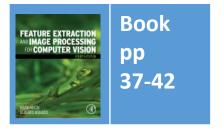
Lecture 2 Image Formation

COMP3204 Computer Vision

What is inside an image?







Content

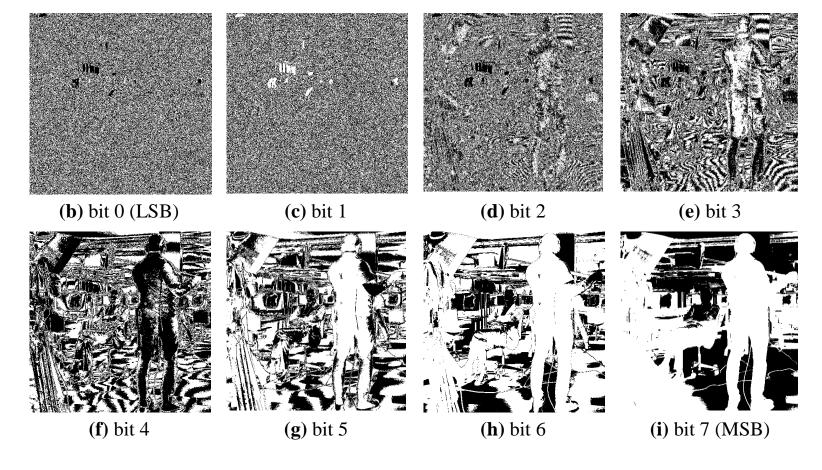
- 1. How is an image formed?
- 2. What restrictions are there on image formation?
- 3. Bonjour Monsieur Fourier

Decomposing an image into its bits

The Most Significant Bit carries the most information where as bit 0 is noise



(a) original image







Effects of differing image resolution







(a) 64×64

(b) 128×128

(c) 256×256



Low resolution lose information but N×N points implies much storage

How do we choose an appropriate value for *N*?

Jean Baptiste Joseph Fourier

- Any periodic function is the result of adding up sine and cosine waves of different frequencies
- Sceptical? Yeah, so were Lagrange and Laplace. Good company eh?
- "Fourier's treatise is one of the very few scientific books that can never be rendered antiquated by the progress of science"
 James Clerk Maxwell 1878
- Fourier 10 Laplace 0 ...



What are 2D waves?

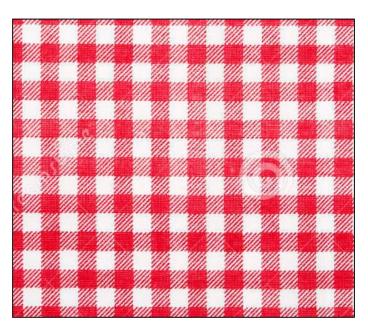
2D waves are along x and y axes simultaneously

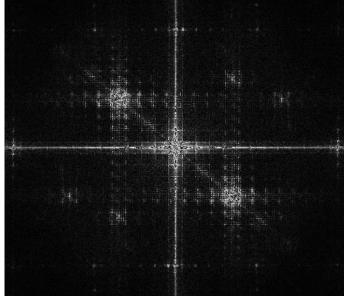




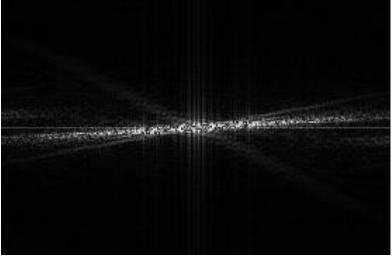
and in terms of frequency

• N.b. colour immaterial (just for visuals)













Step up Monsieur Fourier...

$$Fp(f) = \mathcal{F}(p(t)) = \int_{-\infty}^{\infty} p(t)e^{-jft}dt$$

Whoa! WTF! Where from....

First, we have that the FT is a function a() of a time-variant signal p(t)

The Fourier transform is then

$$Fp = a(p(t))$$

The transform is a function of frequency so

$$Fp(f) = a(p(t))$$

 \mathcal{F} stands for the Fourier transform so

$$Fp(f) = \mathcal{F}(p(t)) = a(p(t))$$

The function a() is actually an integral

$$Fp(f) = \mathcal{F}(p(t)) = \int_{-\infty}^{\infty} p(t) \cos(t) dt$$

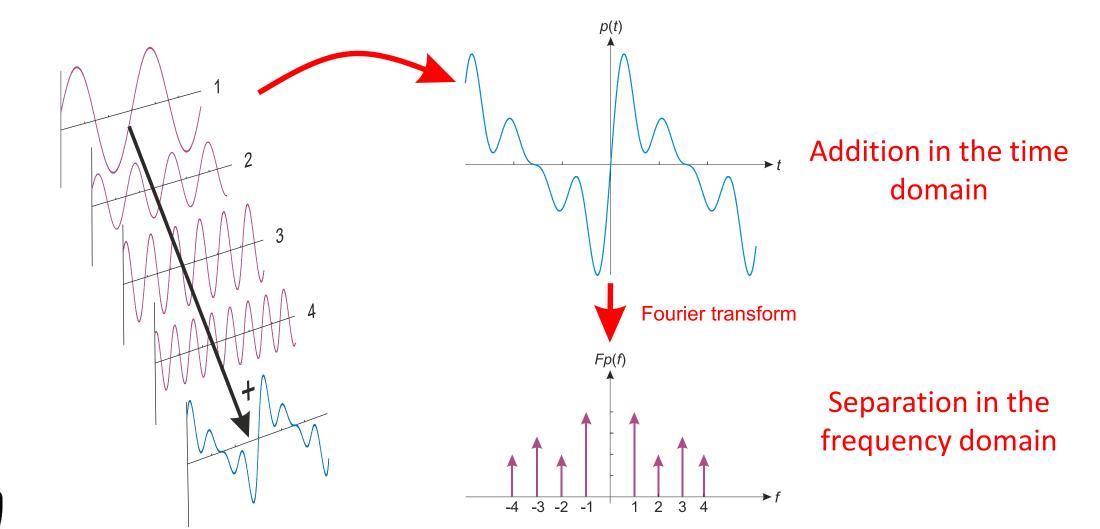
cas(t) describes cosine and sine waves, so

$$Fp(f) = \mathcal{F}(p(t)) = \int_{-\infty}^{\infty} p(t)e^{-jft}dt$$



where cas = 'cos and sin' since $e^{-jft} = \cos(ft) - j\sin(ft)$ and where j is the complex operator $j = \sqrt{-1}$

What does the Fourier transform do?





Fireside time

How did I get into computer vision?

Zut alors! On doit applique ca

• Pulse
$$p(t) = \begin{vmatrix} A & \text{if } -T/2 \le t \le T/2 \\ 0 & \text{otherwise} \end{vmatrix}$$

• Use Fourier
$$Fp(\omega) = \int_{-T/2}^{T/2} Ae^{-j\omega t} dt$$

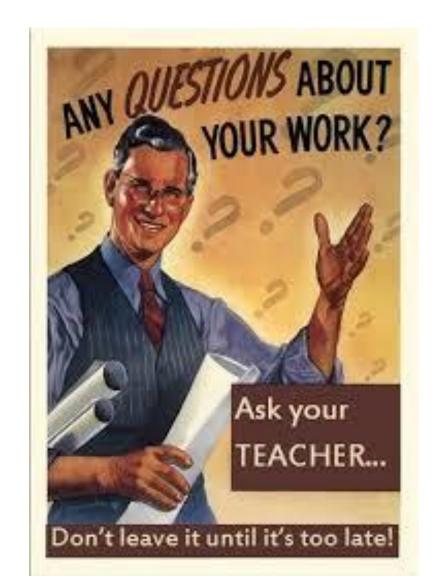
• Evaluate integral
$$Fp(\omega) = -\frac{Ae^{-j\omega T/2} - Ae^{j\omega T/2}}{j\omega}$$

 $Fp(\omega) = \begin{vmatrix} \frac{2A}{\omega} \sin\left(\frac{\omega T}{2}\right) & \text{if } \omega \neq 0 \\ AT & \text{if } \omega = 0 \end{vmatrix}$ And get result





Google "are you frightened of maths"



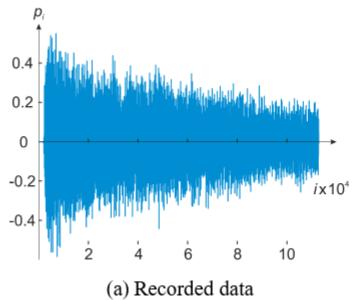


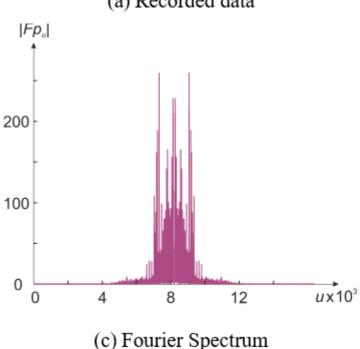
Hard day?

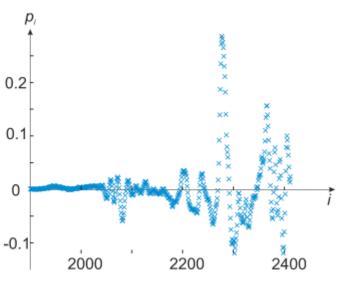
Let's see the Fourier transform of the Hard Day's night chord

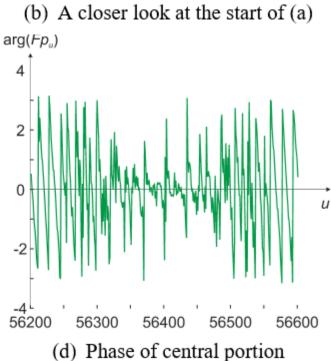


Apparently, the piano is more dominant than expected

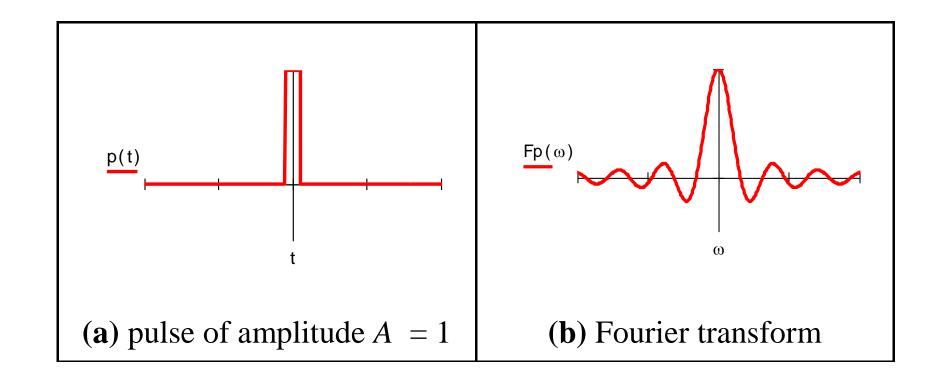






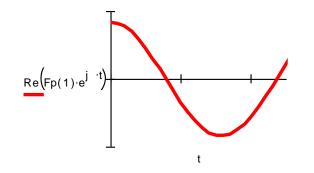


A pulse and its Fourier transform

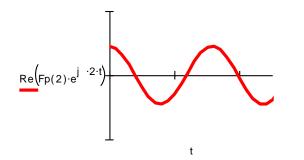




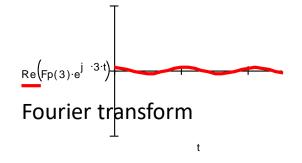
This is the inverse Fourier transform



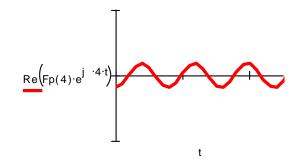
(a) contribution for $\omega = 1$



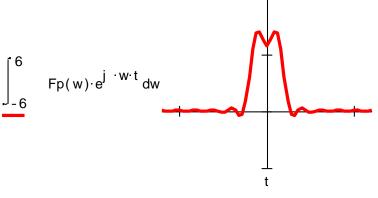
(b) contribution for $\omega = 2$



(c) contribution for $\omega = 3$



(d) contribution for $\omega = 4$

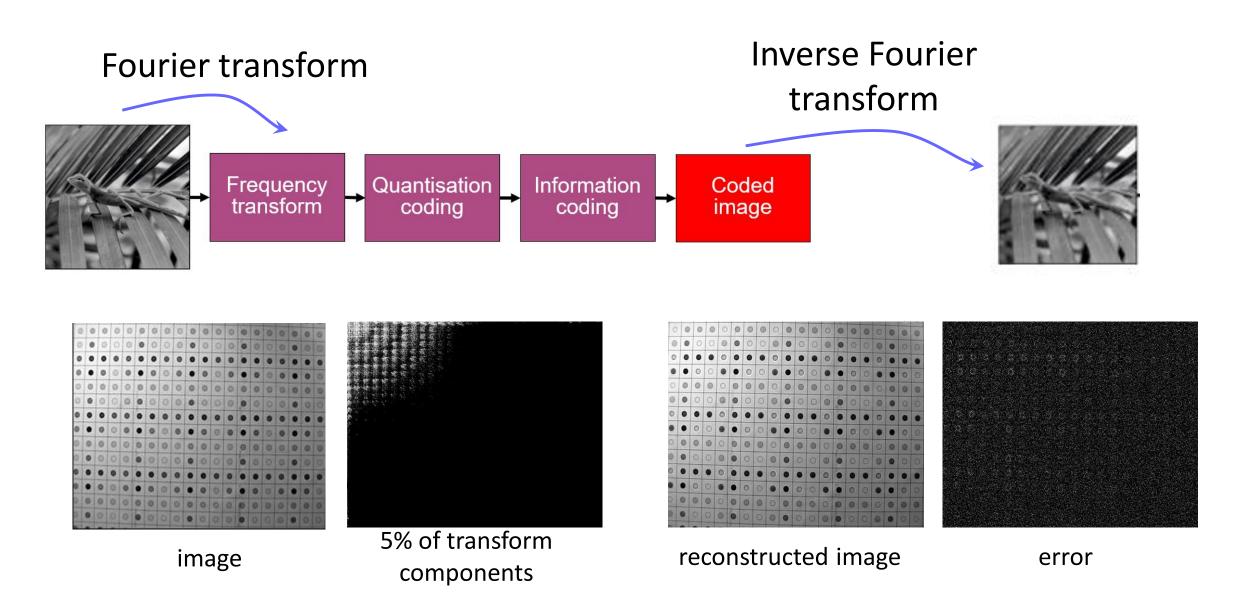


(e) reconstruction by integration



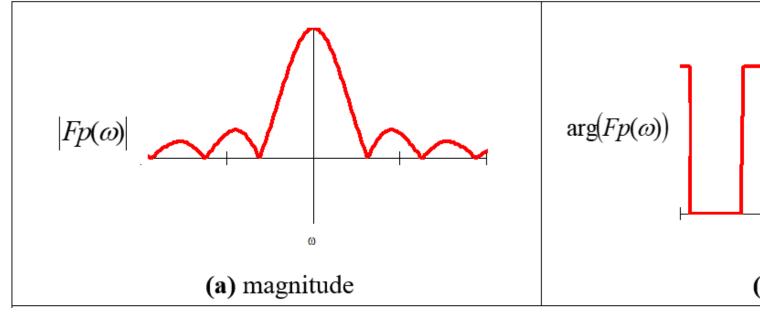


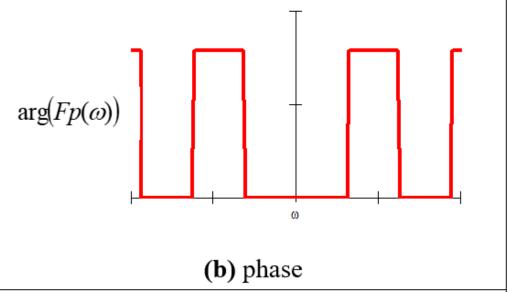
Inverse Fourier transform Is used for reconstruction



Magnitude and phase of Fourier transform of a pulse

$$Fp(\omega) = \int_{-\infty}^{\infty} p(t)e^{-j\omega t}dt = \text{Re}(Fp(\omega)) + j \text{Im}(Fp(\omega))$$









$$|Fp(\omega)| = \sqrt{\operatorname{Re}(Fp(\omega))^{2} + \operatorname{Im}(Fp(\omega))^{2}}$$

$$\arg(Fp(\omega)) = \tan^{-1}\left(\frac{\operatorname{Im}(Fp(\omega))}{\operatorname{Re}(Fp(\omega))}\right)$$

Using Gait as a Biometric, via Phase-Weighted Magnitude Spectra

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Email: dc95r@ecs.soton.ac.uk and msn@ecs.soton.ac.uk

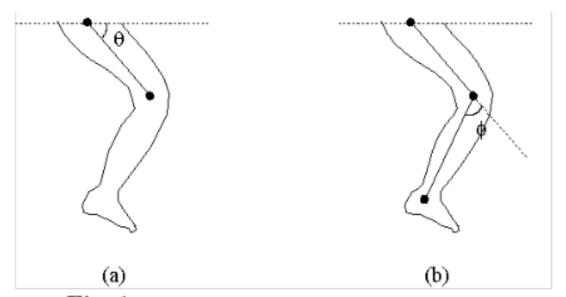


Fig. 1. (a) Hip and (b) Knee rotation angles.

Gait patterns (angle of swinging leg)

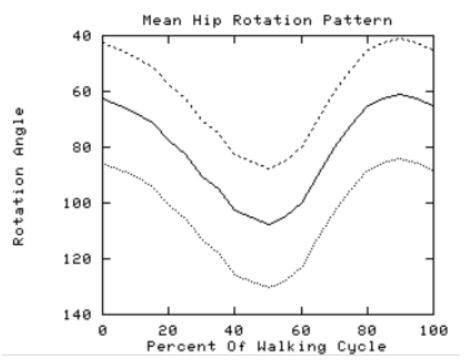


Fig. 2. Variation in Hip Rotation.



Fig. 3. Example Image of Walking Subject.

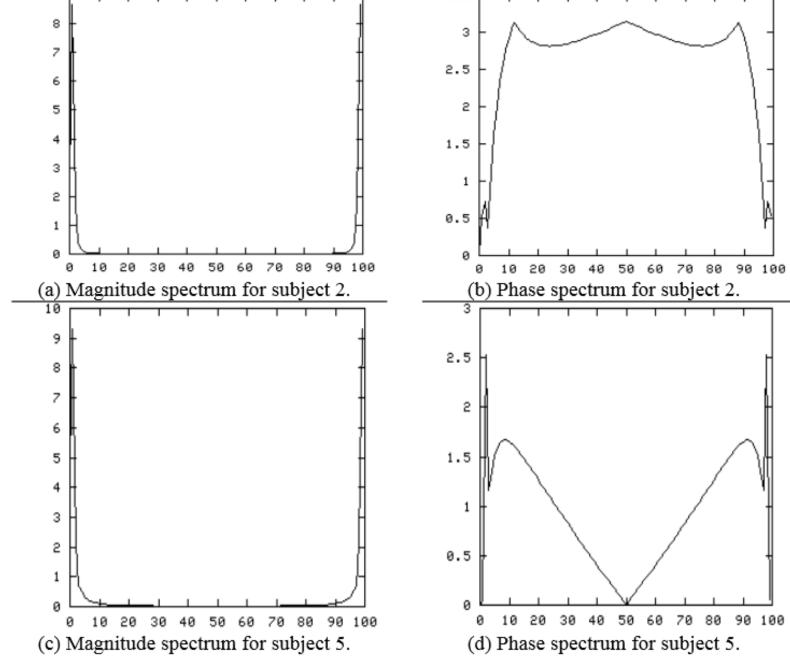
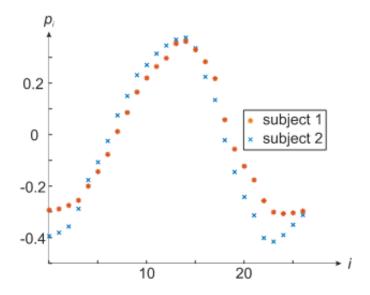
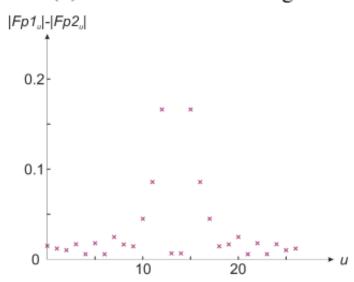


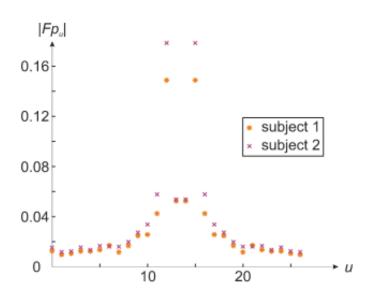
Fig. 6. Phase and Magnitude Gait Spectra.



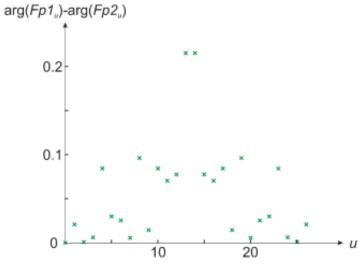
(d) Inclination of left thigh



(f) Difference between magnitudes of DFT



(e) Magnitude of DFT of inclination



(g) Difference between phases of DFT

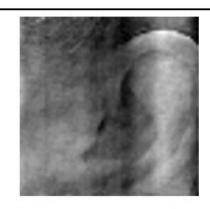
Illustrating the importance of phase



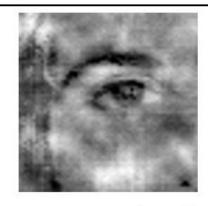
(a) eye image



(b) ear image



(c) reconstruction from magnitude(eye) and phase(ear)



(d) reconstruction from magnitude(ear) and phase(eye)



Takeaway time – four main points

- 1 sampling data is not as simple as it appears
- 2 sampling affects space and brightness
- 3 Fourier allows us to understand frequency

Next, Fourier will allow us to understand sampling



