Feedback Control System Design of Buck Converter

1. Feedback Control Design First Look

In this section, the design of controller system of Buck Converter (DC-DC) is implemented via schematics.

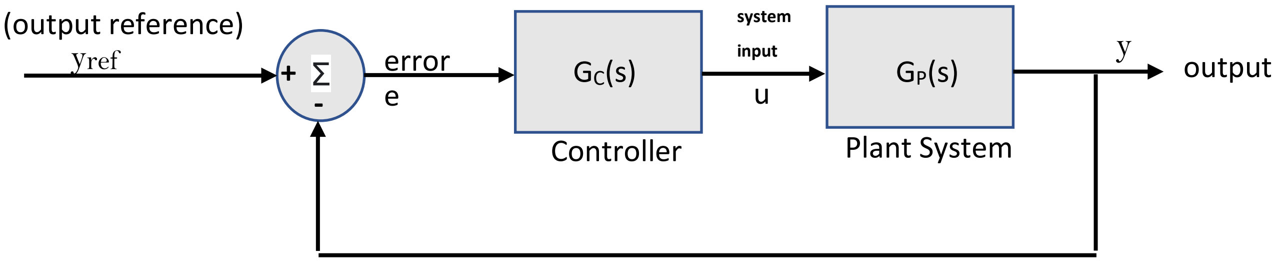


Fig. Feedback Control System Schematic

There are 3 steps to modeling the Plant (Gp(s))

1. Model the dynamic states
2. Simplify & linearize the system
3. Find the transfer function

2. Modeling the Plant System $G_p(s)$

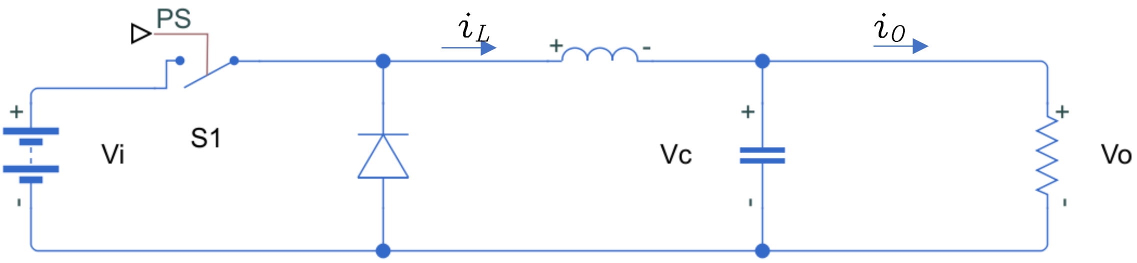


Fig. 1 Buck Converter

$Switch\ S_1\ is\ ON$

$V_L = V_i - V_O$

$Switch\ S_1\ is\ OFF$

$V_L = -V_O$

$$D = \frac{t_{ON}}{T} = \frac{t_{ON}}{t_{ON} + t_{OFF}}$$

How to derive switching circuit into a plant system?

We always start with the dynamic states. Dynamic states are just components that are governed by an equation that has a derivative.

If we look at the schematic for the buck converter above, the two components are governed by differential equations are ''Inductor and Capacitor''.

$$Inductor\ Current\ = \frac{di_L}{dt} = \frac{1}{L}[(V_i - V_C)D + (-V_C)(1-D)]$$

$$Capacitor\ Voltage\ = \frac{dV_C}{dt} = \frac{1}{C}\left[\left(i_L - \frac{V_C}{R}\right)D + \left(i_L - \frac{V_C}{R}\right)(1-D)\right]$$

$$Inductor\ Current\ = \frac{di_L}{dt} = \frac{1}{L}[V_iD\ - V_CD\ - V_C + V_CD)]$$

$$\boxed{\frac{di_L}{dt} = \frac{V_i}{L}D\ - \frac{V_C}{L}}$$

$$Capacitor\ Voltage\ = \frac{dV_C}{dt} = \frac{1}{C}\left[\dot{I}_LD\ - \frac{V_C}{R}D + \dot{I}_L\ - \dot{I}_LD\ - \frac{V_C}{R} + \frac{V_C}{R}D\right]$$

$$\frac{dV_C}{dt} = \frac{1}{C}\left[\dot{I}_L\ - \frac{V_C}{R}\right]$$

$$\boxed{\frac{dV_C}{dt} = \frac{\dot{I}_L}{C} - \frac{V_C}{RC}}$$

3. Simplify & Linearize the System

Inductor Current

$$\frac{di_L}{dt} = -\frac{V_C}{L} + \frac{V_i}{L}D$$

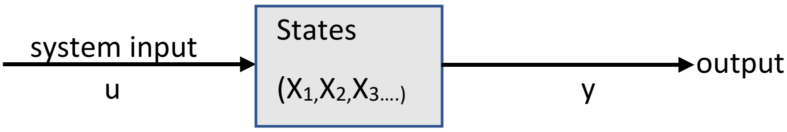
Capacitor Voltage

$$\frac{dV_C}{dt} = \frac{\dot{I}_L}{C} - \frac{V_C}{RC}$$

New Variables:

i_L = x_1 D = u

V_C = x_2 y = V_C = x_2



State Equations

$i_L = x_1$ D = u

$V_C = x_2$ y = V_C = x_2

$$\boxed{\left(\frac{di_L}{dt} = -\frac{V_C}{L} + \frac{V_i}{L}D\right) \longrightarrow \frac{dx_1}{dt} = -\frac{1}{L}x_2 + \frac{V_i}{L}u}$$

$$\boxed{\left(\frac{dV_C}{dt} = \frac{i_L}{C} - \frac{V_C}{RC}\right) \longrightarrow \frac{dx_2}{dt} = \frac{1}{C}x_1 - \frac{1}{RC}x_2}$$

4. Find the Transfer Function

We use Laplace Transform & rearrange the terms.

$$\left.\begin{array}{l}\text{Input} = u \\ \text{Output} = y = x_2\end{array}\right\} \quad T(s) = \frac{y(s)}{u(s)}$$

$$\mathcal{L}\left\{\frac{di_L}{dt} = -\frac{V_C}{L} + \frac{V_i}{L}D\right\} = \mathcal{L}\left\{\frac{dx_1}{dt} = -\frac{1}{L}x_2 + \frac{V_i}{L}u\right\} \rightarrow sX_1 = -\frac{1}{L}X_2 + \frac{V_i}{L}U$$

$$\mathcal{L}\left\{\frac{dV_C}{dt} = \frac{i_L}{C} - \frac{V_C}{RC}\right\} = \mathcal{L}\left\{\frac{dx_2}{dt} = \frac{1}{C}x_1 - \frac{1}{RC}x_2\right\} \rightarrow sX_2 = \frac{1}{C}X_1 - \frac{1}{RC}X_2$$

Consider x_2 = Y

$$sY = \frac{1}{C}X_1 - \frac{1}{RC}X_2$$

$$\frac{1}{C}X_1 = Y\left(s + \frac{1}{RC}\right)$$

$$\boxed{X_1 = CY\left(s + \frac{1}{RC}\right)}$$

$$sX_1 = -\frac{1}{L}Y + \frac{V_i}{L}U$$

$$sCY\left(s + \frac{1}{RC}\right) = -\frac{1}{L}Y + \frac{V_i}{L}U$$

$$sY\left(s + \frac{1}{RC}\right) = -\frac{1}{LC}Y + \frac{V_i}{LC}U$$

$$Y\left(s^2 + \frac{1}{RC}s + \frac{1}{LC}\right) = \frac{V_i}{LC}U$$

$$\boxed{T(s) = \frac{Y(s)}{U(s)} = \frac{\frac{V_i}{LC}}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}}$$

This was our transfer function of $G_P(s)$ plant system.

5. Modeling the Controller System Gc(s)

$$PID: G_c(s) = K_P + K_i\frac{1}{s} + K_D s \ (transfer\ function\ for\ PID\ controller)$$

Multiply PID equation by $\frac{s}{s}$

$$G_c(s) = \left(K_P + K_i\frac{1}{s} + K_D s\right).\frac{s}{s}$$

$$G_c(s) = \frac{(K_D s^2 + K_P s + K_i)}{s}$$

Our total transfer function for the system

$$TF = \frac{Y(s)}{Y_{ref}(s)} = \frac{G_p(s)G_c(s)}{1 + G_p(s)G_c(s)}$$

$G_p(s)$ was calculated in previous page. Put into the above equation:

$$\frac{Y(s)}{Y_{ref}(s)} = \frac{\dfrac{\frac{V_i}{LC}}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}G_c(s)}{1 + \dfrac{\frac{V_i}{LC}}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}G_c(s)}$$

$$\boxed{TF: \frac{Y(s)}{Y_{ref}(s)} = \frac{\frac{V_i}{LC}G_c(s)}{s^2 + \frac{1}{RC}s + \frac{1}{LC} + \frac{V_i}{LC}G_c(s)}}$$

$$G_c(s) = \left(\frac{K_D s^2 + K_P s + K_i}{s}\right)$$

Put $G_c(s)$ eq. into TF eq.

$$= \frac{\frac{V_i}{LC}\left(\frac{K_D s^2 + K_P s + K_i}{s}\right)}{s^2 + \frac{1}{RC}s + \frac{1}{LC} + \frac{V_i}{LC}\left(\frac{K_D s^2 + K_P s + K_i}{s}\right)} \ x \ \frac{s}{s} \ (to\ simplify)$$

$$\frac{Y(s)}{Y_{ref}(s)} = \frac{\frac{V_i}{LC}K_D s^2 + \frac{V_i}{LC}K_P s + \frac{V_i}{LC}K_i}{s^3 + \left(\frac{1}{RC} + \frac{V_i}{LC}K_D\right)s^2 + \left(\frac{1}{LC} + \frac{V_i}{LC}K_P\right)s + \frac{V_i}{LC}K_i}$$

It can be observed from denumerator, the system is having 3^{rd} order equation that means it has 3 poles.

Performance Characteristics:

i) Stability

ii) Rise time, Overshoot, Settling Time