“Calculus 3”

Multi-Variable Calculus

Instructor: Álvaro Lozano-Robledo

Day 3

Any Reminders? Any Questions?

- Class ends at 3:15.
- Slides are being posted on GitHub!
  https://github.com/alozanoroble/MATH-2110Q-Spring-2026
- Videos will be posted on YouTube… but they may lag!
- All requests for make-up quizzes need to go to your TA
- Second quiz (Friday) will be on previous week's material

Today – Lines and Planes!

- Lines
  - Parametric equations of a line
  - Symmetric equation
  - Line segments
- Planes
  - Vector equation
  - Scalar equation
  - Distance to a plane

Questions?

ALVARO: Start the recording!

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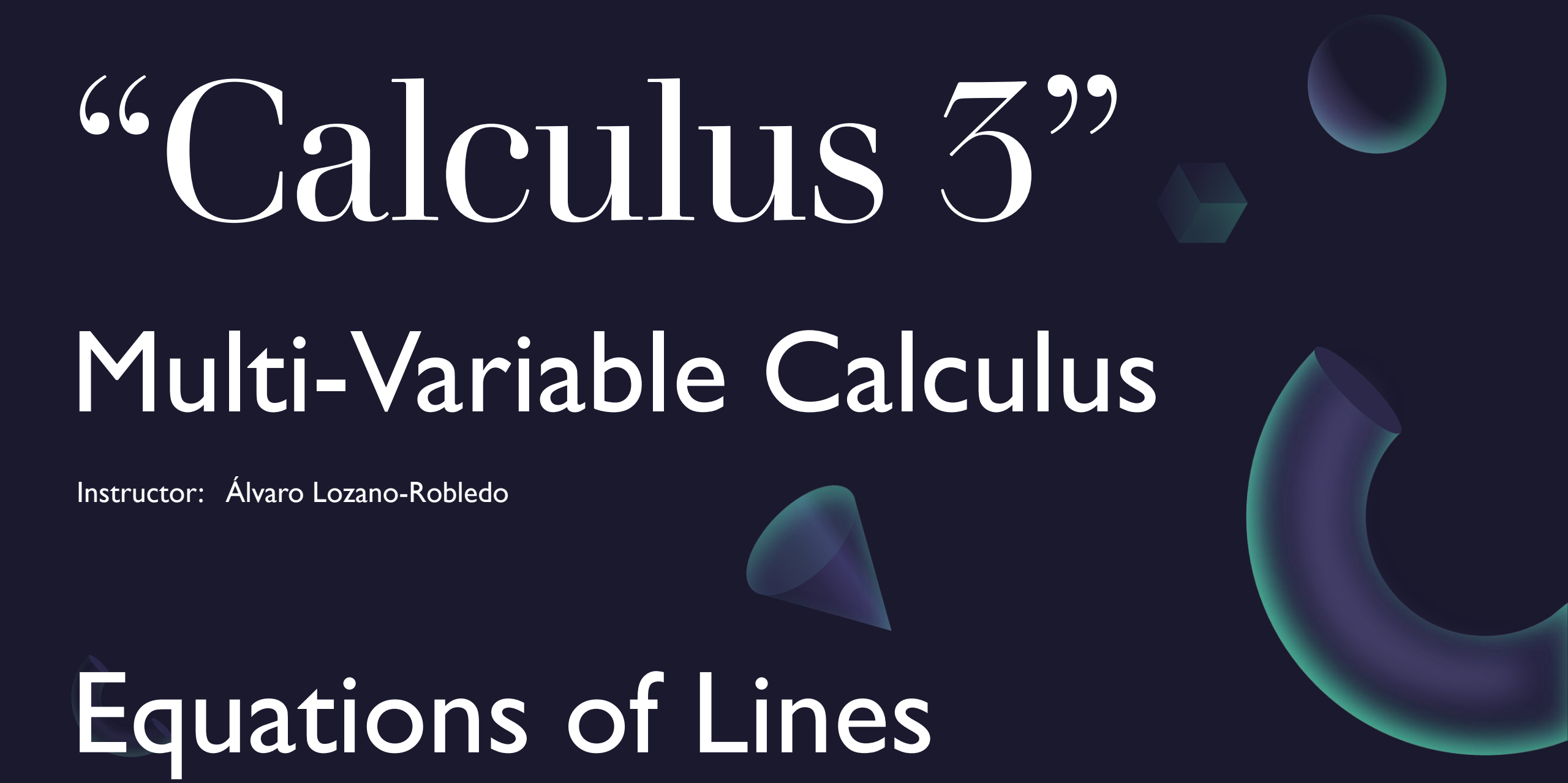
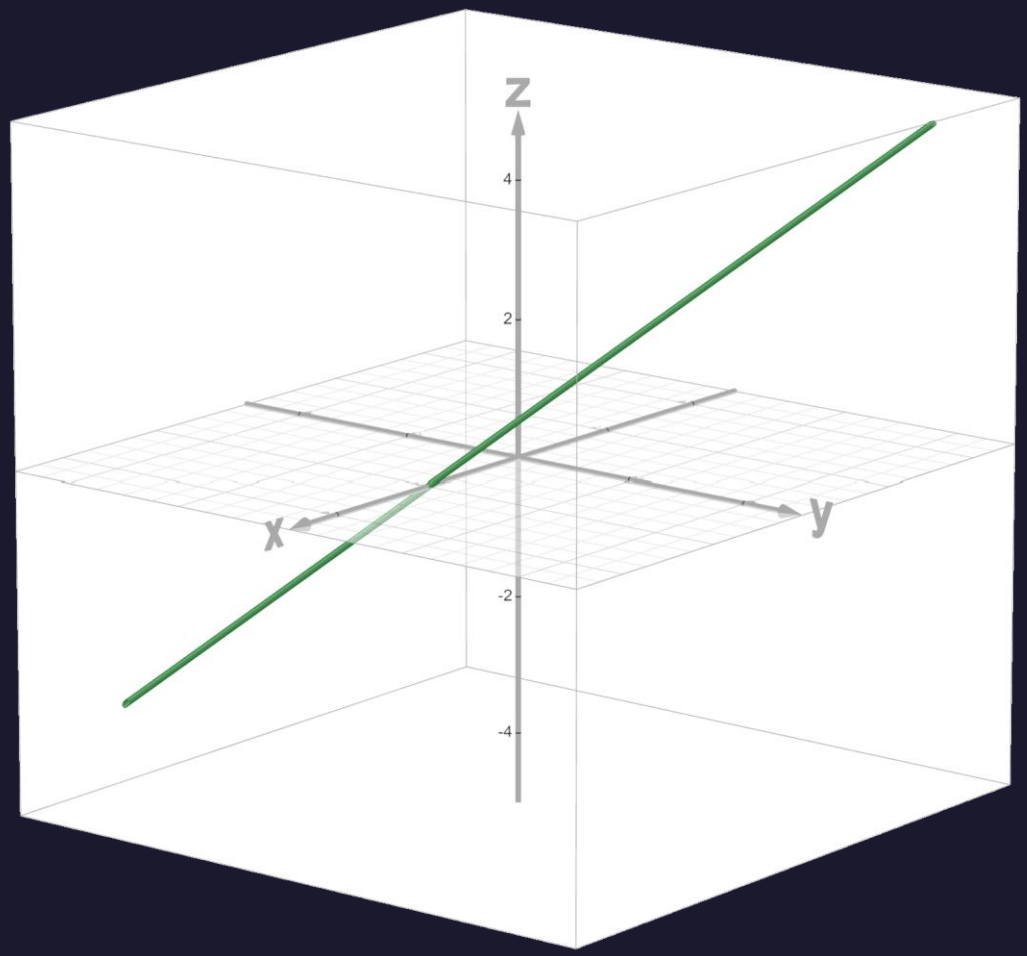


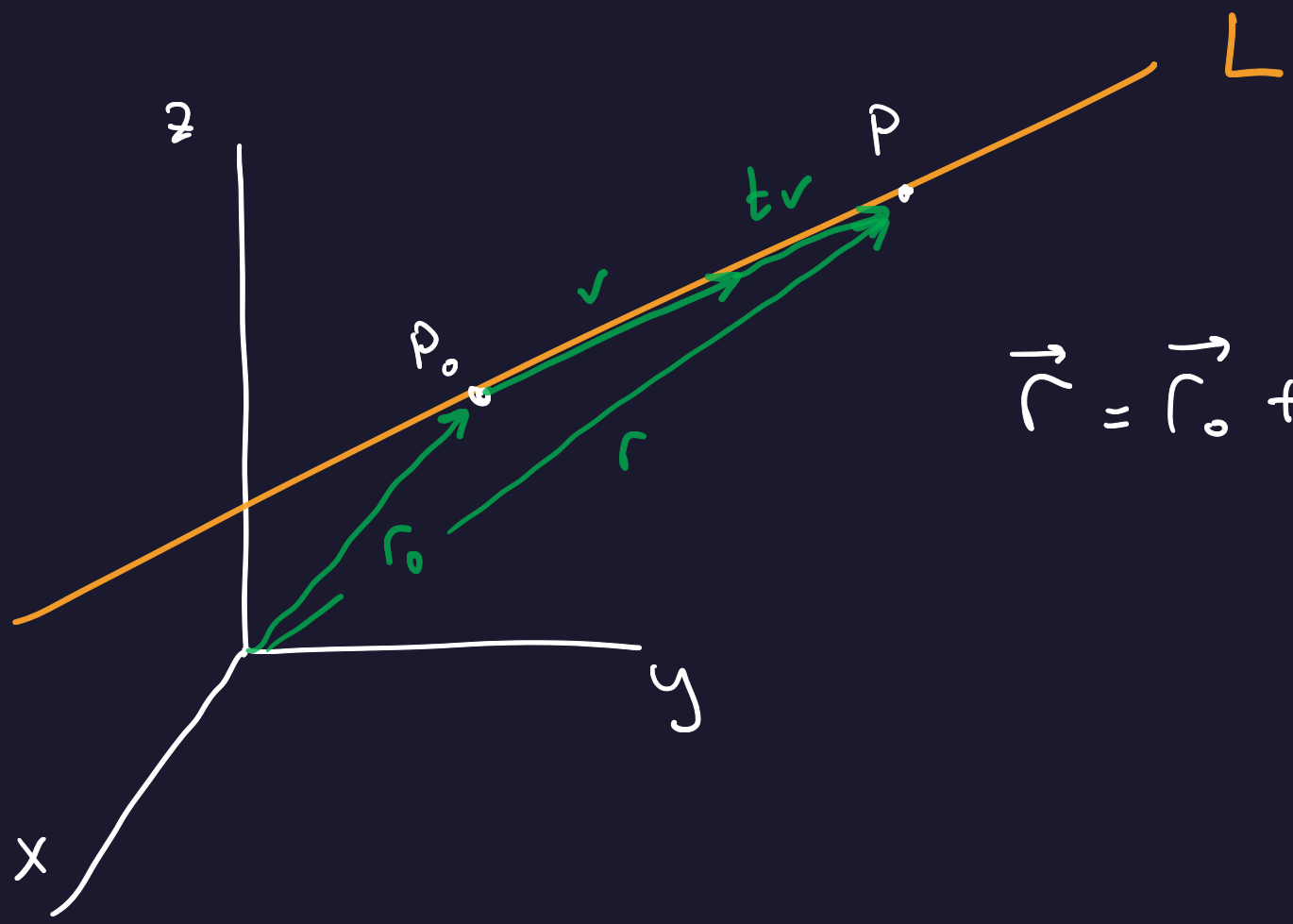
Equations of Lines

Today – Lines!

- Lines
  - Parametric equations of a line
  - Symmetric equation
  - Line segments



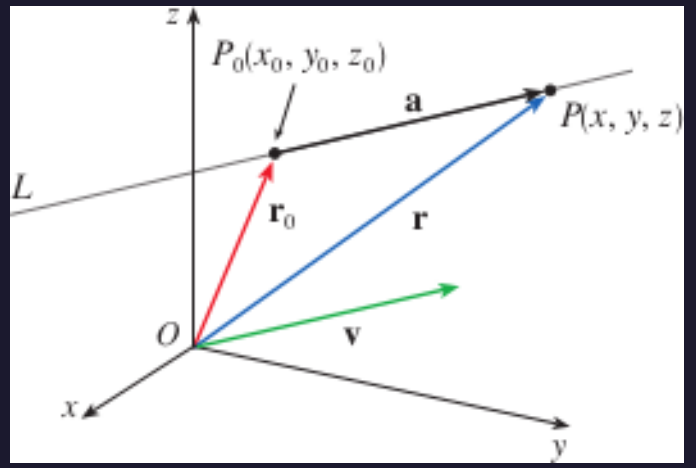
How to describe lines in 3D space?



$$\vec{r} = \vec{r_0} + t\vec{v}$$

Vector and Parametric Equation of a Line

The equation of a line L that passes through $P_0 = (x_0, y_0, z_0)$ and in the direction of the vector $v = (a, b, c)$.



VECTOR EQUATION

$$r = r_0 + t \cdot v$$

$$r_0 = (x_0, y_0, z_0)$$

$$v = (a, b, c)$$

PARAMETRIC EQUATION

$$r = (x_0, y_0, z_0) + t \cdot (a, b, c) = (x_0 + ta, y_0 + tb, z_0 + tc)$$

Example: Find the parametric equation of a line that passes through $P = (1, 2, 3)$ in the direction of $v = (-1, 0, 2)$.

$$r = (1,2,3) + t \cdot (-1,0,2)$$

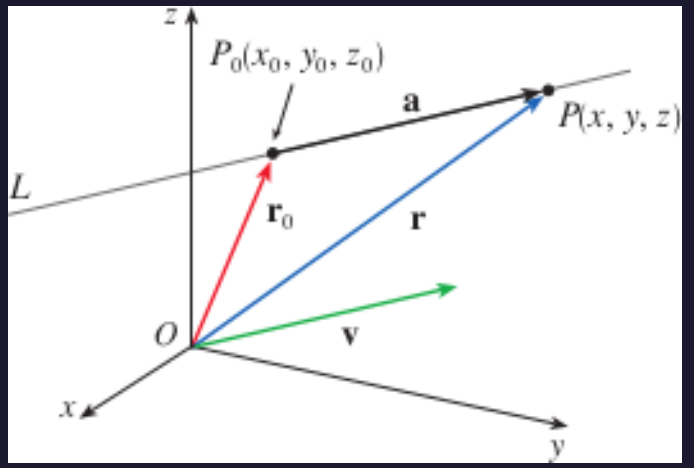
$$L: \begin{cases} x = 1 - t \\ y = 2 + 0 \cdot t \\ z = 3 + 2 \cdot t \end{cases}$$

From Parametric Equations to Symmetric

The equation of a line that passes through $P_0 = (x_0, y_0, z_0)$ and in the direction of the vector $v = (a, b, c)$.

$$r = r_0 + t \cdot v$$

$$L : \begin{cases} x = x_0 + a \cdot t \\ y = y_0 + b \cdot t \\ z = z_0 + c \cdot t \end{cases}$$



$$t = \boxed{\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}}$$

SYMMETRIC EQ'N.

Example: Find the symmetric equation of a line that passes through $P = (1, 2, 3)$ in the direction of $v = (-1, 0, 2)$.

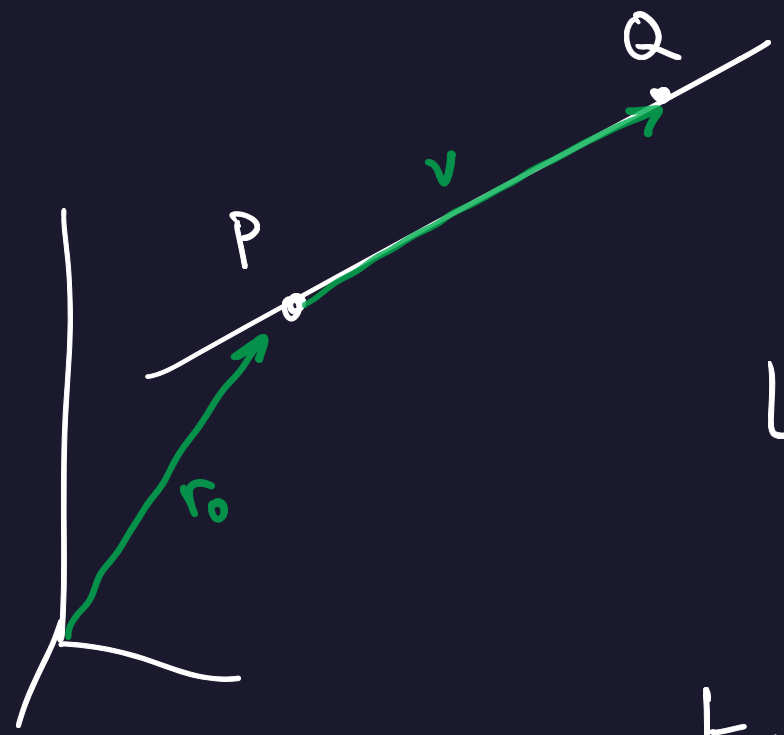
$$\begin{cases} x = 1 - t \\ y = 2 \\ z = 3 + 2t \end{cases}$$

$$t = \frac{x-1}{-1} = \frac{z-3}{2}$$

$$y = 2$$

symm. Eq'n.

Example: Find the parametric and symmetric equation of a line that passes through P = (2, 4, -3) and Q = (3, -1, 1).



$$v = \overrightarrow{PQ} = \vec{Q} - \vec{P} = (3, -1, 1) - (2, 4, -3)$$

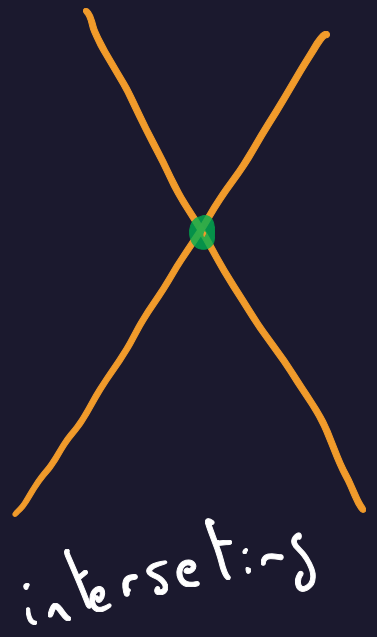
$$= \boxed{(1, 3, 4)} = v$$

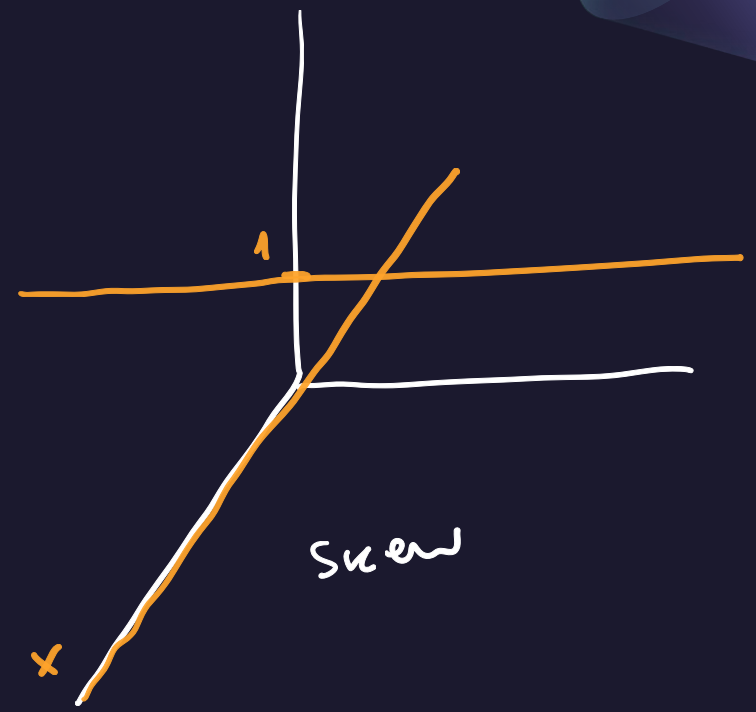
$$L: \begin{cases} x = 2 + t \\ y = 4 + 3t \\ z = -3 + 4t \end{cases} \quad \text{Param.}$$

$$t = \boxed{\frac{x-2}{1} = \frac{y-4}{3} = \frac{z+3}{4}} \quad \text{symm.}$$

Relative Position of Two Lines in Space

Parallel, Intersecting, and Skew Lines





Example: Show that the lines L_1 and L_2 intersect, and find the point of intersection. [Desmos]

L_1 : $x = -2+t, y = 2-2t, z = -1+3t$ and L_2 : $x = -2-s, y = 1+s, z = -2s$.

$$L_1 \cap L_2 = \begin{cases} x = -2+t = -2-s \\ y = 2-2t = 1+s \\ z = -1+3t = -2s \end{cases}$$

$t = -s \Rightarrow 2-2(-s) = 1+s \Rightarrow 2+2s = 1+s \Rightarrow s = -1$

$\Rightarrow t = 1$

$L_1, t = 1 \Rightarrow P = (-1, 0, 2)$

$L_2, s = -1 \Rightarrow P = (-1, 0, 2)$

intersect at $(-1, 0, 2)$

Example: Show that the lines L_1 and L_2 given below are skew lines, that is, they are not parallel and they do not intersect. [Desmos]

$L_1 : x = 1+t, y = -2+3t, z = 4-t$ and $L_2 : x = 2s, y = 3+s, z = -3+4s.$

$v_1 = (1, 3, -1)$ $\quad$ $v_2 = (2, 1, 4)$

not scalar mult. of each other $\Rightarrow$ NOT parallel.

$$\begin{cases} x = 1+t = 2s \\ y = -2+3t = 3+s \\ z = 4-t = -3+4s \end{cases} \Rightarrow t = 2s-1$$

$$\Rightarrow -2+3(2s-1) = 3+s$$

$$-2+6s-3 = 3+s \Rightarrow 5s = 8 \Rightarrow s = \frac{8}{5}$$

$$t = 2\cdot\frac{8}{5} - 1 = \frac{16}{5} - 1 = \frac{11}{5}$$

$$\frac{9}{5} = 4 - \frac{11}{5} \overset{?}{=} -3 + 4\cdot\frac{8}{5} = \frac{17}{5}$$

$\Rightarrow$ DO NOT INTERS. SKEW!

Example: Investigate the relative position of the lines

$L_1 : (1+t, -2t, -1+3t)$ and $L_2 : (2+t, -2-2t, 2+3t)$.

$v_1 = (1, -2, 3)$ $\qquad$ $v_2 = (1, -2, 3)$

$t = 0$ $(1, -2, -1)$ also in L_2?

$$\left.\begin{aligned} 1 &= 2+t \\ -2 &= -2-2t \\ -1 &= 2+3t \end{aligned}\right\} \Rightarrow t = -1$$

$\Rightarrow -2 = -2-2\cdot(-1)$

$-2 \neq 0$ ✗

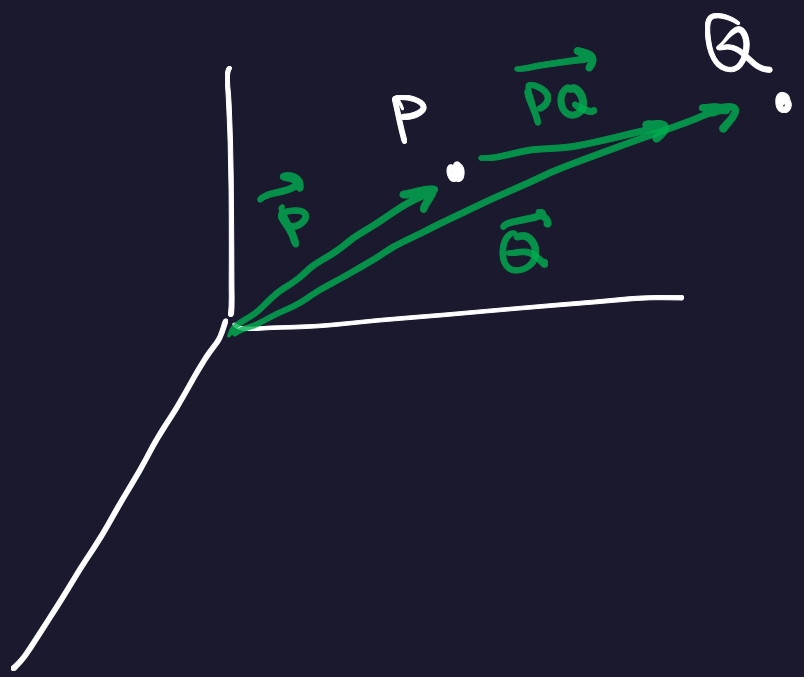
Parallel but

NOT the same line b/c $(1, -2, -1) \in L_1$ but not on L_2.

A Line Segment from P to Q

The equation of a line segment from $P=(x_0, y_0, z_0)$ to $Q=(x_1, y_1, z_1)$.

$L : (1-t)\cdot P + t\cdot Q$ with $0 \le t \le 1$.



$$\begin{aligned} r &= \vec{P} + t\cdot\vec{PQ} \\ &= \vec{P} + t\cdot(\vec{Q}-\vec{P}) \\ &= \vec{P} + t\cdot\vec{Q} - t\cdot\vec{P} \\ &= (1-t)\vec{P} + t\vec{Q} \end{aligned}$$

Example: Find the parametric equation for the segment that goes from $P = (1, 0, 0)$ to $Q = (0, 1, 1)$. [Desmos]

$$L: \quad (1-t)P + tQ \qquad \text{w/} \quad 0 \le t \le 1$$

Line segment

$$(1-t)\cdot(1,0,0) + t\cdot(0,1,1)$$

$$= (1-t, 0, 0) + (0, t, t)$$

$$= \boxed{(1-t, t, t) \qquad 0 \le t \le 1}$$

$$\begin{cases} x = 1-t \\ y = t \\ z = t \end{cases} \qquad 0 \le t \le 1$$

Questions?