

a) users : 9

1, 2, 3, 4, 5, 6, 7, 8, 9

Edges : 14

[1 2] [1 3] [1 4]

[3 2] [3 4]

[4 5] [4 6]

[5 6] [5 7] [5 8]

[6 7] [6 8]

[7 8] [7 9]

b) st, lets find the distance for individual

$D = 1, \{2, 3, 4, 5, 6, 7, 8, 9\}$

$$d(1, 2) = 1$$

$$d(1, 3) = 1$$

$$d(1, 4) = \min(d(1, 3, 4), d(1, 4)) = \min(2, 1) = 1$$

$$d(1, 5) = 2, d(1, 6) = 2, d(1, 7) = 3, d(1, 8) = 3$$

$$d(1, 9) = 4$$

Max of these distances is eccentricity of vertex

$$e(1) = 4$$

for node ②

$$D = 2, \{1, 3, 4, 5, 6, 7, 8, 9\}$$

$$d(2, 1) = 1$$

$$d(2, 3) = 1$$

$$d(2, 4) = \min(d(2, 1, 3, 4), d(2, 3, 4)) \\ = \min(3, 2) = 2$$

$$d(2, 5) = 3$$

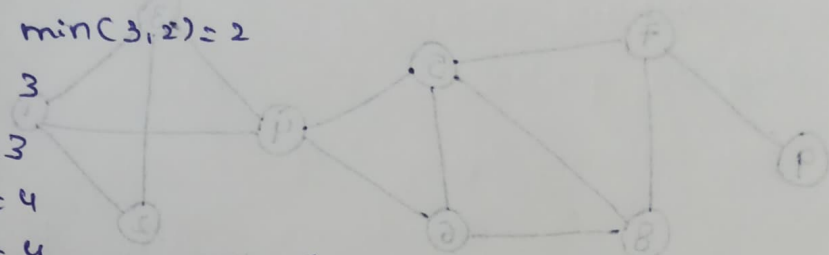
$$d(2, 6) = 3$$

$$d(2, 7) = 4$$

$$d(2, 8) = 4$$

$$d(2, 9) = 5$$

$$e(2) = 5$$



for node ③

$$D = 3, \{1, 2, 4, 5, 6, 7, 8, 9\}$$

$$d(3, 1) = 1$$

$$d(3, 2) = 1$$

$$d(3, 4) = 1$$

$$d(3, 5) = 2$$

$$d(3, 6) = 2$$

$$d(3, 7) = 3$$

$$d(3, 8) = 3$$

$$d(3, 9) = 4$$

$$e(3) = 4$$

for node ④

$$D = 4, \{1, 2, 3, 5, 6, 7, 8, 9\}$$

$$d(4, 1) = 1$$

$$d(4, 2) = 2$$

$$d(4, 3) = 1$$

$$d(4, 5) = 1$$

$$d(4, 6) = 1$$

$$d(4, 7) = 2$$

$$d(4, 8) = 2$$

$$d(4, 9) = 3$$

$$e(4) = 3$$

for node ⑤

$$D = 5, \langle 1, 2, 3, 4, 6, 7, 8, 9 \rangle$$

$$d(5, 1) = 2$$

$$d(5, 2) = 3$$

$$d(5, 3) = 2$$

$$d(5, 4) = 1$$

$$d(5, 6) = 1$$

$$d(5, 7) = 1$$

$$d(5, 8) = 1$$

$$d(5, 9) = 2$$

$$e(5) = 3$$

for node ⑥

$$D = 6, \langle 1, 2, 3, 4, 5, 7, 8, 9 \rangle$$

$$d(6, 1) = 2$$

$$d(6, 2) = 3$$

$$d(6, 3) = 2$$

$$d(6, 4) = 1$$

$$d(6, 5) = 1$$

$$d(6, 7) = 1$$

$$d(6, 8) = 1$$

$$d(6, 9) = 2$$

$$e(6) = 3$$

for node ⑦

$$D = 7, \langle 1, 2, 3, 4, 5, 6, 8, 9 \rangle$$

$$d(7, 1) = 3$$

$$d(7, 2) = 4$$

$$d(7, 3) = 3$$

$$d(7, 4) = 2$$

$$d(7, 5) = 1$$

$$d(7, 6) = 1$$

$$d(7, 8) = 1$$

$$d(7, 9) = 1$$

$$e(7) = 4$$

for node (8)

$$D = 8, \{1, 2, 3, 4, 5, 6, 7, 9\}$$

$$d(8, 1) = 3$$

$$d(8, 2) = 4$$

$$d(8, 3) = 3$$

$$d(8, 4) = 2$$

$$d(8, 5) = 1$$

$$d(8, 6) = 1$$

$$e(8) = 4$$

$$d(8, 7) = 1$$

$$d(8, 9) = 2$$

for node (9)

$$D = 9, \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$d(9, 1) = 4$$

$$d(9, 2) = 5$$

$$d(9, 3) = 4$$

$$d(9, 4) = 3$$

$$d(9, 5) = 2$$

$$d(9, 6) = 2$$

$$d(9, 7) = 1$$

$$d(9, 8) = 2$$

$$e(9) = 5$$

Diameter of the graph = Max of all the
Calculated eccentricities

$$e(1) = 4$$

$$e(2) = 5$$

$$e(3) = 4$$

$$e(4) = 3$$

$$e(5) = 3$$

$$e(6) = 3$$

$$e(7) = 4$$

$$e(8) = 4$$

$$e(9) = 5$$

$$\text{Max} = \{e(2), e(9)\}$$
$$= 5$$

$$\text{Diameter} = 5$$

c) Density of the graph:

$$\text{Density}(D) = \frac{2|E|}{|V|(|V|-1)}$$

$$|E| = 14, |V| = 9$$

$$D = \frac{2 \times 14}{9(9-1)} = \frac{2 \times 14}{9 \times 8} = \frac{28}{72}$$

$$= 0.388$$

$$\text{Density} = 0.388$$

e) periphery of the graph =
nodes whose eccentricity is equal
to the diameter of the graph
Node '2' and Node '9'

f) Clustering Coefficients of each node:

$CC(v)$: $v \rightarrow$ a node

$k_v \rightarrow$ its degree

$N_v \rightarrow$ no. of links b/w
neighbours of v

For Node ①:

$$k_v = 3$$

$$N_v = 2$$

$$CC(1) = \frac{2N_v}{k_v(k_v-1)}$$

$$= \frac{2 \times 2}{3 \times 2} = \frac{2}{3}$$

$$CC(1) = 0.667$$

for Node ② :

$$K_v = 2$$

$$N_v = 1$$

$$CC(2) = \frac{2 N_v}{K_v(K_v - 1)} = \frac{2 \times 1}{2 \times 1}$$

$$CC(2) = 1$$

for Node ③ :

$$K_v = 3$$

$$N_v = 2$$

$$CC(3) = \frac{2 N_v}{K_v(K_v - 1)} = \frac{2 \times 2}{3 \times 2}$$

$$CC(3) = 0.667$$

for Node ④ :

$$K_v = 4$$

$$N_v = 2$$

$$CC(4) = \frac{2 N_v}{K_v(K_v - 1)} = \frac{2 \times 2}{4 \times 3} = \frac{1}{3}$$

$$CC(4) = 0.33$$

for Node ⑤ :

$$K_v = 4$$

$$N_v = 4$$

$$CC(5) = \frac{2 \times 4}{4 \times 3} = \frac{2}{3}$$

$$CC(5) = 0.667$$

for Node ⑥ :

$$K_v = 4$$

$$N_v = 4$$

$$CC(6) = \frac{2 \times 4}{4 \times 3} = \frac{2}{3}$$

$$CC(6) = 0.667$$

for Node ⑦ :

$$K_v = 4$$

$$N_v = 3$$

$$CC(7) = \frac{2 \times 3}{4 \times 3} = \frac{1}{2}$$

$$CC(7) = 0.5$$

for Node ⑧ :

$$K_v = 3$$

$$N_v = 3$$

$$CC(8) = \frac{2N_v}{K_v(K_v-1)} = \frac{2 \times 3}{3 \times 2} = 1$$

$$CC(8) = 1$$

for Node ⑨ :

$$K_v = 1$$

$$N_v = 0$$

$$CC(9) = 0$$

g) Average clustering Coefficient :

$$\frac{\sum_{i=1}^n CC(i)}{n}$$

$$\text{So, } \frac{\sum_{i=1}^9 CC(i)}{9}$$

$$= \frac{0.667 + 1 + 0.667 + 0.33 + 0.667 + 0.667 + 0.667 + 0.667 + 0}{0.5 + 1 + 0.5}$$

$$= \frac{5.478}{9} = 0.6108$$

$$Acc = 0.6108$$

Expected clustering Coefficient :

$$E_{cc} = \frac{3(K-2)}{2(K-1)}$$

$$K = \frac{2E}{N}$$

$$E = 14, N = 9$$

$$= \frac{2(14)}{9}$$

$$E_{cc} = \frac{3(3.1-2)}{2(3.1-1)} = \frac{3(1.1)}{2(2.1)} = \frac{3.3}{4.2} = 0.785$$

2)

a. Erdos - renyi - graph

1. It is also often referred as the ER graph, is a random graph model introduced by mathematicians paul Erdos and Alfred Renyi.
2. It is used to generate random graphs for research and modelling purpose.
3. The Erdos - renyi graph model has two main variations

$$G(n, p), G(n, m)$$

n : number of nodes

m : number of edges

p : probability of Connecting any two nodes

b. Barabasi - albert - graph

1. BA graph is a type of random graph model used in network science and graph theory.
2. It was introduced by Reka Albert and Albert-Laszlo barabasi in 1999.
3. The BA model is designed to capture the "scale-free" property observed in many-real world complex networks.

C.

1. Karate club graph:

It captures the interactions and friendships between members of the club.

2. Zachary's Karate club graph:

It is often used for community detection and social network analysis.

3. Les Misérables graph:

It is a weighted graph where the edges represent interactions between characters, and edge weights may denote the frequency of interaction.

4)

a) specify with nodes and edges represent in network.

nodes - Represent webpages

edges - Represent hyperlinks or connections b/w web pages

b) specify node attributes and edge/link attributes.

node :- URL, domain ; edge :- hyperlink, timestamp

c) Link addition and removal

↓
links are added to webpages when they are created, or updated

d) Hubs in network :

Hubs represent web pages that are have very high number of incoming links

e) Analytical applications :

- page rank analysis

- Community detection

- link prediction

6) Draw the plot of degree distribution and verify powerlaw of any real world example (SNAP)

- choose a dataset from stanford SNAP

- load and create a network from the dataset

- Calculate the degree distribution
- plot it on log-log scale
- fit a powerlaw distribution and check the goodness of fit

Powerlaw degree distribution should appear as a straight line on the log-log plot with a low p-value for goodness of fit test.

