

# Single Phase Half-Wave (Half Bridge), Single Phase Full-Wave (Full Bridge) & Three Phase Full Bridge Rectifiers Design & Simulation Work

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## 1) Single Phase Half-Wave Diode Rectifier

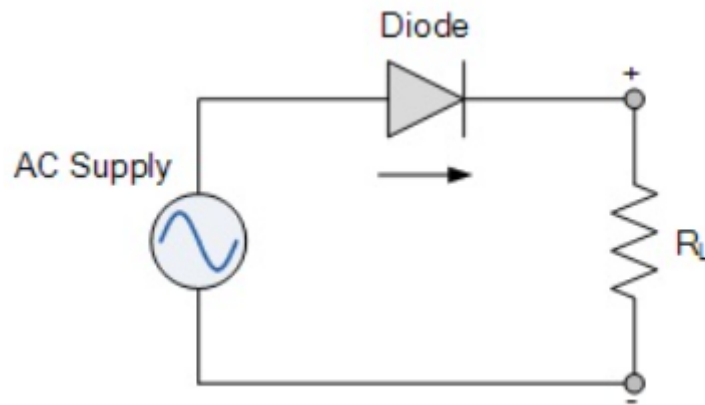


Figure 1 Single Phase Half-Wave Rectifier

Simulate the circuit fed with a resistive load of  $R = 100 \Omega$ . Ignore any non-idealities and take AC source as Turkish electricity grid phase-neutral voltage.

- Plot the output voltage waveform for step times of 1 ns, 0.1 ms and 1 ms.
- Comment on the significance of step time in computer simulations. Which one would you prefer for this question? Comment on the advantages and disadvantages of the selection of step time too small or too large.
- Analytically, calculate the output average voltage and input current THD.
- Using simulation tools, obtain the above quantities in part (c). Compare your analytical results with simulation results. Comment on the results.

MATLAB/Simulink modelling of single-phase half-wave rectifier is:

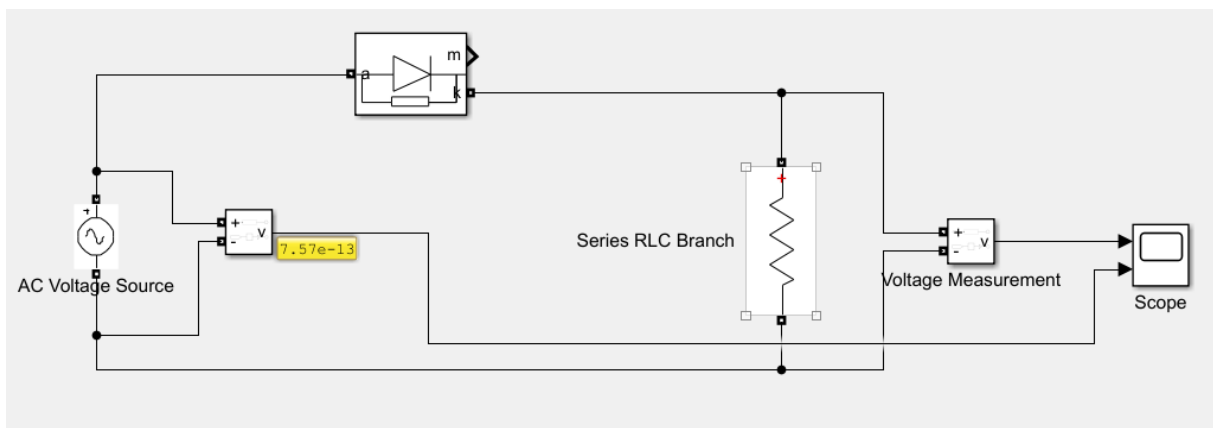


Figure 2 Matlab/Simulink Design of Half-Wave Rectifier

b. Plot the output voltage waveform for step times of 1 ns, 0.1 ms and 1 ms.

- When sample time is 1ns:

Output voltage waveform is:

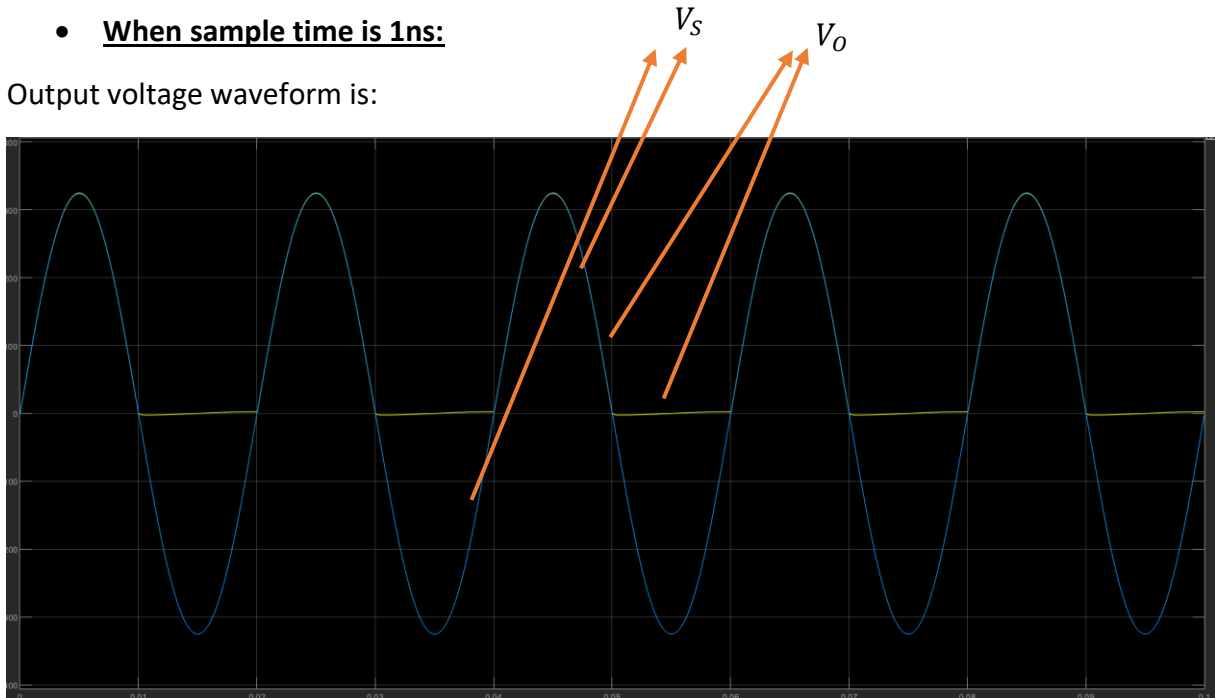


Figure 3 Output Voltage Graphs  $V_s$  &  $V_o$  vs  $t$

As we can see since our sample time is very small, we are getting very accurate results according to graph in above. It is expected. However, simulation time takes too much time to show the results.

- When sample time is 0.1ms:

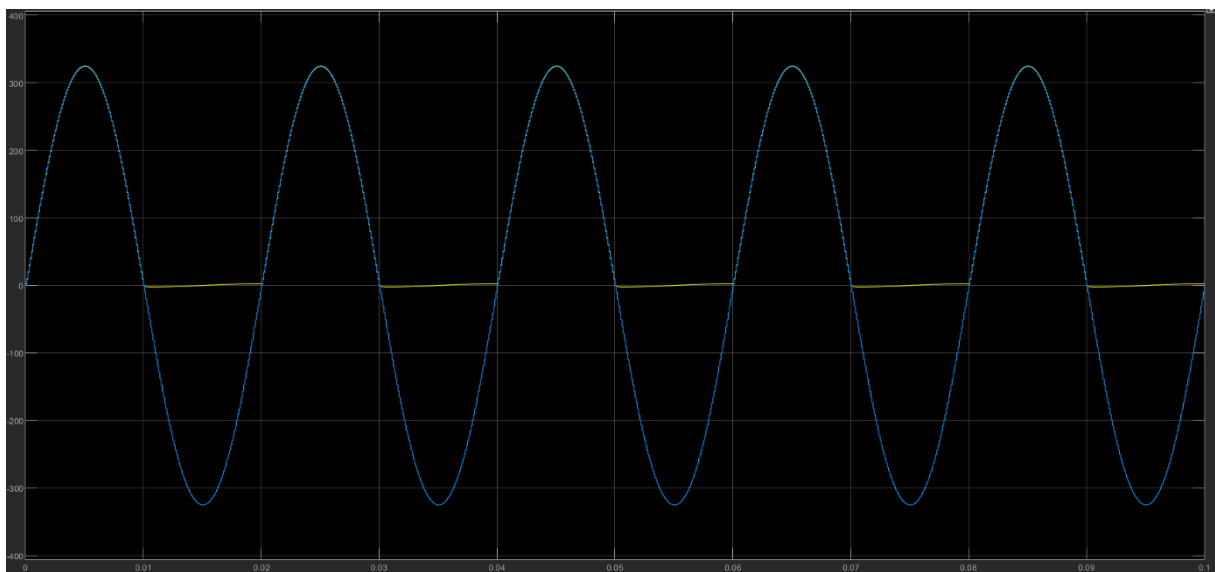


Figure 5 Output Voltage Graphs  $V_s$  &  $V_o$  vs  $t$

- c. Comment on the significance of step time in computer simulations. Which one would you prefer for this question? Comment on the advantages and disadvantages of the selection of step time too small or too large. It is again accurate and fast results as compared to 1ns step time.

- When sample time is 1ms:

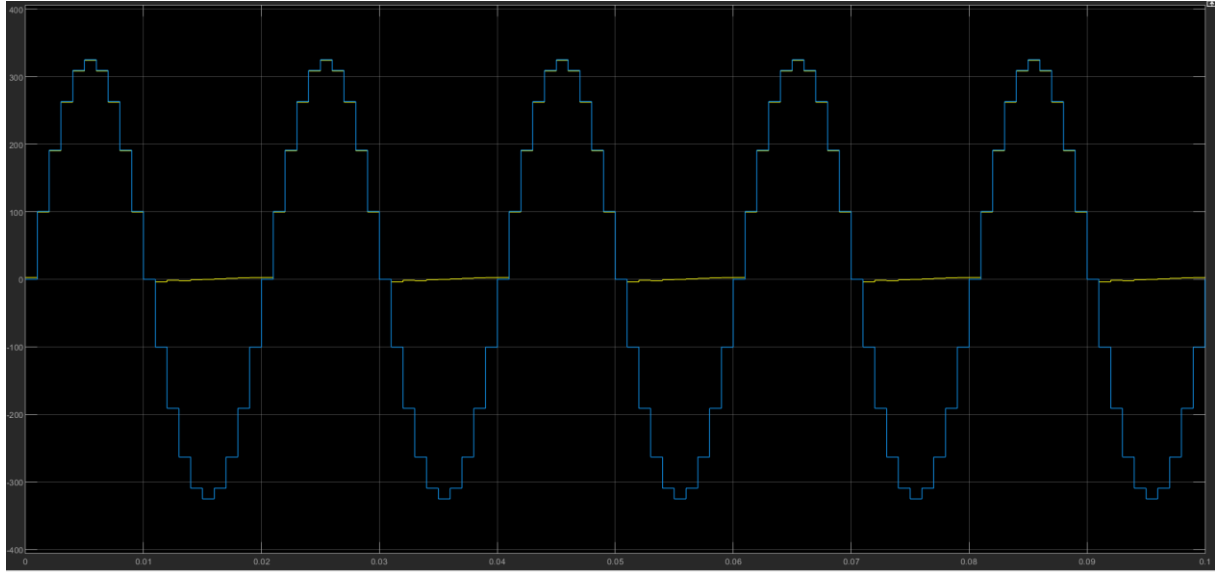


Figure 6. Output Voltage Graphs Vs & Vo vs t

Here we see that our step time is nearly our period that  $\frac{1}{50 \text{ Hz}} = 2 \text{ ms}$ . Therefore, our graph like a step function that gave inaccurate results compared to 1ns and 0.1ms step times. However, it is the fastest computation time.

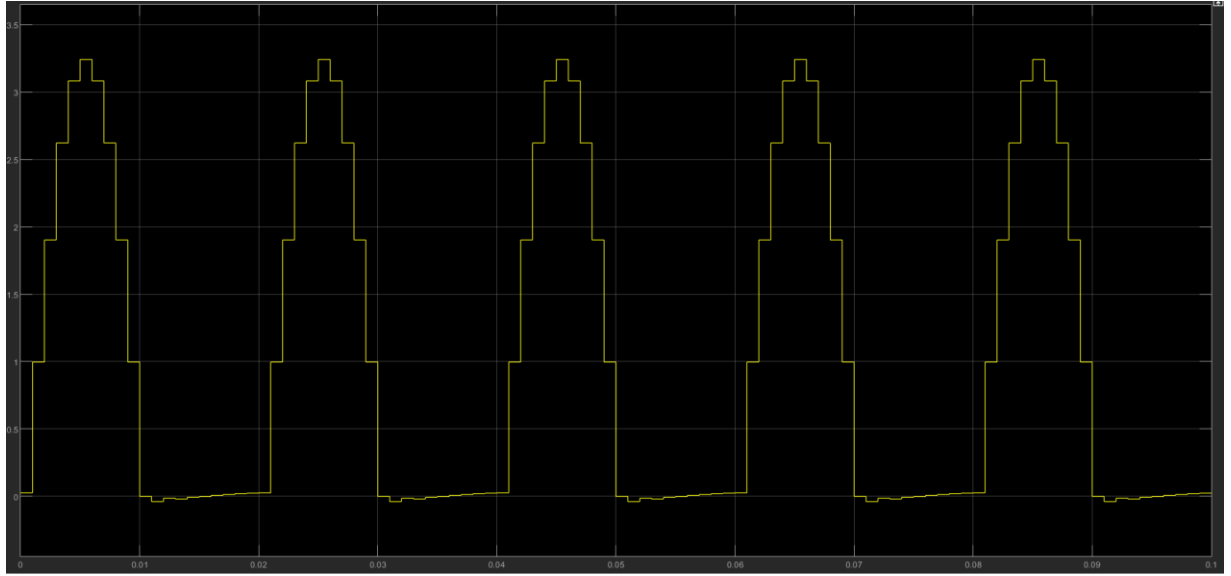
- d. Analytically, calculate the output average voltage and input current THD.

$$V_{O(avg)} = \frac{1}{2\pi} \int_0^\pi \sqrt{2}V_S \sin wt \, dwt$$

$$V_{O(avg)} = \frac{\sqrt{2}V_S}{2\pi} \left[ -\cos \right]_0^\pi$$

$$V_{O(avg)} = \frac{\sqrt{2}V_S}{2\pi} [2] = \frac{\sqrt{2}}{\pi} V_S = 0.45V_S$$

When T sample is 0.001 (1ms), there will be distortion in input current that looks like as figure below:



*Figure 7. Input Current Waveform*

As it can be observed that the distortion in the sinusoidal waveform of the input current is occurred because of sampling time. Because it is not trigonometric function anymore, there will be harmonics that are part of the non-trigonometric function and harmonics are sum of many trigonometric function that can be calculated via Fourier Series. Why it is calculated via Fourier Series instead of Fourier Transform? It is because they are non-trigonometric periodic functions, and non-trigonometric periodic functions are calculated by Fourier Series. On the other hand, Fourier Transform is applied on non-trigonometric non-periodic functions.

Fourier Series is represented as

$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$

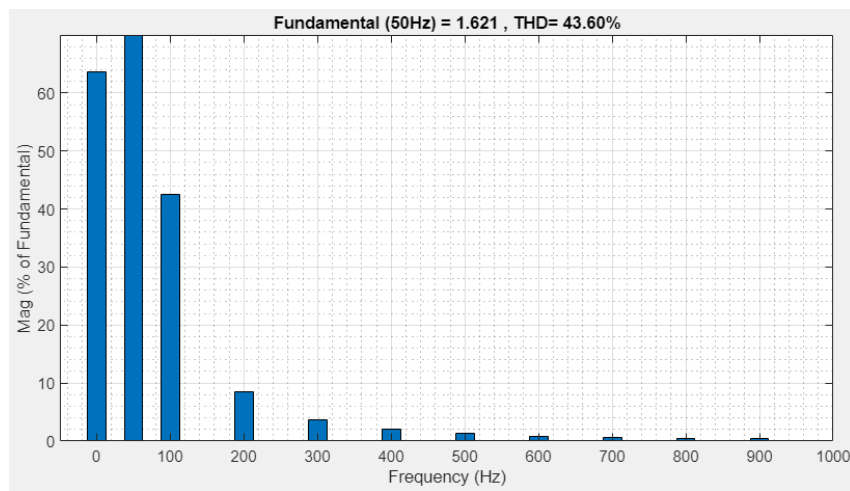
$$a_0 = \frac{1}{T} \int_0^T f(t) dt$$

$$a_n = \frac{2}{T} \int_0^T f(t) \cos n\omega t dt$$

$$b_n = \frac{2}{T} \int_0^T f(t) \sin n\omega t dt$$

$$\text{THD} = \frac{\sqrt{I_S^2 - I_1^2}}{I_1(\text{rms})}$$

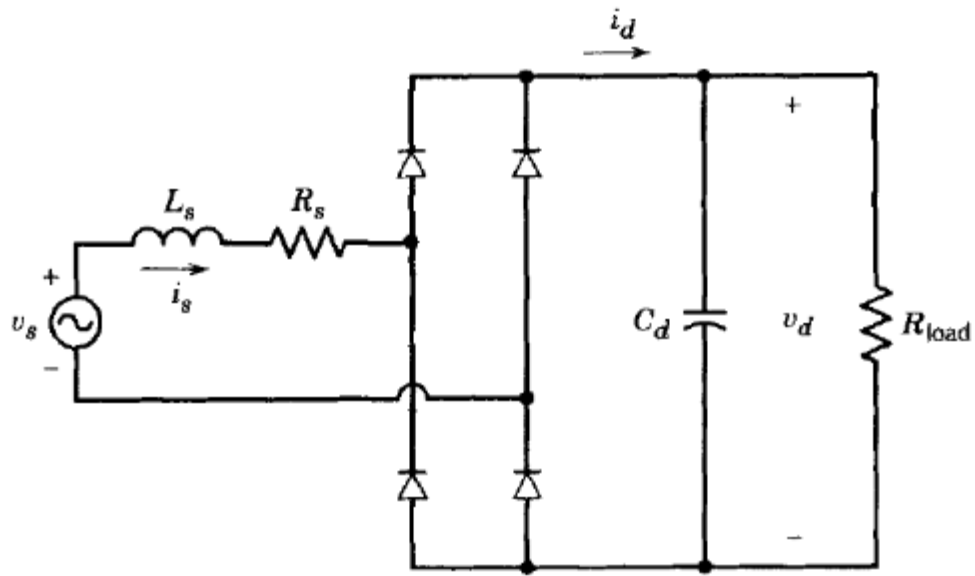
Use powergui tool FFT Analysis that shows the fundamental and other harmonics shown below:



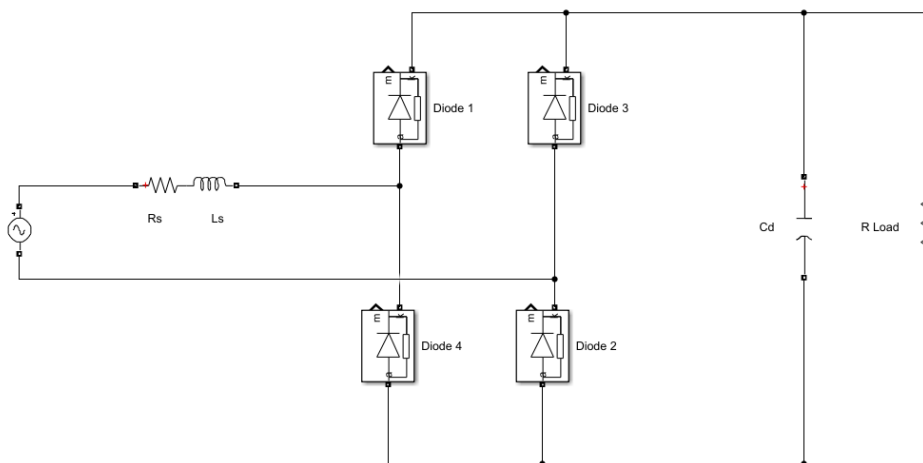
*Figure 8. FFT Analysis of Input Current*

THD is found as 43.6 %.

## 2) Single Phase Full-Wave Diode Rectifier



*Figure 9. Single Phase Full Wave Rectifier Circuit*



*Figure 9. Single Phase Full-Wave Rec. Circuit Schematic*

e. What do  $L_s$  and  $R_s$  represents in practical applications?

$L_s$  and  $R_s$  are representing line impedance and resistance in power systems. As we know that no ideal source is existed itself. There always be some passive components in the systems that we represent as a source resistance and impedance.

f. Using simulation tools, find the minimum output filter capacitance to have 4% output voltage peak-to-peak ripple to mean ratio. Use the values of same AC voltage source in part 1,  $L_s = 1 \text{ mH}$ ,  $R_s = 1 \text{ m}\Omega$  and  $R_{LOAD} = 100 \Omega$ . Assume diodes are ideal.

For this question, first we need to know how to calculate the capacitor value of full-wave rectifier (full bridge rectifier).

For example, let's adding  $1 \text{ mF}$  capacitor to the circuit and observe the voltage waveform of the capacitor.

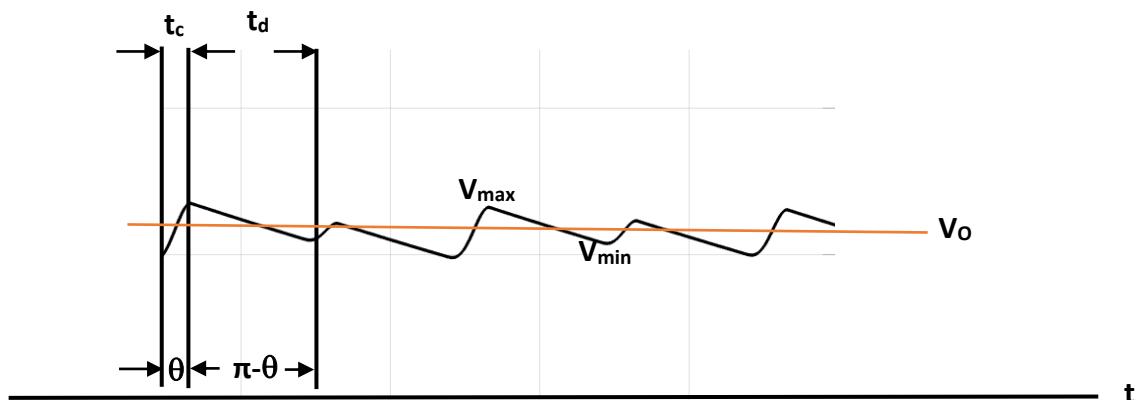


Figure 10. Voltage Waveform of the Capacitor

The energy sum of capacitor is equal to output voltage  $P_o$ .

That means



$$\frac{1}{2} C V_{max}^2 - \frac{1}{2} C V_{min}^2 = P_O \times \frac{\pi - \theta}{\pi} \times \frac{T_S}{2}$$

$$T_S = \frac{1}{f_S}$$

\* As it is known that frequency is doubled in Full-Wave Rectifier.

$$\begin{aligned} \frac{1}{2} C [V_{max}^2 - V_{min}^2] &= \frac{1}{2} C [(V_{max} + V_{min}) + (V_{max} - V_{min})] \\ &= C \left[ \underbrace{\frac{V_{max} + V_{min}}{2}}_{V_O} + \underbrace{\frac{V_{max} - V_{min}}{2}}_{\Delta V_r} \right] \end{aligned}$$

$$C \cdot V_O \cdot \Delta V_r = P_O \cdot \left( \frac{\pi - \theta}{\pi} \right) \cdot \frac{1}{2 f_S} \quad \Rightarrow \quad C \cdot V_O \cdot \Delta V_r = V_O \cdot I_O \cdot \left( \frac{\pi - \theta}{\pi} \right) \cdot \frac{1}{2 f_S}$$

We leave C alone in the equation.

$$C = \frac{\cancel{V_O} \cdot I_O \cdot \left( \frac{\pi - \theta}{\pi} \right) \cdot \frac{1}{2 f_S}}{\cancel{V_O} \cdot \Delta V_r}$$

$$C = \frac{I_O}{2 f_S \cdot \Delta V_r} \cdot \left( \frac{\pi - \theta}{\pi} \right)$$

In design specifications it is asked that output filter capacitance to have 4% output voltage peak-to-peak ripple to mean ratio. To find the capacitor value, the current value and angle values are needed. The output current ( $I_o$ ) can be get from simulation results:



Figure 11. The Output Current Graph when  $C$  is not added

$I_o$  is measured 2A.

And when we add  $I_o$  into the equation that derived above section:

$$C = 42 \mu F$$

When the capacitor is added to the circuit and the results will:

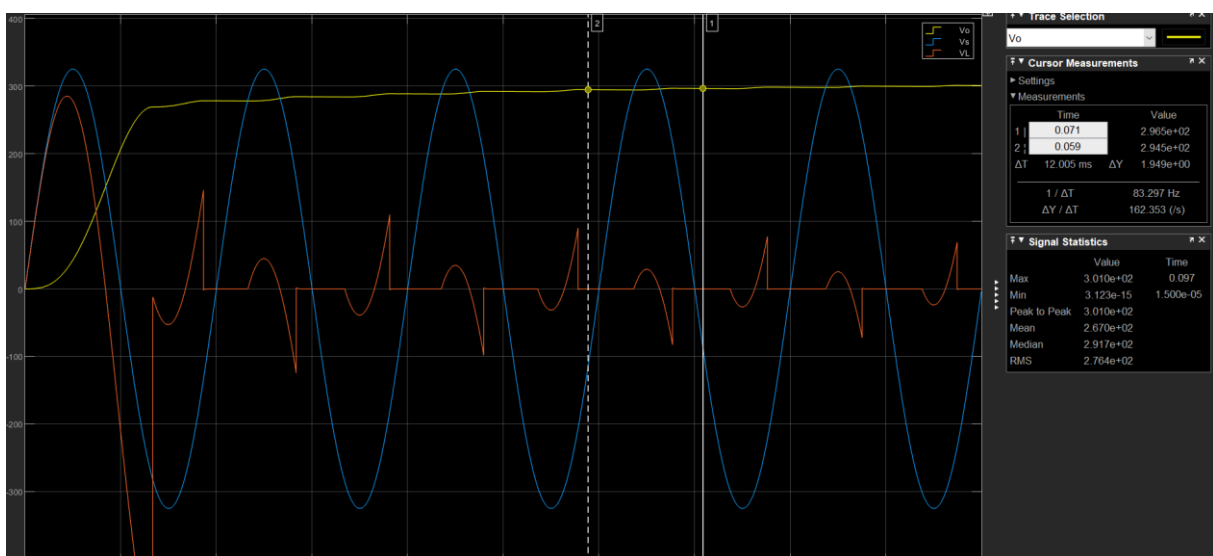


Figure 12. Output Voltage Waveform of  $V_s$ ,  $V_o$  &  $V_L$

It can be seen that output voltage waveform became straight line as compared to Full-Wave Rectifier without capacitor. Also, ripple voltage is less than 0.4%.

g. Plot the output voltage, input voltage and current at the same graph.

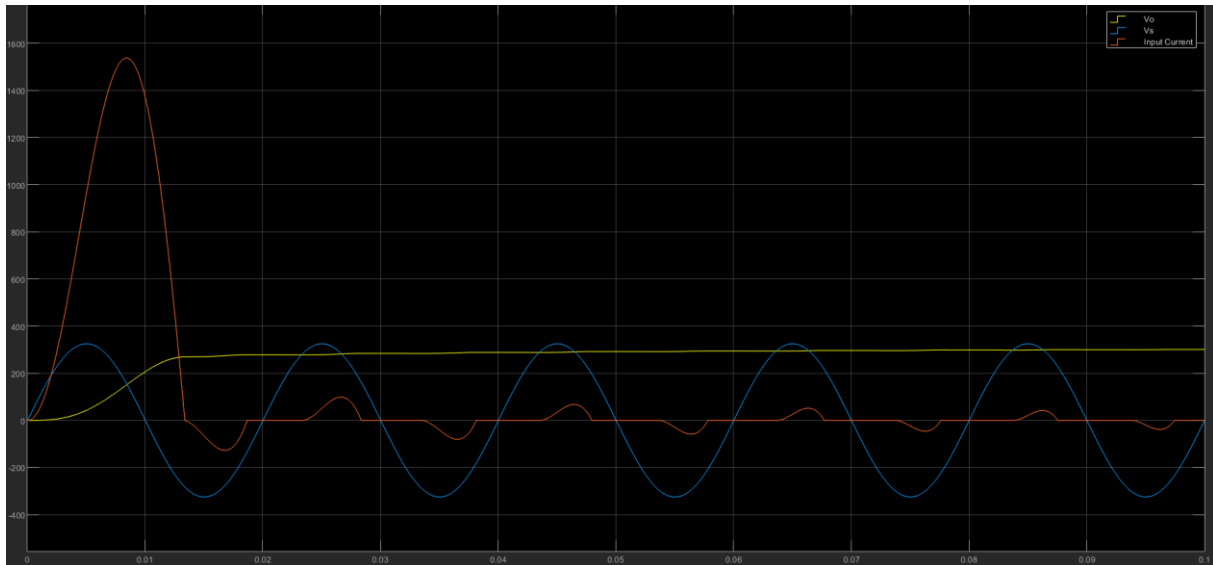


Figure 13. Input Current, Voltage & Output Voltage

h. Measure average of output voltage, input current THD and power factor.

Output voltage:

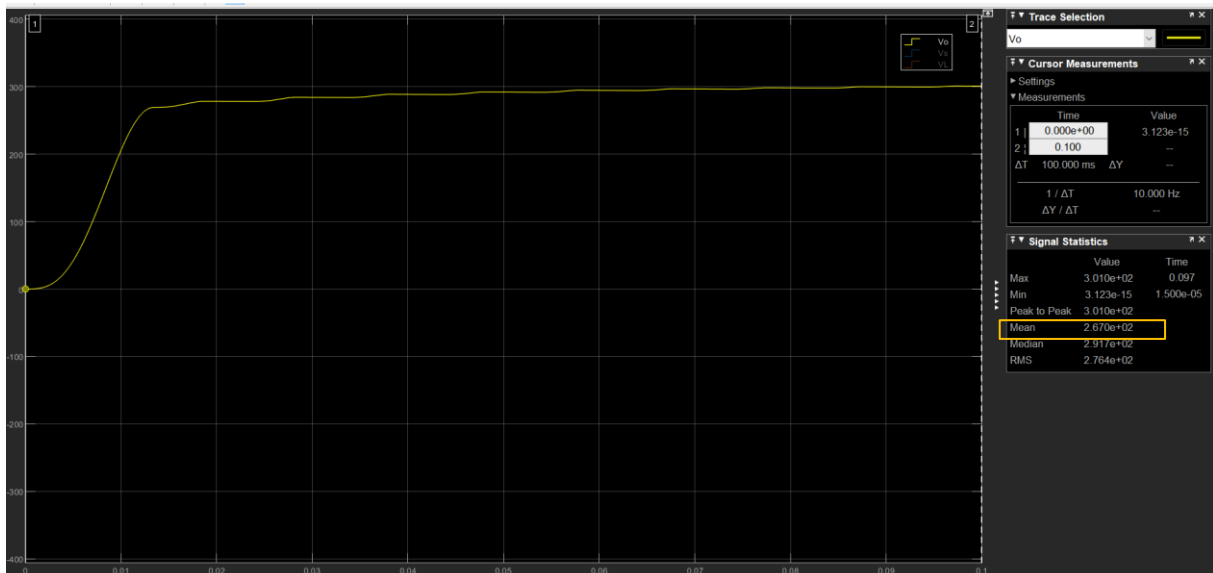


Figure 14. Output Voltage Waveform

Given specific time, our average output voltage is calculated by MATLAB is:

$$V_{O(avg)} = 267V.$$

### Input Current THD:

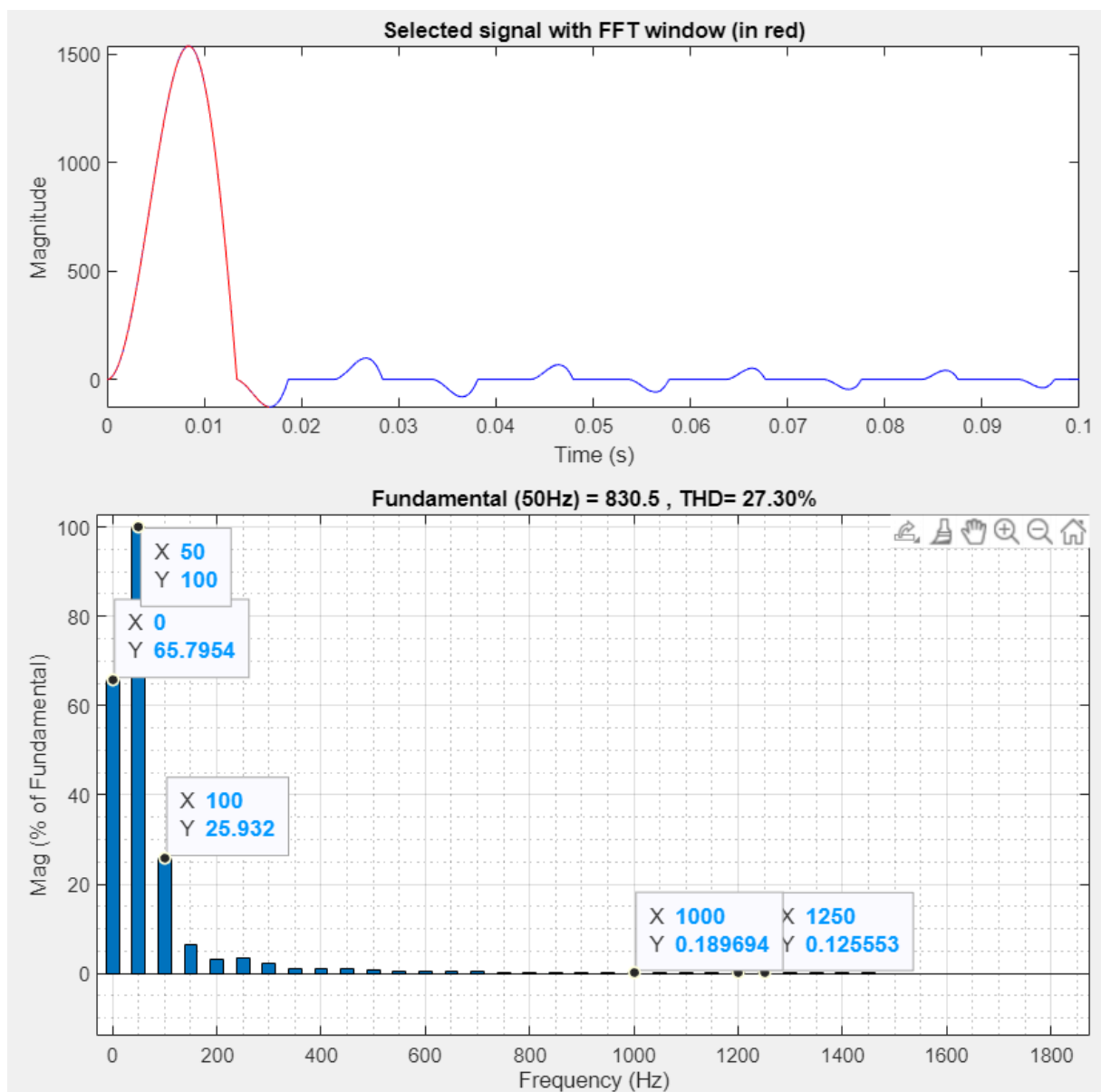


Figure 15. Fundamental Frequency and THD of Input Current Waveform

Power Factor:

$$\text{Power Factor (PF)} = \frac{\text{True Power}}{\text{Apparent Power}}$$

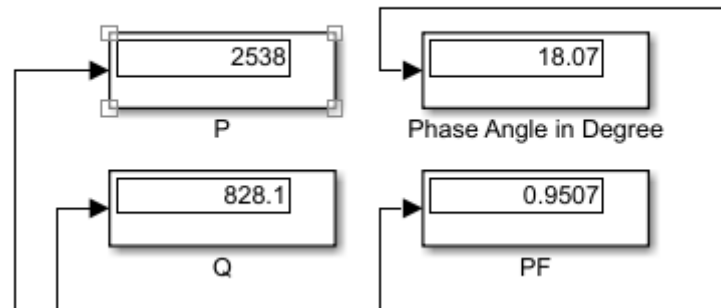


Figure 16 P,Q, Phase Angle (Theta) and PF

- i. Simulate the same circuit without  $R_s$  and  $L_s$  and observe and state the differences. What is the effect of these two?

When  $L_s$  and  $R_s$  effects are removed, the output graph will be more straight line and no distortion as it is shown in the below graph:

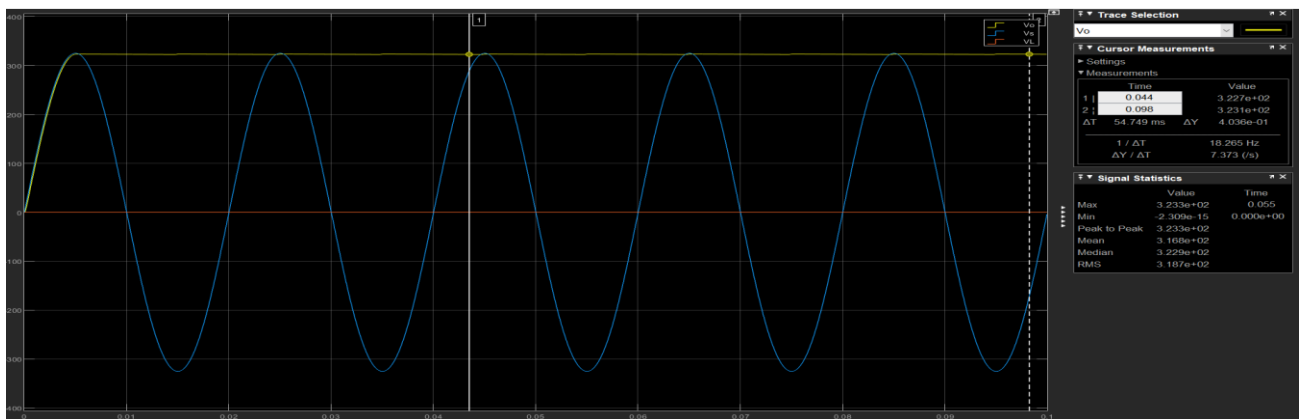


Figure 17. Output Voltage Graph w/out  $R_s$  and  $L_s$

### 3) Three Phase Full Bridge Diode Rectifier

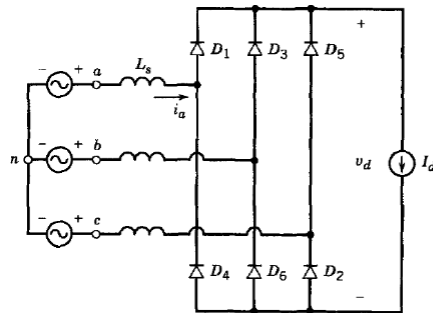


Figure 18. 3-Phase Full Bridge Rectifier Circuit

- j. Simulate the circuit for  $L_s=0$ ,  $V_{an}$ ,  $V_{bn}$  and  $V_{cn}$  are balanced three phase voltages connected to the same grid as before, ideal diodes and  $I_d=50$  A. Plot the output voltage and phase A input current on the same graph.

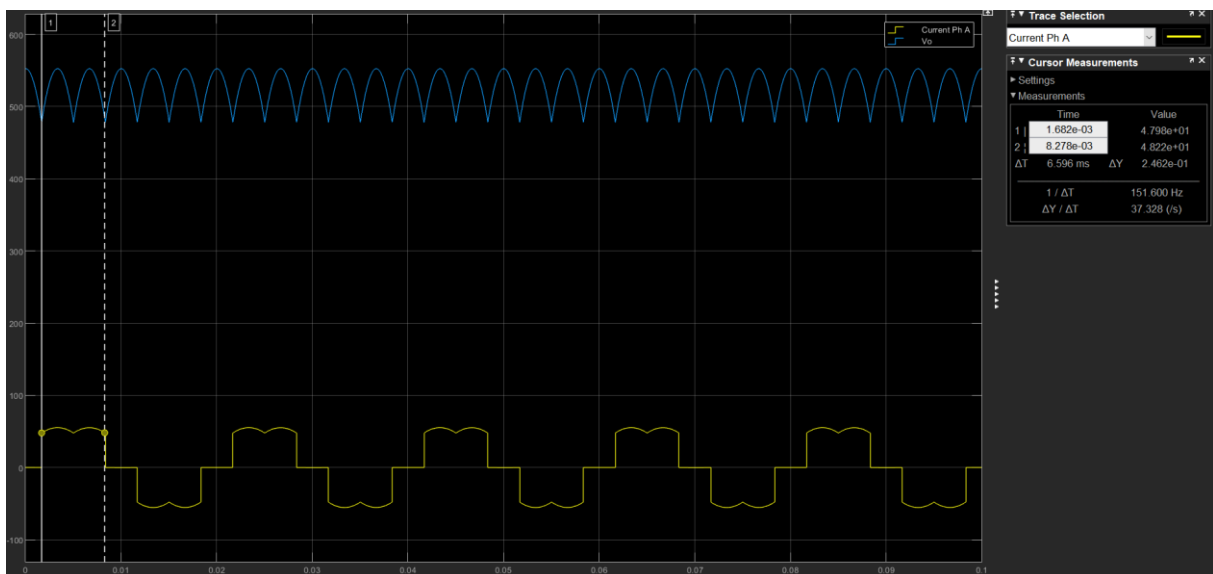


Figure 19. Phase A Input Current and Output Voltage Waveform

k. Analytically, calculate the output voltage average and compare with simulation results.

Comment on any differences.

$$V_{O(avg)} = \frac{1}{\frac{\pi}{6}} \int_0^{\frac{\pi}{6}} (\sqrt{3} \sqrt{2} V_S \cos wt \, dw t)$$

$$V_{O(avg)} = \frac{6\sqrt{6}}{\pi} V_S \int_0^{\frac{\pi}{6}} \cos wt \, dw t$$

$$V_{O(avg)} = \frac{6\sqrt{6}}{\pi} \times V_S \times \left[ \sin wt \right]_0^{\frac{\pi}{6}}$$

$$V_{O(avg)} = \frac{6\sqrt{6}}{\pi} \times V_{S_{RMS}} \times \left[ \sin\left(\frac{\pi}{6}\right) - \sin 0 \right]$$

$$V_{O(avg)} = V_{S_{RMS}} \times \frac{6\sqrt{6}}{\pi} \left[ \frac{1}{2} \right]$$

$$V_{O(avg)} = \frac{3\sqrt{6}}{\pi} V_{S_{RMS}}$$

$$V_{S_{RMS}} = 230V \text{ in Turkey}$$

$$V_{O(avg)} = \frac{3\sqrt{2}}{\pi} V_{S_{L-L}} V_{S_{L-L(RMS)}} \text{ or } 1.35 V_{S_{L-L(RMS)}}$$

$$V_{S_{1\emptyset-RMS}} = 230V \quad \& \quad V_{S_{L-L(RMS)}} = 398V \quad \text{in Turkey}$$

$$V_{S_{1\emptyset-PEAK}} = 325V \quad \& \quad V_{S_{L-L(PEAK)}} = 563V \quad \text{in Turkey}$$

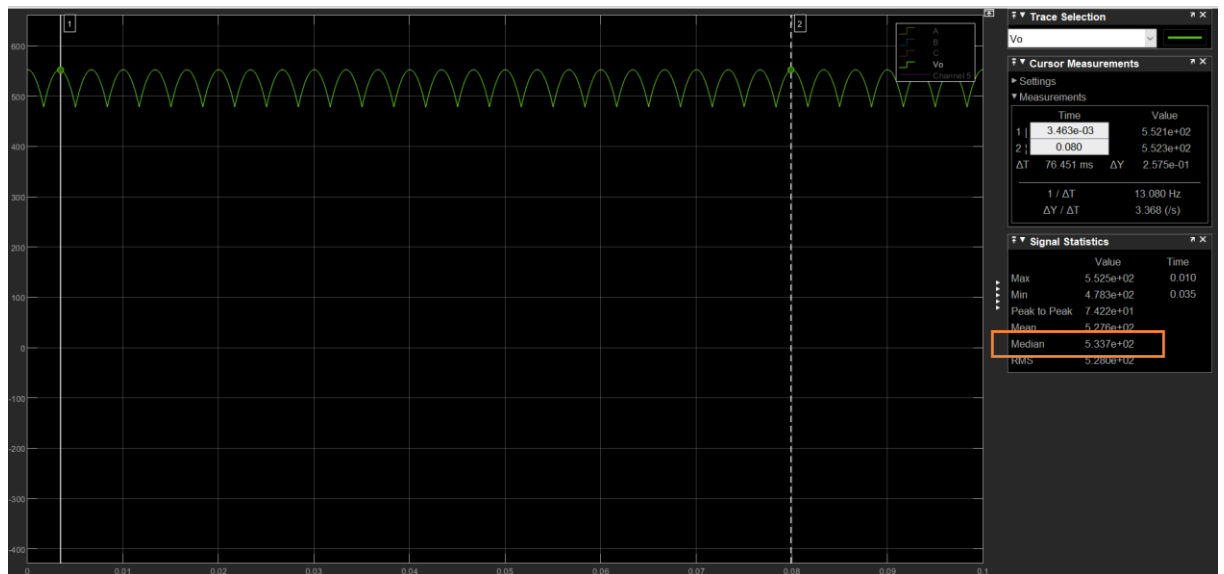


Figure 10. Output Voltage Waveform of 3-Phase Diode Rectifier

As it is shown in the figure above, the mean (average) value of the output voltage waveform is 533.7V

Analytical calculation of the output voltage in 3-Phase Diode Rectifier is:

$$V_{O(avg)} = \frac{3\sqrt{2}}{\pi} V_{S_{L-L}} V_{S_{L-L}(RMS)} \text{ or } 1.35 V_{S_{L-L}(RMS)}$$

$$V_{O(avg)} = 1.35 \times 398 = 537.3V$$

The difference is between analytical and simulational result is as voltage drop losses due to the diode.



- I. Perform the harmonics analysis, i.e. find the harmonic content of output voltage and input current up to 30th harmonic of line frequency.

Input Current:

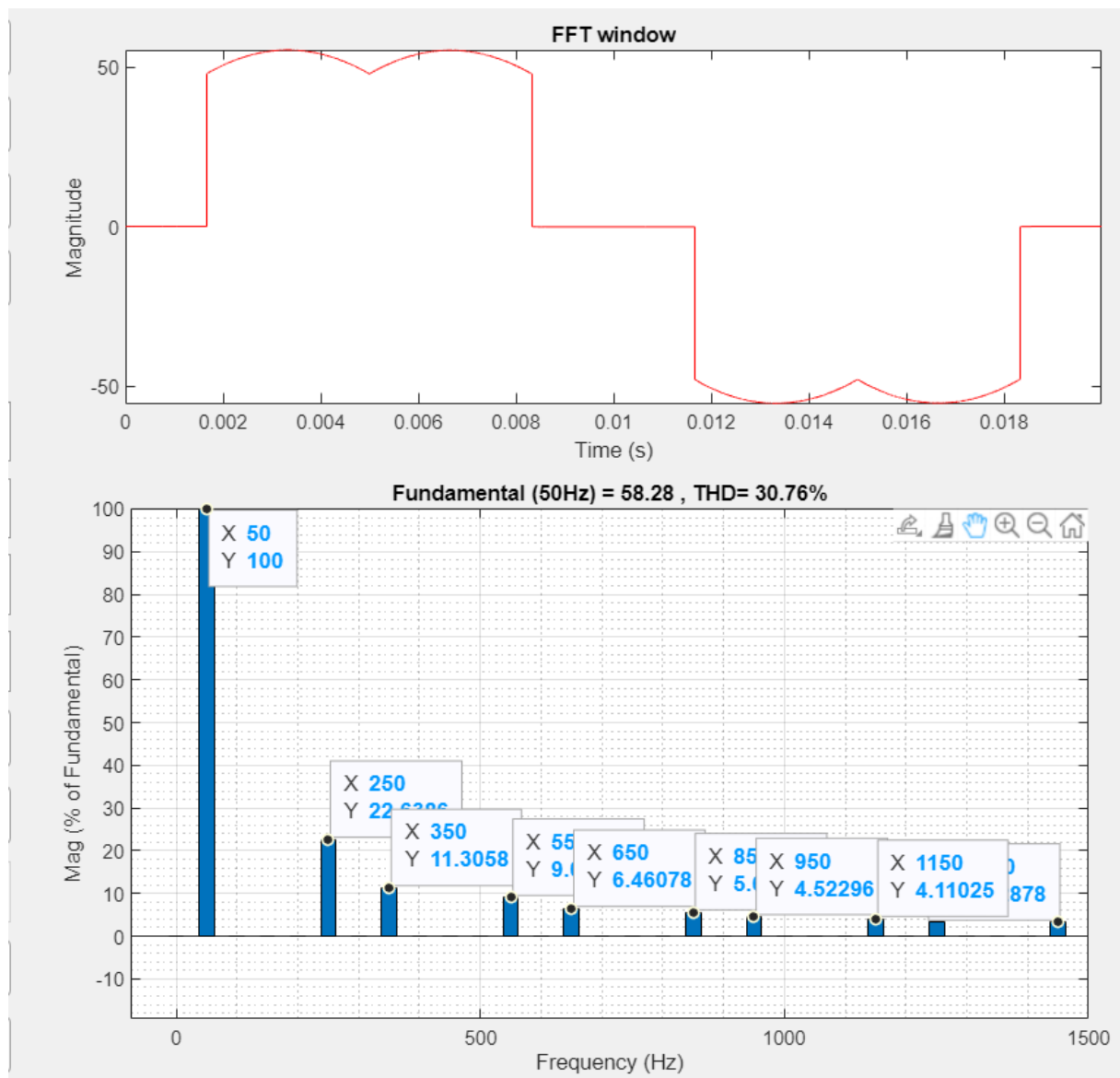


Figure 21. Input Current Harmonics and THD

It can be observed that in current harmonics we have fundamental, 5<sup>th</sup>, 7<sup>th</sup>, 11<sup>th</sup> harmonics.

### Output Voltage:

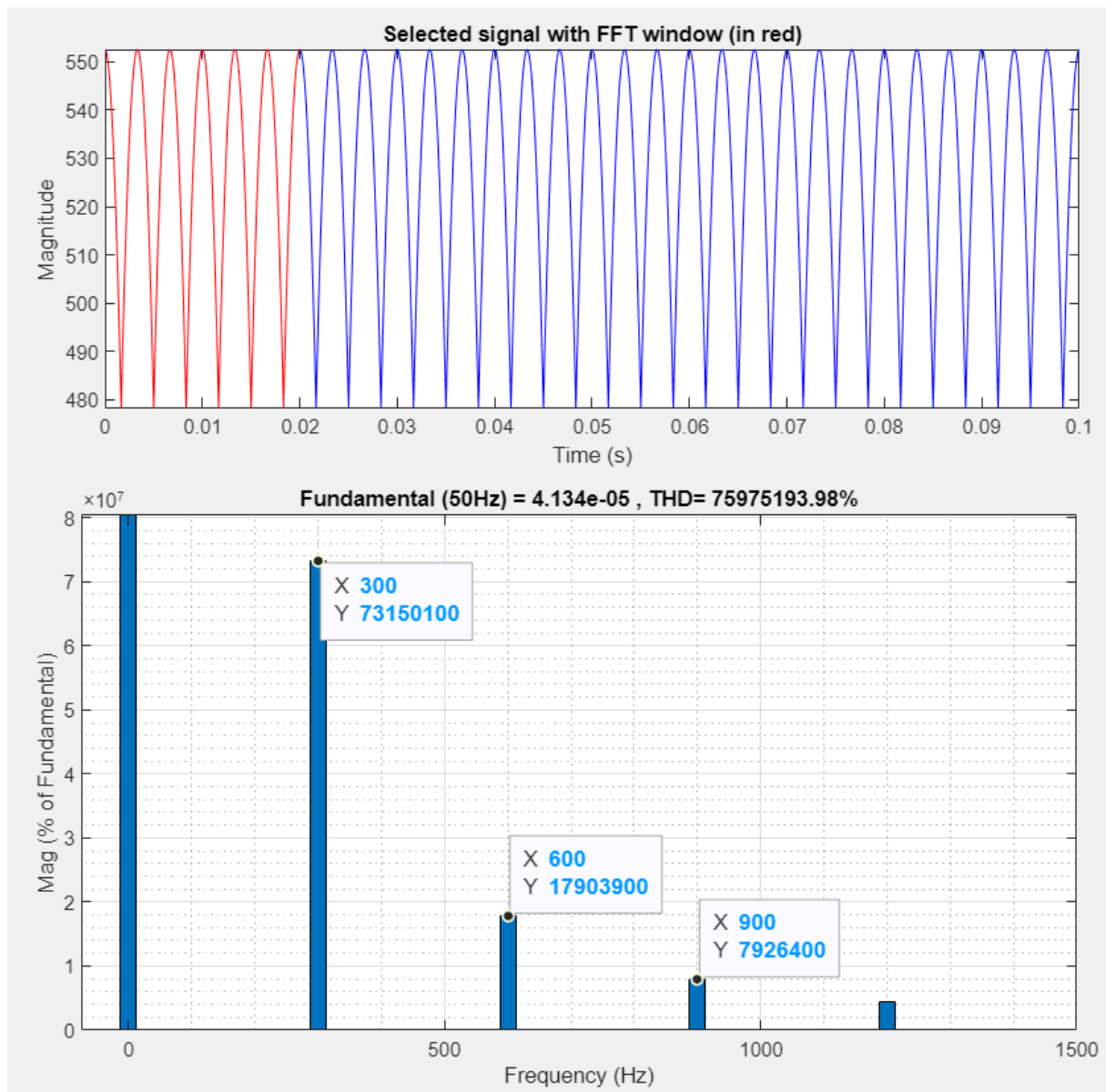


Figure 22. Output Voltage Harmonics & THD

- m. Simulate the same rectifier for  $L_s=1$  mH and plot the output voltage and input current.  
Observe the effect of line inductance and comment on your observations.

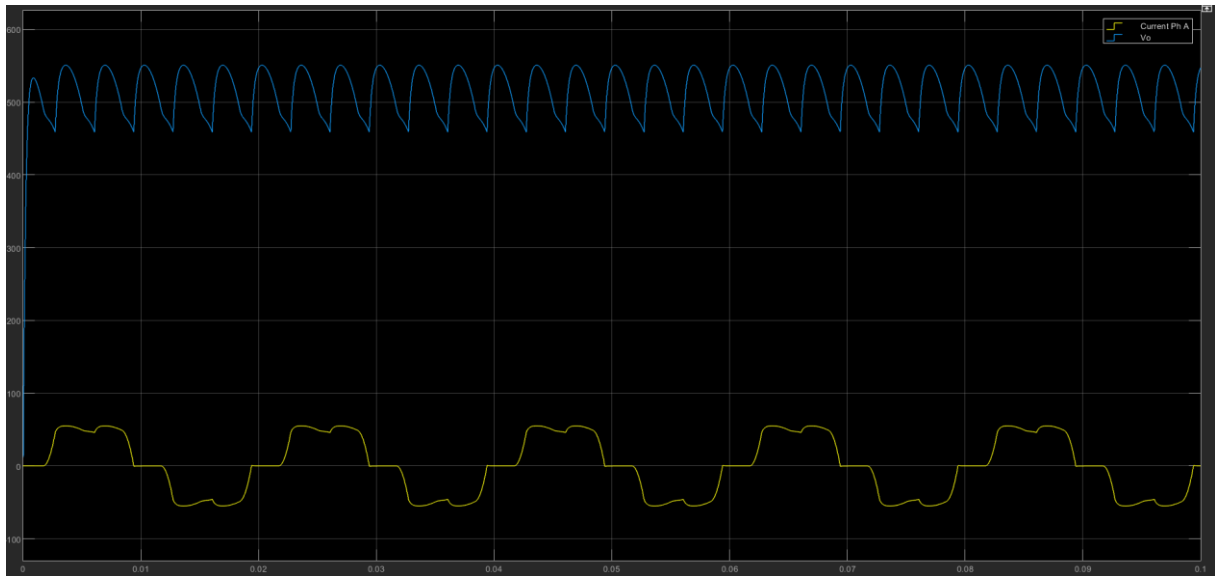


Figure 23. Output Voltage & Input Current Waveforms with Line Inductance

It can be easily observed that line inductance affects the waveforms in manner of distortion. There occurs commutation due to the line inductance because it delays the current and cause current positive and negative cycle happens at the same time during commutation.

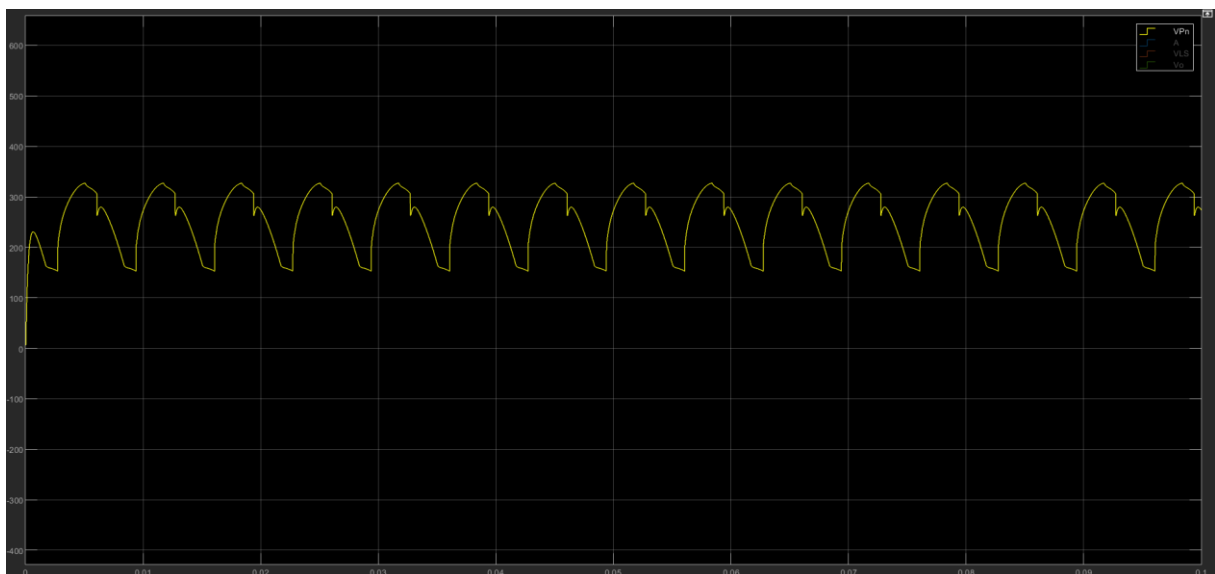


Figure 24. Commutation Effects

a) Repeat part (b) with  $L_s=1\text{mH}$ .

$$\Delta V_O = \frac{6\omega L_S I_O}{2\pi} \rightarrow \text{Average voltage lost due comm.}$$

$$V_O = V_{O_0} - \Delta V_O = \frac{3\sqrt{2}}{\pi} V_{l-l} - \frac{3}{\pi} \omega L_S I_O \rightarrow \text{Output voltage with comm.}$$

Output avg voltage was calculated in previous section that is 537.7V

With commutation the output voltage is reduced to 519V

So our commutation loss is  $533.7 - 519 = 14.7\text{V}$

Let's calculate in analytically:

$$\Delta V_O = \frac{6\omega L_S I_O}{2\pi} = \frac{6 \times 2\pi \times 50 \times 1 \times 10^{-3} \times 5.12}{2\pi} = ?$$

n. Compare the harmonic content of the input current for different  $L_s$  values of 0, 1 mH, 10 mH by checking IEEE 519-2014 standard. Comment on the differences. What kind of problems can having high input current THD cause to the grid?

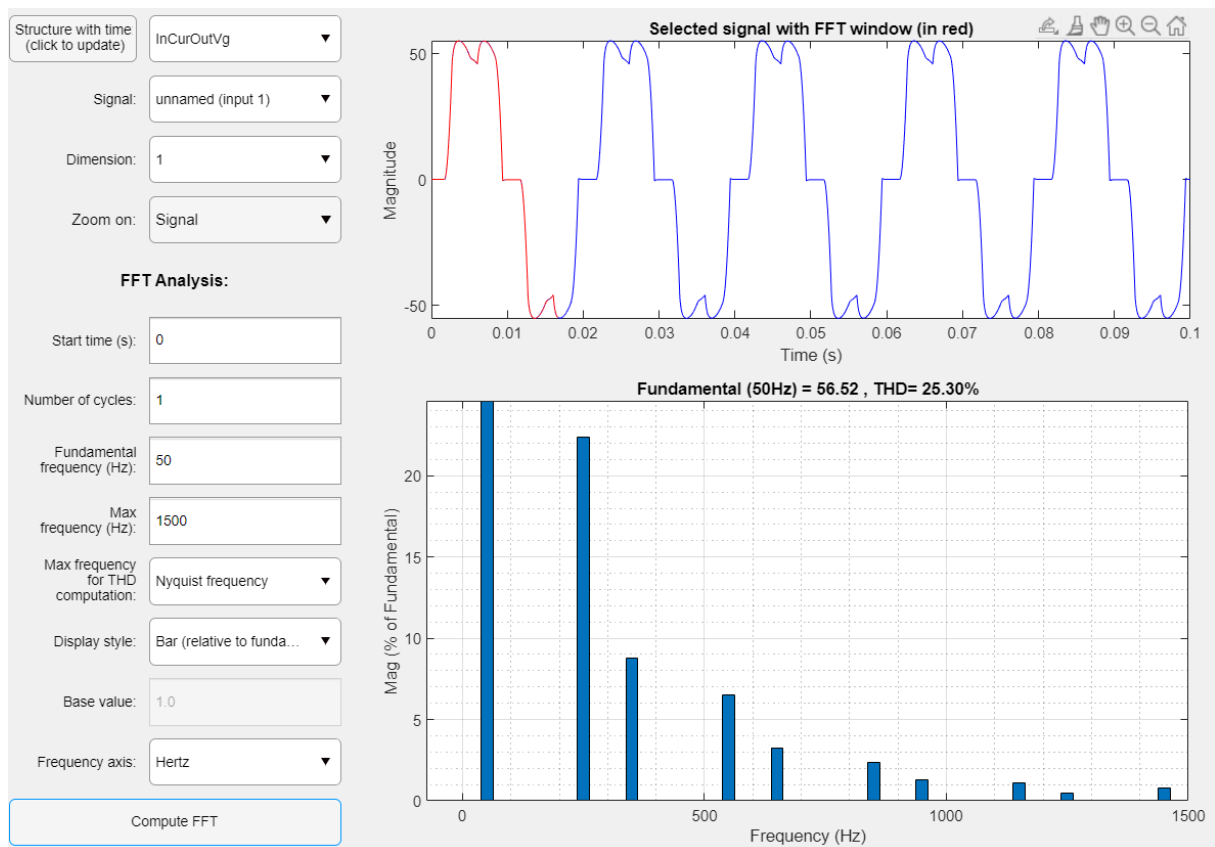


Figure 25.11 THD when  $L_s = 1\text{mH}$

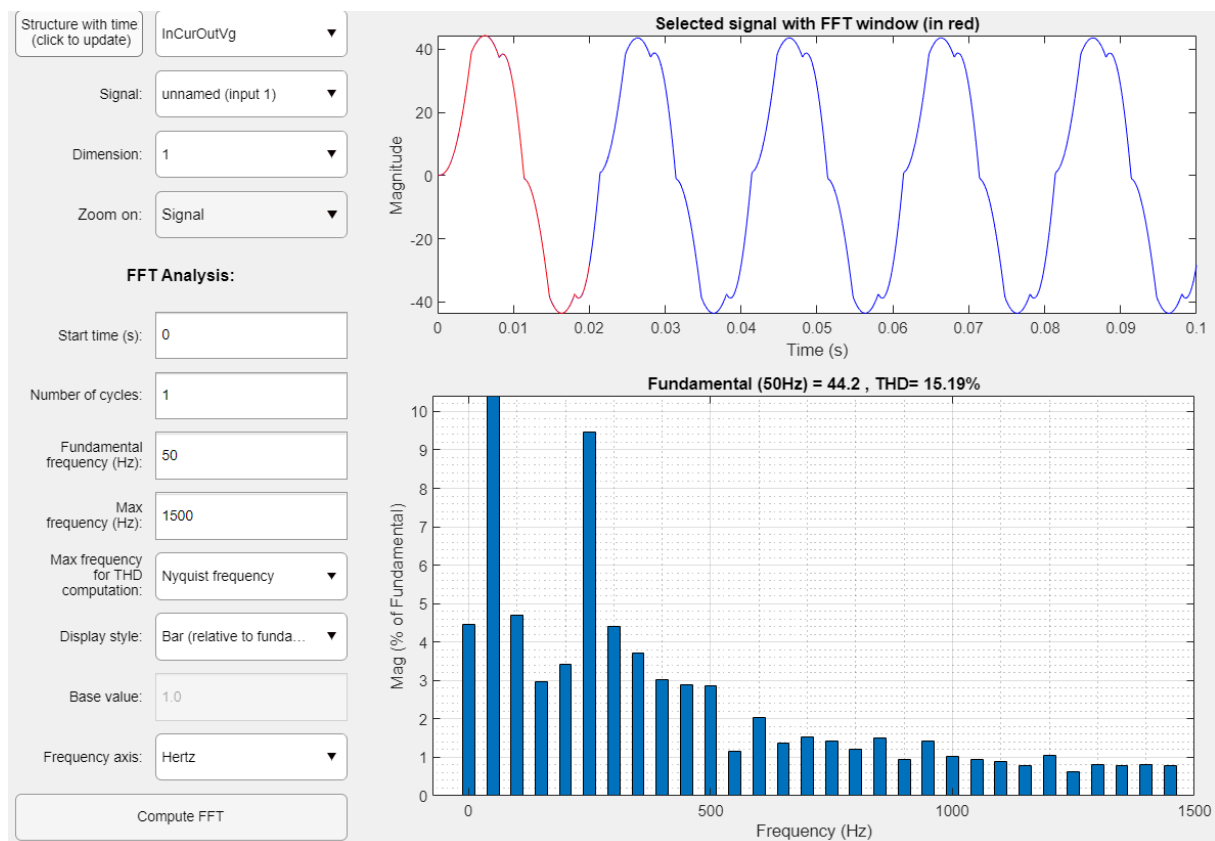


Figure 26. THD when  $L_s = 10\text{mH}$

Result: When  $L_s$  value gets higher, the input current will reduce and THD becomes smaller.