Who will win the Superbowl LX?

“Calculus 3”

Multi-Variable Calculus

Instructor: Álvaro Lozano-Robledo

Day 4

Any Reminders? Any Questions?

- Class ends at 3:15.
- Slides are being posted on GitHub!
  https://github.com/alozanoroble/MATH-2110Q-Spring-2026
- Videos will be posted on YouTube… but they may lag!
- All requests for make-up quizzes need to go to your TA
- Second quiz (Friday) will be on previous week's material

Today – Lines and Planes!

- Lines
  - Parametric equations of a line
  - Symmetric equation
  - Line segments
- Planes
  - Vector equation
  - Scalar equation
  - Distance to a plane

Questions?

ALVARO: Start the recording!

“Calculus 3”

Multi-Variable Calculus

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Review of Lines and Planes

Example: Find the parametric equation of a line that passes through $P = (1, 0, 2)$ in the direction of $v = (-1, 1, 1)$.

Example: Find the parametric equation for the segment that goes from

$P = (1, 2, 3)$ to $Q = (1, 0, 1)$.

Example: Show that the lines L_1 and L_2 intersect, and find the point of intersection.

$L_1 : x = -2+t, y = 2-2t, z = -1+3t$ and $L_2 : x = -2-s, y = 1+s, z = -2s.$

Example (LET'S FIX IT!): Investigate the relative position of the lines

L_1 : (1+t, -2t, -1+3t) and L_2 : (2+t, -2-2t, 2+3t).

Example: Find the equation of a plane that goes through $P = (1, 2, 3)$ and it is perpendicular to $n = (1, -1, 2)$.

Example: Find the equation of a plane that goes through $P = (1, 2, 3)$, $Q = (1, 0, 1)$, and $R = (0, -1, 1)$.

The planes $2(x - 2) - 4(y + 1) - 2(z - 1) = 0$ and $x - 2y - z = 3$ are...

Example: The planes

$2(x - 2) - 4(y + 1) - 2(z - 1) = 0$ and $x - 2y - z = 3$

are…

Questions?

ALVARO: Start the recording!

“Calculus 3”

Multi-Variable Calculus

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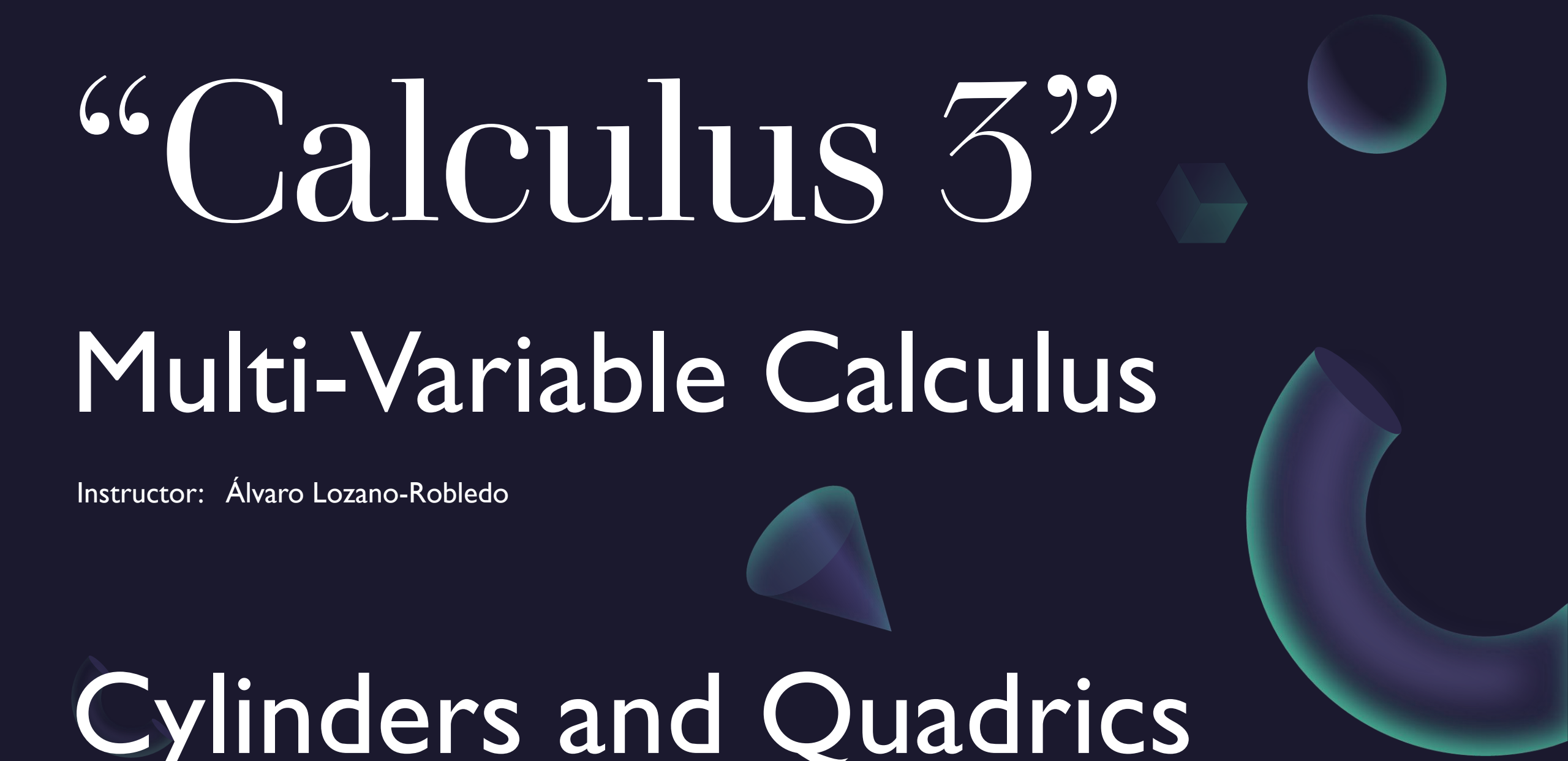
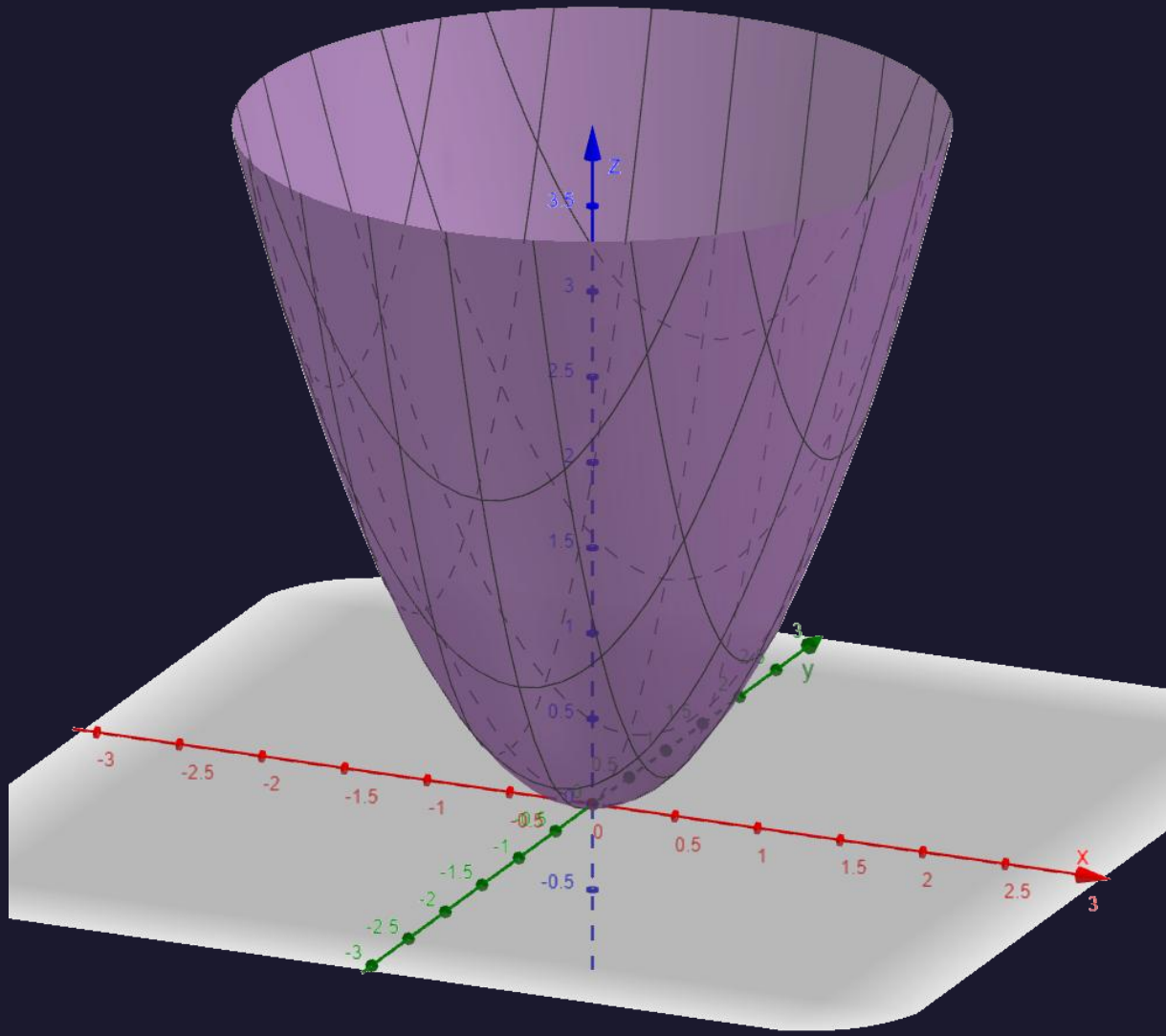


Cylinders and Quadrics

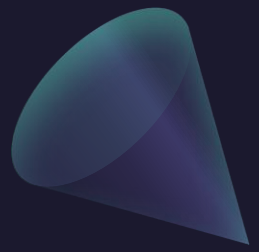
Today – Quadrics!

- Cylinders
- Quadric Surfaces
- Ellipsoids, Paraboloids, Hyperboloids.
- Sketching a Quadric Surface



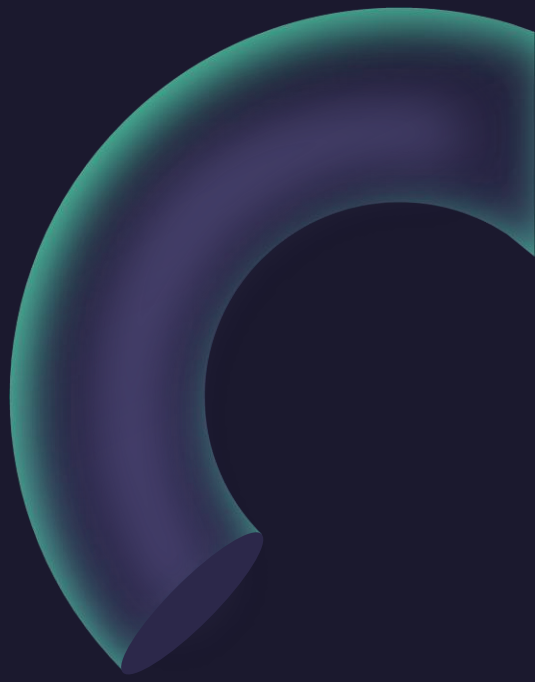
“Cylinders”

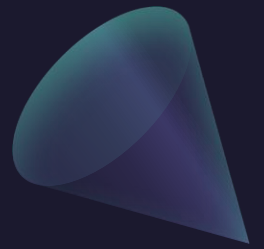
A “cylinder” is a surface that consists of all lines (called “rulings”) that are parallel to a given line and pass through a given plane curve.



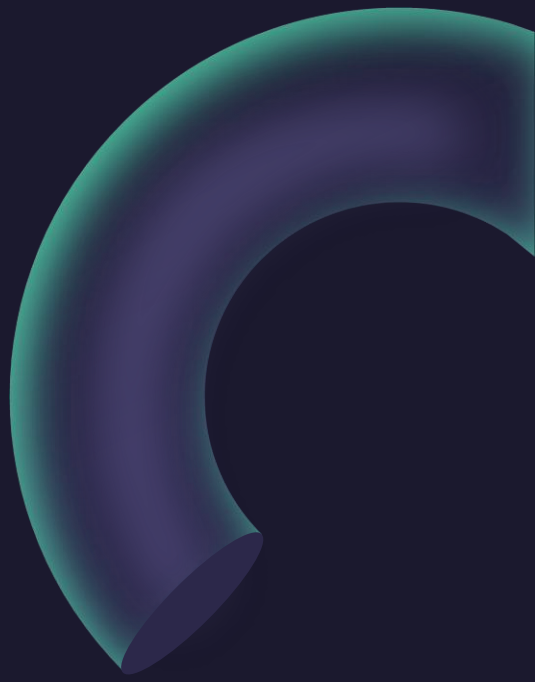
Recall: Circles, Ellipses, Parabolas and Hyperbolas (Conic Sections)

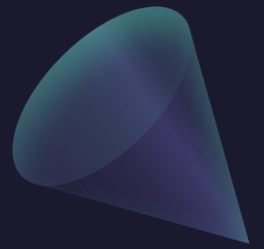
Example: Sketch the surfaces $x^2 + y^2 = 1$ and $x^2 + z^2 = 1$.





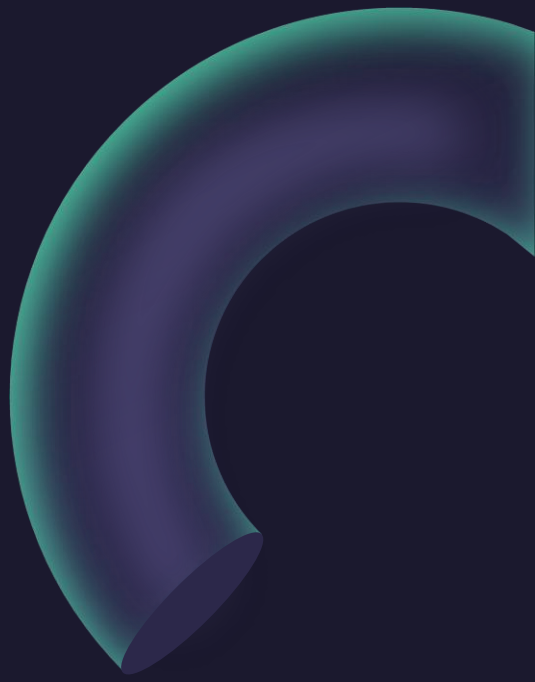
Example: Sketch the surfaces $y + x^2 = 1$ and $z - y^2 = -1$.

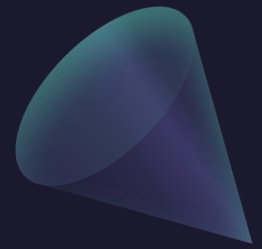




The equation x^2 = 1 + y^2 describes...

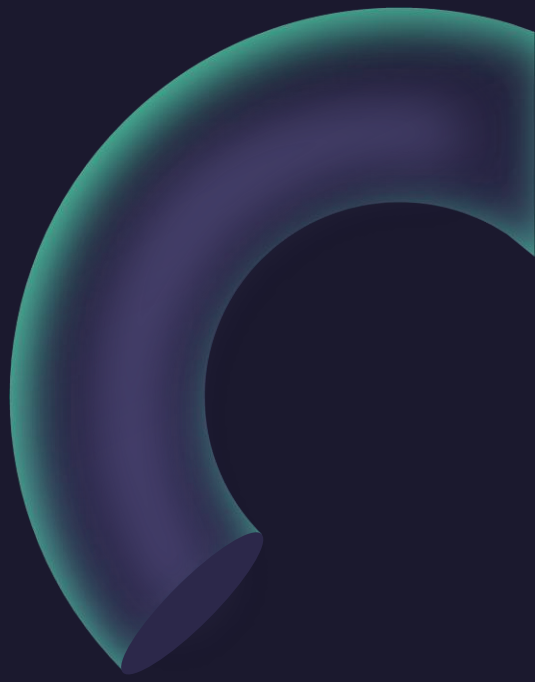
Example: Sketch the surface $x^2 - y^2 = 1$.

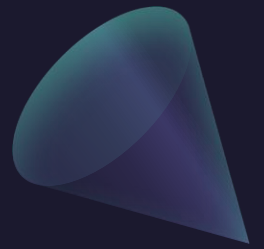




Example: Sketch the surface

$$x^2 + (y - z)^2 = 1 \quad \text{, i.e., } \quad x^2 + y^2 - 2yz + z^2 = 1.$$





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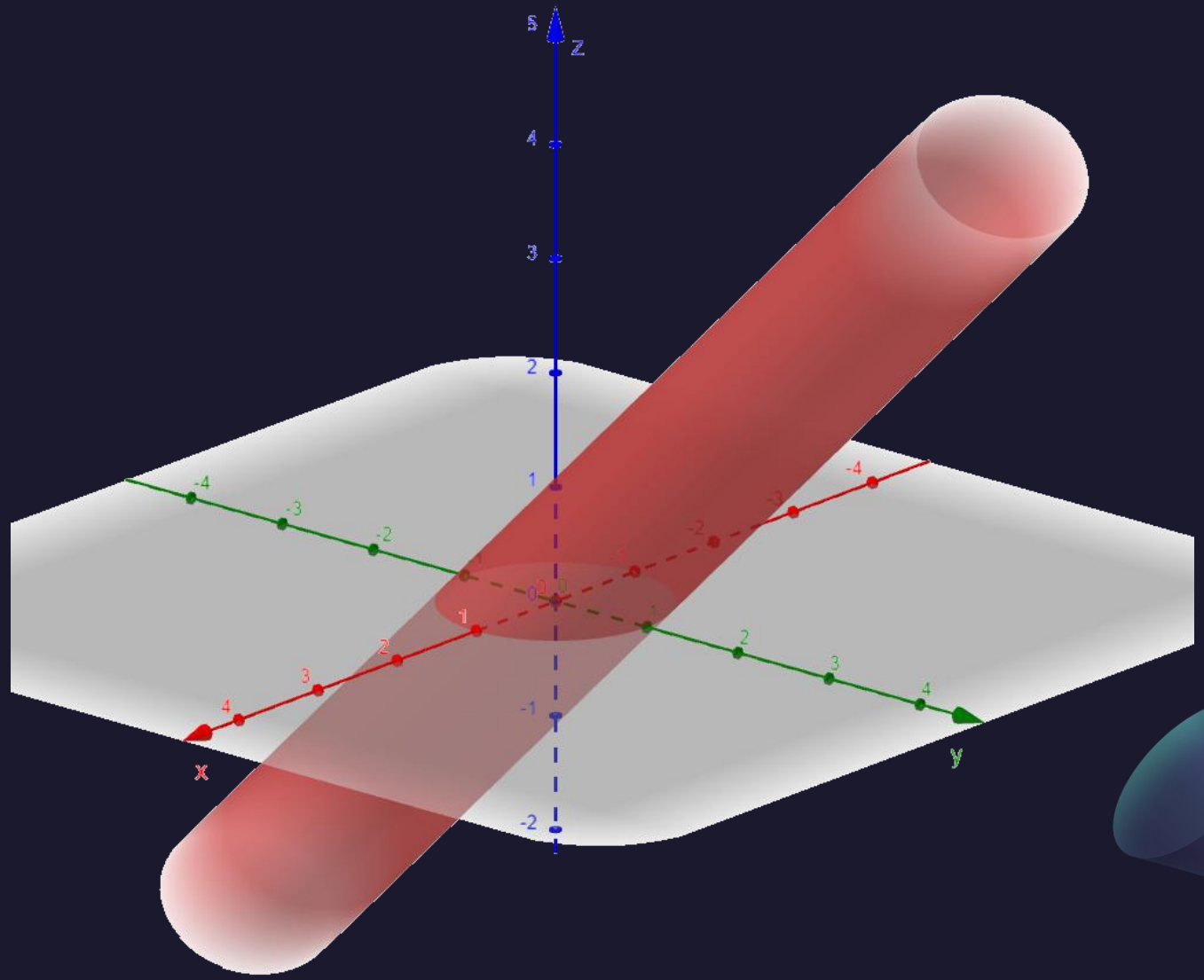


Quadric Surfaces

A **quadric surface** is the graph of a second-degree equation in three variables x, y, and z. The most general such equation is

$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + Gx + Hy + Iz + J = 0$$

where $A, B, C, \ldots, J$ are constants, but by translation and rotation it can be brought into one of the two *standard forms*

$$Ax^2 + By^2 + Cz^2 + J = 0$$

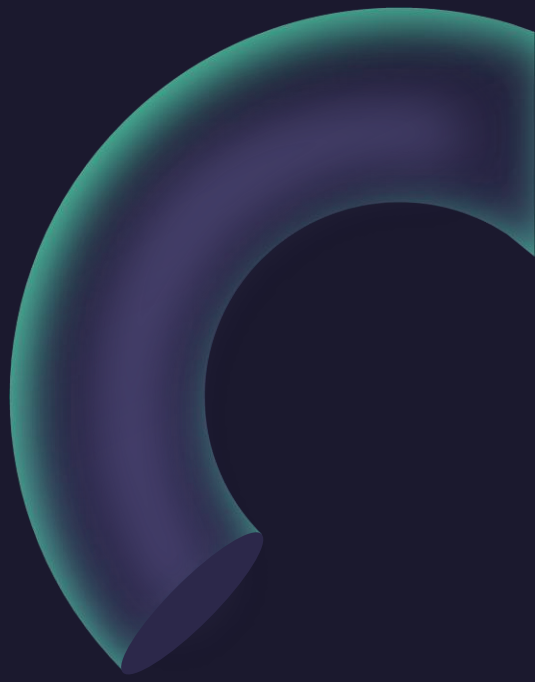
or

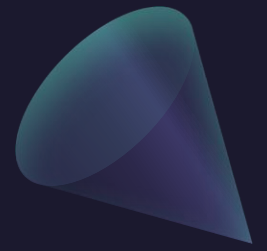
$$Ax^2 + By^2 + Iz = 0$$

How to sketch a quadric surface?

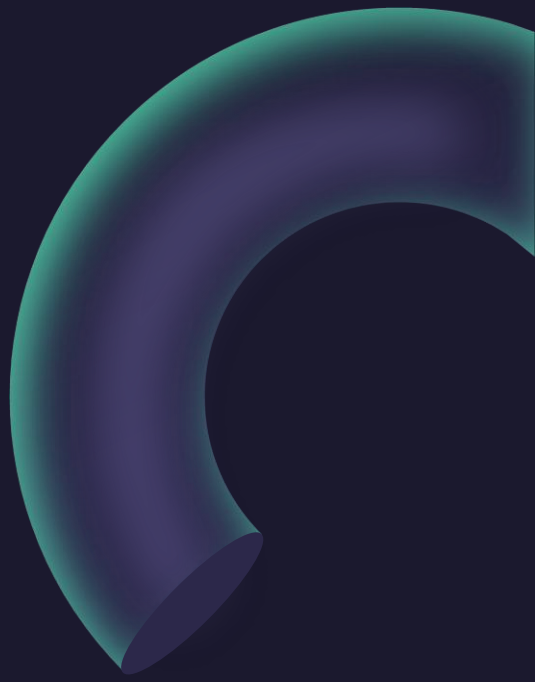
Traces or Cross Sections of a Surface

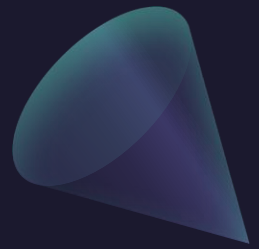
Example: Sketch the surface $x^2 + 2y^2 + 3z^2 = 1$.



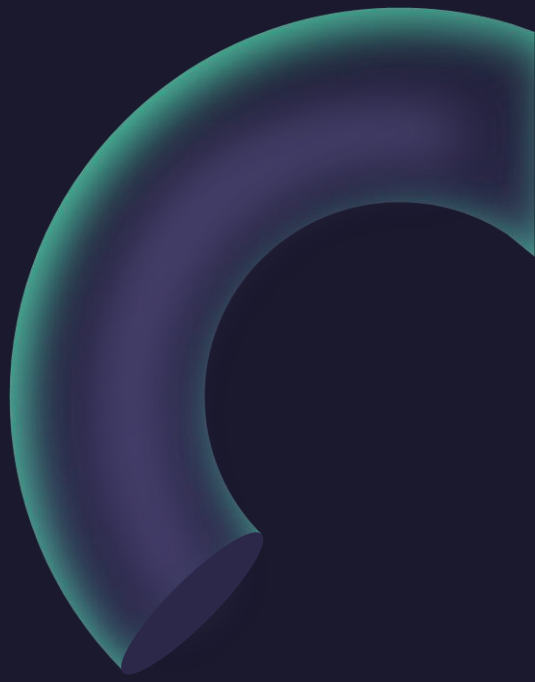


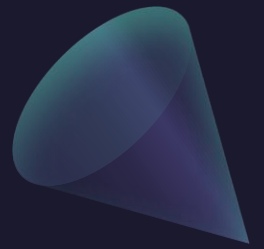
Example: Sketch the surface $z = 4x^2 + y^2$.



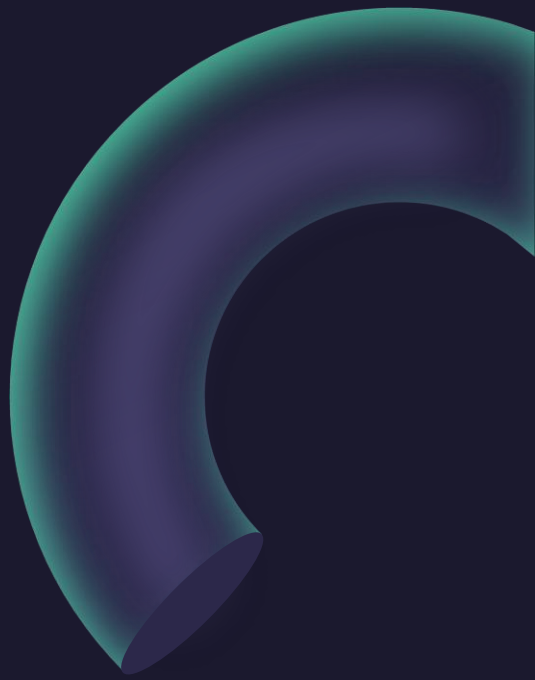


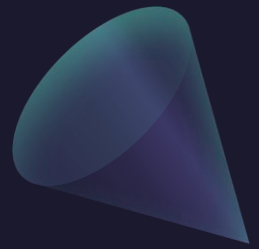
Example: Sketch the surface $z = x^2 - y^2$.



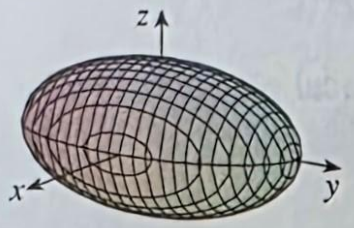
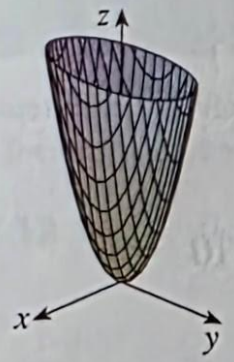


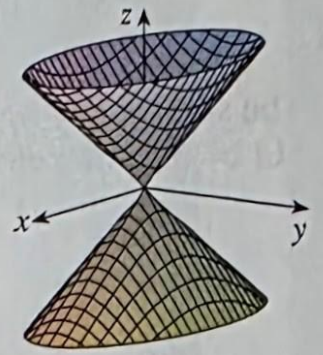
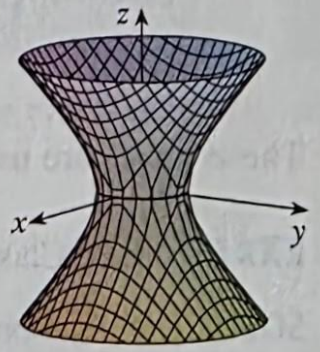
Example: Sketch the surface $x^2 + y^2 - z^2 = 1$.





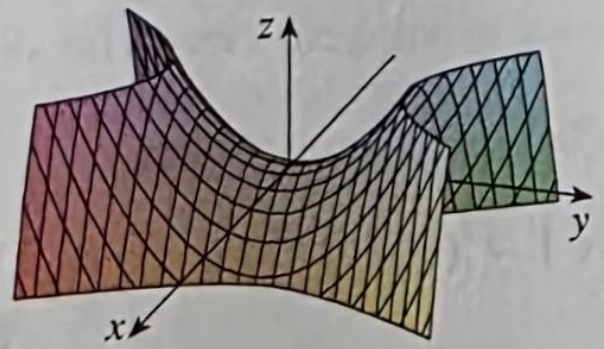
Types of Quadrics (1)

Surface	Equation
Ellipsoid 	$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ All traces are ellipses. If $a=b=c$, the ellipsoid is a sphere.
Elliptic Paraboloid 	$\frac{z}{c}=\frac{x^2}{a^2}+\frac{y^2}{b^2}$ Horizontal traces are ellipses. Vertical traces are parabolas. The variable raised to the first power indicates the axis of the paraboloid.

Surface	Equation
Cone 	$\frac{z^2}{c^2}=\frac{x^2}{a^2}+\frac{y^2}{b^2}$ Horizontal traces are ellipses. Vertical traces in the planes $x=k$ and $y=k$ are hyperbolas if $k\neq 0$ but are pairs of lines if $k=0$.
Hyperboloid of One Sheet 	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ Horizontal traces are ellipses. Vertical traces are hyperbolas. The axis of symmetry corresponds to the variable whose coefficient is negative.

Types of Quadrics (2)

Hyperbolic Paraboloid



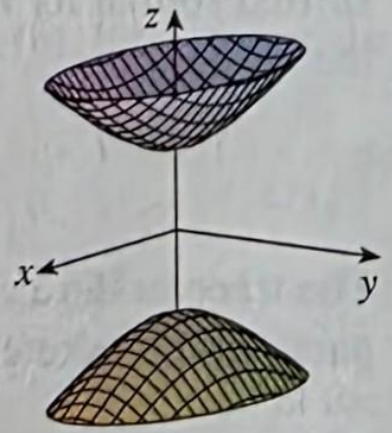
$$\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$

Horizontal traces are hyperbolas.

Vertical traces are parabolas.

The case where $c < 0$ is illustrated.

Hyperboloid of Two Sheets



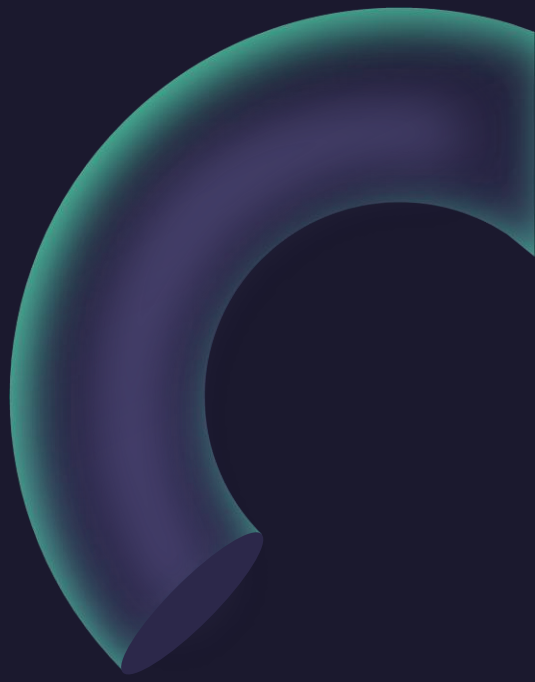
$$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

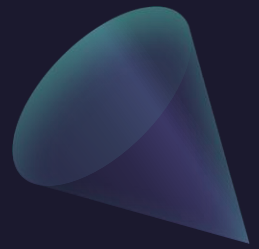
Horizontal traces in $z = k$ are ellipses if $k > c$ or $k < -c$.

Vertical traces are hyperbolas.

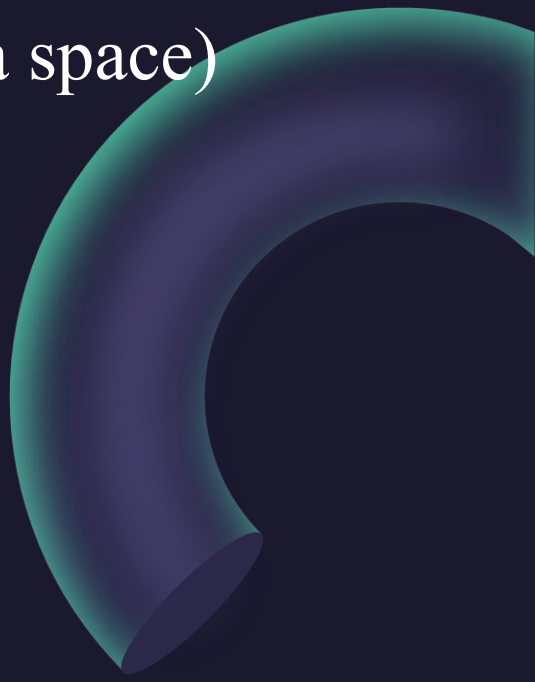
The two minus signs indicate two sheets.

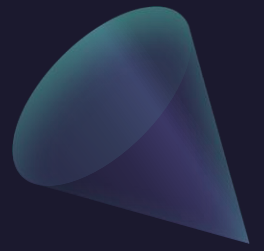
Example: Sketch the surface $x^2 + 2z^2 - 6x - y + 10 = 0$.





Example: Sketch the surface $x^2 + 2z^2 - 6x - y + 10 = 0$. (Extra space)

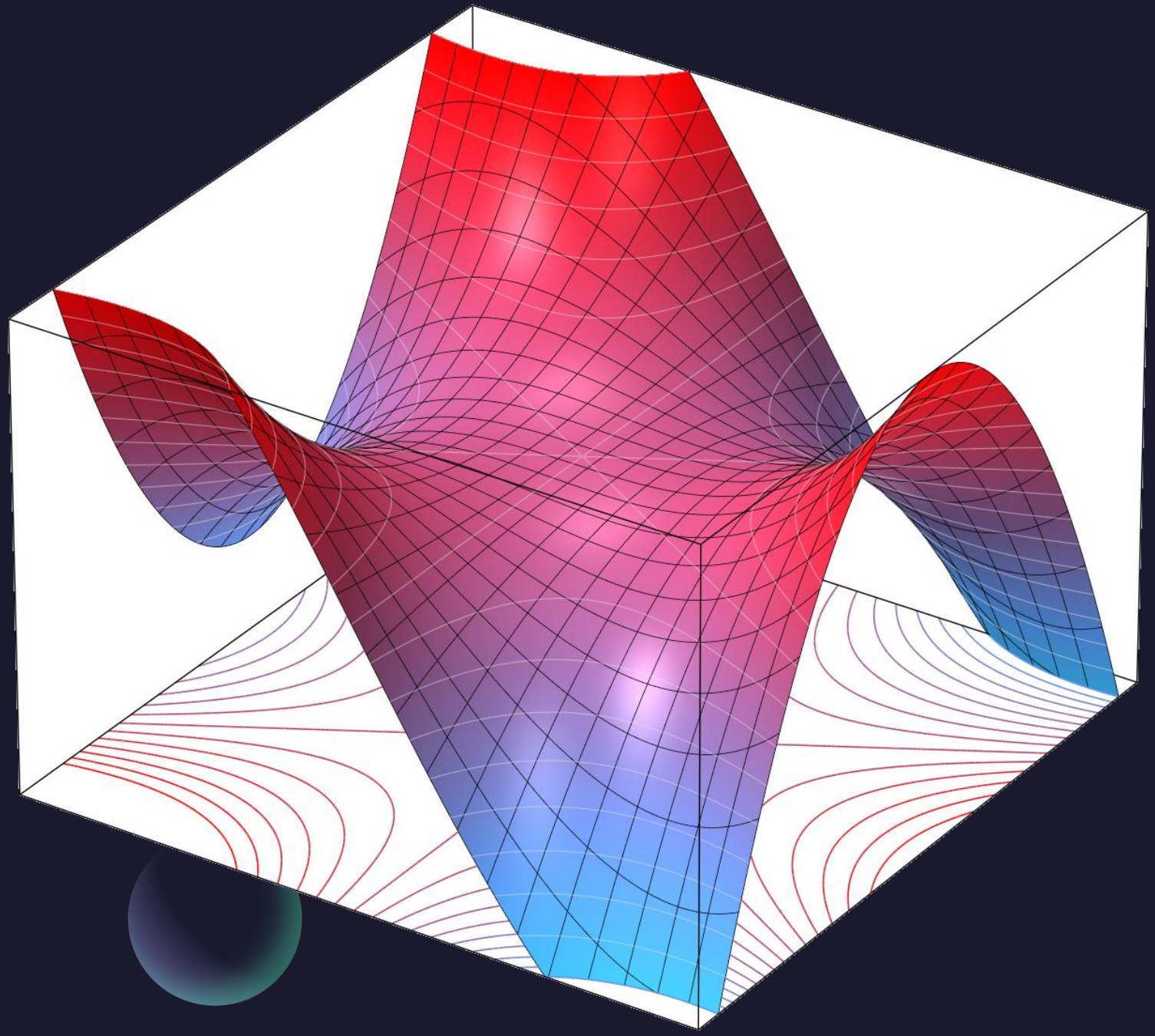




Questions?

Thank you

Until next time.



ALVARO: Start the recording!

“Calculus 3”

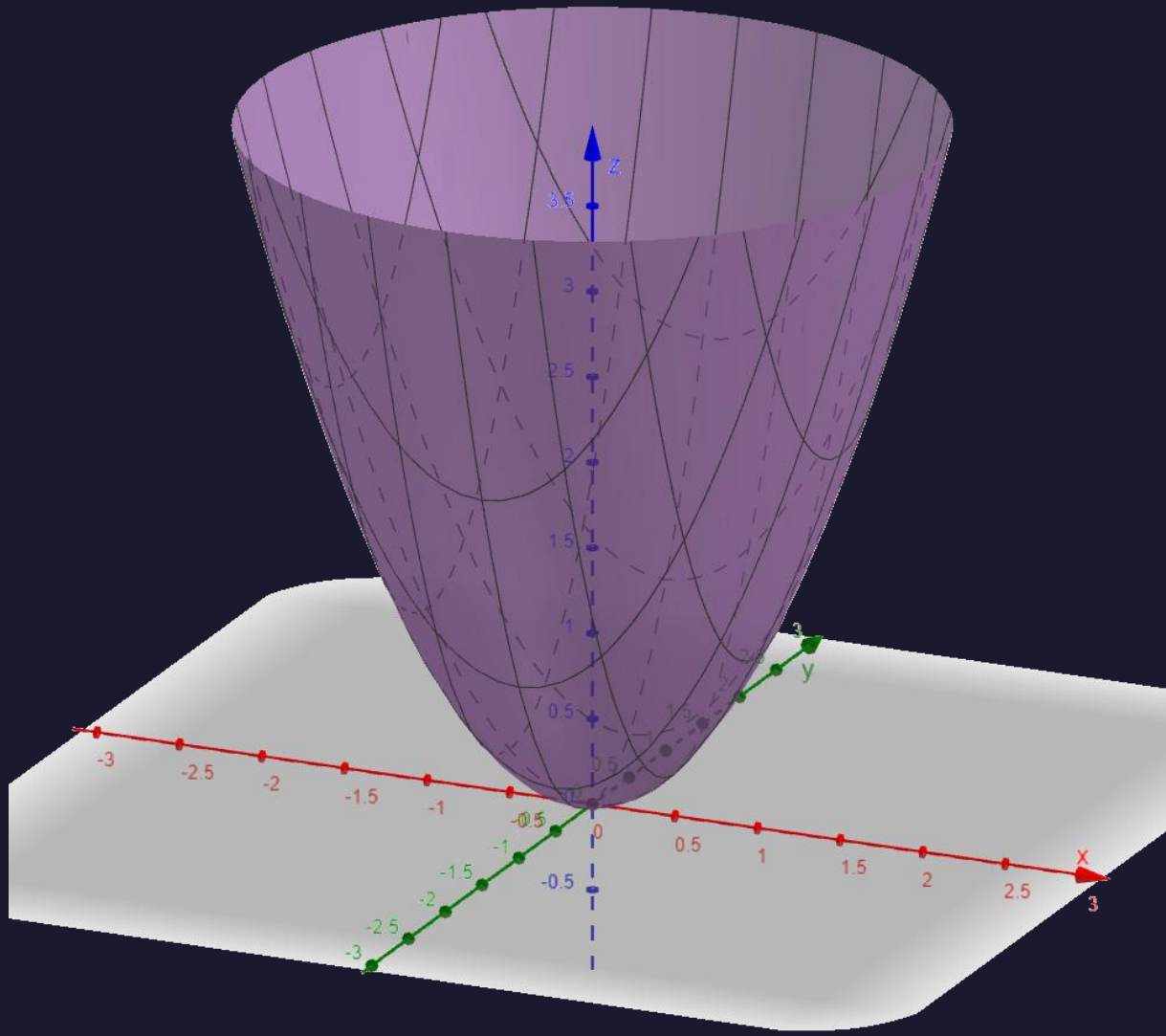
Multi-Variable Calculus

Instructor: Álvaro Lozano-Robledo

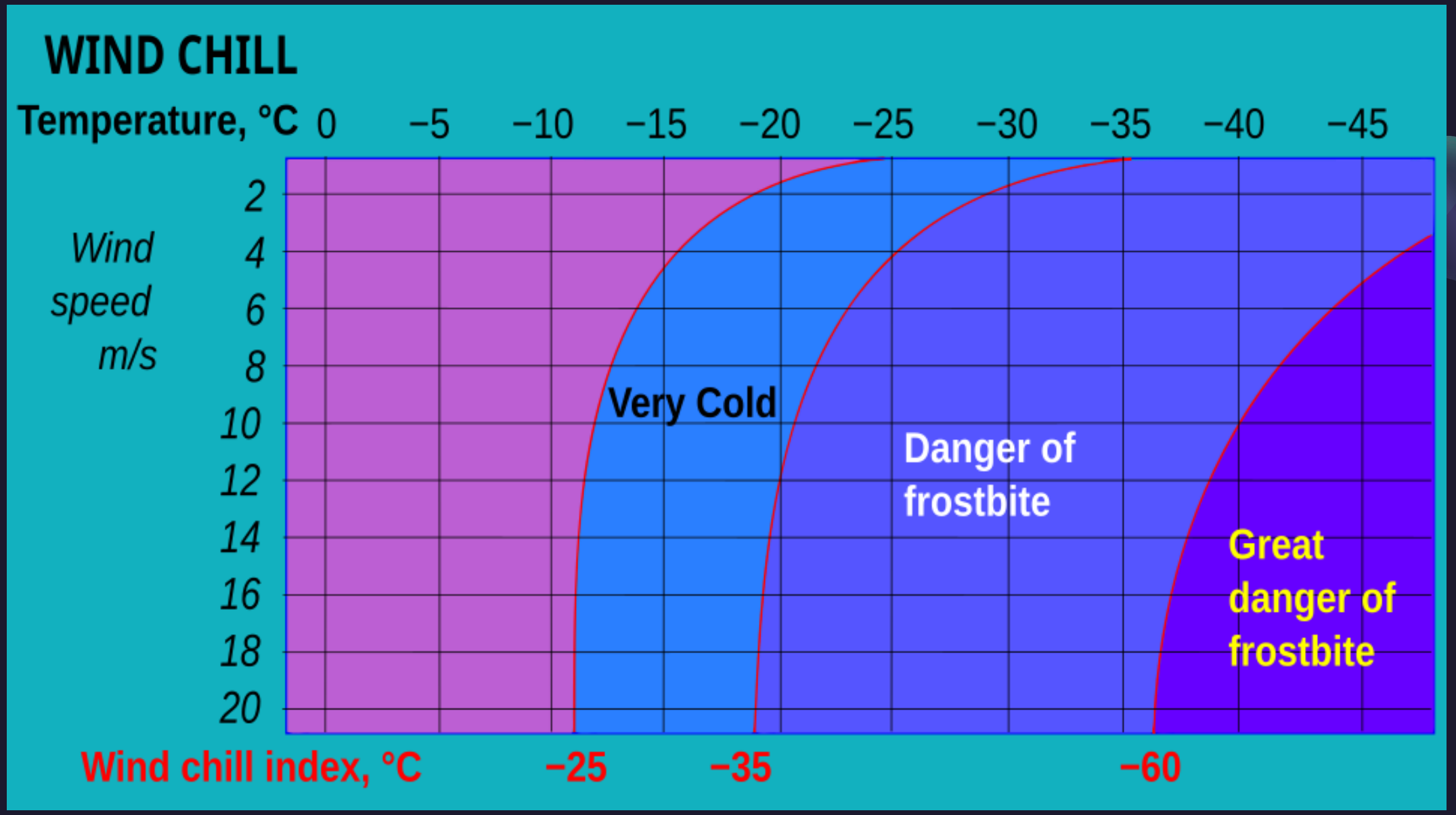
Functions of Several Variables

Today – Functions!

- Functions of Two Variables
- Domain and Range
- Graphs
- Level Curves
- Functions of More Than Two Variables



Functions of Two Variables



Functions of Two Variables

The standard wind chill formula for Environment Canada is:[3]

$$T_{\mathrm{wc}} = 13.12 + 0.6215T_{\mathrm{a}} - 11.37v^{+0.16} + 0.3965T_{\mathrm{a}}v^{+0.16},$$

where T_{wc} is the wind chill index, based on the Celsius temperature scale; T_{a} is the air temperature in degrees Celsius; and v is the wind speed at 10 m (33 ft) standard anemometer height, in kilometres per hour.[11]

When the temperature is −20 °C (−4 °F) and the wind speed is 5 km/h (3 mph), the wind chill index is −24. If the temperature remains at −20 °C and the wind speed increases to 30 km/h (19 mph), the wind chill index falls to −33.

The equivalent formula in US customary units is:[12][3]

$$T_{\mathrm{wc}} = 35.74 + 0.6215T_{\mathrm{a}} - 35.75v^{+0.16} + 0.4275T_{\mathrm{a}}v^{+0.16},$$

where T_{wc} is the wind chill index, based on the Fahrenheit scale; T_{a} is the air temperature in degrees Fahrenheit; and v is the wind speed in miles per hour.[13]

Functions of Two Variables, Domain, and Range

Definition

A **function f of two variables** is a rule that assigns to each ordered pair of real numbers (x, y) in a set D a unique real number denoted by $f(x, y)$. The set D is the **domain** of f and its **range** is the set of values that f takes on, that is,

$\{f(x, y) \mid (x, y) \in D\}$.

Functions of Two Variables, Domain, and Range

Example: Sketch the domain of the function, and evaluate at (3,2)

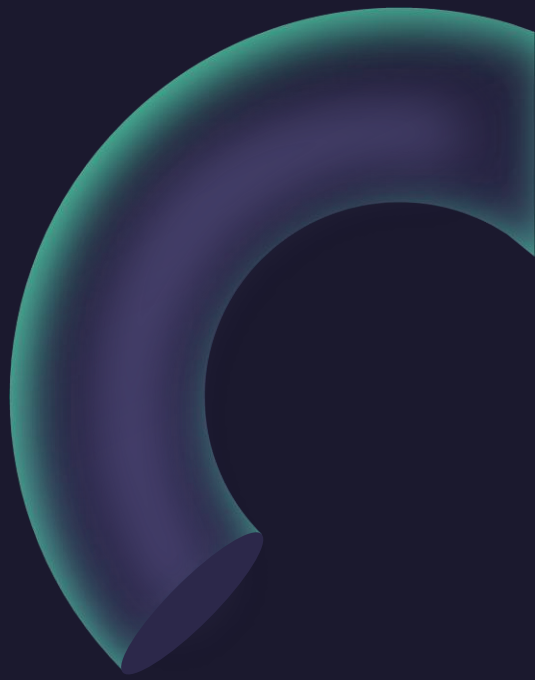
$$f(x,y) = \frac{\sqrt{x+y+1}}{x-1}$$

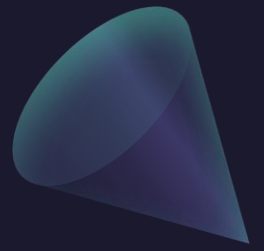
Example: Sketch the domain of the function, and evaluate at (3,2)

$$f(x,y) = x \cdot \ln(y^2 - x)$$

Example: Find the domain and range of the function

$$f(x,y) = \sqrt{9 - x^2 - y^2}$$





Questions?

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Until next time.