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Electromagnetic guidance based on orbital dynamics for nano satellite docking

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Abstract

Nanosatellites has been rising in popularity due to their efficient size, ability to utilize orbital environment and performing the large variety of space missions for a cheaper price. They would gain even more functionality when joined with other nanosatellites to form more complex structure or service each other. This would require rendezvous and docking processes that pose certain challenges for small sized satellites. Modern satellites often use the combination of the electromagnetic actuators and gas thrusters to guide towards each other. This would make nanosatellites dependent on the consumable amount of propellant. Therefore, our thesis proposes the rendezvous and docking strategy for nanosatellites that utilized only electromagnetic interaction. Two main issues will be addressed with the use of electromagnetic actuators: power consumption and docking accuracy. Power consumption must be minimized due to the energy restrictions of the small satellites. Thus, this thesis provides a strategy of using orbital motions of satellites to use less electrical energy compared to contemporary guidance methods. Second, using electromagnets result in highly nonlinear dynamics that can hinder the accuracy of docking due to the size restrictions of the satellite. Therefore, control method was developed to maintain the rendezvous orbit and docking accuracy.

This thesis provides electromagnetic rendezvous strategy for two satellites equipped with electromagnetic actuators. We have focused on the orbital dynamics of the guidance and assumed an active attitude control. The success of the guidance method and power consumption are shown and evaluated as results of numerical simulations performed by the AOCS.

Chapter 1. Introduction

1.1 Motivation

Traditionally, rendezvous and docking operations were used for the purposes of space exploration and on-orbit servicing. Concepts as in-orbit assembly of telescope and debris removal are expanding the application field of such operations and small satellites.

Nanosatellites are the class of small satellites with a mass of less than 10kg. The most popular among them, CubeSat is made of usually 1 to 6 U's (Each U is a cubic box of 10x10x10cm size). Their small size holds numerous advantages such as using earth's magnetic field for active stabilization and dumping the momentum. Moreover, they can be launched to the space as an additional cargo if there is an empty space in a spaceship. Low development and launch costs make them especially attractive to small companies and student groups. As electronic technology is advancing and moving towards miniaturization, capabilities of the nanosatellites are rising. One of the recent examples is Autonomous Assembly of Reconfigurable Space Telescope using a group of nanosatellites (Underwood,2014).

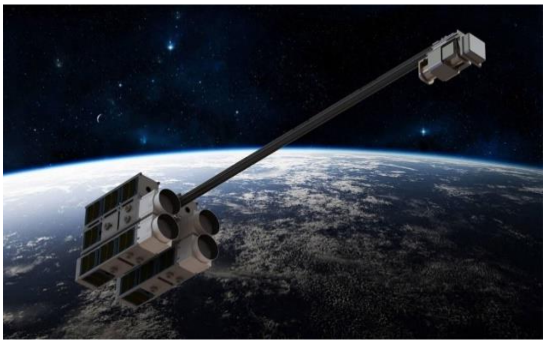


Image.1 AAReST mission Credit: Sergio Pellegrino/Caltech [1]

However, consumable propellants limit the potential of nanosatellite's operation in space. Due to their size, they cannot store large amount of propellants such as cold gas. Therefore, we were motivated by the phenomena of self -adjusting and joining that happens between the magnetic substances. Utilizing their natural characteristics to propose a propellantless guidance method is the interest behind this thesis.

Moreover, nanosatellites cannot generate high amount of force with magnet, especially if rendezvous distance is relatively large. Electromagnets consume high amount of power to operate at full capacity. Therefore, proposing an energy efficient method is also the priority of this thesis paper.

This thesis takes inspiration from the idea of electromagnetic docking and using relative orbital motions of satellites. It provides a feasible way of implementing these ideas in a scenario with acceptable assumptions. Successful rendezvous and docking missions using this method were demonstrated with numerical simulations.

1.2 Previous relative work

Guidance:

There are numerous ways of guiding the satellite and controlling, each having a various level of complexity. One of them is the control the guidance by using image recognition tools to locate the target satellite (Camille, 2018). It uses the shape of an object or LED markers to calculate the distance between. This method of navigation relies on processing of visual data and can have issues with lighting conditions. When it comes to the position change, combination of gas thrusters and electromagnets are often used. Gas thrusters are prone to plume impingement and can cause costly disturbances to docking process. Therefore, it is better to propose a navigation method that relies only on electromagnetic actuators that do not use consumable propellants.

Docking:

With an advent of nanosatellites, there were many propositions of docking and grasping mechanisms. Some systems such as ARCADE (Olivieri, 2017) employed the classical probe/drogue mechanism and reduced it in size and cost. Another one known as SAM represents the semi-androgynous docking method that has mechanical grabbing mechanism where any of two satellites can act as a drogue and probe.

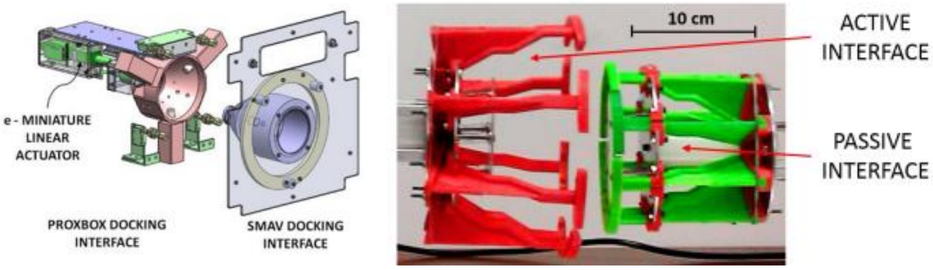


Image.2 ARCADE (left) and SAM (right) docking mechanisms [2]

These methods still leave the docking mechanism relatively complicated for the nanosatellites. Due to the complexity of mechanical joints and other parts, they are under the risk of damage during the transportation (vibrations) and launch processes. Therefore, we need a simpler yet effective way to dock and maintain the grip for nanosatellites. Our proposal of using the permanent magnet and electromagnet on satellites allows using the natural attraction forces between them to perform docking. For the purpose of a release and separation, electromagnet can change its orientation by applying current to coils in different direction.

Electromagnetic force:

Magnetic force and torque are used in space missions mainly for attitude control of satellites in LEO, electromagnetic formation flight (EMFF) and electromagnetic docking. The electromagnetic attitude control system is based on the interaction between a group of orthogonal magnetic coils and the geomagnetic field (Ashun, 2017). As a result, these coils can generate corresponding torques to actively control the satellite attitude. The EMFF uses the magnetic force to control positioning between the satellites. However, the docking operation is more affected by the nonlinearity problem: electromagnetic force increases with distance to the fourth power as relative distance between spacecraft decreases. Thus, in electromagnetic docking missions, robust control capability is cardinal (HUO, 2012).

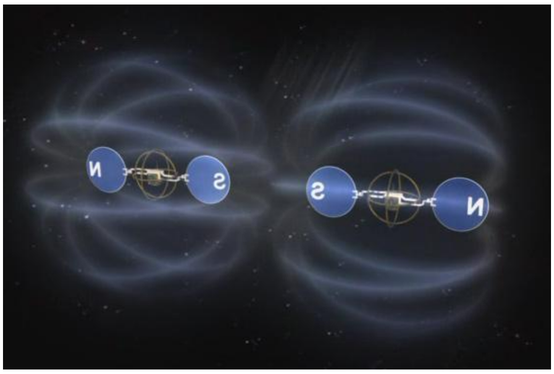


Image.3 EMFF credit: Benjamin Schweighart/ MIT-SSL [3]

1.3 Overview of the Research and thesis

This thesis paper proposes and evaluates the idea of using only electromagnetic actuators to initiate and control the rendezvous and docking process. The electric coil in the target satellite applies a short-term pulling force to the permanent magnet on the chaser satellite, altering its orbit so they will eventually join. Changing the orbital motion of the chaser satellite and forming the special rendezvous trajectory saves the power consumption compared to the direct pulling methods. Control method based on Sliding Mode Control is applied to keep the chaser satellite on the guidance track. Thus, nonlinearity of the force, unmodeled parameters of the system and external disturbances can be neglected. As a result of extensive testing on the numerical simulations, the feasibility of this guidance method was confirmed.

The structure of this thesis is constructed as such:

In Chapter 2, the strategy of the guidance was introduced in detail. The entire mission is divided into basic steps that are explained as transformation from stationary orbit to the rendezvous orbit. The description of those orbits constitutes the main part of this chapter.

In Chapter 3, far field model of the electromagnetic force was presented and evaluated. This model is crucial for description of the system dynamics and the controller design.

In Chapter 4, the role of the control function is explained in detail. It features the steps for Sliding Mode Controller design to keep the chaser satellite on the docking orbit.

In Chapter 5, simulation results of different cases with or without controller are presented and discussed. The power consumption simulation is performed for our rendezvous method to compare it with contemporary rendezvous methods.

In Chapter 6, main findings of this thesis have been discussed and concluded. Some suggestions on future work are also present.

Chapter 2. Modeling the orbital motion

To analyze the rendezvous process, forming a dynamic model of relative motion is essential. Relative motion of satellites has been well studied and there are many models that can describe it. A set of linearized differential equations, known as Clohessy Wiltshire (CW) equations (Clohessy and Wiltshire 1960), is well-known classical way of describing the relative motion of two satellites in near-circular orbits. This model has the form of differential equations that makes it very favorable for controller design.

In addition, there are numerous versions of this model that account for eccentricity and perturbation, offering a more complex system. However, as we are focused on satellite rendezvous and want to keep the control law simple, we will choose the fundamental form of the CW equations.

2.1 Description of the coordinate system (LVLH)

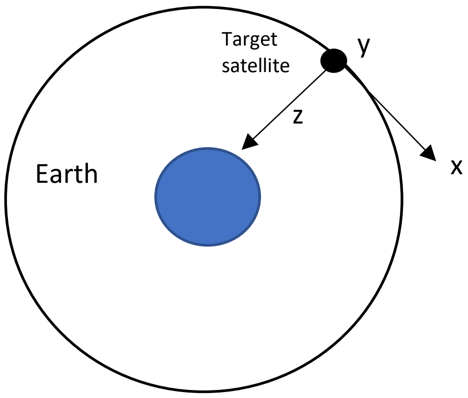


Fig.1 Local Vertical Local Horizontal frame [4]

The +z axis is directed towards the center of the Earth, the +y axis is turned to the negative orbit normal and the +x axis is a tangent to the orbit completing a right-handed frame.

If we assume that orbit of target is circular, motion of chaser satellite in LVLH is descried as:

$$\begin{cases} \ddot{x}_o + 2n\dot{z}_o = 0 \\ \ddot{y}_o + n^2 y_o = 0 \\ \ddot{z}_o + 2n\dot{x}_o - 3n^2 z_o = 0 \end{cases} \tag{1}$$

Here, x_o, y_o and z_o are position of the chaser satellite from the origin (target satellite), “o” indicating the LVLH frame. $\dot{x}_o, \dot{y}_o$ and $\dot{z}_o$ are the relative velocity components, $n = \sqrt{\frac{\mu}{r^3}}$ is the mean motion, μ denotes gravitational coefficient and r denotes radius of satellite orbit.

2.2 Explanation of the stationary orbit

In LVLH frame, the chaser satellite is rotating around the target satellite on a stationary orbit, where it does not require any external force to maintain the orbit. The formula that describes the following orbit can be derived in such a way (Y.Yamada, 2018):

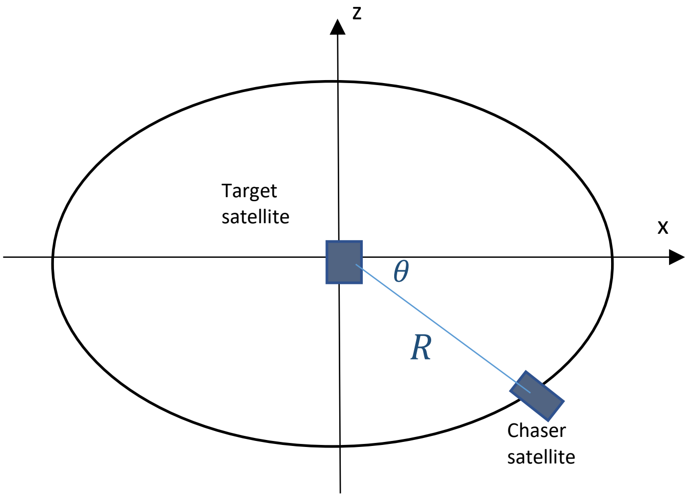


Fig.2 Stationary Orbit [5]

First of all, me must obtain the analytical solution of the equation (1):

$$
\begin{aligned}
x_o(t) &= (6nz_{o0} - 3\dot{x}_{o0})t + \left(x_{o0} + \frac{2\dot{z}_{o0}}{n}\right) - \left(6z_{o0} - \frac{4\dot{x}_{o0}}{n}\right)\sin(nt) - \frac{2\dot{z}_{o0}}{n}\cos(nt) \\
y_o(t) &= \frac{\dot{y}_{o0}}{n}\sin(nt) + y_{o0}\cos(nt) \\
z_o(t) &= \left(4z_{o0} - \frac{2\dot{x}_{o0}}{n}\right) + \frac{\dot{z}_{o0}}{n}\sin(nt) - \left(3z_{o0} - \frac{2\dot{x}_{o0}}{n}\right)\cos(nt) \\
\dot{x}_o(t) &= (6nz_{o0} - 3\dot{x}_{o0}) - (6nz_{o0} - 4\dot{x}_{o0})\cos(nt) + 2\dot{z}_{o0}\sin(nt) \\
\dot{y}_o(t) &= \dot{y}_{o0}\cos(nt) - ny_{o0}\sin(nt) \\
\dot{z}_o(t) &= \dot{z}_{o0}\cos(nt) + (3nz_{o0} - 2\dot{x}_{o0})\sin(nt)
\end{aligned} \tag{2}
$$

We can see how variables of x_o and z_o depend from each other and the presence of a rotational motion in the x-z orbit. Value of y_o does not depend on another axis and the center of the motion does not move along y axis. Therefore, we have decided to concentrate on x-y plane to observe rotational motion.

In order to form the stationary orbit, center of the rotational motion must be located in the origin of the LVLH frame. For that, following conditions must be fulfilled(Y.Yamada, 2018):

$$\dot{x}_{00} = 2nz_{00}\text{c} \qquad\qquad \dot{z}_{00} \quad nx_{00}$$

Then equations (2) will transform into (ignoring y-axis):

$$\begin{aligned} x(t) &= 2z_0 \sin(nt) + x_0 \cos(nt) \\ z(t) &= -\tfrac{x_0}{2} \sin(nt) + z_0 \cos(nt) \\ \dot{x}(t) &= 2nz_0 \cos(nt) - nx_0 \sin(nt) \\ \dot{z}(t) &= -\frac{nx_0}{2} \cos(nt) + nz_0 \sin(nt) \end{aligned} \tag{3}$$

These equations describe the position of the chaser satellite at any point of the stationary orbit on the x-z frame.

2.3 Explanation of the rendezvous orbit

When permanent magnet of the chaser and electromagnet of target satellite align to the single line, electromagnet generates a short term strong force that changes the orbital motion of the chaser satellites. With an altered orbital motion, chaser draws closer to the target satellite resulting in docking.

We have decided to use the strategy of rendezvous orbit as shown above to minimize the energy consumption and take advantage of the orbital motion itself.

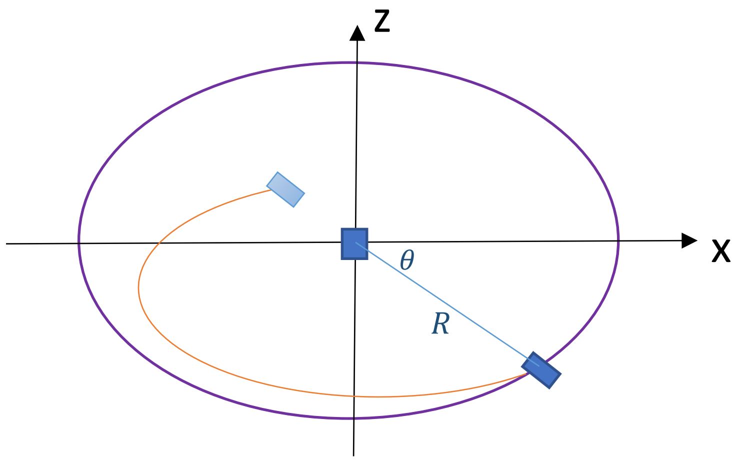


Fig.3 Rendezvous orbit shown with orange color [6]

Starting point of the rendezvous orbit is described as:

$$\begin{aligned} x_{00} &= R\cos\theta \\ z_{00} &= -\tfrac{R}{2}\sin\theta \\ \dot{x}_{00} &= -nR\sin(\theta) - v_m\cos(\theta) \\ \dot{z}_{00} &= -\frac{nR}{2}\cos(\theta) + v_m\sin(\theta) \end{aligned} \tag{4}$$

Now if we substitute these equations into the set of equations (2), we can obtain the position and velocity of the target satellite at any time on the rendezvous orbit (Y.Yamada, 2018):

$$x_0(t_s,\, v_m) = 3v_m t_s \cos(\theta) + \frac{2v_m}{n}\sin(\theta) - \left(R\sin(\theta) + \frac{4v_m}{n}\cos(\theta)\right)\sin(nt_s) + \left(R\cos(\theta) - \frac{2v_m}{n}\sin(\theta)\right)\cos(nt_s)$$

$$z_0(t_s,\, v_m) = \frac{2v_m}{n}\cos(\theta) - \left(\frac{R}{2}\cos(\theta) - \frac{v_m}{n}\sin(\theta)\right) * \sin(nt_s) - \left(\frac{R}{2}\sin(\theta) + \frac{2v_m}{n}\cos(\theta)\right)\cos(nt_s) \tag{5}$$

By setting the values of $x_0(t_s,\, v_m)$ and $z_0(t_s,\, v_m)$ to our target position (docking point), we can find the time of rendezvous (t_s) and how chaser satellite's velocity change (v_m). This can be done by numerical simulation where we can obtain numerous combinations of t_s and v_m. Then, we must choose the most suitable amongst them by evaluating amount of the time spend for docking and energy requirement for electromagnetic actuator. The lower time suggests fast docking, however, too high value of v_m might be not attainable due to actuator power limitations.

Chapter 3. Modeling the Magnetic Force

To study the dynamics and derive the dynamics equations of this system, we must first develop the theoretical model of the magnetic forces generated. Based on this model, the corresponding force equations needed for generating the full dynamic model are presented.

3.1 Explanation of the electromagnet from satellite

The target satellite is equipped with a ferromagnetic material that is wrapped around with a copper wire. This device is widely found on nanosatellites and serves as a mean to control attitude on LEO. It can generate a magnetic force that pulls the permanent magnet of the chaser satellite. Therefore, all the force that is required for maintaining the rendezvous orbit of the chaser is generated from the target satellite. For this reason, we declare the magnetic moment generated by target satellite (μ_T) as a control variable.

Magnetic moment from such device is $\mu_T = niS$, where *n* is number of coil wrappings, *i* is the current and *S* is the cross-section area of the coil.

Let's consider the magnetic force generated by two magnetic moments. We assume that there are two magnets based on the coil. The magnetic field created by a coil through which a current I1 is passing is given by the Biot - Savart law (Fabacher, 2015):

$$B_1(s) = \frac{\mu_0 N_1 i_1}{4\pi} \oint \frac{dl \times \hat{r}}{\|r\|^2} \tag{6}$$

- **μ0** - the magnetic permeability of the void
- **N1** - the number of turn in coil 1,
- **i1** - the current in coil 1 (A),
- **dl** - the elementary vector on the coil,
- **r** - the vector from a point on the coil to the point considered,
- $\hat{\boldsymbol{r}}$ - the elementary vector from a point on the coil to the point considered,
- **s** - the vector from coil center to the point considered.

The force on element of conductor $\boldsymbol{dl_2}$ through which passes a current i_2 and surrounded by a magnetic field B_1 is:

$$dF = i_2 \boldsymbol{dl_2} \times B_1 \tag{7}$$

Integrating the (7) on the coil gives us:

$$F_{1/2} = N_2 i_2 \oint \boldsymbol{dl_2} \times B_1 \tag{8}$$

Now if we insert the equation **(6):**

$$F_{\frac{1}{2}}(s) = \frac{N_1 i_1 N_2 i_2}{4\pi} \oint \oint \frac{dl \times \hat{r}}{\|r\|^2} \times \boldsymbol{dl_2} \tag{9}$$

Called “close field” expression, it stands valid even if the distance between magnetic actuators is very close. However, is not easy to use for our purposes due to its complex structure.

3.2 Explanation of the far field model

As we have mentioned before, the exact solution of the magnetic field equations contains integrals that cannot be solved analytically. This poses a serious challenge to model the force between magnets accurately. In order to address the time limitation of our research and keep the controller model simpler, we decided to use the far field model that will be explained in this chapter.

The first order expansion of the Taylor series of “close field” model is known as the far-field model (or magnetic dipole assumption). This model provides an analytical solution and it is easy to implement. However, as magnetic devices get closer, the reliability of the model decreases.

$$F = \frac{3\mu_0}{4\pi d^4}\left\{(\mu_C \hat{d})\mu_T + (\mu_T \hat{d})\mu_C + (\mu_C * \mu_T)\hat{d} - 5(\mu_C \hat{d})(\mu_T \hat{d})\hat{d}\right\} \tag{10}$$

Where $\boldsymbol{\mu_0}$is permeability in vacuum, d is distance between magnetic substances, $\boldsymbol{\mu_C}$ and $\boldsymbol{\mu_T}$ are magnetic moments from chaser satellite and target satellite respectively.

In order to form a control law, we need our controller variable $\boldsymbol{\mu_T}$ to be outside of the scopes { }. For that reason, $\boldsymbol{\Psi_C}$ is introduced to the equation.

Now force can be expressed as $= -\frac{3\mu_0}{4\pi}\Psi_C \mu_T$, where (Fabacher, 2015):

$$\begin{aligned}\Psi_C(\mu_C, d) = \left(\frac{5}{d^7}\right)(\mu_{Co} d_o)\begin{bmatrix} d_{ox}^2 & d_{ox}d_{oy} & d_{ox}d_{oz} \\ d_{oz}d_{oy} & d_{oy}^2 & d_{oy}d_{oz} \\ d_{ox}d_{oz} & d_{oy}d_{oz} & d_{oz}^2 \end{bmatrix} + \frac{(\mu_{Co} d_o)}{d^5} I_{3x3} \\ + \frac{1}{d^5}\begin{bmatrix} 2\mu_{Cox}d_{ox} & \mu_{Cox}d_{oy} + \mu_{Coy}d_{ox} & \mu_{Cox}d_{oy} + \mu_{Coy}d_{ox} \\ \mu_{Cox}d_{oy} + \mu_{Coy}d_{ox} & 2\mu_{Coy}d_{oy} & \mu_{Coy}d_{oz} + \mu_{Coz}d_{oy} \\ \mu_{Cox}d_{oy} + \mu_{Coy}d_{ox} & \mu_{Coy}d_{oz} + \mu_{Coz}d_{oy} & 2\mu_{Coz}d_{oz} \end{bmatrix}\end{aligned} \tag{11}$$

With this new simplification, the forces can be easily calculated for any dipole configuration. This is much simpler than the double integral equation, but these equations are accurate in the far field.

3.3 Short time force

As you have seen in chapter 2, a change in the velocity (V_m) of the chaser satellite is caused by a short-term force applied from target satellite. The relation between that force (F_{MTC}), its duration (t_m) and change of velocity are shown below (Y.Yamada, 2018):

$$V_m = \int_0^{t_m} \frac{F}{M} \approx \frac{Ft_m}{M} \tag{12}$$

Where, $M = \frac{m_t+m_c}{m_t m_c}$: m_t and m_c masses of target and chaser satellite respectively.

When permanent magnet and electromagnet are aligned to the same direction, the equation of the force can be simplified as:

$$F = \frac{3\mu_0|\mu_c||\mu_t|}{2\pi d^4}\hat{d} \tag{13}$$

Inserting the equation (12) we find the exact relationship between the variables:

$$V_m = \frac{m_t m_c}{m_t + m_c}\frac{3\mu_0|\mu_c||\mu_t|}{2nd^4}t_m \tag{14}$$

3.4 Translational Dynamics

In order, to derive control method, first we need the equation of our non-linear dynamical system:

$\ddot{\mathrm{r}} = \begin{bmatrix}\ddot{\mathrm{x}}\\ \ddot{\mathrm{y}}\\ \ddot{\mathrm{z}}\end{bmatrix}$ is a state vector, and μ_{Tb} is an input vector.

General form of the state equation:

$$\ddot{\boldsymbol{r}}_{\boldsymbol{o}} = \mathbf{F}(\mathbf{r}_o, \dot{\mathbf{r}}_\mathbf{o}) + \mathbf{G}(\mathbf{d_o}, \boldsymbol{\mu}_\mathbf{Co})\boldsymbol{\mu}_\mathbf{Tb} \tag{15}$$

Applying the orbit equations and equations of the far field force:

$$\ddot{\boldsymbol{r}}_{\boldsymbol{o}} = \begin{bmatrix} -\mathbf{2n\dot{z}_o} \\ -\mathbf{n^2y_0} \\ -\mathbf{2n\dot{x}_0 + 3n^2z_o} \end{bmatrix} + \left(\frac{\boldsymbol{m_t + m_c}}{\boldsymbol{m_t m_c}}\right)\boldsymbol{\Psi_C}(\boldsymbol{\mu_{Co}}, \boldsymbol{d_o})\boldsymbol{C_{Tbo}} \tag{16}$$

Where C_{Tbo} is a direction cosine matrix from body frame to the LVLH frame.

Chapter 4. Control method

4.1 Why we need control

Our control objective is to ensure that the system state tracks a desired trajectory (rendezvous orbit) despite environmental disturbances and uncertainties present in the dynamic model. As you can see, if we do not have any control method, chaser satellites go off the rendezvous orbit.

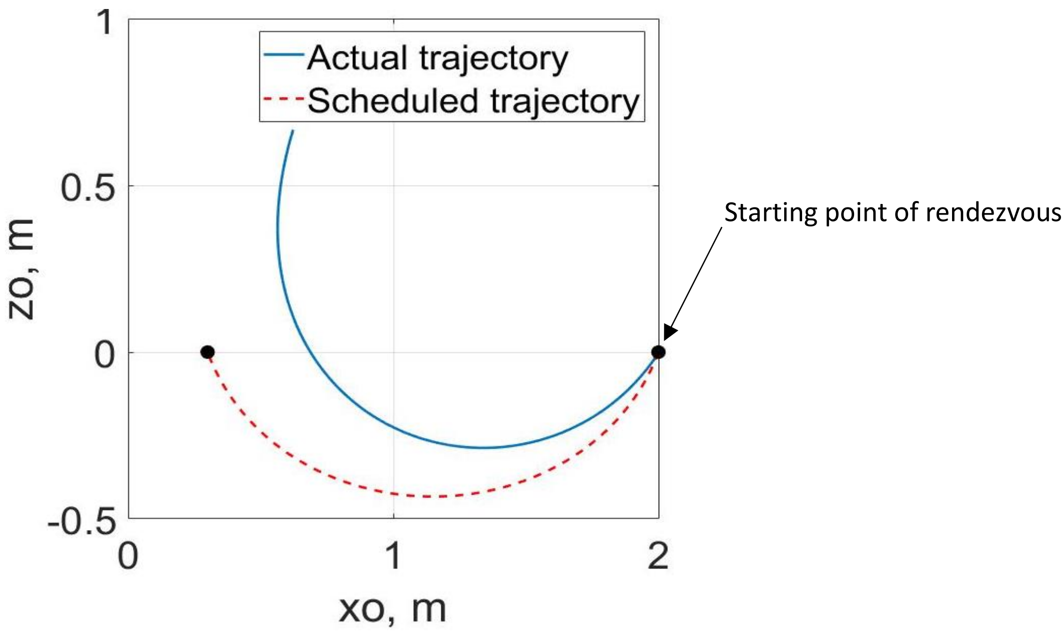
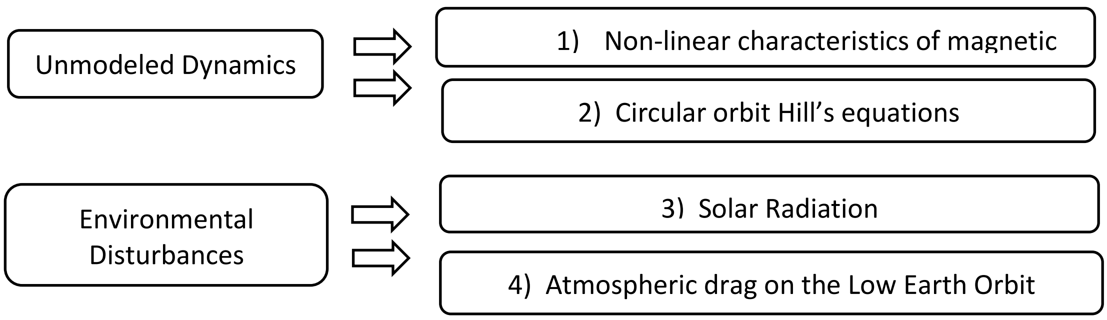


Fig.4 (this is the case 1 rendezvous, details of simulation can be found in the next chapter) [7]

There are two main reasons that can cause actual trajectory of rendezvous to be different from the scheduled one: unmodeled dynamics and environmental disturbances.



1. We have employed the far field model of the magnetic force to develop control law. This model works well for the magnets that are located on a relatively far distance from each other. However, it does not describe the "close case" interactions very well. Electromagnetic force

goes up with distance to the fourth power as the relative distance between Target and Chaser spacecraft increases.

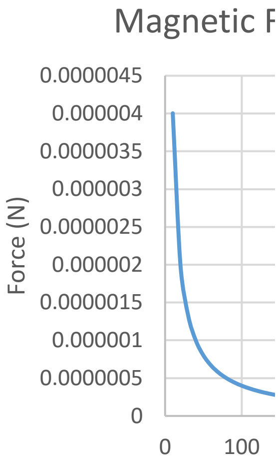


Fig.5 Force between two 20 ampere-meter poles [8]

It is hard to obtain the exact force on close distances with our model, that is why the real value of the force will always be different from the predicted one.

2. Another challenge comes from the simplification of the orbital motion equation. It implies circular orbit and does not account for perturbation due to the earth's oblateness. Although it is easier to use for our application, it has deviations from the real orbital motion model.

3. Solar radiation pressure is a pressure that is applied to the surface of the spacecraft by the sun. It has a crucial impact on the bodies that are located on the outer space. Even if the amount of pressure is negligibly small, its total effect over a time is worth considering. It is given by a fundamental formula:

$$\Delta F_{\mathrm{sp}} = P\left(C_{\mathrm{a}} + 2C_{\mathrm{s}} + \frac{5}{3}C_{\mathrm{ds}}\right)\Delta S \tag{17}$$

whereC_{a}, C_{s} and C_{ds} are coefficients of absorption, specular reflection and diffuse reflection respectively. S is surface area.

4. On the Low Earth Orbit, the effect of air particles can also have a significant effect and alter the spacecraft's path. This effect will increase in magnitude as satellite's altitude goes lower. The atmospheric drag force is given as:

$$\Delta F_{\mathrm{ad}} = \frac{1}{2}\rho C_{\mathrm{d}} v^2 \Delta S \tag{18}$$

where C_d is a coefficient of drag, ρ is density of air at the given altitude and v is a velocity of the object.

4.2 Control Strategy

We use Sliding Mode Control as our control method because it is insensitive to parameter variations and disturbances. This resolves our issues of not having an exact model of system dynamics and presence of disturbances.

The design process for sliding mode control consists from two steps:

1. Choosing a sliding surface (**s**) so that **s** = 0 gives rise to a "stable" differential equation, where solution of error (**e**) will achieve zero in finite time. System dynamics will be restricted to the equations of the surface.
2. Defining the control law and feedback gains, so that sliding surface would be obtained in the finite time.

4.3 Sliding surface design

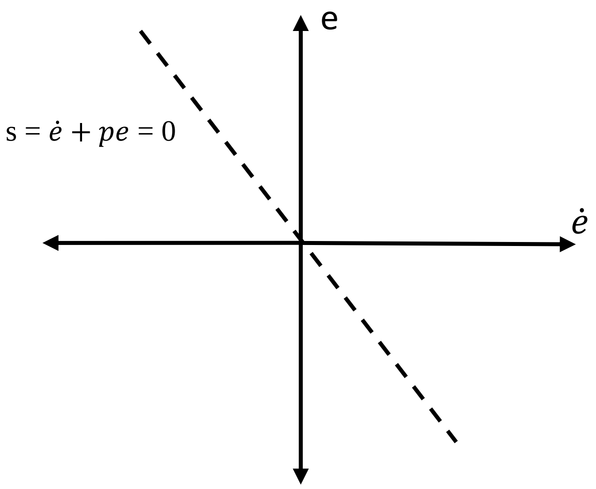


Fig.6 Sliding surface [9]

Now we define the error vector as a difference between scheduled rendezvous orbit $\boldsymbol{r_{ot}}$ and actual rendezvous orbit $\boldsymbol{r_o}$.

$$\boldsymbol{e} = \boldsymbol{r_{ot}} - \boldsymbol{r_o} \tag{19}$$

Having our error vector determined, we define sliding surface using the formula below, where p is gain of the sliding surface .

$$s = (\frac{d}{dt} + p)^k \int_o^t e(t)dt \tag{20}$$

As we have a second order system, we will have first order switching function. This means k=1:

$$s = \dot{e} + pe = 0 \tag{21}$$

where p is the control gain. Value of the p gain will define the tradeoff between performance and robustness of control.

4.4 Control function

Our task is to design a state-feedback control law to stabilize the dynamical system.

There are two parts of the control: corrective control and equivalent control.

$$\mu(t) = \mu_c(t) + \mu_{eq}(t) \tag{22}$$

$\mu_c(t)$, compensates the deviations from the sliding surface to reach the sliding surface

$\mu_{eq}(t)$, makes the derivative of the sliding surface equal to zero to stay on the sliding surface

In order to check the existence of the sliding mode and prove the stability of the ODE (s = $\dot{e} + pe$) , we can use Lyapunov function:

$$\mathbf{V} = \frac{\mathbf{s^T s}}{\mathbf{2}} \quad (\dot{V} = s^T\dot{s}) \tag{23}$$

For a system to be stable, two conditions must be true: $\lim_{s\to\infty} V = \infty$ and $\dot{V} < 0$

As first one is easy to see, we will concentrate on the second condition:

$$\dot{V} = s^T(\dot{e} \dot{+} pe) \tag{24}$$

where $\dot{s} = \ddot{e} + p\dot{e} = (\ddot{r}_{ot} - \ddot{r}_o) + p(\dot{r}_{ot} - \dot{r}_o)$,

If we substitute the dynamic system equation into the equation above:

$$\dot{s} = \ddot{e} + p\dot{e} = (\boldsymbol{F}(\boldsymbol{r_{0t}}, \dot{\boldsymbol{r}}_{\boldsymbol{ot}}) + \boldsymbol{G}(\boldsymbol{d_{ot}}, \boldsymbol{\mu_{Cot}})\boldsymbol{\mu_{Tbt}}) - ((\boldsymbol{F}(\boldsymbol{r_0}, \dot{\boldsymbol{r}}_{\boldsymbol{o}}) + \boldsymbol{G}(\boldsymbol{d_o}, \boldsymbol{\mu_{Co}})\boldsymbol{\mu_{Tb}})) + p(\dot{\boldsymbol{r}}_{\boldsymbol{ot}} - \dot{\boldsymbol{r}}_{\boldsymbol{o}})$$

As error is zero when chaser satellite is at the rendezvous trajectory, $\boldsymbol{\mu_{Tbt}}$ is 0**.** Therefore:

$$\dot{s} = \boldsymbol{F}(\boldsymbol{r_{0t}}, \dot{\boldsymbol{r}}_{\boldsymbol{ot}}) - \boldsymbol{F}(\boldsymbol{r_0}, \dot{\boldsymbol{r}}_{\boldsymbol{o}}) + \boldsymbol{G}(\boldsymbol{d_o}, \boldsymbol{\mu_{Co}})\boldsymbol{\mu_{Tb}} + p(\dot{\boldsymbol{r}}_{\boldsymbol{ot}} - \dot{\boldsymbol{r}}_{\boldsymbol{o}}).$$

Now, let's substitute the $\dot{s}$ into Lyapunov function to obtain:

$$\dot{V} = s^T\big(\boldsymbol{F}(\boldsymbol{r_{0t}}, \dot{\boldsymbol{r}}_{\boldsymbol{ot}}) - \boldsymbol{F}(\boldsymbol{r_0}, \dot{\boldsymbol{r}}_{\boldsymbol{o}}) + \boldsymbol{G}(\boldsymbol{d_o}, \boldsymbol{\mu_{Co}})\boldsymbol{\mu_{Tb}} + p(\dot{\boldsymbol{r}}_{\boldsymbol{ot}} - \dot{\boldsymbol{r}}_{\boldsymbol{o}})\big) < 0.$$

To satisfy the requirement, we select the control law as:

$$\boldsymbol{\mu_{Tb}} = -\frac{\boldsymbol{F}(\boldsymbol{r_{0t}}, \dot{\boldsymbol{r}}_{\boldsymbol{ot}}) - \boldsymbol{F}(\boldsymbol{r_0}, \dot{\boldsymbol{r}}_{\boldsymbol{o}}) + \boldsymbol{p}(\dot{\boldsymbol{r}}_{\boldsymbol{ot}} - \dot{\boldsymbol{r}}_{\boldsymbol{o}})}{\boldsymbol{G}(\boldsymbol{d_o}, \boldsymbol{\mu_{Co}})} + \boldsymbol{\mu_c} \tag{25}$$

$$\boldsymbol{\mu_{eq}}(\mathbf{t}) = \frac{\mathbf{F}(\mathbf{r_{0t}}, \dot{\mathbf{r}}_{\mathbf{ot}}) - \mathbf{F}(\mathbf{r_0}, \dot{\mathbf{r}}_{\mathbf{o}}) + \mathbf{p}(\dot{\mathbf{r}}_{\mathbf{ot}} - \dot{\mathbf{r}}_{\mathbf{o}})}{\mathbf{G}(\mathbf{d_o}, \boldsymbol{\mu}_{\mathbf{Co}})} \tag{26}$$

$$\boldsymbol{\mu_c}(\boldsymbol{t}) = \boldsymbol{k}\frac{\boldsymbol{s_i}}{|\boldsymbol{s_i}| + \boldsymbol{\delta}} \tag{27}$$

Here, p, k and δ are gains of the sliding mode control.

Chapter 5. Simulation results

These simulations were performed in AOCS (Attitude and Orbit Control Simulator) made by Inamori Laboratory. It simulates the motion of two satellites using the numerical simulation. Disturbances from sun radiation pressure and atmospheric drag were included. During the simulation, we assume the active altitude control to be present.

In the graphs below, origin of the graph is the location of the target satellite. The dot on the right side corresponds to the initial location (center of body) of the chaser satellite in LVLH frame. There are two lines connecting the dots. The red one is the Rendezvous orbit, is a trajectory that chaser satellite must follow in order to successfully perform docking. The blue one is the actual trajectory that chaser satellite will follow as a result of disturbances and applied control method (Sliding Mode Control). The dot on the right side represents the center of chaser satellite after docking, therefore it is located in 0.3 meters from the origin as length of each satellite is 0.3 meters. If the blue line closely matches the red line and meet with their endpoints, the docking is considered to be successful. In this case, success criteria is the accuracy error of 0.5 cm.

5.1 Simulation of accurate rendezvous across many different distances

Here is the simulation of the rendezvous operation with initial distance of satellites for 4 meters. Parameters are shown below:

Weights of each satellite	3 kg
Maximum magnetic moment input from the electromagnet	$80\ \mathrm{Am}^2$
Magnetic moment of the permanent magnet	$50\ \mathrm{Am}^2$
Initial position of the chaser	$(2,\ 0.0,\ 0.0)\ \mathrm{m}$
Final position of the chaser	$(0.3,\ 0.0,0.0)\ \mathrm{m}$
Altitude of the satellites	Around 700km
Time of applied force	34s
Gains of the control $p,\ k,\ \delta$	0.005, 0.01, 0.2
Time of rendezvous	8416

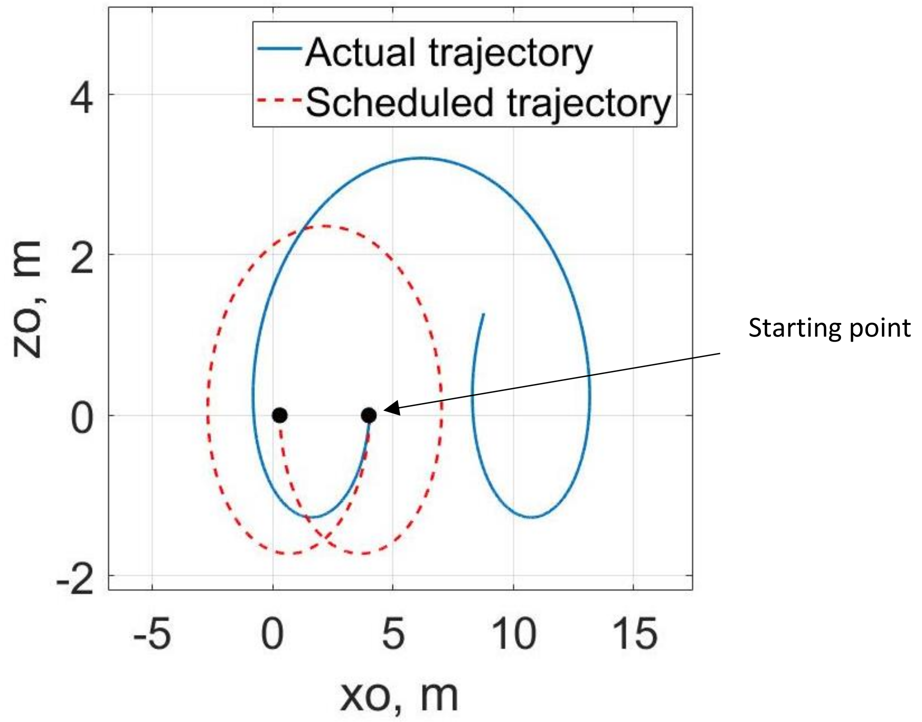


Fig.7 Rendezvous trajectory of the chaser satellite without the applied control [10]

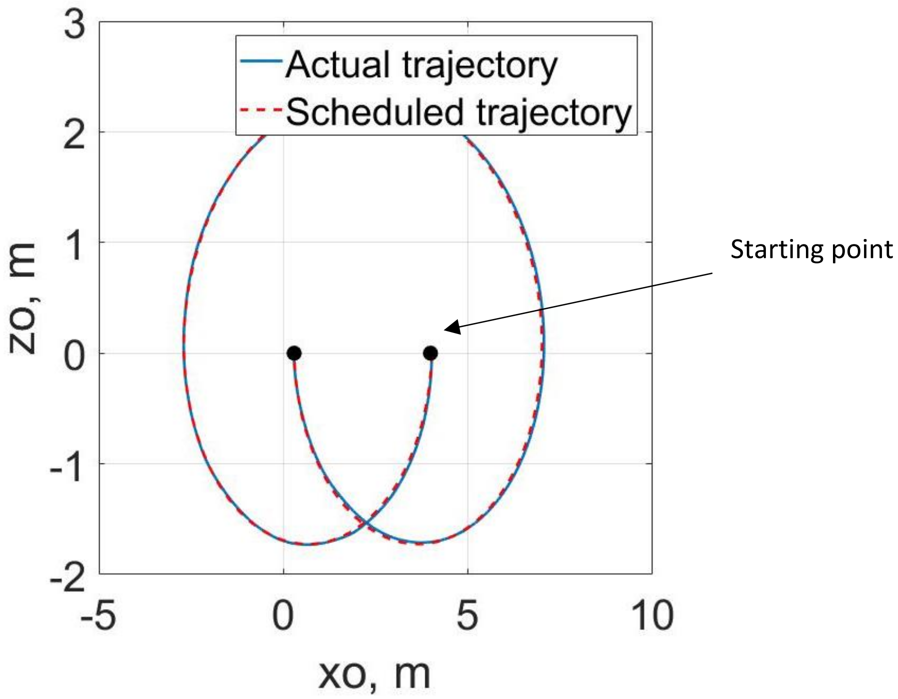


Fig.8 Rendezvous trajectory of the chaser satellite with an applied control [11]

Here is the simulation of the rendezvous operation with initial distance of satellites for 2 meters. Parameters are shown below:

Weights of each satellite	3 kg
Maximum magnetic moment input from the electromagnet	$20\ \mathrm{Am}^2$
Magnetic moment of the permanent magnet	$50\ \mathrm{Am}^2$
Initial position of the chaser	$(2,\ 0.0,\ 0.0)$ m
Final position of the chaser	$(0.3,\ 0.0,0.0)$ m
Altitude of the satellites	Around 700km
Time of applied force	1s
Gains of the control $p,\ k,\ \delta$	0.005, 0.01, 0.2
Time of rendezvous	1509s

Starting point

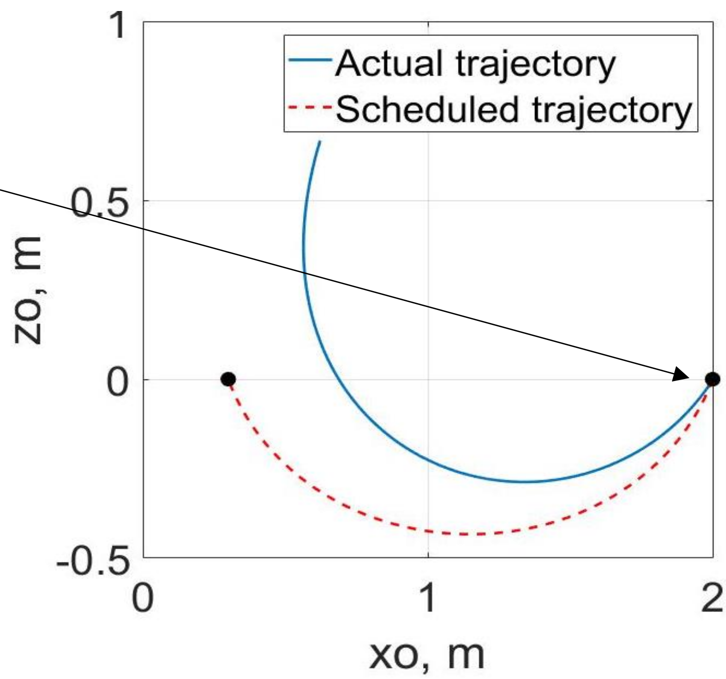


Fig.9 Rendezvous trajectory of the chaser satellite without the applied control [12]

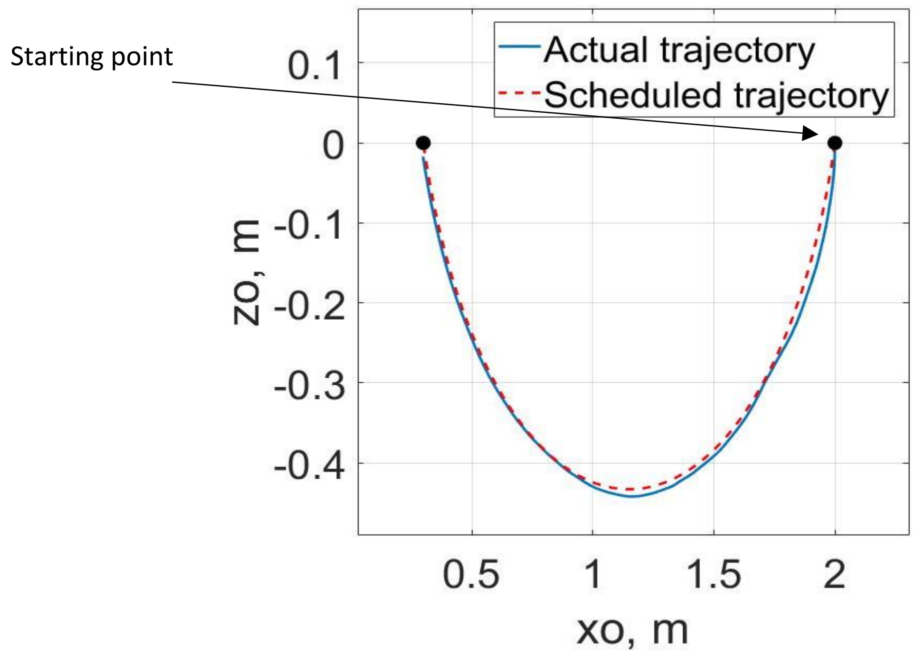


Fig.10 Rendezvous trajectory of the chaser satellite with an applied control [13]

Simulation results have confirmed that the most important error sources were unmodeled parameters from the equations of force and orbits. This was checked by running simulators without controller and environmental disturbances. Then the effects of environmental disturbances were investigated. Among the disturbances, air drag was the most influential. However, effect of both disturbances was very small to notice. That is due to the short duration time of the rendezvous orbit.

5.2 Simulation showing the energy efficiency of the rendezvous path

In this simulation, we have compared our rendezvous strategy with the method of “direct” pulling without using the difference in orbital motions. In that case, a strong electromagnetic force is applied to the chaser satellite to bring it closer to the target in shorter time. The target state is not the scheduled rendezvous orbit as before, but the location of the satellite with electromagnet. In our rendezvous method, relatively smaller power is required to generate the guiding path and maintain it for the chaser satellite. This will be demonstrated in the example below. Here, we use the same parameters as the case 2 (docking from 2 meters):

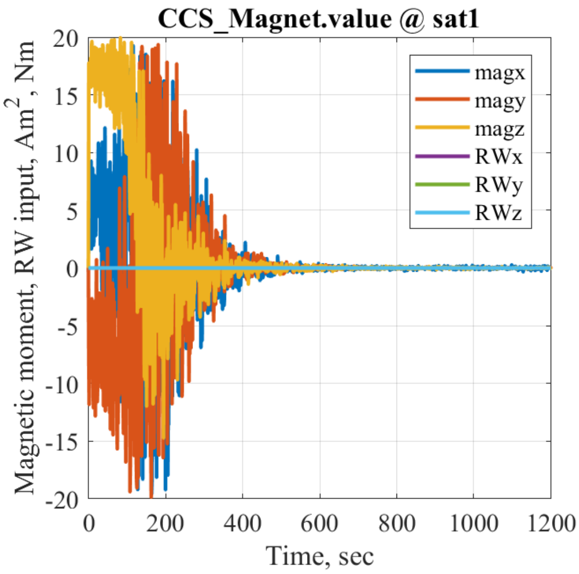


Fig.11 Without using the relative orbital motion [14]

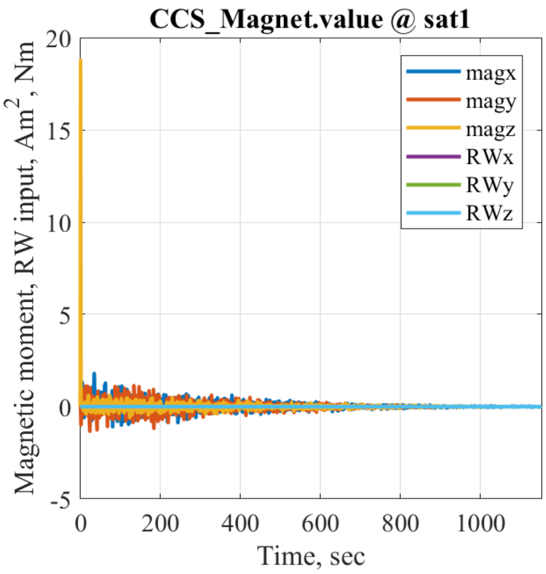


Fig.12 With using the relative orbital motion [15]

	Without using orbital motion	**With using orbital motion**
Magnetic moment generated before docking (time integration of magnetic moment)	$7441Am^2s$	$459Am^2s$

As you can see, not using orbital motion can result in a quicker guidance process, however the amount the magnetic moment generated is almost 16 times more. We decided to display the power consumption in the amount of magnetic moment generated so that we don’t have to specify the details of the electromagnet. If we show it in the current (I), the value will depend on the area of the electric coil (S) and number of the turns for wounded coil (n).

Magnetic moment intensity = $\mu = nIS$

Discussion

Assumptions:

We have accepted numerous fair assumptions during our research. One of them is that satellites are represented as a dot, which is based on their center of mass. During the rendezvous design and docking conditions design, size of the satellites was accounted as 1X3 U cubesats. During the disturbances modeling, we have also considered the surface are of the satellites. We have considered the sizes of satellites when dimensions were cardinal. However, when applying the electromagnetic force and controlling trajectory, satellites were modeled as magnetic dipole dots. This was compensated, by choosing the docking end conditions that accounted for this condition. Although, accuracy of the results would have been better if we tried to use geometric model of the satellite, designing and modelling process would be too costly for us in terms of resources. Moreover, simulation results show a very high precision of docking, thus there is no need in an accurate geometric satellite model.

We have also assumed that there is an active attitude controller that always aligns the directions of the satellites during the rendezvous operation. Only when satellites' magnets face each other, they will work in full potential. Even the slightest change in angles can result in significant reduction of force between them. We did not have enough time to design the electromagnetic attitude controller for rendezvous based on SMC. Its development and combination with orbital controller can be the topic of further research.

Result:

The essential reduction in power consumption as a result of the orbital motion manipulation was confirmed in numerical simulations. They were measured in the amount of the magnetic moment generated, as the supply of current would depend on the characteristics of the electromagnet. However, using the proportional relation between the value of magnetic moment and current we can see the same results. Our expectations about the operation of the Sliding Mode controller were also fulfilled. It has neglected the incomplete model dynamics and external disturbances to the system. As a result, chaser satellite maintained its rendezvous orbit and achieved docking in numerical simulations.

Future work:

As we mentioned before, addition of the attitude control for the satellites would be significant improvement to our research. Furthermore, modeling not only one, but multiple actuators for each satellite would be very practical. That would also extend the possible missions by satellites.

Investigating separation performance using the same control method would be very beneficial. That would complete our research and could be used for self-assembly of the space structures. It is important to investigate the ways of separating two satellites and putting them into the stationary orbit that was discussed in this paper.

Conclusion

The rendezvous and docking method for nanosatellites that employs the relative orbital motion between them and electromagnetic actuators has been proposed and evaluated. This method relies only on the magnetic force to perform the guidance of satellites toward each other. It does not consume a limited propellant such as gas, thus not limiting the number of docking missions that can be done. It was shown that taking advantage of the orbital motions between them requires only a short amount of strong initial force and then small controlling force. This has proven to be less power consuming compared the direct pulling method without using the advantages of orbital motion difference. Sliding Mode Control based control law was developed to make sure that chaser satellite will stay on the scheduled rendezvous trajectory. It has addressed the issues of nonlinear characteristics of our model equations, incomplete dynamic modeling and environmental disturbances. In order to confirm the feasibility and the accuracy of the method, we have conducted numerical simulations. Simulations were performed under various parameters to evaluate the proposed method in different situations.

The results of the research could be used in all kinds of space missions that would employ rendezvous and docking between two satellites. Those include but not limited to:

- Orbit service could benefit from the improvement of controllability and zero propellant consumption.
- Multiple nano satellites can dock efficiently to unite into bigger satellite, obtaining more functionality.

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