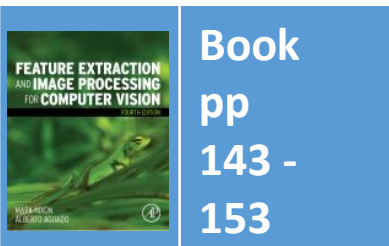


# Lecture 6 Edge Detection

COMP3204 Computer Vision

**What are edges and how do we find them?**



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# Content

1. Differentiation/ differencing can be used to find edges of features
2. How can we improve the differencing process?

# Edge detection

What is an **edge**? It's **contrast**



(a) original image



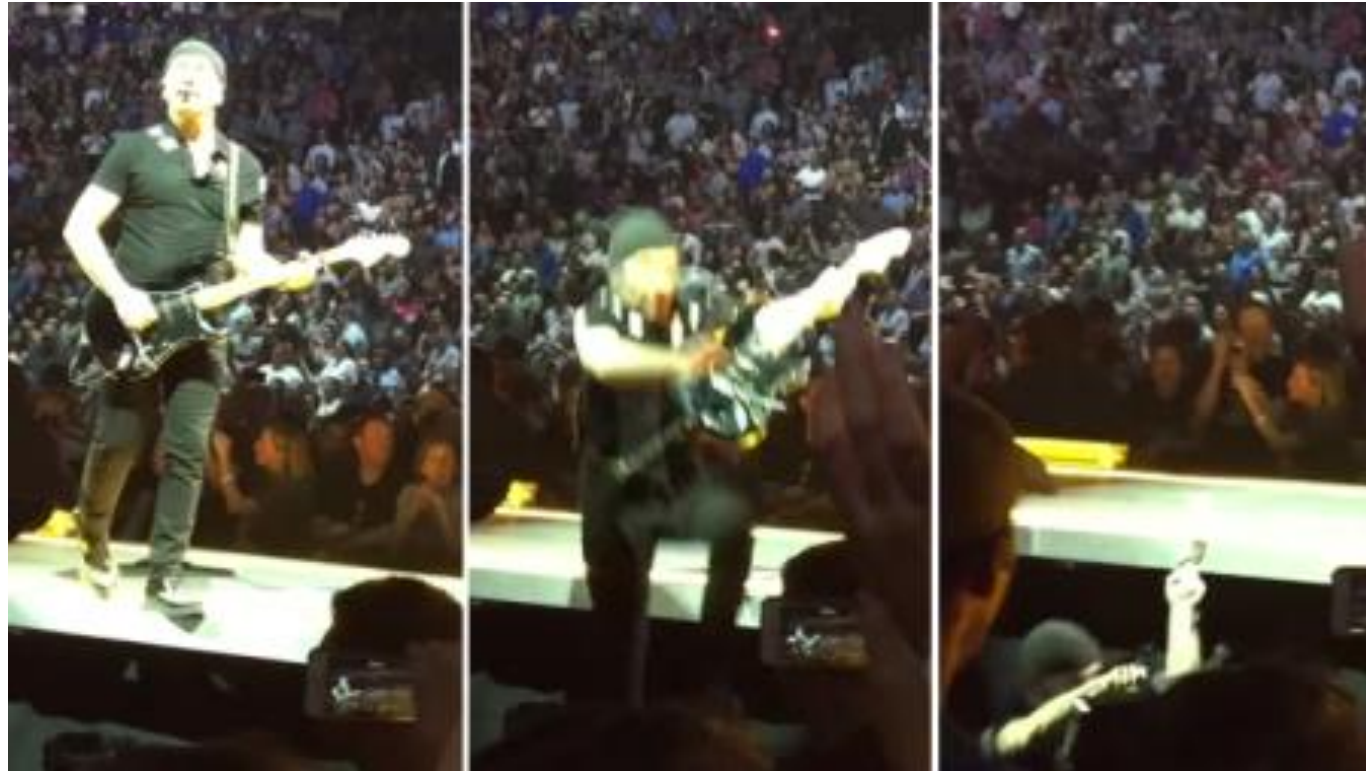
(b) Sobel edge magnitude



(c) thresholded magnitude

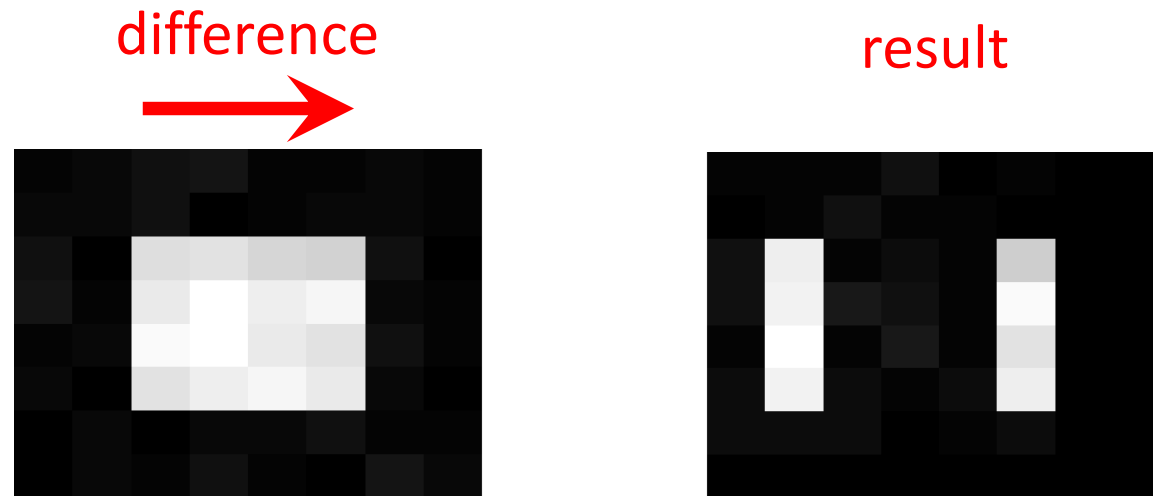


# U2's Edge can't detect edges



<http://metro.co.uk/2015/05/15/the-edge-falls-off-the-edge-of-the-stage-in-spectacular-style-during-u2s-world-tour-5199503/>

# Horizontal differencing

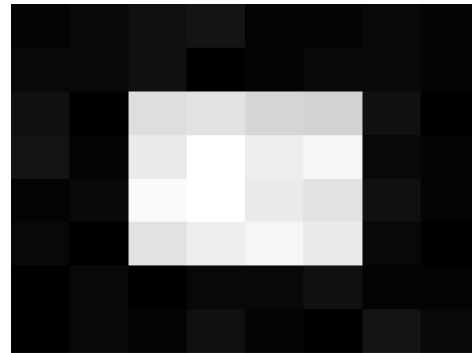


Horizontal differencing detects vertical edges



# Vertical differencing

difference



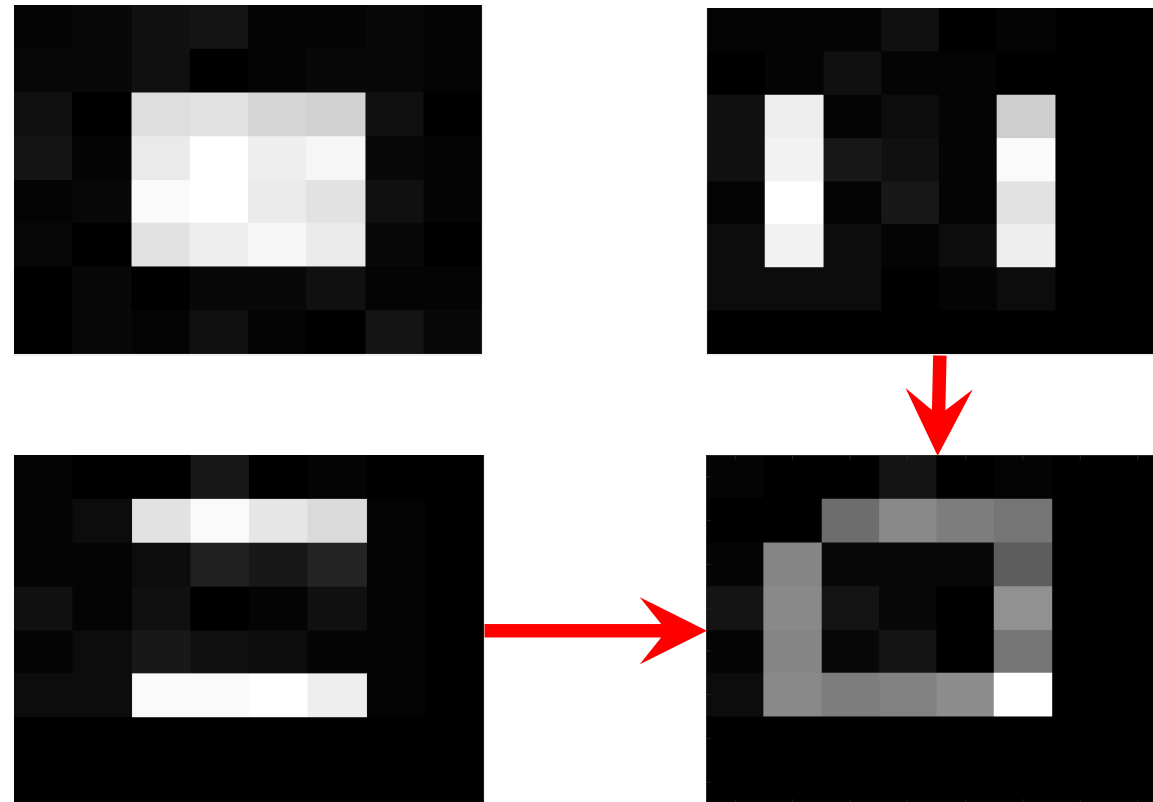
result



Vertical differencing detects horizontal edges



# First order edge detection



Addition of  
horizontal  
and vertical



# First order edge detection

- **vertical** edges,  **$E_x$**

$$E_x_{x,y} = |P_{x,y} - P_{x+1,y}|$$

- **horizontal** edges,  **$E_y$**

$$E_y_{x,y} = |P_{x,y} - P_{x,y+1}|$$

- **vertical and horizontal** edges

$$E_{x,y} = |2 \times P_{x,y} - P_{x+1,y} - P_{x,y+1}|$$





# First order edge detection

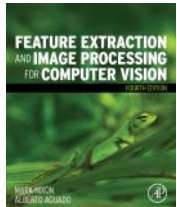
# Template

2	-1
-1	0

## Code

```
function edge = basic_difference(image)

for x = 1:cols-2 %address all columns except border
    for y = 1:rows-2 %address all rows except border
        edge(y,x)=abs(2*image(y,x)-image(y+1,x)-image(y,x+1)); % Eq. 4.4
    end
end
```



How can we **improve** it?

# Taylor series – evaluate $f(t + \Delta t)$

First approximation, original value

$$f(t + \Delta t) = f(t)$$

Second approximation, add gradient

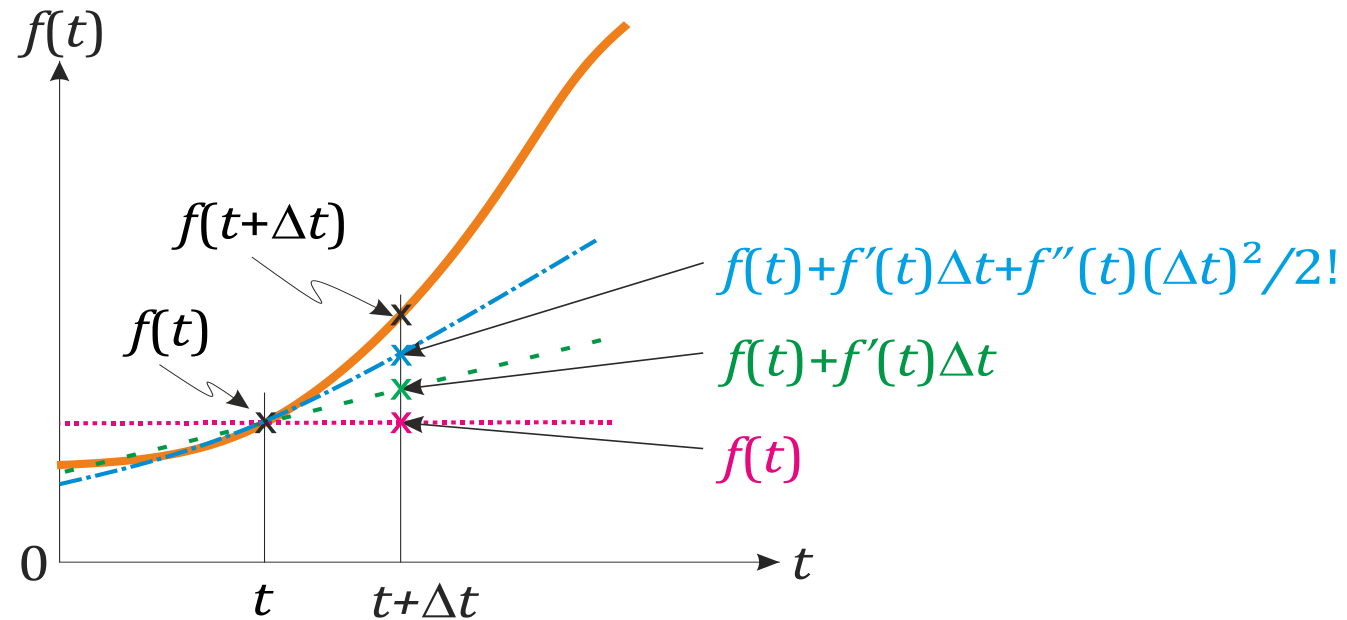
$$f(t + \Delta t) = f(t) + f'(t)\Delta t$$

Third approximation, add  $f''$

$$f(t + \Delta t) = f(t) + f'(t)\Delta t + \frac{f''(t)}{2!}(\Delta t)^2$$

Taylor series

$$f(t + \Delta t) = f(t) + f'(t)\Delta t + \frac{f''(t)}{2!}(\Delta t)^2 + \frac{f'''(t)}{3!}(\Delta t)^3 + \dots + \frac{f^n(t)}{n!}(\Delta t)^n$$



# Edge detection maths

Taylor expansion for  $f(x + \Delta x)$   $f(x + \Delta x) = f(x) + \Delta x \times f'(x) + \frac{\Delta x^2}{2!} \times f''(x) + O(\Delta x^3)$  **A**

By rearrangement, 
$$f'(x) = \frac{f(x + \Delta x) - f(x)}{\Delta x} - O(\Delta x)$$

This is equivalent to  $\mathbf{E}\mathbf{x}_{x,y} = \left| \mathbf{P}_{x,y} - \mathbf{P}_{x-1,y} \right|$

Expand  $f(x - \Delta x)$   $f(x - \Delta x) = f(x) - \Delta x \times f'(x) + \frac{\Delta x^2}{2!} \times f''(x) - O(\Delta x^3)$  **B**

$$\mathbf{A} - \mathbf{B} \quad f'(x) = \frac{f(x + \Delta x) - f(x - \Delta x)}{2\Delta x} - O(\Delta x^2) \quad \mathbf{E_{xx}}_{x,y} = \left| \mathbf{P}_{x+1,y} - \mathbf{P}_{x-1,y} \right|$$

If  $\Delta x < 1$ , this error is clearly smaller





# Fireside time

Where is computer vision going?

Where is biometrics going?

IEEE TRANSACTIONS ON  
PATTERN ANALYSIS AND  
MACHINE INTELLIGENCE



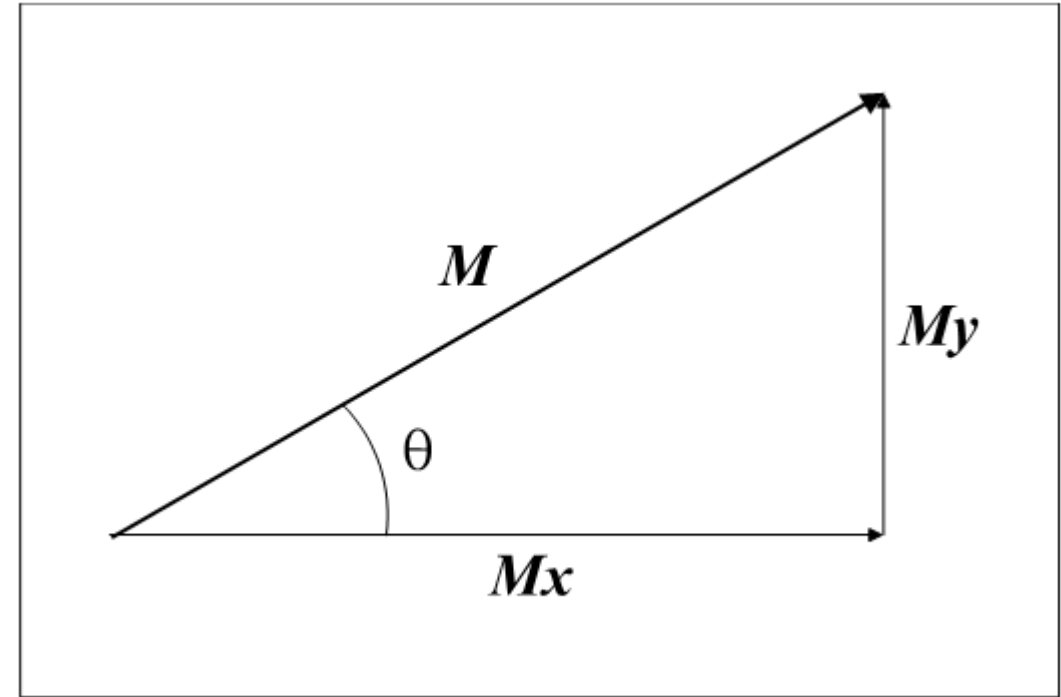
IEEE TRANSACTIONS ON BIOMETRICS,  
BEHAVIOR, AND IDENTITY SCIENCE

# Edge Detection in Vector Format

Vectors have **magnitude** (strength) and **direction**

$$M = \text{magnitude} = \sqrt{M_x^2 + M_y^2}$$

$$\theta = \text{direction} = \tan^{-1} \left( \frac{M_y}{M_x} \right)$$



# Templates for 3×3 Prewitt operator

**Average** improved horizontal and vertical operators over 3 rows/  
columns to give **Prewitt** templates

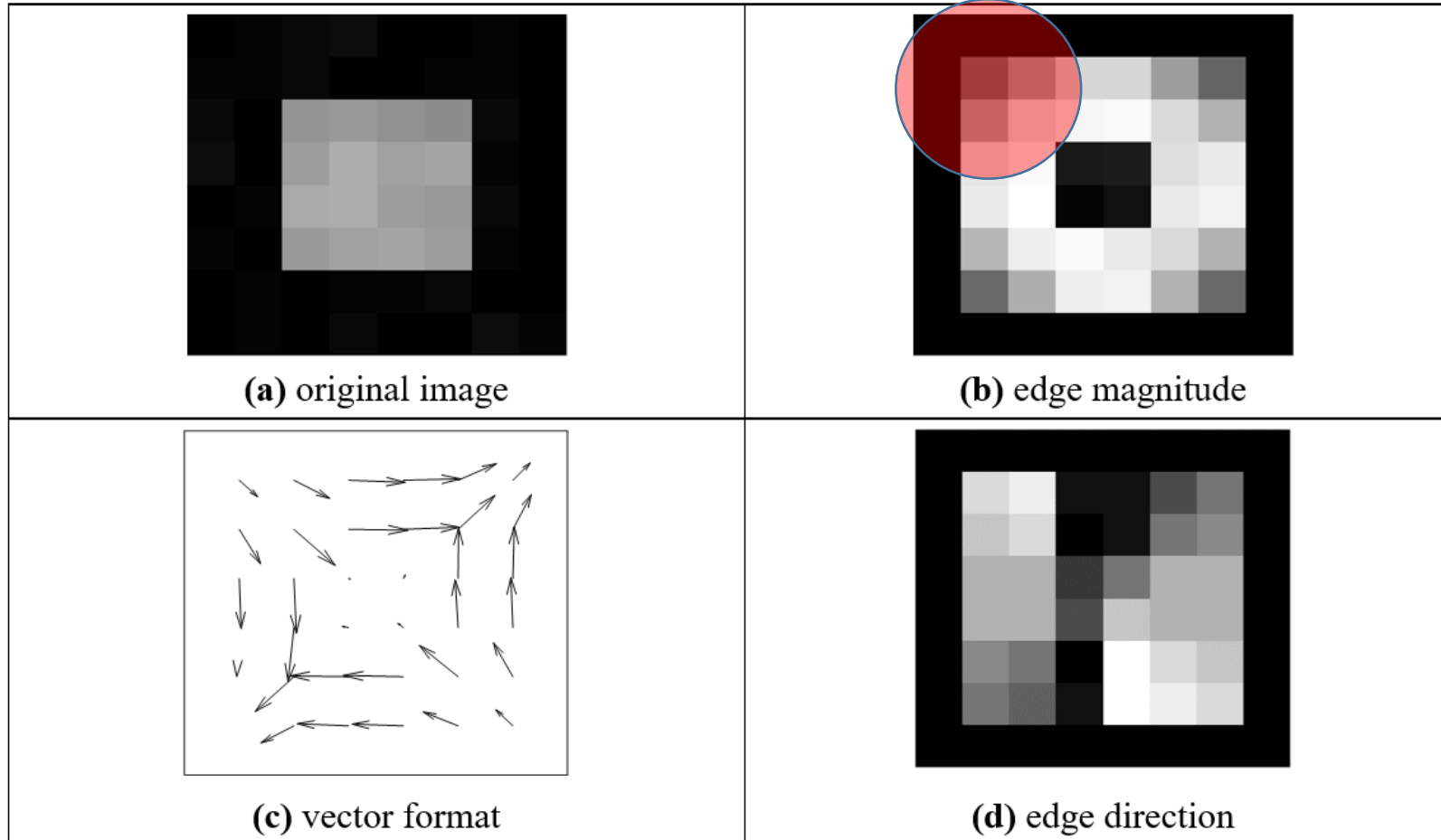
<table><tr><td>1</td><td>0</td><td>-1</td></tr><tr><td>1</td><td>0</td><td>-1</td></tr><tr><td>1</td><td>0</td><td>-1</td></tr></table> <p>(a) Mx</p>	1	0	-1	1	0	-1	1	0	-1	<table><tr><td>1</td><td>1</td><td>1</td></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>-1</td><td>-1</td><td>-1</td></tr></table> <p>(b) My</p>	1	1	1	0	0	0	-1	-1	-1
1	0	-1																	
1	0	-1																	
1	0	-1																	
1	1	1																	
0	0	0																	
-1	-1	-1																	

Edge magnitude and direction calculated for **centre** point



# Applying the Prewitt Operator

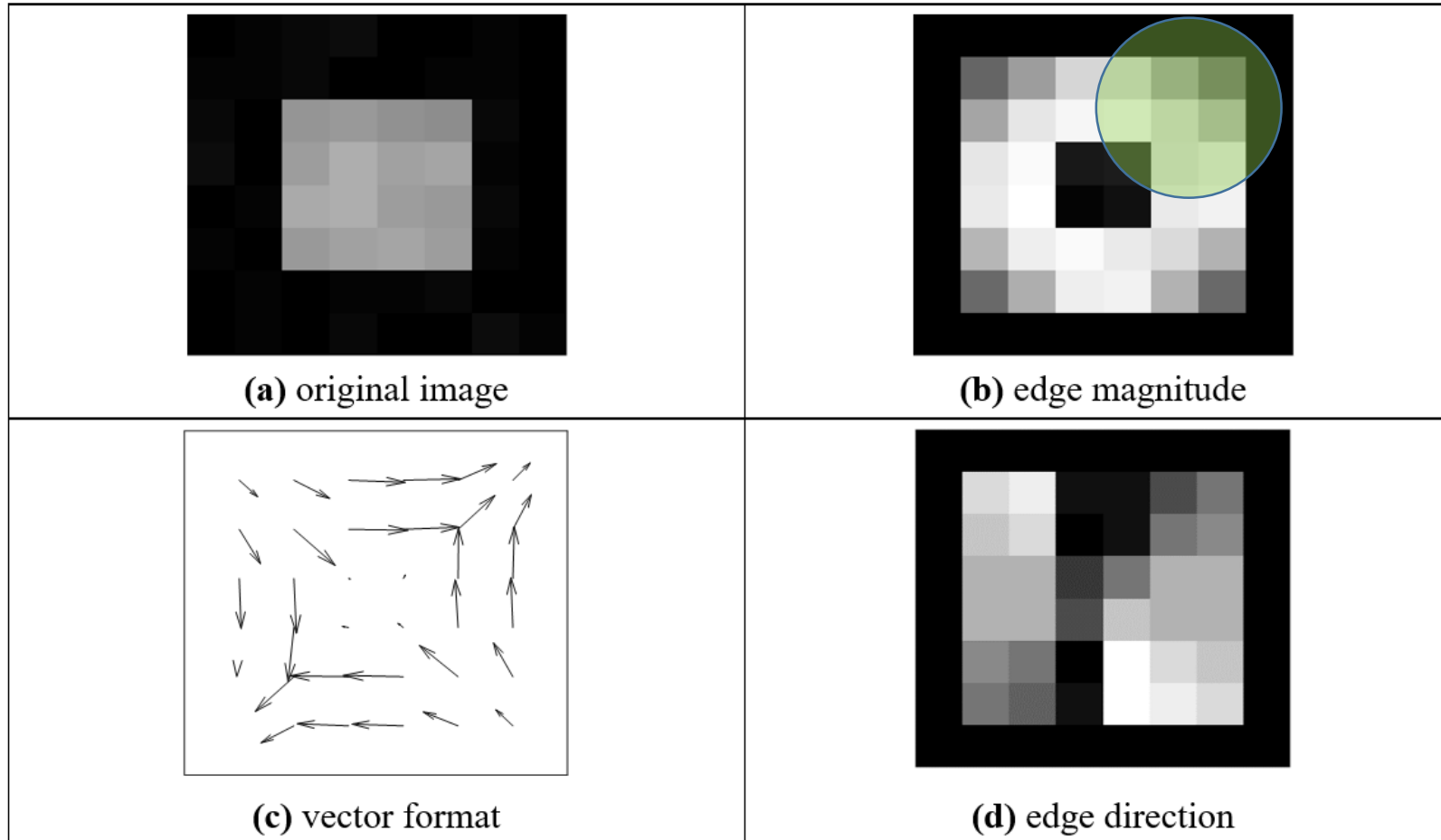
No missing points





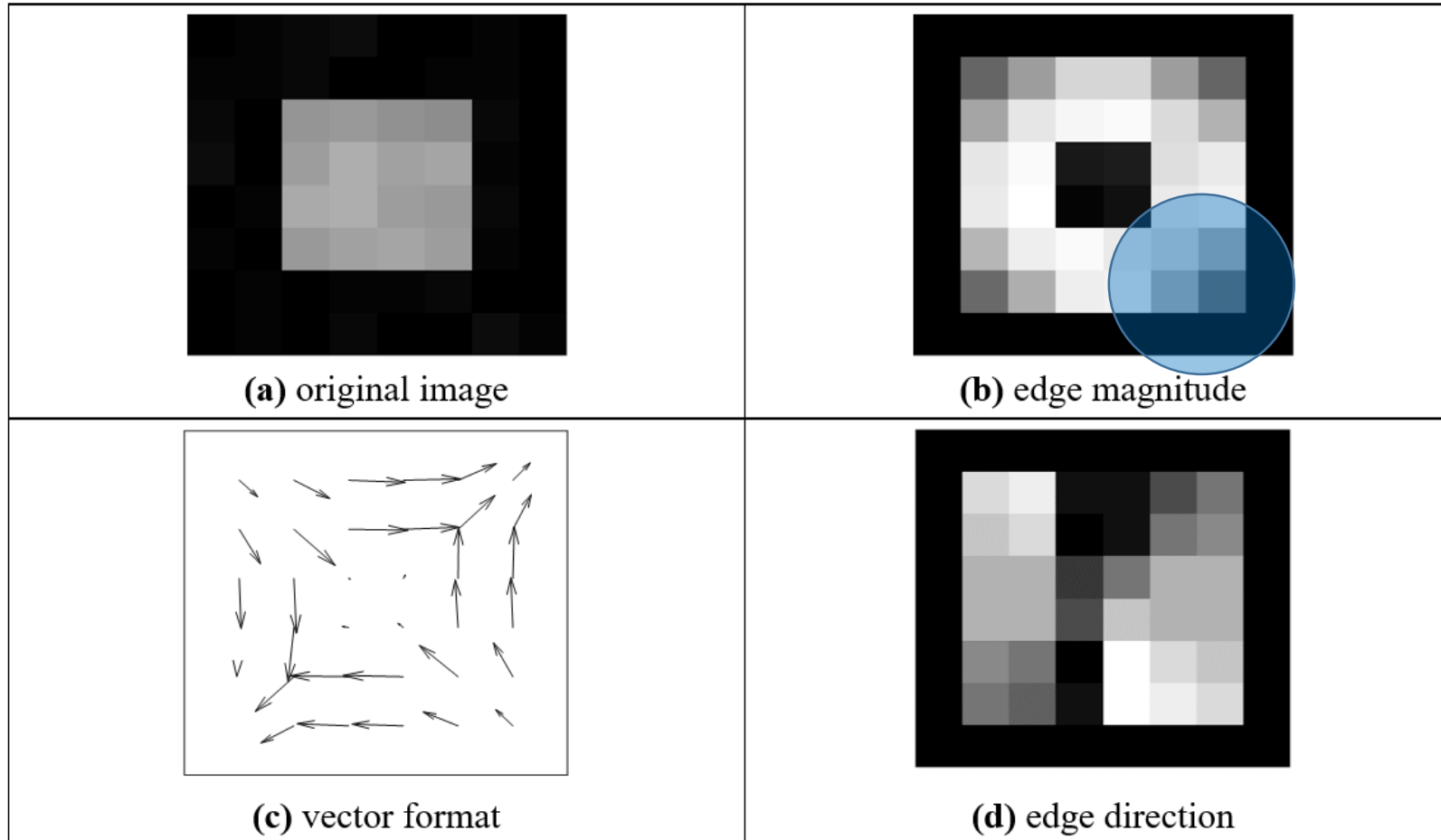
# Applying the Prewitt Operator

Blurred edges



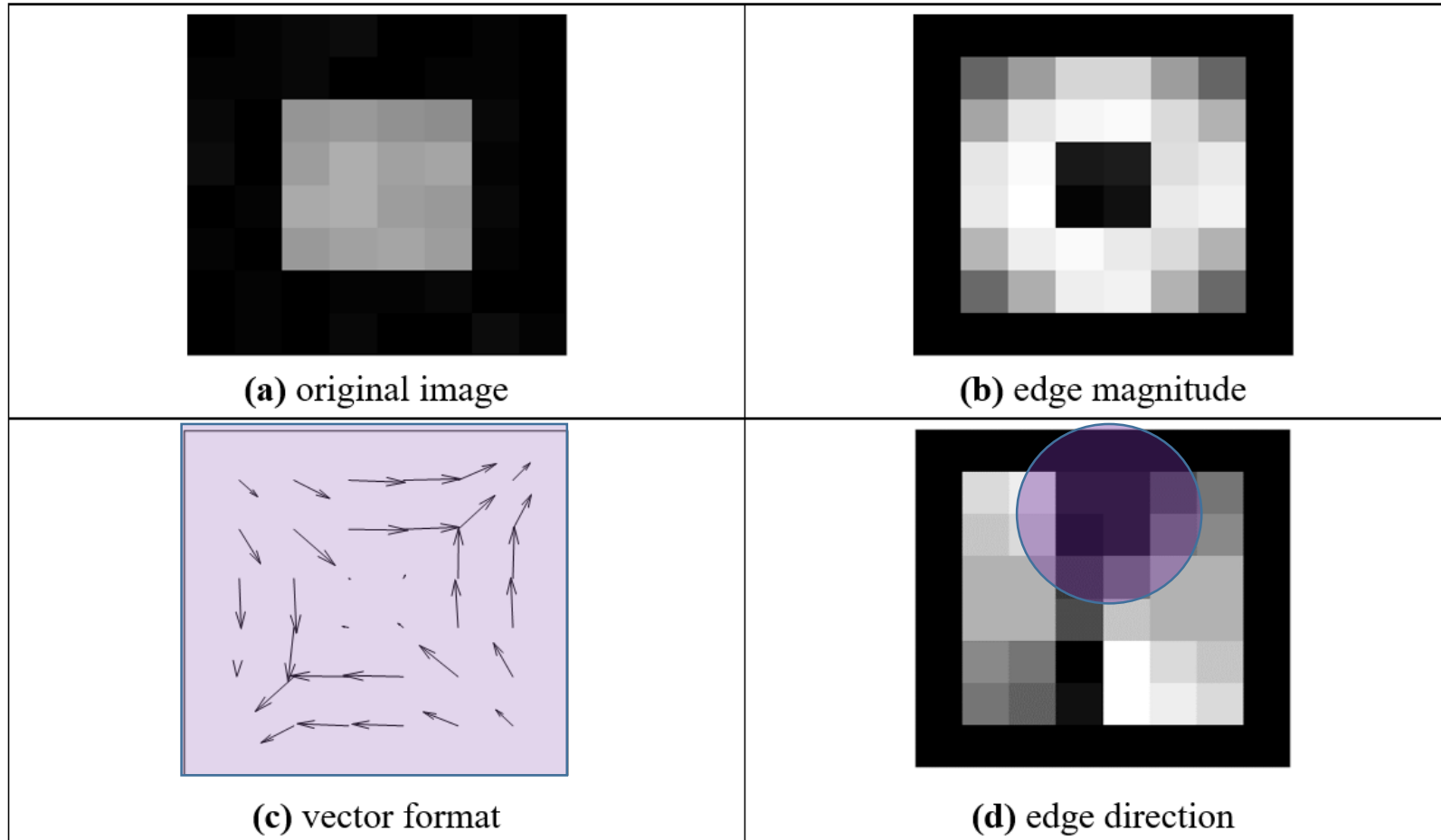
# Applying the Prewitt Operator

No double points

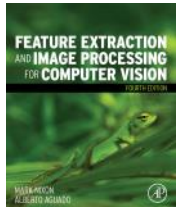


# Applying the Prewitt Operator

Displaying gradients as an image communicates nothing



So use vectors



# Templates for Sobel operator

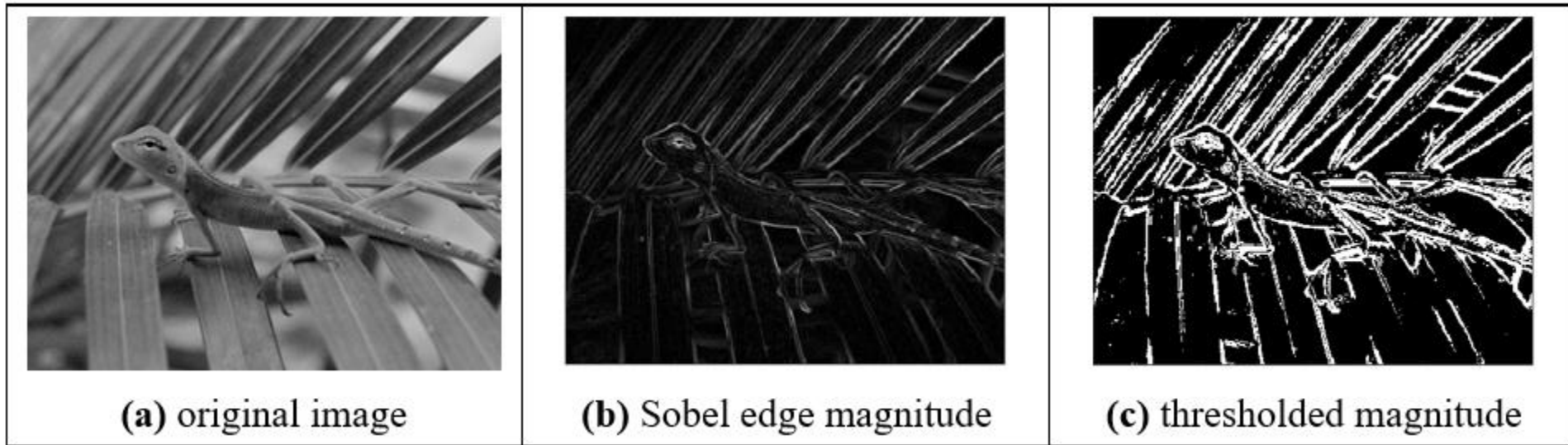
**Sobel** is most popular basic operator  
Double the centre coefficients of Prewitt

<table><tr><td>1</td><td>0</td><td>-1</td></tr><tr><td>2</td><td>0</td><td>-2</td></tr><tr><td>1</td><td>0</td><td>-1</td></tr></table> <p>(a) M<sub>x</sub></p>	1	0	-1	2	0	-2	1	0	-1	<table><tr><td>1</td><td>2</td><td>1</td></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>-1</td><td>-2</td><td>-1</td></tr></table> <p>(b) M<sub>y</sub></p>	1	2	1	0	0	0	-1	-2	-1
1	0	-1																	
2	0	-2																	
1	0	-1																	
1	2	1																	
0	0	0																	
-1	-2	-1																	

WHY?



# Applying Sobel operator



# Generalising Sobel - use Pascal's triangle

## 1. Averaging

# Window size

2

1

1

3

1

2

1

## Sobel 3x3

4

1

3

3

1

5

1

4

6

4

1

## Sobel 5x5

## 2. Differencing

# Window size

2

1

-1

3

1

0

-1

## Sobel 3x3

4

1

1

-1

-1

5

1

2

0

-2

-1

## Sobel 5x5



# Generalised Sobel

Generated by:  $\text{averaging} * (\text{differencing})^T$

```
>> s=Sobel_templates(5)
```

```
s(:, :, 1) =
```

1	2	0	-2	-1
4	8	0	-8	-4
6	12	0	-12	-6
4	8	0	-8	-4
1	2	0	-2	-1

**COURSEWORK!!!!**



# Takeaway time

- 1 – differencing detects contrast and thus **edges**
  - 2 - can **improve** the differencing process (by maths!!)
  - 3 – **Sobel** is a good general purpose operator
- We shall go to more sophisticated methods, coming up next.

