Social Network Analysis (CSE 329) AP20110010494

Assignment - 2 (unit - 2)

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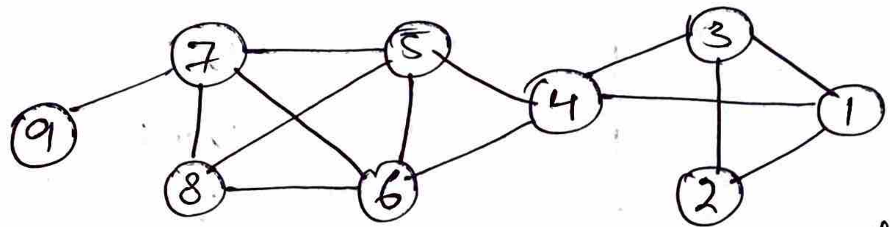
1. 

a. betweeness centrality of node '4' (Lets ~~consider~~ exclude the endpoint in our calculations):
$C_b(4) = ?$

Pairs	σ_{uw} (shortest path existing)	$\sigma_{uw}(v)$ [does it pass through '4']	$\frac{\sigma_{uw}(v)}{\sigma_{uw}}$
(9,7)	1	0	0
(9,8)	1	0	0
(9,6)	1	0	0
(9,5)	1	0	0
(9,3)	2	1	0.5
(9,2)	4	1	0.25
(9,1)	2	1	0.5
(8,7)	1	0	0
(8,6)	1	0	0
(8,5)	1	0	0
(8,3)	2	1	0.5
(8,2)	4	1	0.25
(8,1)	2	1	0.5

Pairs	σ_{uw} (shortest path existing)	$\sigma_{uw}(v)$ [does it pass through '4']?	$\frac{\sigma_{uw}(v)}{\sigma_{uw}}$
(7,6)	1	0	0
(7,5)	1	0	0
(7,3)	2	1	0.5
(7,2)	4	1	0.25
(7,1)	2	1	0.5
(6,5)	1	0	0
(6,3)	1	1	1
(6,2)	2	1	0.5
(6,1)	1	1	1
(5,3)	1	1	1
(5,2)	2	1	0.5
(5,1)	1	1	1
(3,2)	1	0	0
(3,1)	1	0	0
(2,1)	1	0	0

$$\sum \frac{\sigma_{uw}(v)}{\sigma_{uw}} = C_b(4)$$

$$\Rightarrow \boxed{C_b(4) = 8.75}$$

b. Degree centrality

→ undirected graph : $C_D(i) = \frac{d(i)}{n-1}$; n = n.o of nodes.

$C_D(9) = \frac{1}{8} = 0.125$

$C_D(8) = \frac{4}{8} = 0.5$

$C_D(7) = \frac{4}{8} = 0.5$

$C_D(5) = \frac{4}{8} = 0.5$

$C_D(6) = \frac{4}{8} = 0.5$

$C_D(4) = \frac{4}{8} = 0.5$

$C_D(3) = \frac{3}{8} = 0.375$

$C_D(2) = \frac{2}{8} = 0.25$

$C_D(1) = \frac{3}{8} = 0.375$.

* 5 nodes are tied with the highest Degree centrality
nodes: 8, 7, 5, 6, 4 all have a CD of 0.5.

* Lowest Degree centrality : node : 9 ; CD of 0.125
2nd Lowest CD : node : 2 ; CD of 0.25
3rd Lowest CD : node : 3, 1 ; CD of 0.375

c. Closeness centrality

→ undirected graph: $C_C(i) = \frac{n-1}{\sum_{j=1}^{n} d(i,j)}$ n = n.o of nodes.

$C_C(3) = \frac{8}{1+1+1+2+2+3+3+4}$

$C_C(3) = \frac{8}{17} \Rightarrow \boxed{C_C(3) = 0.4706}$ //

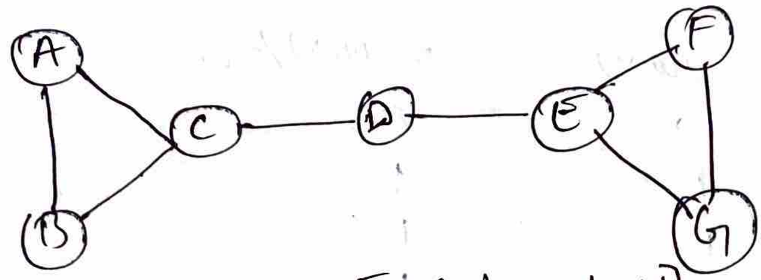
$C_C(4) = \frac{8}{1+2+1+1+1+2+2+3}$

$C_C(4) = \frac{8}{13} \Rightarrow \boxed{C_C(4) = 0.61538}$ //

Observation :

* Node '4' significantly has a higher closeness centrality of "0.61538" than Node '3' which has a closeness centrality of "0.4706".
* This suggests that Node '4' is a highly central Node in the network.
* It is closely connected to other nodes, and the average distance from Node '4' to all other nodes in the network is relatively shorter compared to other nodes.
* Node '4' can be considered a key player in the network's information flow or influence.

2.



a. unnormalized [exclude end point]

(i) $C_B(A) = ?$

Pairs	σ_{uw}	$\sigma_{uw}(v)$	$\sigma_{uw}(v)/\sigma_{uw}$
BC	1	0	0
BD	1	0	0
BE	1	0	0
BF	1	0	0
BG	1	0	0
CD	1	0	0
CE	1	0	0
CF	1	0	0
CG	1	0	0
DE	1	0	0
DF	1	0	0
DG	1	0	0
EF	1	0	0
EG	1	0	0
FG	1	0	0

$\boxed{C_b(A) = 0}$

* Since this graph is symmetric; the nodes which are in the mirror image of "A" will also have the same "C_b" value.

Therefore, $\boxed{C_b(A) = C_b(B) = C_b(F) = C_b(G) = 0}$

(ii) Cb(C) = ?

Pairs	σuw	σuw(v)	σuw(v)/σuw.
AB	1	0	0
AD	1	1	1
AE	1	1	1
AF	1	1	1
AG	1	1	1
BD	1	1	1
BE	1	1	1
BF	1	1	1
BG	1	1	1
DE	1	0	0
DF	1	0	0
DG	1	0	0
EF	1	0	0
EG	1	0	0
FG	1	0	0

$\boxed{Cb(C) = 8}$

* since, 'C' is symmetric to Node 'E'.
* Therefore, $\boxed{Cb(C) = Cb(E) = 8}$

(iii) Cb(D) = ?

Pairs	σuw	σuw(v)	σuw(v)/σuw.
AB	1	0	0
AC	1	0	0
AE	1	1	1
AF	1	1	1
AG	1	1	1
BC	1	0	0
BE	1	1	1
BF	1	1	1
BG	1	1	1
CE	1	1	1
CF	1	1	1
CG	1	1	1
EF	1	0	0
EG	1	0	0
FG	1	0	0

$\boxed{Cb(D) = 9}$

Node: 'D', 'C', 'E' have the highest betweeness Centrality.
(9) (8) (8)

b. Considering End points.

(i) Cb(A) = ?

Pairs	σ_{uw}	$\sigma_{uw}(v)$	$\sigma_{uw}(v) / \sigma_{uw}$.
AB	1	1	1
AC	1	1	1
AD	1	1	1
AE	1	1	1
AF	1	1	1
AG	1	1	1
BC	1	0	0
BD	1	0	0
BE	1	0	0
BF	1	0	0
BG	1	0	0
CD	1	0	0
CE	1	0	0
CF	1	0	0
CG	1	0	0
DE	1	0	0
DF	1	0	0
DG	1	0	0
EF	1	0	0
EG	1	0	0
FG	1	0	0

$\boxed{Cb(A) = 6}$

* Therefore,

$\boxed{Cb(A) = Cb(B) = Cb(F) = Cb(G) = 6}$

(ii) $C_b(C) = ?$

Pairs	σ_{uw}	$\sigma_{uw}(v)$	$\sigma_{uw}(v) / \sigma_{uw}$
AB	1	0	0
AC	1	1	1
AD	1	1	1
AE	1	1	1
AF	1	1	1
AG	1	1	1
BC	1	1	1
BD	1	1	1
BE	1	1	1
BF	1	1	1
BG	1	1	1
CD	1	1	1
CE	1	1	1
CF	1	1	1
CG	1	1	1
DE	1	0	0
DF	1	0	0
DG	1	0	0
EF	1	0	0
EG	1	0	0
FG	1	0	0

$\boxed{C_b(C) = 14}$

* Therefore, $\boxed{C_b(C) = C_b(E) = 14}$

(iii) $C_B(D) = ?$

Pairs	σ_{uw}	$\sigma_{uw}(v)$	$\sigma_{uw}(v) / \sigma_{uw}$
AB	1	0	0
AC	1	0	0
AD	1	1	1
AE	1	1	1
AF	1	1	1
AG	1	1	0
BC	1	0	1
BD	1	1	1
BE	1	1	1
BF	1	1	1
BG	1	1	1
CD	1	1	1
CE	1	1	1
CF	1	1	1
CG	1	1	1
DE	1	1	1
DF	1	1	1
DG	1	1	1
EF	1	0	0
EG	1	0	0
FG	1	0	0

$\boxed{C_B(D) = 15}$

Node : D, C, E have the highest betweenness Centrality.
(15) (14) (14)

C. Not considering End points
+
Already Computed in 'a' bit.

d. Normalized, not considering end points.

Since, this is an undirected network: $\frac{(n-1)(n-2)}{2}$, $n=7$

* Divide the C(b) with →

$C_b'(A) = \frac{0}{\frac{(7-1)(7-2)}{2}} = 0 \Rightarrow \boxed{C_b(A) = 0}$

similarly,

$\boxed{C_b(B) = 0}$

$\boxed{C_b(F) = 0}$

$\boxed{C_b(G) = 0}$

$C_b(C) = \frac{8}{\frac{(6)(5)}{2}} = \frac{8 \times 2}{6 \times 5} = \frac{16}{30} = 0.533 \Rightarrow \boxed{C_b(C) = 0.533}$

similarly,

$\boxed{C_b(E) = 0.533}$

$C_b(D) = \frac{9 \times 2}{6 \times 5} = \frac{18}{30} \Rightarrow \boxed{C_b(D) = 0.6}$

e. Considering source nodes as {A, B, C, D} and target nodes as {E, F, G}.

$C_b(A) = ?$

Pairs	σ_{uw}	$\sigma_{uw}(v)$	$\sigma_{uw}(v)/\sigma_{uw}$
AE	1	1	1
AF	1	1	1
AG	1	1	1
BE	1	0	0
BF	1	0	0
BG	1	0	0
CE	1	0	0
CF	1	0	0
CG	1	0	0
DE	1	0	0
DF	1	0	0
DG	1	0	0

$\boxed{C_b(A) = 3}$

$\boxed{C_b(B) = C_b(A) = C_b(F) = C_b(G) = 3}$

$C_b(C) = ?$

Pairs	σ_{uw}	$\sigma_{uw}(v)$	$\sigma_{uw}(v)/\sigma_{uw}$
AE	1	1	1
AF	1	1	1
AG	1	1	1
BE	1	1	1
BF	1	1	1
BG	1	1	1
CE	1	1	1
CF	1	1	1
CG	1	1	1
DE	1	0	0
DF	1	0	0
DG	1	0	0

$\boxed{C_b(C) = 9}$

$\boxed{C_b(C) = C_b(E) = 9}$

$C_b(D) = ?$

Pairs	σ_{uw}	$\sigma_{uw}(v)$	$\sigma_{uw}(v)/\sigma_{uw}$
AE	1	1	1
AF	1	1	1
AG	1	1	1
BE	1	1	1
BF	1	1	1
BG	1	1	1
CE	1	1	1
CF	1	1	1
CG	1	1	1
DE	1	1	1
DF	1	1	1
DG	1	1	1

$\boxed{C_b(D) = 12}$

f. Consider edges

(i) Edge (A B)

pairs	σ_{uw}	$\sigma_{uw}(E)$	$\sigma_{uw}(E)/\sigma_{uw}$
AB	1	1	1
AC	1	0	0
AD	1	0	0
AE	1	0	0
AF	1	0	0
AG	1	0	0
BC	1	0	0
BD	1	0	0
BE	1	0	0
BF	1	0	0
BG	1	0	0
CD	1	0	0
CE	1	0	0
CF	1	0	0
CG	1	0	0
DE	1	0	0
DF	1	0	0
DG	1	0	0
EF	1	0	0
EG	1	0	0
FG	1	0	0

$\boxed{C_b(E_{AB}) = 1}$

* since this graph is symmetric:

$\boxed{* \ C_b(E_{AB}) = C_b(E_{FG}) = 1}$

(iii) Edge (AC)

Pairs	σ_{uw}	$\sigma_{uw}(E)$	$\sigma_{uw}(E)/\sigma_{uw}$
AB	1	0	0
AC	1	1	1
AD	1	1	1
AE	1	1	1
AF	1	1	1
AG	1	1	1
BC	1	0	0
BD	1	0	0
BE	1	0	0
BF	1	0	0
BG	1	0	0
CD	1	0	0
CE	1	0	0
CF	1	0	0
CG	1	0	0
DE	1	0	0
DF	1	0	0
DG	1	0	0
EF	1	0	0
EG	1	0	0
FG	1	0	0

$\boxed{C_b(E_{AC}) = 5}$

* since this graph is symmetric.

$\boxed{* \; C_b(E_{AC}) = C_b(E_{BC}) = C_b(E_{FE}) = C_b(E_{GE}) = 5}$

(iii) Edge (CD)

pairs	σ_{uw}	$\sigma_{uw}(E)$	$\sigma_{uw}(E)/\sigma_{uw}$
AB	1	0	0
AC	1	0	0
AD	1	1	1
AE	1	1	1
AF	1	1	1
AG	1	1	1
BC	1	0	0
BD	1	1	1
BE	1	1	1
BF	1	1	1
BG	1	1	1
CD	1	1	1
CE	1	1	1
CF	1	1	1
CG	1	1	1
DE	1	0	0
DF	1	0	0
DG	1	0	0
EF	1	0	0
EG	1	0	0
FG	1	0	0

$\boxed{C_b(E_{CD}) = 12}$

* since this graph is symmetric.

* $\boxed{C_b(E_{CD}) = C_b(E_{DE}) = 12}$.

9. (i) Edge (AB)

pairs	σ_{uw}	$\sigma_{uw}(E)$	$\sigma_{uw}(E)/\sigma_{uw}$
AE	1	0	0
AF	1	0	0
AG	1	0	0
BE	1	0	0
BF	1	0	0
BG	1	0	0
CE	1	0	0
CF	1	0	0
CG	1	0	0
DE	1	0	0
DF	1	0	0
DG	1	0	0

$\boxed{C_b(E_{AB}) = 0}$

$\boxed{\therefore \; C_b(E_{AB}) = C_b(E_{FG}) = 0}$

(ii) Edge (AC)

pairs	σ_{uw}	$\sigma_{uw}(E)$	$\sigma_{uw}(E)/\sigma_{uw}$
AE	1	1	1
AF	1	1	1
AG	1	1	1
BE	1	0	0
BF	1	0	0
BG	1	0	0
CE	1	0	0
CF	1	0	0
CG	1	0	0
DE	1	0	0
DF	1	0	0
DG	1	0	0

$\boxed{C_b(E_{AC}) = 3}$

$\therefore \boxed{C_b(E_{AC}) = C_b(E_{BC}) = C_b(E_{FE}) = C_b(E_{GE}) = 3}$.

(iii) Edge (CD)

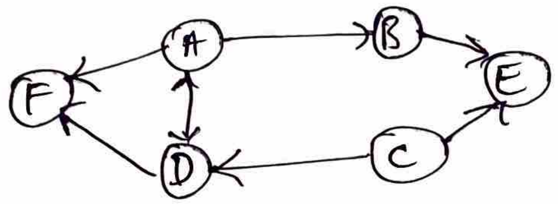
pairs	σ_{uw}	$\sigma_{uw}(E)$	$\sigma_{uw}(E)/\sigma_{uw}$.
AE	1	1	1
AF	1	1	1
AG	1	1	1
BE	1	1	1
BF	1	1	1
BG	1	1	1
CE	1	1	1
CF	1	1	1
CG	1	1	1
DE	1	0	0
DF	1	0	0
DG	1	0	0
			$\boxed{C_b(E_{CD}) = 9}$

~~* Same, CD~~

* So, $\boxed{C_b(E_{CD}) = C_b(E_{DE}) = 9}$ //

3. Directed Connected graph

Ans:



Iteration 1:

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array}\begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \times \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \\ 2 \\ 0 \\ 0 \end{bmatrix} \approx \begin{bmatrix} 0.7075 \\ 0.2358 \\ 0.4717 \\ 0.4717 \\ 0 \\ 0 \end{bmatrix}$$

normalize $= \sqrt{9+1+4+4} = \sqrt{18} \approx 4.24$

Iteration 2:

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0.7075 \\ 0.2358 \\ 0.4717 \\ 0.4717 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.7075 \\ 0 \\ 0.4717 \\ 0.7075 \\ 0 \\ 0 \end{bmatrix} \approx \begin{bmatrix} 0.637 \\ 0 \\ 0.4265 \\ 0.637 \\ 0 \\ 0 \end{bmatrix}$$

normalize $= \sqrt{(0.7075)^2 + (0.4717)^2 + (0.7075)^2} \approx 1.106$

Iteration 3:

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0.637 \\ 0 \\ 0.4265 \\ 0.637 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.637 \\ 0 \\ 0.637 \\ 0.637 \\ 0 \\ 0 \end{bmatrix} \approx \begin{bmatrix} 0.5775 \\ 0 \\ 0.5775 \\ 0.5775 \\ 0 \\ 0 \end{bmatrix}$$

normalize $= 1.103$

* since the normalized value is stabilized after 3rd iteration. we stop the iterating process and assign the eigen centrality vectors.

$(EC)_A = 0.5775$

$(EC)_B = 0$

$(EC)_C = 0.5775$

$(EC)_D = 0.5775$

$(EC)_E = 0.$

$(EC)_F = 0.$

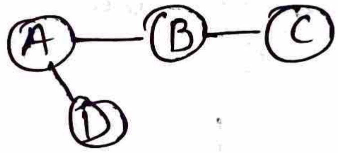
* Here we observe that $(EC)_B$, $(EC)_E$ and $(EC)_F$ are all '0'.

* It is pretty obvious that Node: E and F have a EC of '0', as they don't have any outdegree.

* In case of node 'B', since its only outdegree is pointing to node 'E', which has no outdegree. 'EC' of 'B' will also be '0' as the iterations progresses.

* From this we understand that, the Eigenvector Centrality depends on the neighbours to which the node is connected also.

4. Ans:- Consider the graph,



1. Eigenvector Centrality :

* start with an initial vector [1, 1, 1, 1]

Iteration 1 :

* A's neighbours are B and D.

$$C_A = \frac{1}{\lambda}[B(0) + D(0)] = \frac{1}{2}[1+1] = 1.$$

* B's neighbours are A and C.

$$C_B = \frac{1}{\lambda}[A(0) + C(0)] = \frac{1}{2}[1+1] = 1.$$

* C's neighbours are B.

$$C_C = \frac{1}{\lambda}[B(0)] = \frac{1}{1} * 1 = 1.$$

* D's neighbours are A.

$$C_D = \frac{1}{\lambda}[A(0)] = \frac{1}{1} * 1 = 1.$$

After Iteration 1 : our vector (v) = $\begin{bmatrix}1\\1\\1\\1\end{bmatrix}$.

* No matter how many times you iterate this, it will stay $\begin{bmatrix}1\\1\\1\\1\end{bmatrix}$.

So, [1, 1, 1, 1] are eigen centrality values for A, B, C, D.

2. Betweenness Centrality

$C_b(A) = ?$

pair	σ_{uw}	$\sigma_{uw}(v)$	$\sigma_{uw}(v)/\sigma_{uw}$
BC	1	0	0
BD	1	1	1
CD	1	1	1

$C_b(A) = 2$

$C_b(B) = ?$

pair	σ_{uw}	$\sigma_{uw}(v)$	$\sigma_{uw}(v)/\sigma_{uw}$
AC	1	1	1
AD	1	0	0
CD	1	1	1

$C_B(B) = 2$

$C_b(C) = ?$

pair	σ_{uw}	$\sigma_{uw}(v)$	$\sigma_{uw}(v)/\sigma_{uw}$
AB	1	0	0
AD	1	0	0
BD	1	0	0

$C_b(C) = 0$

$C_b(D) = ?$

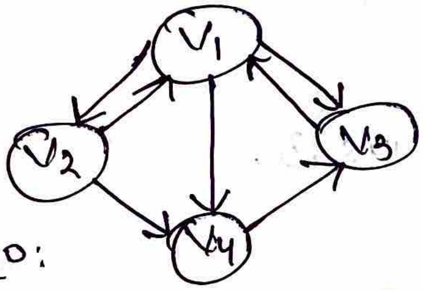
pair	σ_{uw}	$\sigma_{uw}(v)$	$\sigma_{uw}(v)/\sigma_{uw}$
AB	1	0	0
BC	1	0	0
AC	1	0	0

$C_b(D) = 0$

* Here, we achieve different 'C_b''s. $\boxed{C_b(A) = C_b(B) = 2}$ and $\boxed{C_b(C) = C_b(D) = 0}$.

5. **Basic Page Rank**

Ans:



Iteration 0:

Initially ; Page Rank (PR) of all nodes $= \frac{1}{n} = \frac{1}{4} = 0.25$.

$PR(V1) = PR(V2) = PR(V3) = PR(V4) = 0.25$

Iteration 1:

$PR(V1) = \frac{PR(V2)}{C(V2)} + \frac{PR(V3)}{C(V3)} \Rightarrow \frac{0.25}{2} + \frac{0.25}{1}$

$= 0.125 + 0.250$

$\boxed{PR(V1) = 0.375}$

$PR(V2) = \frac{PR(V1)}{C(V1)} = \frac{0.375}{3} = 0.125$

$\boxed{PR(V2) = 0.125}$

$PR(V3) = \frac{PR(V1)}{C(V1)} + \frac{PR(V4)}{C(V4)} = \frac{0.375}{3} + \frac{0.25}{1}$

$= 0.125 + 0.250$

$\boxed{PR(V3) = 0.375}$

$PR(V4) = \frac{PR(V1)}{C(V1)} + \frac{PR(V2)}{C(V2)} = \frac{0.375}{3} + \frac{0.125}{2}$

$= 0.125 + 0.0625$

$\boxed{PR(V4) = 0.1875}$

Iteration 2:

$$PR(V_1) = \frac{PR(V_2)}{C(V_2)} + \frac{PR(V_3)}{C(V_3)} = \frac{0.125}{2} + \frac{0.375}{1} = 0.0625 + 0.3750$$

$$\boxed{PR(V_1) = 0.4375}$$

$$PR(V_2) = \frac{PR(V_1)}{C(V_1)} = \frac{0.4375}{3} = 0.14583.$$

$$\boxed{PR(V_2) = 0.14583}$$

$$PR(V_3) = \frac{PR(V_1)}{C(V_1)} + \frac{PR(V_4)}{C(V_4)} = \frac{0.4375}{3} + \frac{0.1875}{1}$$

$$= 0.14583 + 0.1875$$

$$\boxed{PR(V_3) = 0.3333}$$

$$PR(V_4) = \frac{PR(V_1)}{C(V_1)} + \frac{PR(V_2)}{C(V_2)} = \frac{0.4375}{3} + \frac{0.14583}{2}$$

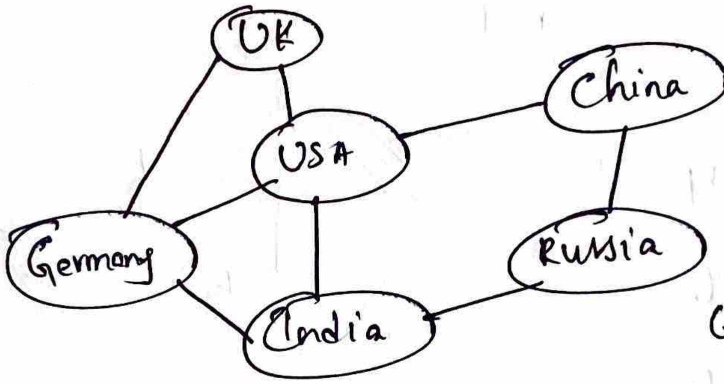
$$= 0.14583 + 0.0729$$

$$\boxed{PR(V_4) = 0.2187495}$$

Iterations	PR(V1)	PR(V2)	PR(V3)	PR(V4)
Iteration 0	0.25	0.25	0.25	0.25
Iteration 1	0.375	0.125	0.375	0.1875
Iteration 2	0.4375	0.14583	0.3333	0.2187495

6.

Ans:



G.

a. Adjacency Matrix:

$$A = \begin{array}{c} \\ USA \\ UK \\ Ger \\ Ind \\ China \\ Russia \end{array} \begin{array}{c} \begin{array}{cccccc} USA & UK & Ger & Ind & Chn & Rus \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

for a Matrix to be symmetric $A = A^T$.

$$A^T = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

we observe that $\boxed{A = A^T}$.

$\therefore$ The Adjacency Matrix of this Graph (G) is symmetric.

b. Degree centrality $\quad C_D = \frac{d(i)}{n-1}$

$C_D(USA) = \frac{4}{5} \Rightarrow 0.8$

$C_D(UK) = \frac{2}{5} \Rightarrow 0.4$

$C_D(Ger) = \frac{3}{5} \Rightarrow 0.6$

$C_D(Ind) = \frac{3}{5} \Rightarrow 0.6$

$C_D(Chn) = \frac{2}{5} \Rightarrow 0.4$

$C_D(Rus) = \frac{2}{5} \Rightarrow 0.4$

Closseness Centrality $\quad C_C = \frac{n-1}{\sum_{j=1} d(i,j)}$

$C_C(USA) = \frac{5}{1+1+1+1+2} = \frac{5}{6} \Rightarrow 0.833$

$C_C(UK) = \frac{5}{1+1+2+2+3} = \frac{5}{9} \Rightarrow 0.555$

$C_C(Ger) = \frac{5}{1+1+1+2+2} = \frac{5}{7} \Rightarrow 0.714$

$C_C(Ind) = \frac{5}{1+2+1+2+1} = \frac{5}{7} \Rightarrow 0.714$

$C_C(China) = \frac{5}{1+2+2+2+1} = \frac{5}{8} \Rightarrow 0.625$

$C_C(Russia) = \frac{5}{2+3+2+1+1} = \frac{5}{9} \Rightarrow 0.555$

Betweeness Centrality. (excluding the end point).

(1) Cb(USA) = ?

Pairs	σ_{uw} [shortest paths]	$\sigma_{uw}(v)$ does it pass	$\sigma_{uw}(v)/\sigma_{uw}$
(UK, Ger)	1	0	0
(UK, Ind)	2	1	0.5
(UK, Chn)	1	1	1
(UK, Rus)	3	1	0.33
(Ger, Ind)	1	0	0
(Ger, Rus)	1	0	0
(Ger, Chn)	1	1	1
(Ind, Rus)	1	0	0
(Ind, Chn)	2	1	0.5
(Chn, Rus)	1	0	0
			3.33

Cb(USA) = 3.33

(2) Cb(UK) = ?

Pairs	σ_{uw}	$\sigma_{uw}(v)$	$\sigma_{uw}(v)/\sigma_{uw}$
(USA, Ger)	1	0	0
(USA, Ind)	1	0	0
(USA, Chn)	1	0	0
(USA, Rus)	2	0	0
(Ger, Ind)	1	0	0
(Ger, Chn)	1	0	0
(Ger, Rus)	1	0	0
(Ind, Chn)	2	0	0
(Ind, Rus)	1	0	0
(Chn, Rus)	1	0	0
			0

Cb(UK) = 0

(3) Cb(Ger) = ?

Pairs	σ_{uw}	$\sigma_{uw}(v)$	$\sigma_{uw}(v)/\sigma_{uw}$
(UK, USA)	1	0	0
(UK, Ind)	2	1	0.5
(UK, Chn)	1	0	0
(UK, Rus)	3	1	0.33
(USA, Ind)	1	0	0
(USA, Chn)	1	0	0
(USA, Rus)	2	0	0
(Ind, Chn)	2	0	0
(Ind, Rus)	1	0	0
(Chn, Rus)	1	0	0
			0.83

$\boxed{Cb(Ger) = 0.83}$

(4) Cb (Ind) = ?

Pairs	σ_{uw}	$\sigma_{uw}(v)$	$\sigma_{uw}(v)/\sigma_{uw}$
(UK, Ger)	1	0	0
(UK, USA)	1	0	0
(UK, Chn)	1	0	0
(UK, Rus)	3	1	0.33
(USA, Ger)	1	0	0
(USA, Chn)	1	0	0
(USA, Rus)	2	1	0.5
(Ger, Chn)	1	0	0
(Ger, Rus)	1	1	1
(Chn, Rus)	1	0	0
			1.83

$\boxed{Cb(Ind) = 1.83}$

(5) Cb (China) = ?

Pairs	σ_{uw}	$\sigma_{uw}(v)$	$\sigma_{uw}(v) / \sigma_{uw}$
(UK, USA)	1	0	0
(UK, Ger)	1	0	0
(UK, Ind)	2	0	0
(UK, Rus)	3	1	0.33
(USA, Ger)	1	0	0
(USA, Ind)	1	0	0
(USA, Rus)	2	1	0.5
(Ind, Ger)	1	0	0
(Ind, Rus)	1	0	0
(Ger, Rus)	1	0	0

$\boxed{Cb(\text{China}) = 0.83}$

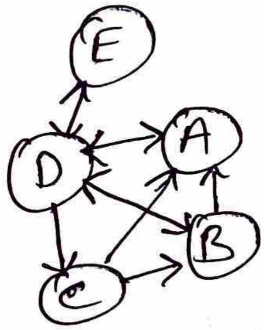
(6) Cb (Rus) = ?

Pairs	σ_{uw}	$\sigma_{uw}(v)$	$\sigma_{uw}(v) / \sigma_{uw}$
(UK, USA)	1	0	0
(UK, Ger)	1	0	0
(UK, Ind)	2	0	0
(UK, Chn)	1	0	0
(USA, Ger)	1	0	0
(USA, Ind)	1	0	0
(USA, Chn)	1	0	0
(Ger, Ind)	1	0	0
(Ger, Chn)	1	0	0
(Ind, Chn)	2	1	0.5

$\boxed{Cb(\text{Russia}) = 0.5}$

7. Citation network

Ans: * The Centrality Measure that we're going to use for this network is the Katz centrality.



Lets consider this to be our citation network.

Let,
α (scaling factor) = 0.5
β (Constant term) = 1.

1. Author A:

* A is cited by Authors B, C, D.

$$(C_{Katz})_A = \alpha\,[C(B)+C(C)+C(D)] + \beta.$$
$$= 0.5\,[C(B)+C(C)+C(D)] + \beta.$$

2. Author B:

* B is cited by Authors C, D.

$$(C_{Katz})_B = \alpha\,[C(C)+C(D)] + \beta.$$
$$= 0.5\,[C(C)+C(D)] + \beta.$$

3. Author C:

* C is cited by Author D.

$$(C_{Katz})_C = \alpha\,[C(D)] + \beta$$
$$= 0.5\,[C(D)] + \beta.$$

4. Author D :

$(C_{KaltE})_D = 0.5[C(A) + C(B) + C(E)] + 1$

* 'D' is cited by Author A, B, E.

5. Author E :

$(C_{KaltE})_E = 0.5[C(D)] + 1.$

* 'E' is cited by Author D.

Iteration 0

Initially assign,

$C(A) = C(B) = C(C) = C(D) = C(E) = 1.$

Iteration 1

$C(A) = 0.5[1+1+1] + 1$

$\boxed{C(A) = 2.5}$

$C(B) = 0.5[1+1] + 1$

$\boxed{C(B) = 2}$

$C(C) = 0.5[1] + 1$

$\boxed{C(C) = 1.5}$

$C(D) = 0.5[2.5+2+1] + 1$

$\boxed{C(D) = 3.75}$

$C(E) = 0.5[3.75] + 1.$

$\boxed{C(E) = 2.875}$

Iteration 2

$C(A) = 0.5[2+1.5+3.75]+1$

$\boxed{C(A) = 4.625}$

$C(B) = 0.5[1.5+3.75]+1$

$\boxed{C(B) = 3.625}$

$C(C) = 0.5[3.75]+1$

$\boxed{C(C) = 2.875}$

$C(D) = 0.5[4.625+3.625+2.875]+1$

$\boxed{C(D) = 6.5625}$

$C(E) = 0.5[6.5625]+1.$

$\boxed{C(E) = 4.28125}$

Let's stop our iterations here.

Iterations	C(A)	C(B)	C(C)	C(D)	C(E)
0	1	1	1	1	1
1	2.5	2	1.5	3.75	2.875
2	4.625	3.625	2.875	6.5625	4.28125