Lecture 2 Image Formation

COMP3204 Computer Vision

What is inside an image?

Book
pp
37-42

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School of Electronics and Computer Science

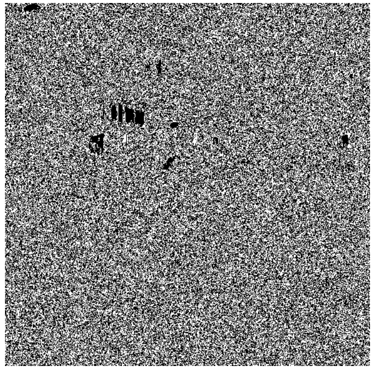
Content

1. How is an image formed?
2. What restrictions are there on image formation?
3. Bonjour Monsieur Fourier …..

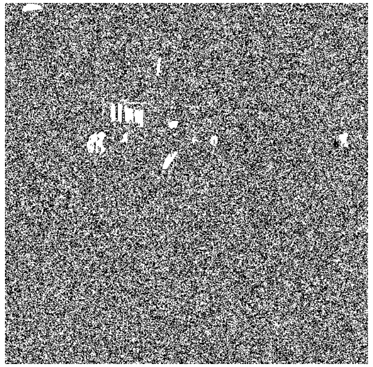
Decomposing an image into its bits

The Most Significant Bit carries the most information where as bit 0 is noise

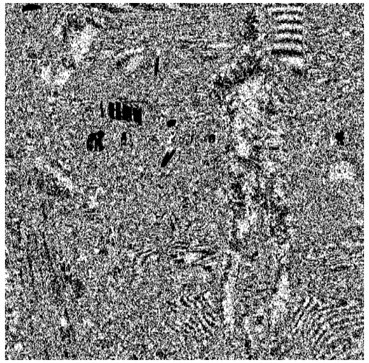
(a) original image



(b) bit 0 (LSB)

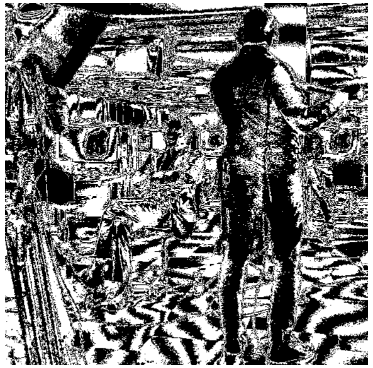


(c) bit 1

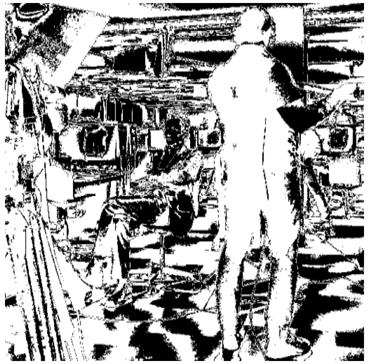


(d) bit 2

(e) bit 3



(f) bit 4



(g) bit 5

(h) bit 6

(i) bit 7 (MSB)

… and here, bit 4 is the lighting

Effects of differing image resolution

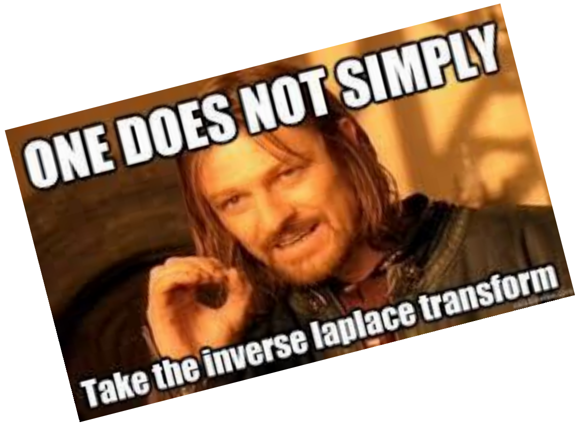
(a) 64×64 **(b)** 128×128 **(c)** 256×256

Low resolution lose information but *N*×*N* points implies much storage

How do we choose an appropriate value for *N*?

Jean Baptiste Joseph Fourier

- Any periodic function is the result of adding up sine and cosine waves of different frequencies
- Sceptical? Yeah, so were Lagrange and Laplace. Good company eh?
- "Fourier's treatise is one of the very few scientific books that can never be rendered antiquated by the progress of science" James Clerk Maxwell 1878
- Fourier 10 Laplace 0 …

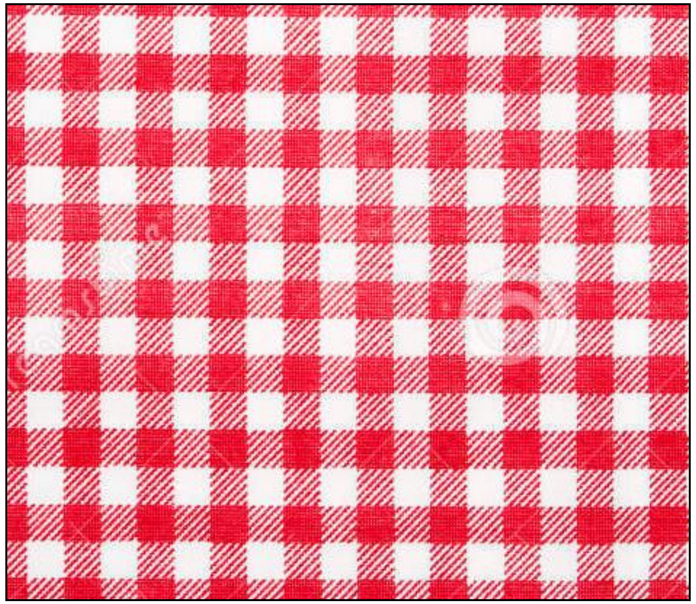


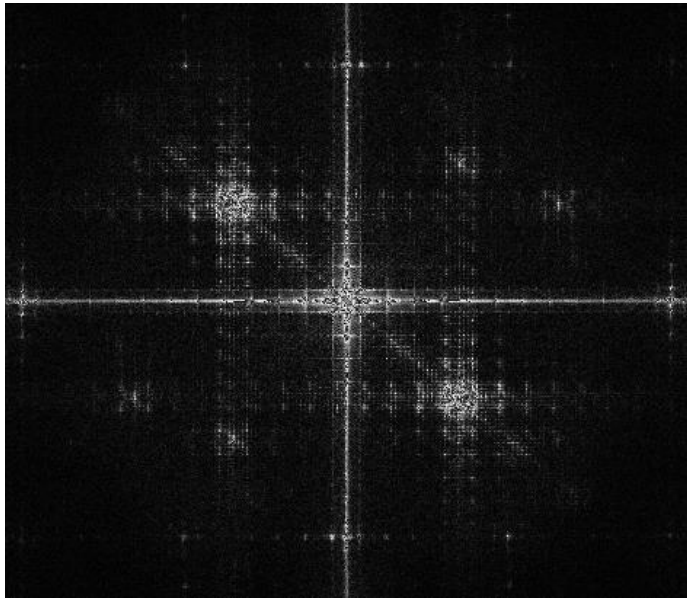
What are 2D waves?

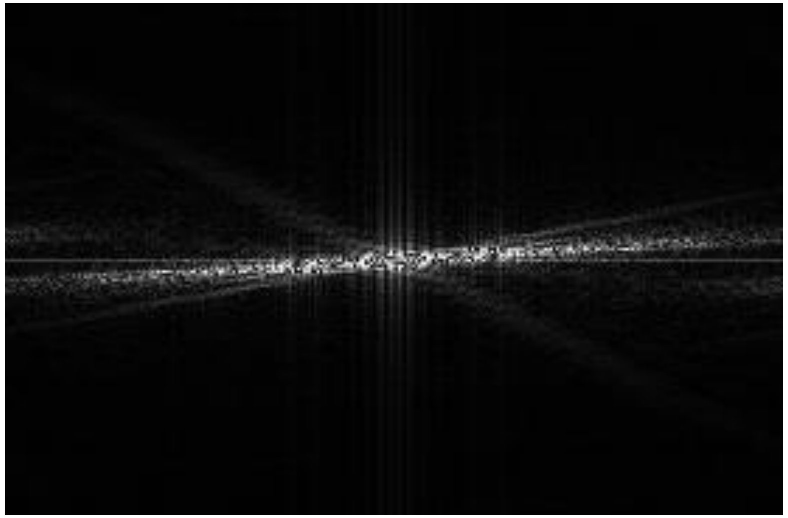
2D waves are along x and y axes simultaneously

and in terms of frequency

- N.b. colour immaterial (just for visuals)







Step up Monsieur Fourier…

$$Fp(f) = \mathcal{F}\big(p(t)\big) = \int_{-\infty}^{\infty} p(t)e^{-jft}dt$$

Whoa! WTF! Where from….

First, we have that the FT is a function $a(\quad)$ of a time-variant signal $p(t)$

The Fourier transform is then $\qquad Fp = a\big(p(t)\big)$

The transform is a function of frequency so $\qquad Fp(f) = a\big(p(t)\big)$

$\mathcal{F}$ stands for the Fourier transform so $\qquad Fp(f) = \mathcal{F}\big(p(t)\big) = a\big(p(t)\big)$

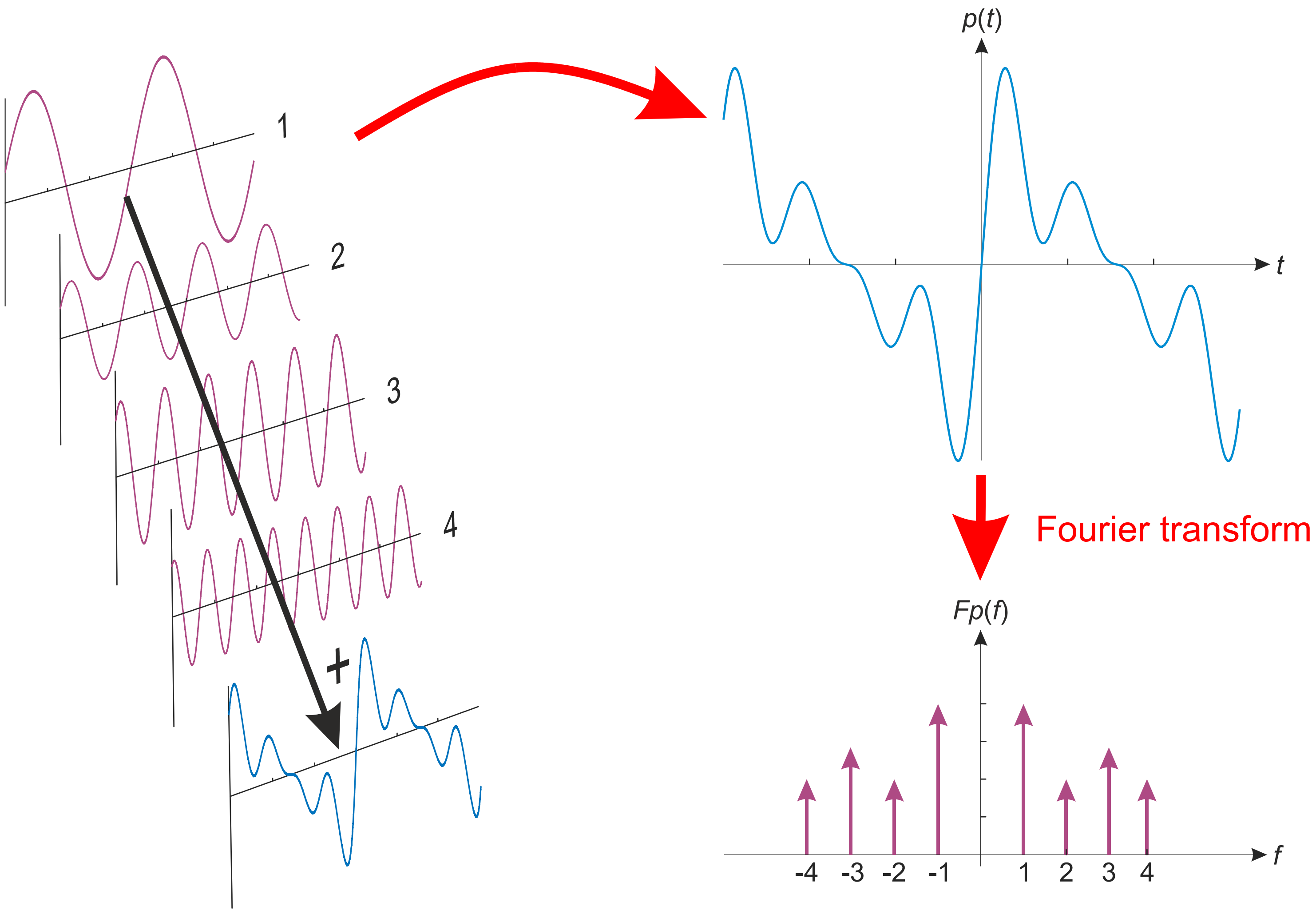
The function $a(\quad)$ is actually an integral $\qquad Fp(f) = \mathcal{F}\big(p(t)\big) = \int_{-\infty}^{\infty} p(t)\mathrm{cas}(t)dt$

$\mathrm{cas}(t)$ describes cosine and sine waves, so $\qquad Fp(f) = \mathcal{F}\big(p(t)\big) = \int_{-\infty}^{\infty} p(t)e^{-jft}dt$

where cas = ‘cos and sin’ since $e^{-jft} = \cos(ft) - j\sin(ft)$,

where j is the complex number $j = \sqrt{-1}$

What does the Fourier transform do?



Addition in the time domain

Separation in the frequency domain

Fireside time

How did I get into computer vision?

Zut alors! On doit applique ca

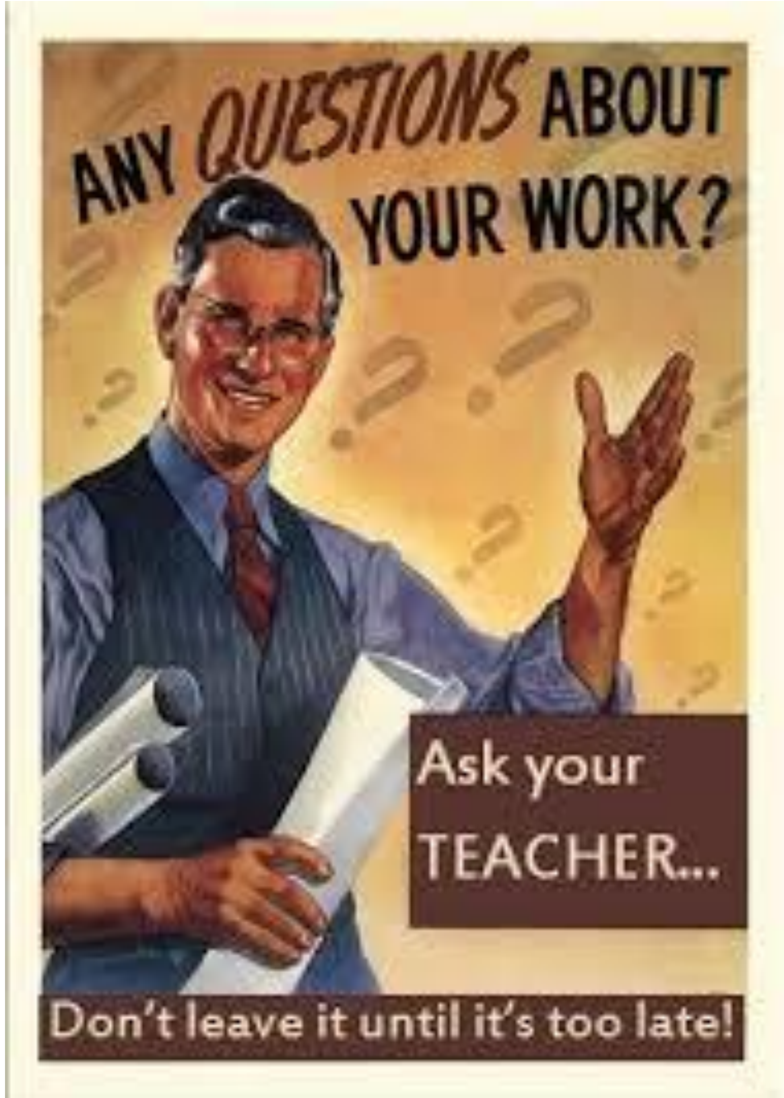
- Pulse $$p(t) = \begin{cases} A & \text{if} \quad -T/2 \le t \le T/2 \\ 0 & \text{otherwise} \end{cases}$$

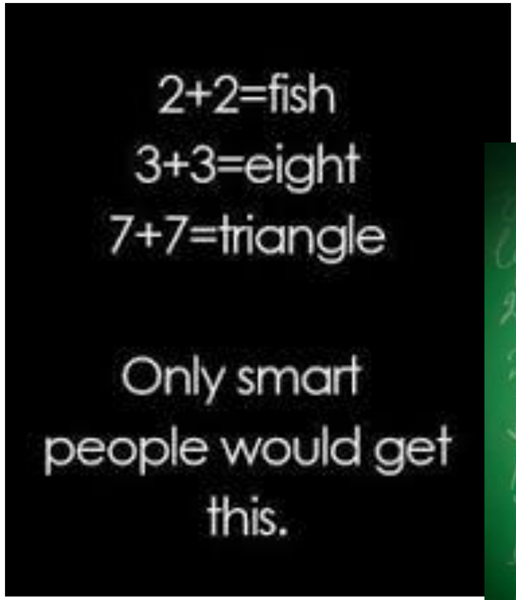
- Use Fourier $$Fp(\omega) = \int_{-T/2}^{T/2} Ae^{-j\omega t}\,dt$$

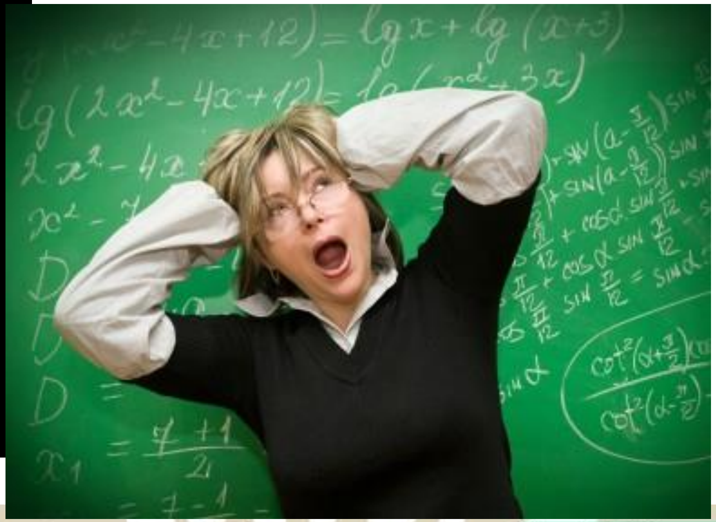
- Evaluate integral $$Fp(\omega) = -\frac{Ae^{-j\omega T/2} - Ae^{j\omega T/2}}{j\omega}$$

- And get result $$Fp(\omega) = \begin{cases} \dfrac{2A}{\omega}\sin\left(\dfrac{\omega T}{2}\right) & \text{if} \quad \omega \ne 0 \\ AT \quad \text{if} \quad \omega = 0 \end{cases}$$

Google “are you frightened of maths”

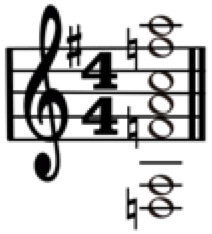




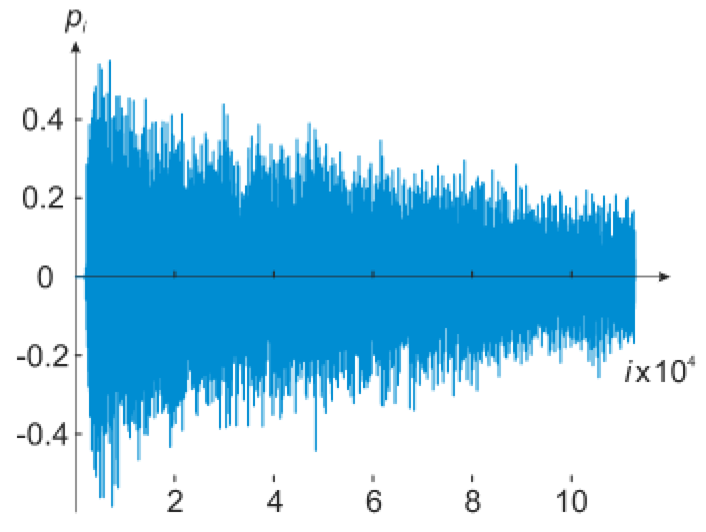


Hard day?

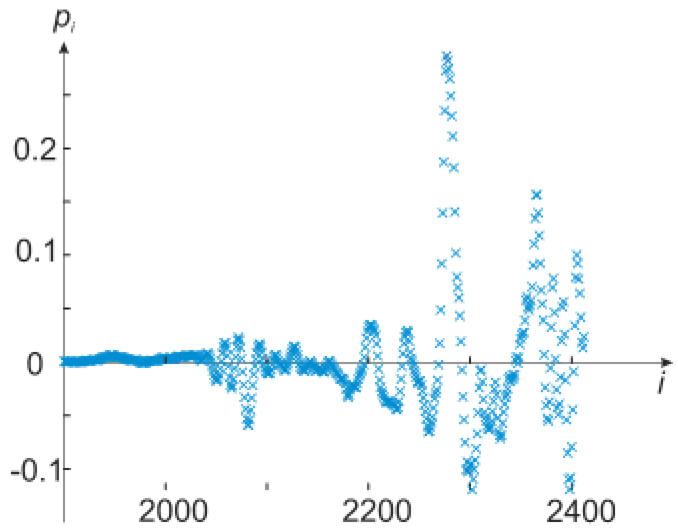
Let's see the Fourier transform of the Hard Day's night chord



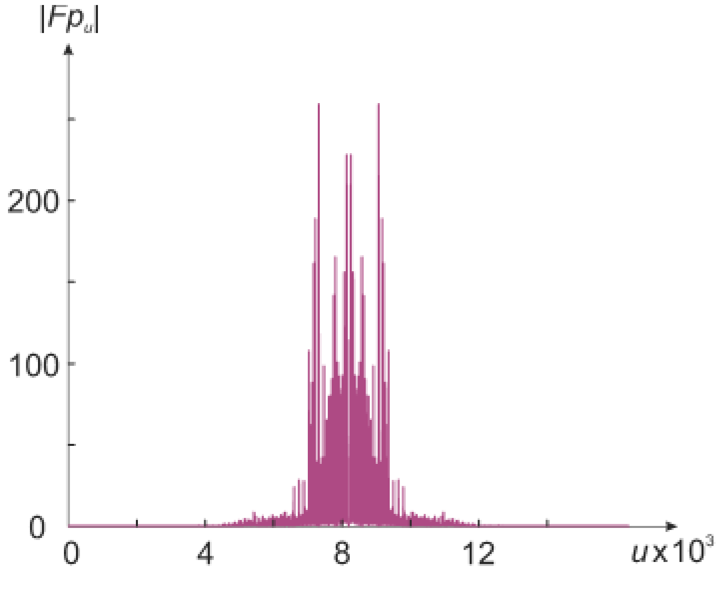
Apparently, the piano is more dominant than expected



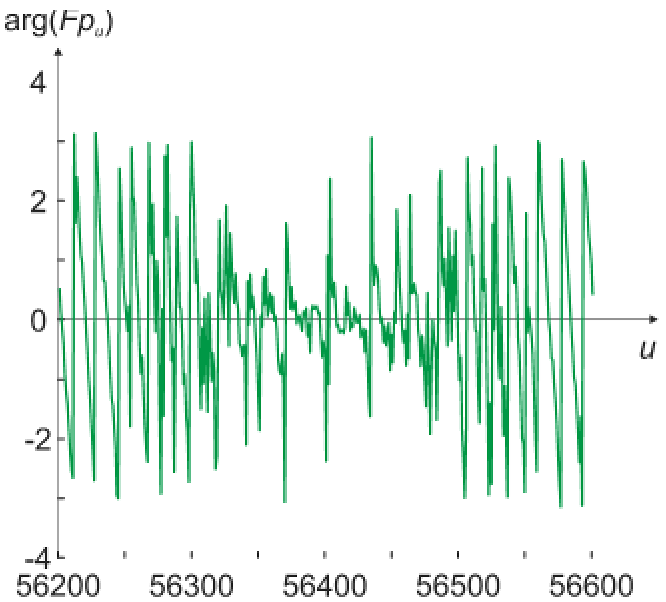
(a) Recorded data



(b) A closer look at the start of (a)

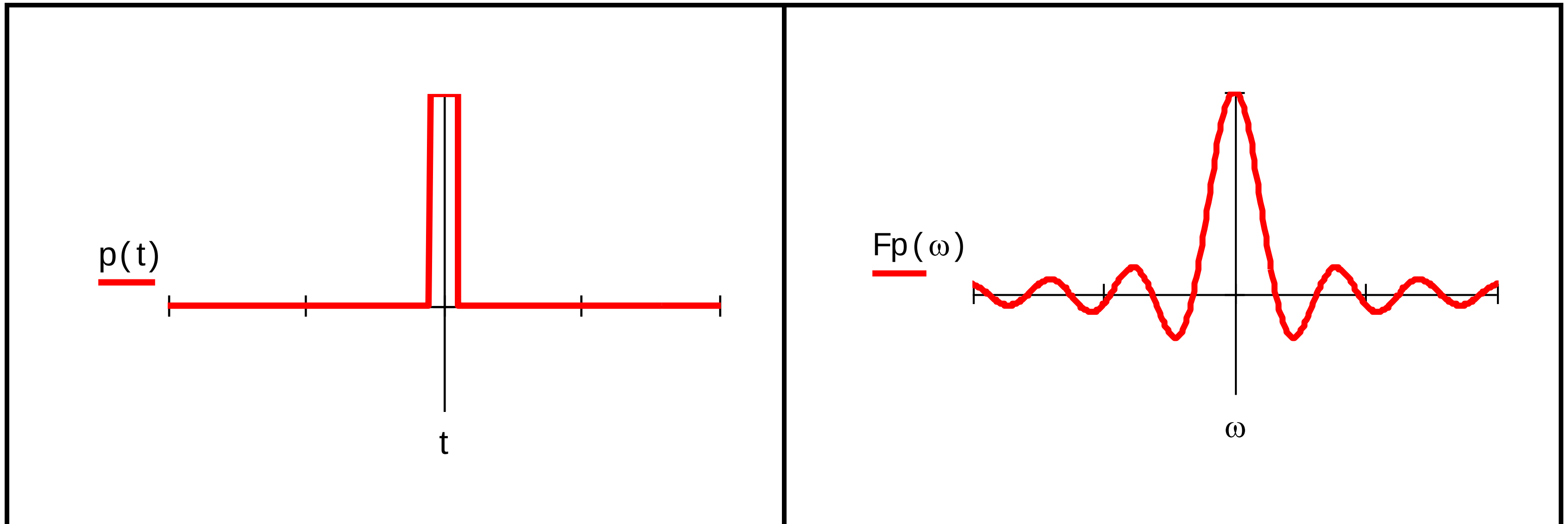


(c) Fourier Spectrum



(d) Phase of central portion

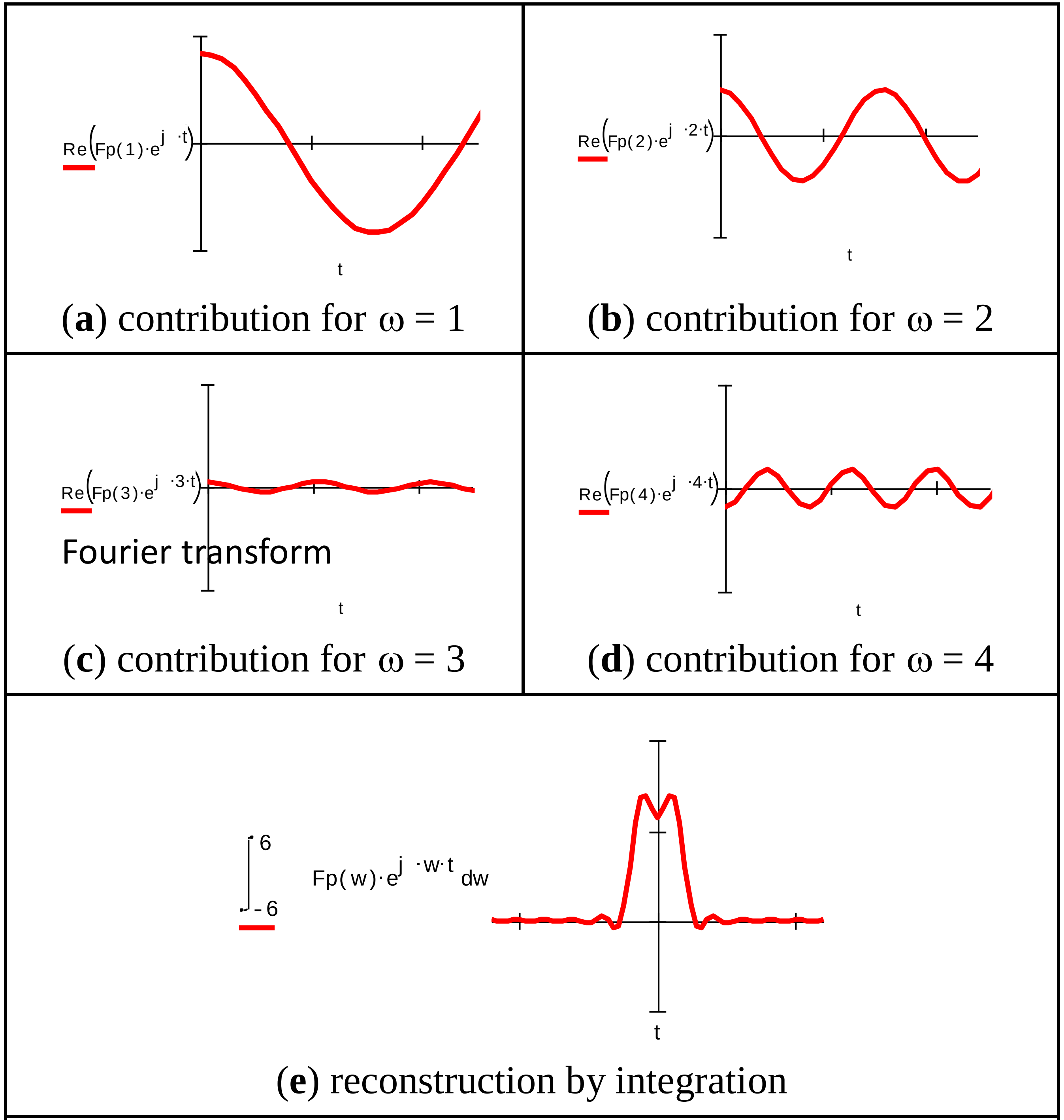
A pulse and its Fourier transform



(a) pulse of amplitude $A = 1$ **(b)** Fourier transform

Reconstructing a signal from its Fourier transform

This is the **inverse Fourier transform**



(**a**) contribution for ω = 1

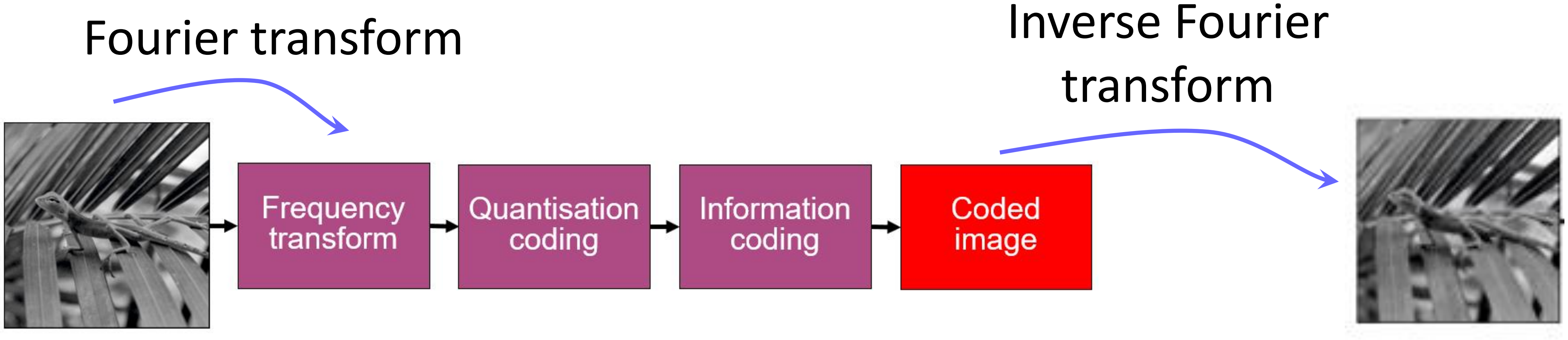
(**b**) contribution for ω = 2

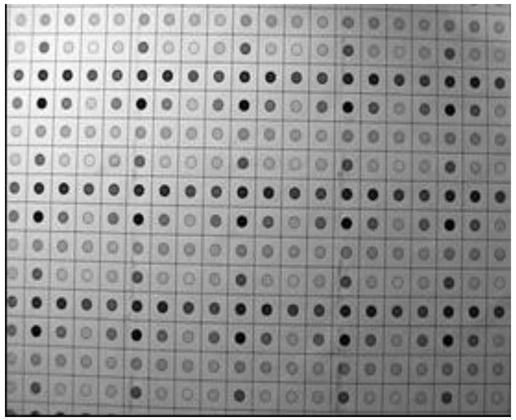
(**c**) contribution for ω = 3

(**d**) contribution for ω = 4

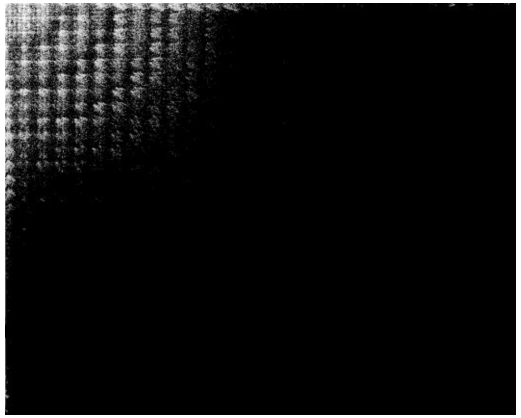
(**e**) reconstruction by integration

Inverse Fourier transform is used for reconstruction

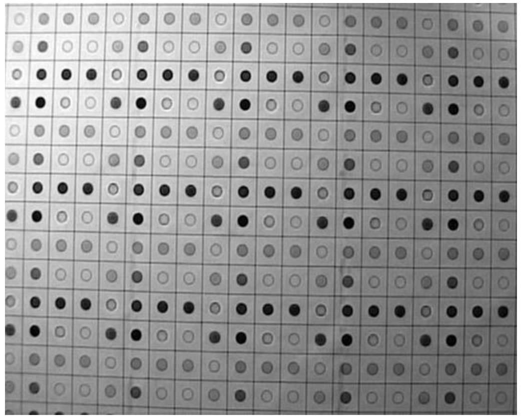




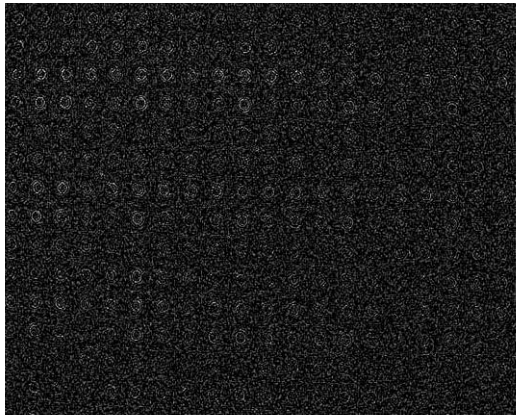
image



5% of transform components



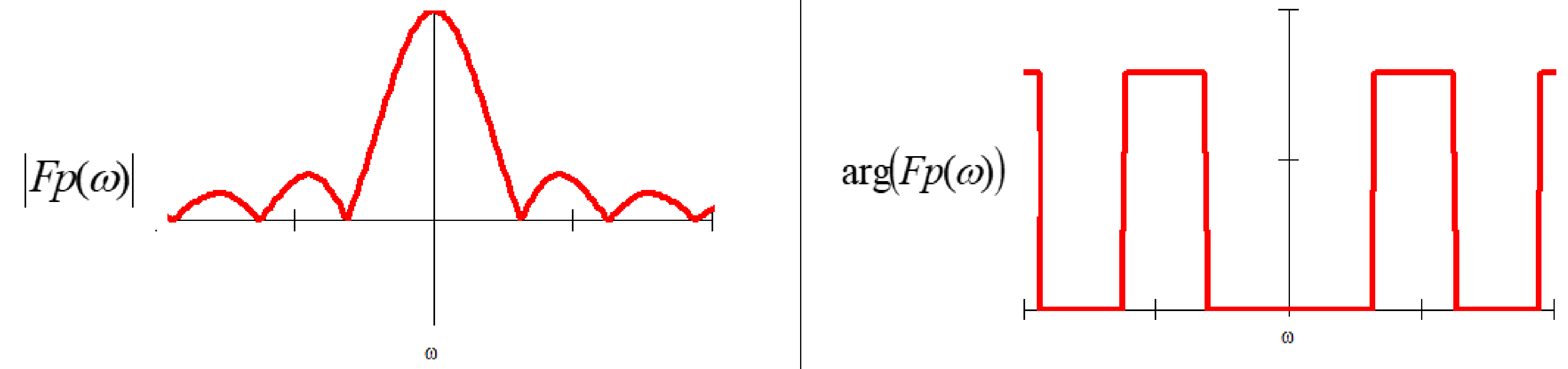
reconstructed image



error

Magnitude and phase of Fourier transform of a pulse

$$Fp(\omega) = \int_{-\infty}^{\infty} p(t)e^{-j\omega t}dt = \mathrm{Re}\left(Fp(\omega)\right) + j\,\mathrm{Im}\left(Fp(\omega)\right)$$



(a) magnitude

(b) phase

$$\left|Fp(\omega)\right| = \sqrt{\mathrm{Re}\left(Fp(\omega)\right)^2 + \mathrm{Im}\left(Fp(\omega)\right)^2}$$

$$\arg\left(Fp(\omega)\right) = \tan^{-1}\left(\frac{\mathrm{Im}\left(Fp(\omega)\right)}{\mathrm{Re}\left(Fp(\omega)\right)}\right)$$

Using Gait as a Biometric, via Phase-Weighted Magnitude Spectra

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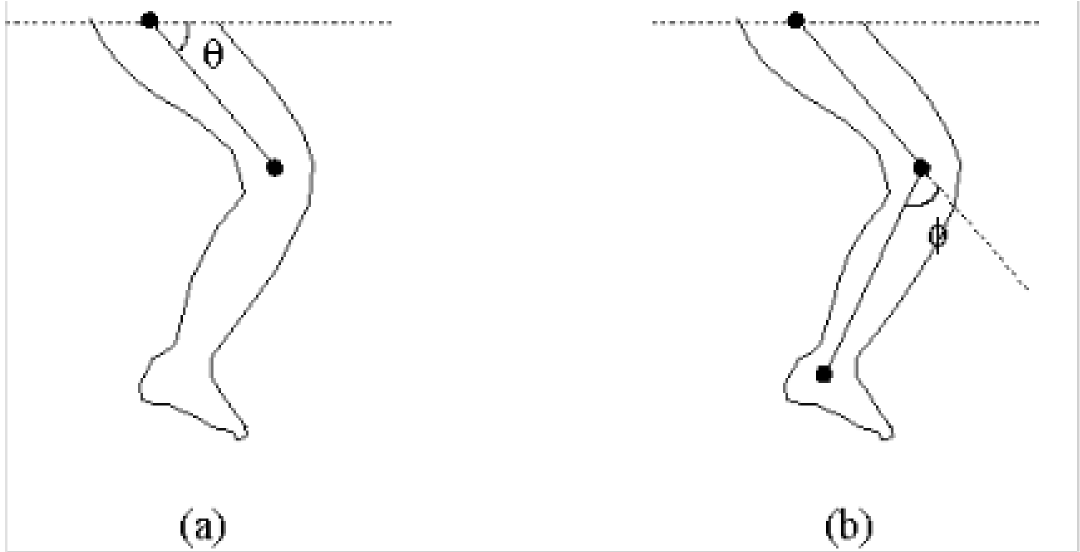


Fig. 1. (a) Hip and (b) Knee rotation angles.

http://eprints.soton.ac.uk/250040/1/cunado_avbpa.pdf

Gait patterns (angle of swinging leg)

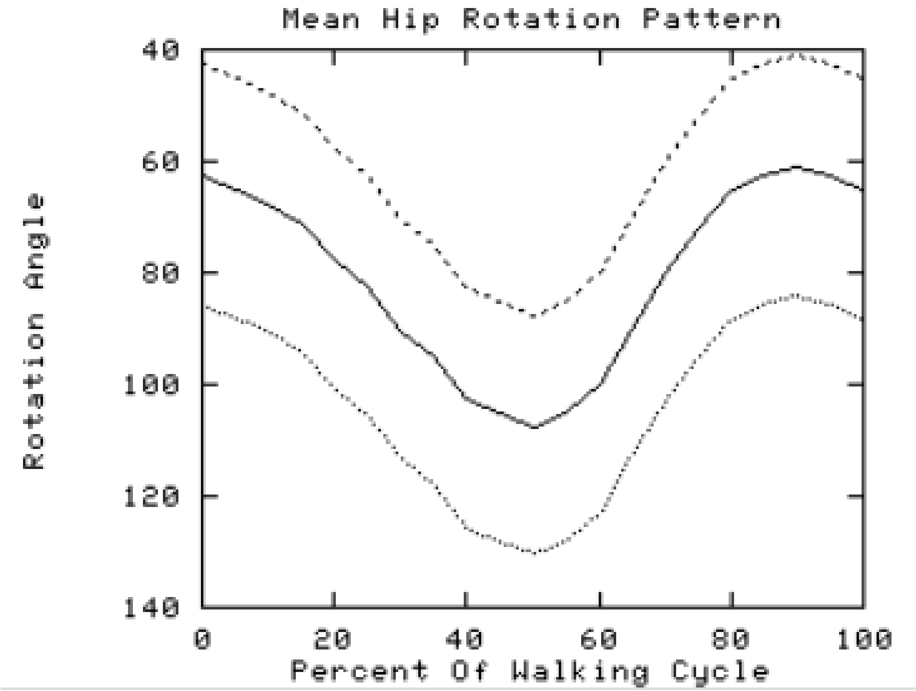
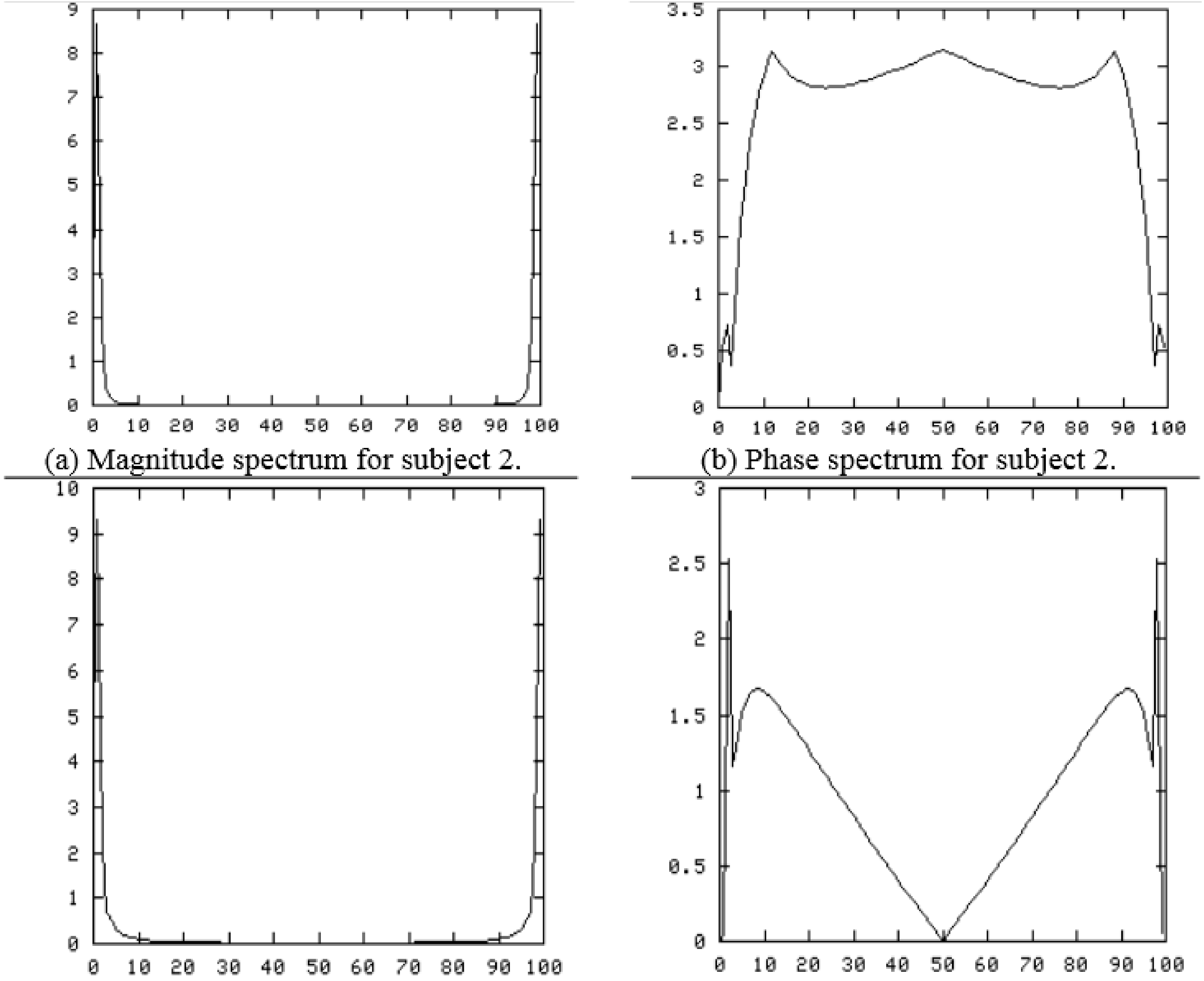


Fig. 2. Variation in Hip Rotation.

Fig. 3. Example Image of Walking Subject.

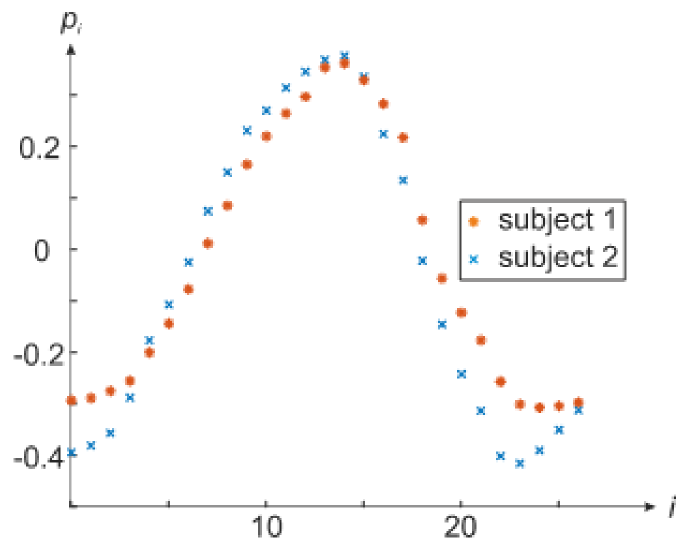
Gait magnitude and phase



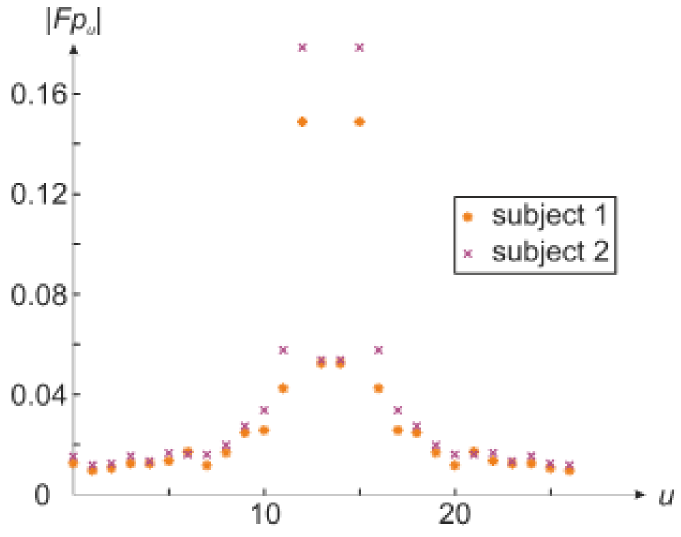
(a) Magnitude spectrum for subject 2. (b) Phase spectrum for subject 2.

(c) Magnitude spectrum for subject 5. (d) Phase spectrum for subject 5.

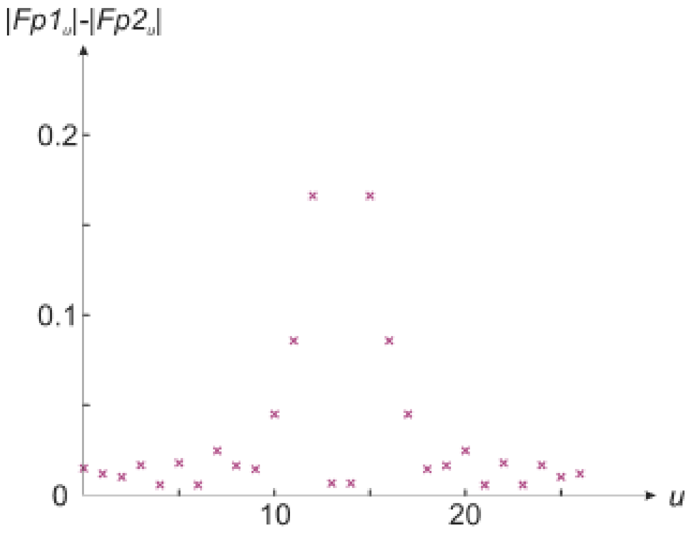
Fig. 6. Phase and Magnitude Gait Spectra.



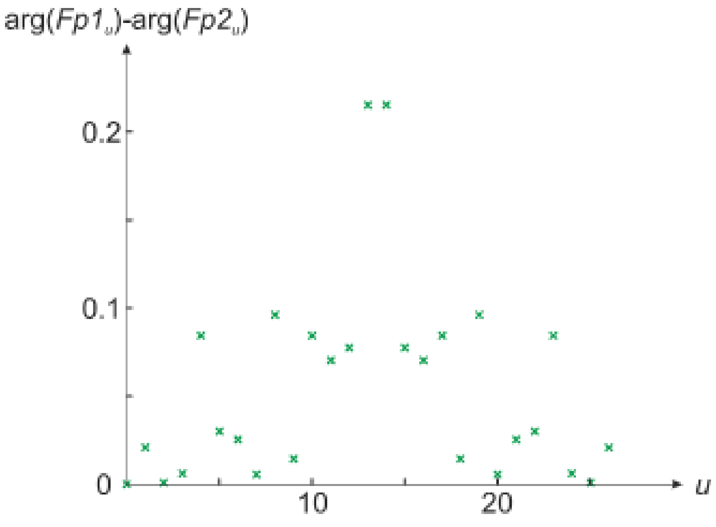
(d) Inclination of left thigh



(e) Magnitude of DFT of inclination

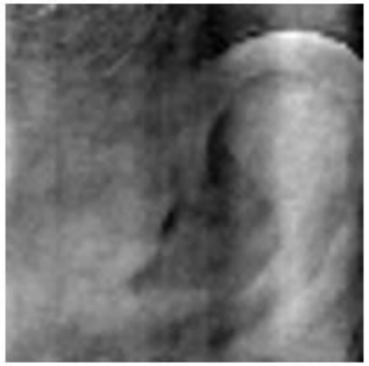
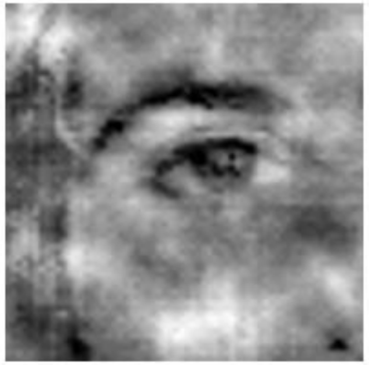


(f) Difference between magnitudes of DFT



(g) Difference between phases of DFT

Illustrating the importance of phase

			
(a) eye image	**(b)** ear image	**(c)** reconstruction from magnitude(eye) and phase(ear)	**(d)** reconstruction from magnitude(ear) and phase(eye)

Takeaway time –main points

1 – sampling data is not as simple as it appears

2 – sampling affects space and brightness

3 – Fourier allows us to understand frequency

4 – Fourier allows for coding and more

Next, Fourier will allow us to understand sampling