2 - 6x - y + 10 = 0$.

Questions?

Thank you

Until next time.