Magnetic Design Guidance for Power Electronics

By

Eren Alpaslan

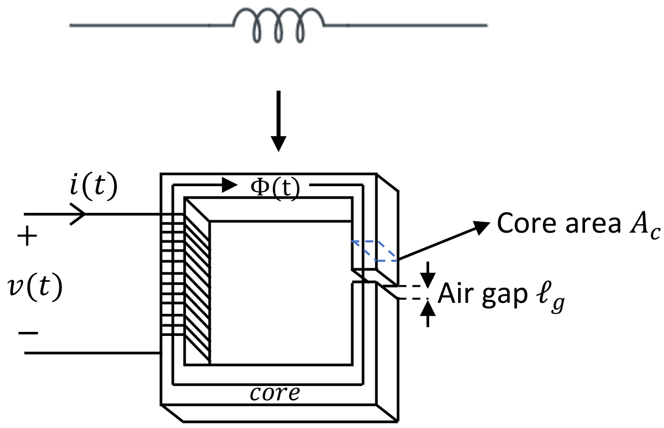
Chapter 1 **Abstract**

The document aims to guide in designing magnetic components for power electronics.

Chapter 2 **Subjects**

Contents

1.1. A simple inductor



Faraday's law:

For each turn of wire:

$$v_{turn}(t) = \frac{d\Phi}{dt}$$

Total winding voltage is

$$v(t) = N\frac{d\Phi}{dt}$$

Express flux (Φ) in terms of flux density $\Phi = BA_c$

$$v(t) = NA_c\frac{dB(t)}{dt}, where\ B = \mu H$$

$$v(t) = N\mu A_c\frac{dH(t)}{dt}$$

Ampere's law state that:

$$\oint H d\ell = Ni(t)$$

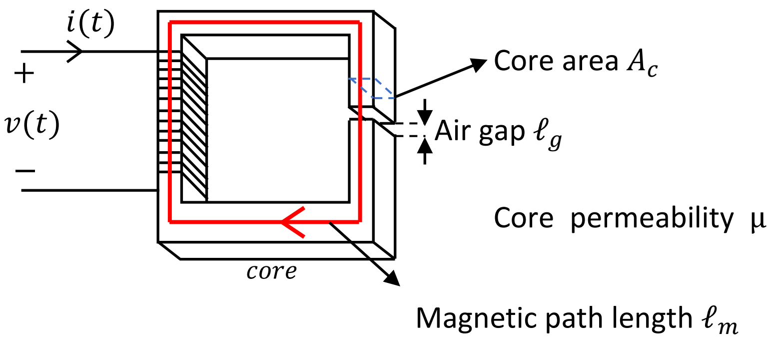
$$H = \frac{Ni(t)}{\ell_m}$$

$$v(t) = N\mu A_c\frac{d\frac{Ni(t)}{\ell_m}}{dt} = \frac{\mu N^2 A_c}{\ell_m}\frac{di(t)}{dt}$$

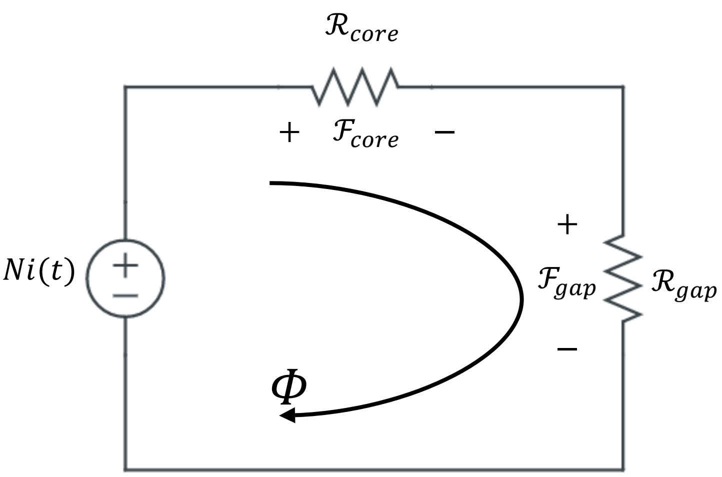
$$v(t) = N\frac{d\Phi}{dt} = L\frac{di}{dt} = \frac{\mu N^2 A_c}{\ell_m}\frac{di(t)}{dt}$$

$$N\frac{d\Phi}{dt} = L\frac{di}{dt} \qquad \rightarrow \qquad \frac{L}{N} = \frac{\Phi}{I} \qquad \rightarrow \frac{\frac{\mu N^2 A_c}{\ell_m}}{N} = \frac{BA_c}{I}$$

$$\boxed{B = \frac{\mu NI}{\ell_m} \qquad (Tesla)}$$



Circuit representation of above magnetic component is



$\mathcal{R}_{core}$

$$\mathcal{R}_{\text{core}} = \frac{\ell_{\text{core}}}{\mu_{\text{core}} A_c}$$

$\mathcal{R}_{gap}$ = *reluctance of the air*

$Ni = \mathcal{F}_{total} = \mathcal{F}_{core} + \mathcal{F}_{gap}$

$$\mathcal{R}_{\text{gap}} = \frac{\ell_g}{\mu_o A_c}$$

$Ni = \Phi(\mathcal{R}_{\text{core}} + \mathcal{R}_{\text{gap}})$

$$\Phi = \frac{Ni}{(\mathcal{R}_{\text{core}} + \mathcal{R}_{\text{gap}})}$$

Faraday's law:

$$v(t) = \frac{Nd\Phi}{dt}$$

Substitute for Φ:

$$v(t) = \frac{Nd\frac{Ni}{(\mathcal{R}_{\text{core}} + \mathcal{R}_{\text{gap}})}}{dt}$$

$$v(t) = \frac{N^2\frac{di}{dt}}{\mathcal{R}_{\text{core}} + \mathcal{R}_{\text{gap}}}$$

Here, inductance is

$$\boxed{L = \frac{N^2}{\mathcal{R}_{\text{core}} + \mathcal{R}_{\text{gap}}} \qquad (H)}$$

$$MMF(\mathcal{F}) = \Phi\mathcal{R} = Ni$$

$$\Phi = BA_c \rightarrow \mathcal{F} = BA_c\mathcal{R} = Ni$$

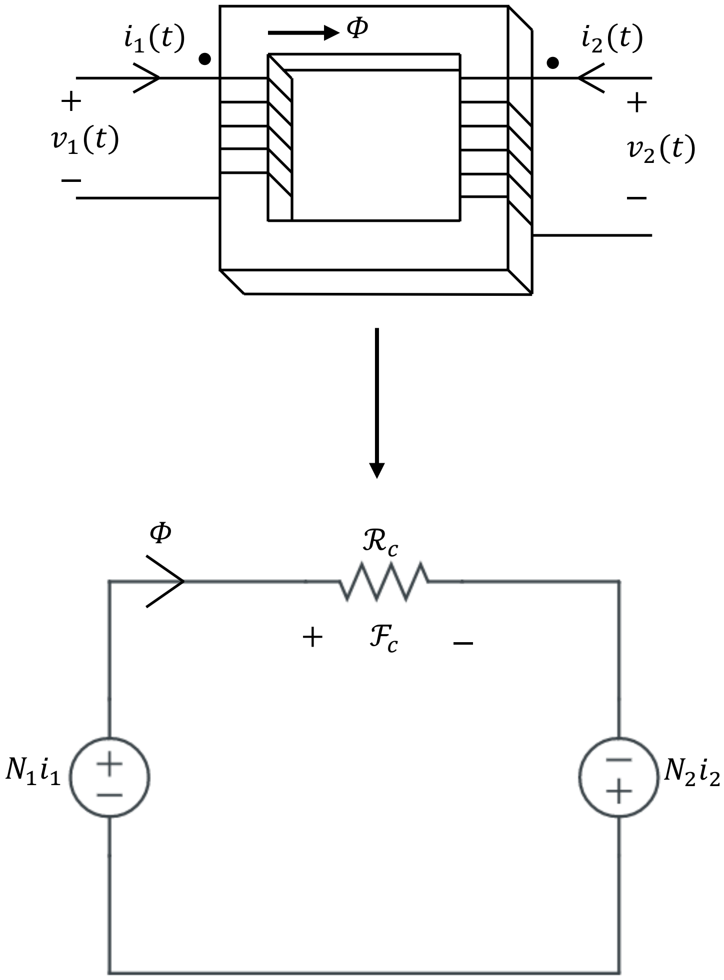
$$\boxed{I_{sat} = \frac{B_{sat}A_c}{N}(\mathcal{R}_c + \mathcal{R}_g)}$$

<u>Effects of air gap:</u>

- decrease inductance
- increase saturation current

NOTE: inductance is less dependent on core permeability

1.2. The Ideal Transformer Model



Two windings, no air gap

$$\mathcal{R}_c = \frac{\ell_m}{\mu A_c}$$

$$\mathcal{F}_c = N_1 i_1 + N_2 i_2$$

$$\Phi \mathcal{R} = N_1 i_1 + N_2 i_2$$

In ideal transformer, the core reluctance $\mathcal{R}_c$ approaches to 0, so $MMF(\mathcal{F}_c = \Phi\mathcal{R})\ approaches\ to\ 0$.

$$0 = N_1 i_1 + N_2 i_2$$

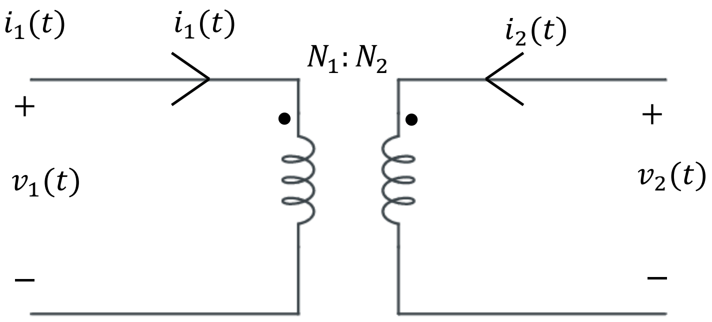
$$v_1 = N_1 \frac{d\Phi}{dt} \ \&\ v_2 = N_2 \frac{d\Phi}{dt}$$

Eliminate Φ:

$$\frac{d\Phi}{dt} = \frac{v_1}{N_1} = \frac{v_2}{N_2}$$

In ideal xfmr:

$$\frac{v_1}{N_1} = \frac{v_2}{N_2}$$



$Ideal\ Transformer$

The Magnetizing Inductance:

In case where core reluctance is not zero:

$$\Phi\mathcal{R} = N_1 i_1 + N_2 i_2 \qquad\qquad v_1 = N_1 \frac{d\Phi}{dt}$$

To eliminate Φ, substitute Φ equation to v_1 equation:

$$\underbrace{v_1 = N_1 \frac{d\Phi}{dt} = \frac{{N_1}^2}{\mathcal{R}} \frac{d}{dt}\left[i_1 + \frac{N_2}{N_1} i_2\right]}$$

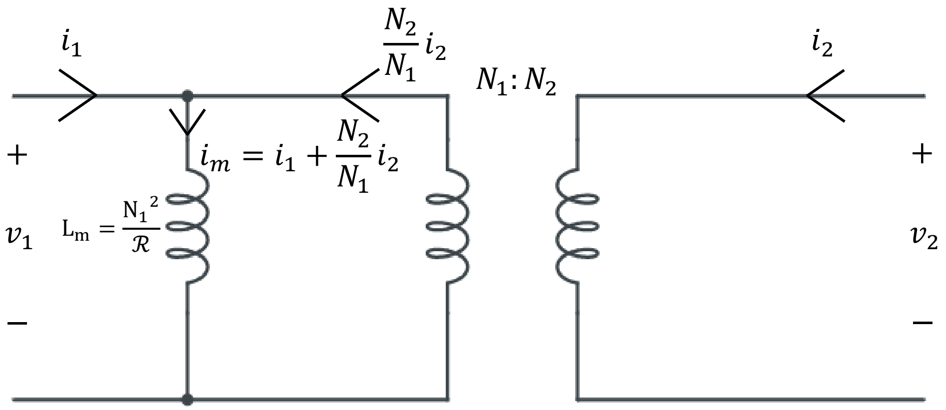
This equation is of the form

$$v_1 = L_m \boxed{\frac{di_m}{dt}}$$

$$L_m = \boxed{\frac{{N_1}^2}{\mathcal{R}}}$$

$$v_1 = \boxed{\frac{{N_1}^2}{\mathcal{R}}} \frac{d}{dt} \boxed{\left[i_1 + \frac{N_2}{N_1} i_2\right]}$$

$$\boxed{i_m = i_1 + \frac{N_2}{N_1} i_2}$$



Magnetizing current $i_m = i_1 + \frac{N_1}{N_2} i_2$ depends on the integral of the applied voltage

$$\boxed{i_m(t) = \frac{1}{L_m} \int v_1 (t) dt}$$

$$\frac{L}{N} = \frac{\Phi}{I} = \frac{BA_c}{I}$$

$$\boxed{B(t) = \frac{1}{N_1 A_c} \int v_1 (t) dt}$$

Flux density becomes large and core will be saturated when the applied volt-seconds λ_1 are too large, where

$$\lambda_1 = \int_{t_1}^{t_2} v_1(t) dt$$

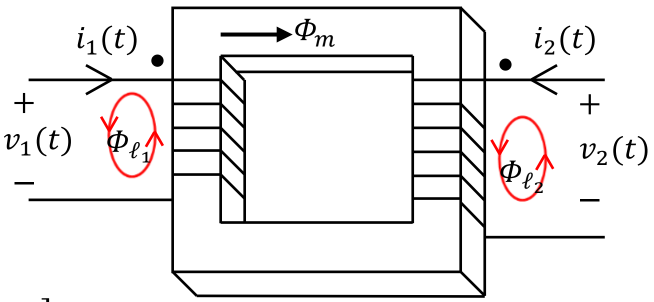
NOTE: Saturation of core is related with applied voltage rather than applied current.

- If your inductor is saturated, you can fix that by removing turns or adding an airgap

- In the case of transformer, neither those will work. In fact, we add turns to reduce flux density.

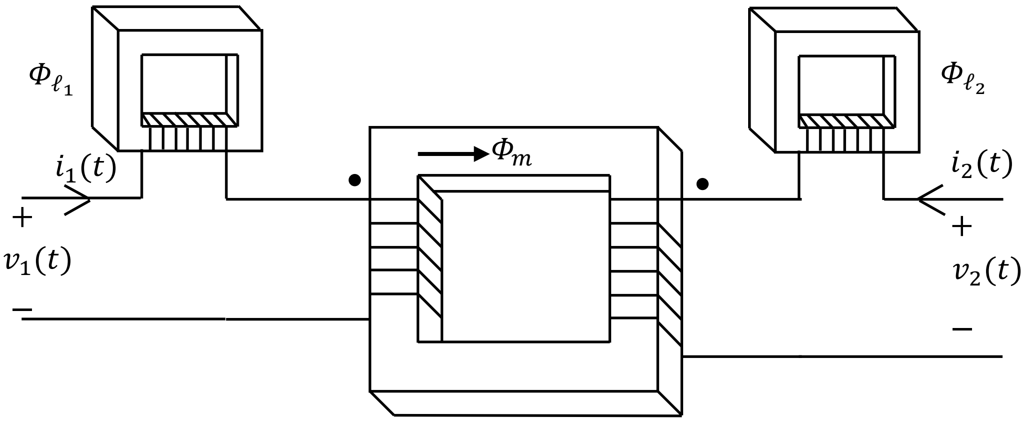
1.3. Leakage Inductance

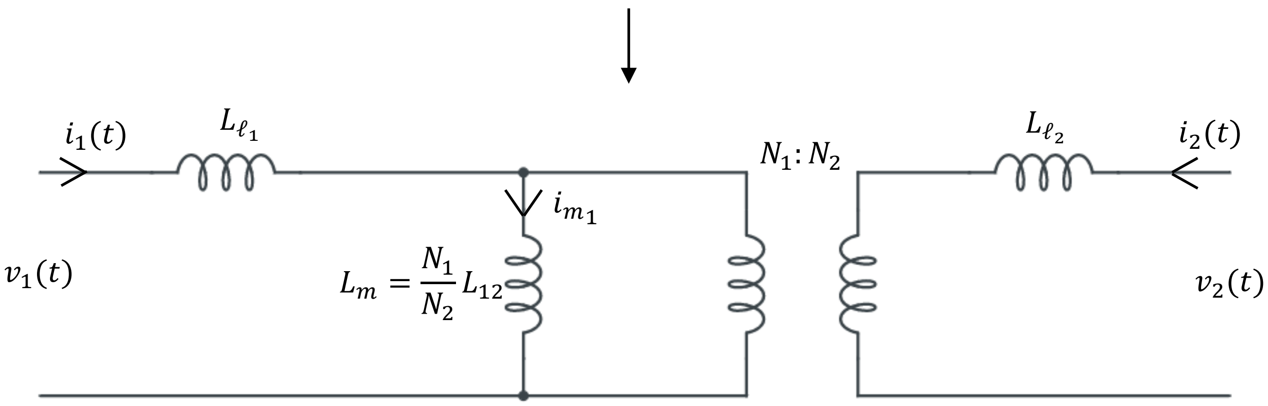
Leakage flux is the flux that passing one winding but not the other. That is, flux is not following the path which is supposed to follow.



$$v_1 = N_1 \frac{d}{dt}\left[\left[\Phi_{\ell_1} + \Phi_m\right]\right]$$

$$v_2 = N_2 \frac{d}{dt}\left[\left[\Phi_{\ell_2} + \Phi_m\right]\right]$$





$$\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix}$$

Mutual inductance L_{12}

$$L_{12} = \frac{N_1 N_2}{\mathcal{R}} = \frac{N_2}{N_1} L_m$$

Self-inductances L_{11} & L_{22}

$$L_{11} = L_{\ell_1} + \frac{N_1}{N_2} L_{12}$$

Effective turns ratio: $n_e = \sqrt{\frac{L_{22}}{L_{11}}}$

$$L_{22} = L_{\ell_2} + \frac{N_2}{N_1} L_{12}$$

Coupling coefficient: $k = \frac{L_{12}}{\sqrt{L_{11} L_{22}}}$

1.4. Loss Mechanism in Magnetic Devices

- Low-Frequency losses
    - DC copper loss
    - Core loss, hysteresis loss
- High-Frequency losses
    - Core loss: classical eddy current losses
    - Eddy current losses in ferrite cores
- High-Frequency copper loss: Proximity Effect
    - Proximity effect: high-frequency limit
    - MMF diagrams, losses in a layer and the losses in basic multilayer windings
    - Effect of PWM waveform harmonics

1.4.1. Core Loss

Energy per cycle W, flowing into n turn winding of an inductor, excited by periodic waveforms of frequency *f*:

$$W = \int_{\text{one cycle}} v(t)i(t)dt$$

Relate winding voltage and current to core. B and H via Faraday's Law and Ampere's Law:

$$\left.v(t) = N\frac{d\Phi}{dt}\right\} \text{Faraday's Law}$$

$$v(t) = NA_c\frac{dB}{dt}$$

$$\left.Ni(t) = H\ell_m\right\} \text{Ampere's Law}$$

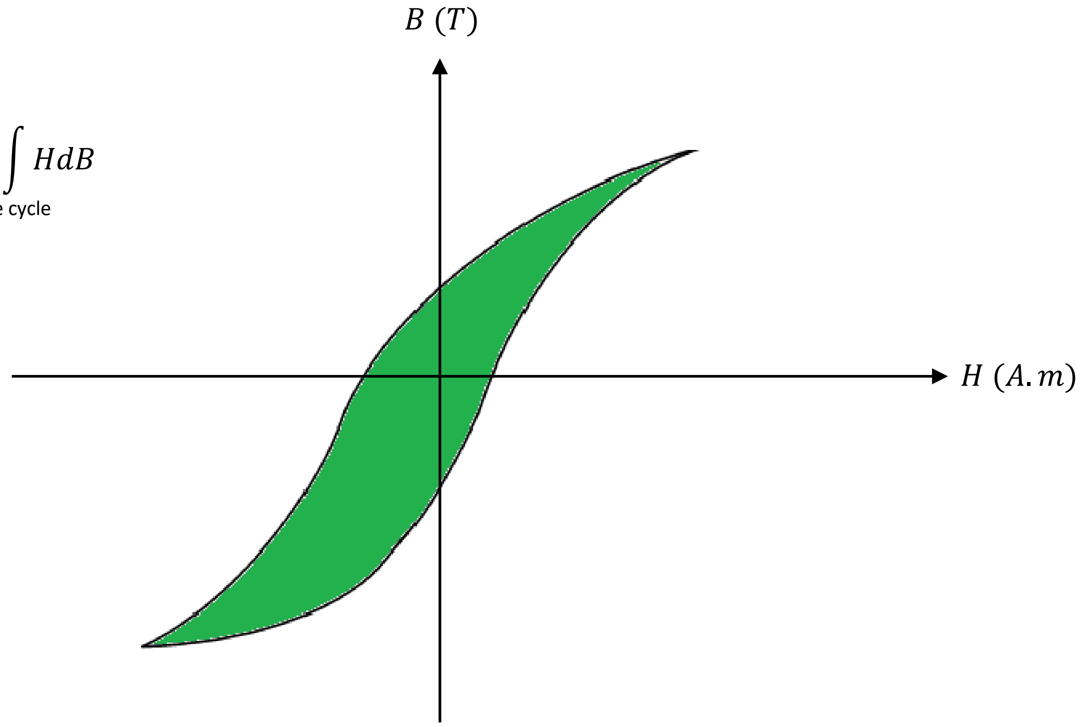
$$i(t) = \frac{H\ell_m}{N}$$

$$W = \int_{\text{one cycle}} \underbrace{\left(NA_c\frac{dB}{dt}\right)}_{v(t)}\underbrace{\left(\frac{H\ell_m}{N}\right)}_{i(t)} dt$$

$$\boxed{W = \underbrace{\boxed{A_c\ell_m}}_{\text{Volume of the core material}}\int_{\text{one cycle}} HdB}$$

1.4.2. Core Loss: Hysteresis Loss

$$W = A_c \ell_m \int_{\text{one cycle}} H dB$$



The term $A_c\ell_m$ is the volume of the core, while the integral is the area of the B-H loop as in graph above.

$$(Energy\ lost\ per\ cycle) = (core\ volume) \times (area\ of\ the\ B - H\ loop)$$

$$P_H = f(A_c\ell_m) \int HdB$$

* Hysteresis loss is directly proportional to applied frequency.

- Dependence on maximum flux density (B): How does the area of B-H loop depend on maximum flux density?

Steinmetz's equation explains:

Hysteresis loss per unit volume:

$P_H = K_H f B_{max}{}^{\alpha}$

Total hysteresis loss:

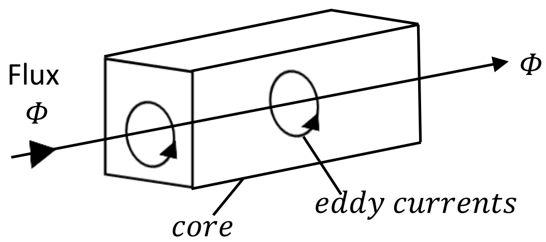
$P_H = K_H f B_{max}{}^{\alpha} \times (core\ volume)$

$$\boxed{P_H = K_H f B_{max}{}^{\alpha} \times A_c \ell_m}$$

*The parameters K_H & α are determined experimentally.

1.4.3. Core loss: Eddy current loss

Magnetic core materials are reasonably good conductors of electric current. Hence, according to Lenz's Law, magnetic fields withing the core induce currents (eddy currents) to flow within the core. The eddy currents flow such that they tend to generate flux which opooses changes in the core flux $\Phi(t)$. The eddy currents tend to prevent flux from penetrating the core.



$Eddy\ current\ loss\ = i^2(t)R$

Layers of lamination method is used to prevent eddy current to shift to next layer (It increases capacitance).

Also, classical Steinmetz's equation for eddy current loss

$P_E = K_E f^2 B_{max}{}^2 \times (core\ volume)$

$$\boxed{P_E = K_E f^2 {B_{max}}^2 \times A_c \ell_m}$$

Ferrite core material is capacitive, this causes eddy current power loss to increase at f^4

Core Materials

Core Type	B_{sat}	Relative Core Loss	Applications
Laminations iron, Silicon steel	1.5-2.0 Tesla	High	50-60Hz transformer, inductor
Powdered cores powdered iron, Molypermalloy	0.6-0.8 Tesla	Medium	1kHz xfmr, 100kHz filter inductor
Ferrite Manganese-zinc Nickel-zinc	0.25-0.5 Tesla	Low	20kHz-1MHz xfmr, AC inductors

1.4.4. Penetration Depth (Skin Depth)

For sinusoidal currents: current density is an exponentially decaying function of distance into the conductor with characteristics length δ known as the *penetration depth* or *skin depth*.

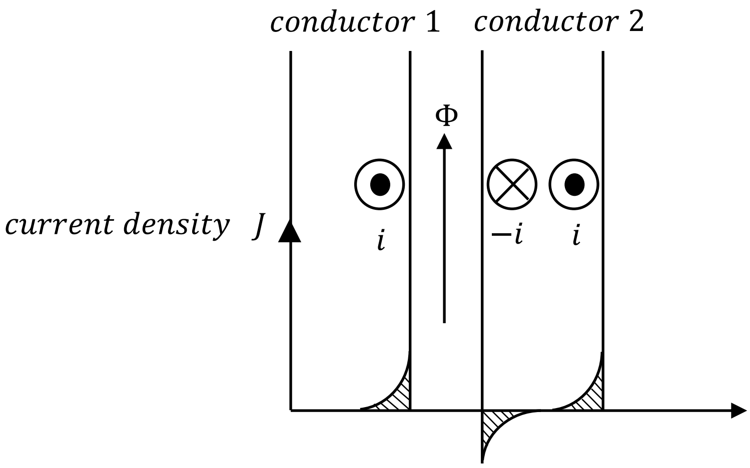
$$\delta = \sqrt{\frac{\rho}{\pi \mu f}}\ cm, \qquad \delta = \frac{7.5}{\sqrt{f}}\ cm\ at\ room\ temperature$$

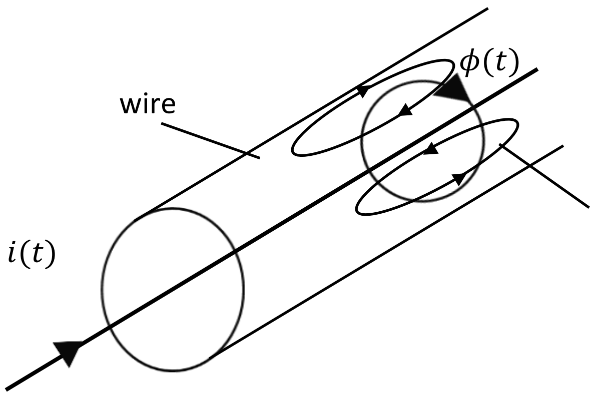
Current density (J)

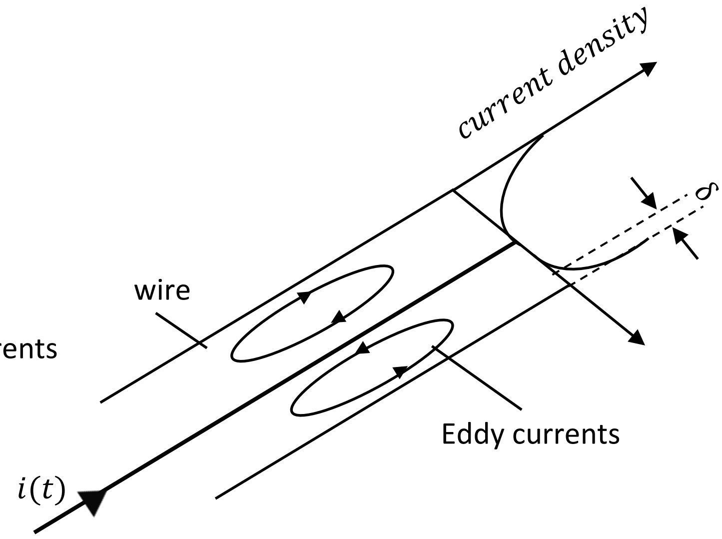
$$J = J_0 e^{-X/\delta}$$ where x is the distance into wire.

1.4.5. Proximity Effect

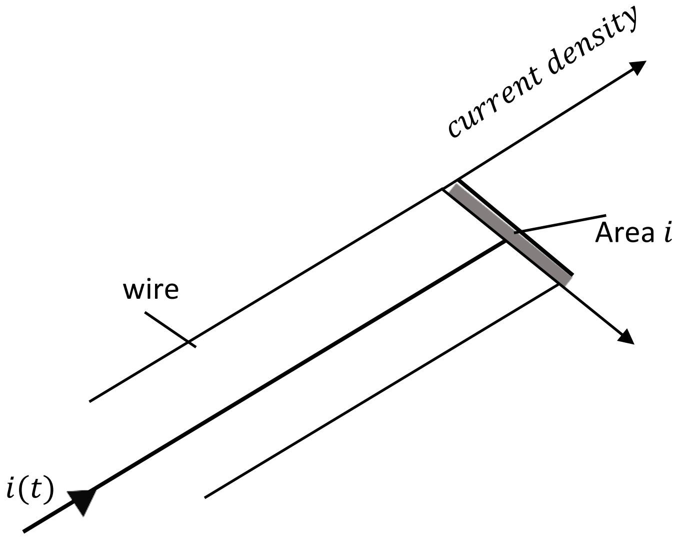
As current in a conductor induces eddy currents in adjacent conductors by a process called the proximity effect. This causes a significant power loss in the windings of high-frequency transformers and AC inductors.



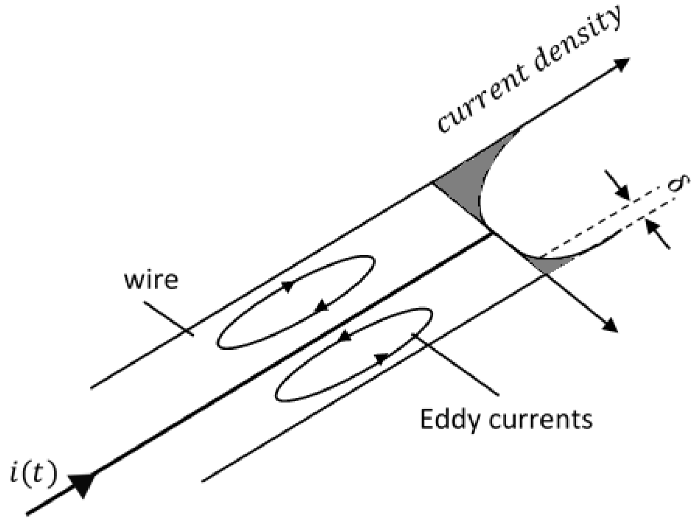




DC: Uniform current distribution



AC: Skin effect changes distribution



$$P_{cu} = i^2 R_{dc}$$

$$R_{dc} = \frac{\rho \ell}{A_w}$$

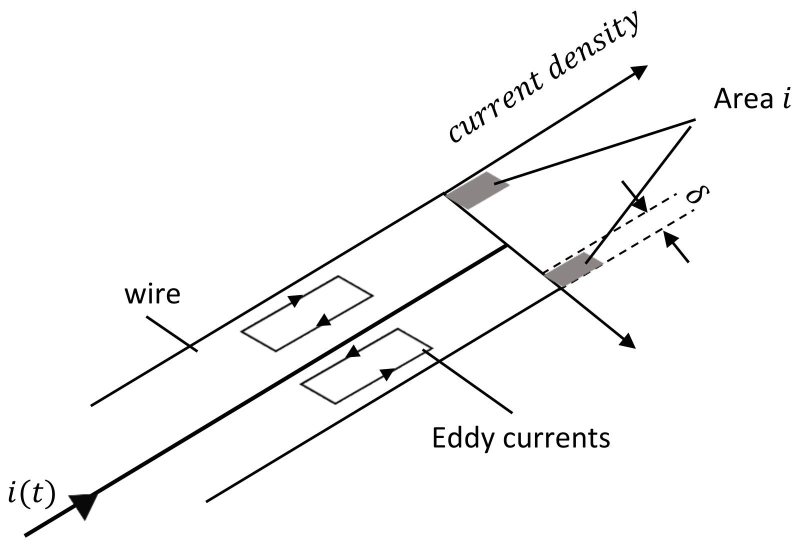
$$P_{cu} = i^2 R_{ac}$$

$$R_{ac} = \frac{\rho \ell}{A_{eff}}$$

$$R_{ac} = R_{dc} \frac{A_w}{A_{eff}}$$

It can be shown that A_{eff} is area of annulus having outside the diameter equal to wire diameter, and thickness equal to skin depth δ

AC: Copper pipe analog



1.4.6. Litz Wire

A way to increase conductor area while low proximity losses.

- Many strands of small-gauge wire are bundled together and externally connected in parallel.
- Strands are twisted, or transposed so that each strand passes equally through each position on inside & outside of bundle. This prevent circulation of currents between strands.
- Strand diameter should be sufficiently smaller than skin depth.
- The Litz wire bundle itself is composed of multiple layers.

Advantage: When properly sized, can significantly reduce proximity loss.

Dissadvantage: Increased cost & decreased amount of copper within core window.

1.5. Filter Inductor Design

A. Filter Inductor design constraints

B. A first-pass design procedure

C. Multiple winding magnetics design using the K_g method

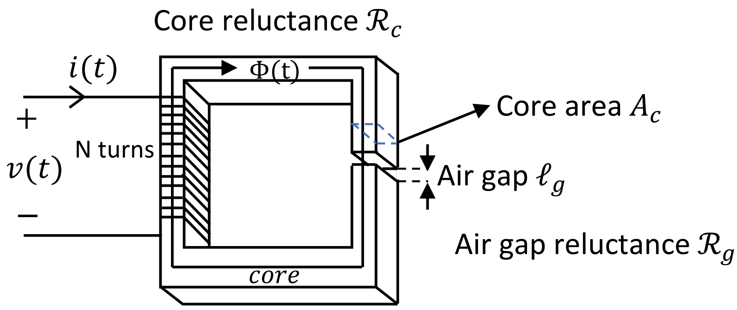
D. Summary of key points

1.5.1. Filter inductor design constraints

Objectives:

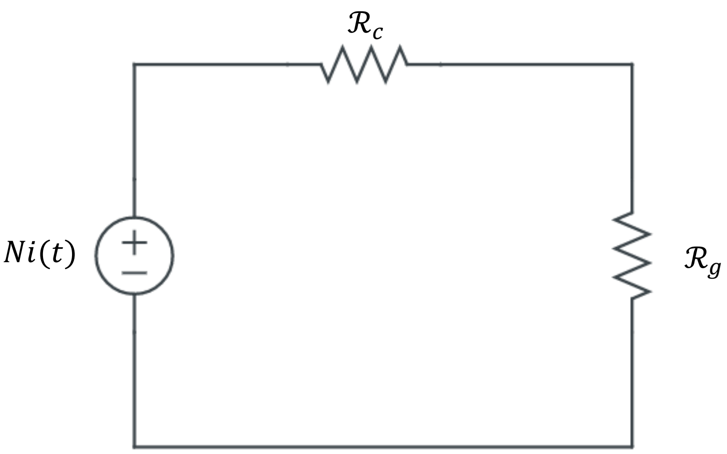
- Design an inductor having a given inductance L_s which carries worst-case current I_{max} without saturating, and which has a given winding resistance R, equivalently exhibits worst-case copper loss of $P_{cu} = {I_{rms}}^2 R$

  (example: filter inductor in CCM buck converter)



$$\mathcal{R}_c = \frac{\ell_c}{\mu_c A_c}$$

$$\mathcal{R}_g = \frac{\ell_g}{\mu_o A_c}$$



$$Ni = \phi(\mathcal{R}_c + \mathcal{R}_g)$$

Usually $\mathcal{R}_c << \mathcal{R}_g$

$$Ni = \phi\mathcal{R}_g$$

Given a peak winding current I_{max}, it is desired to operate the core flux density at a peak value B_{max}. The value of B_{max} is chosen to be less than the worst-case saturation flux density B_{sat} of the core.

1.5.2. First constraint

$$Ni = \phi \mathcal{R}_g$$

$$Ni = BA_c\mathcal{R}_g$$

Let $i = I_{max}$ & $B = B_{max}$

$$NI_{max} = B_{max}A_c\mathcal{R}_g = B_{max}A_c\frac{\ell_g}{\mu_o A_c}$$

$$\boxed{NI_{max} = B_{max}\frac{\ell_g}{\mu_o}} \longrightarrow \text{Constraint } 1$$

Unknowns:

$\ell_g = air\ gap\ length$

$N = number\ of\ turns$

1.5.3. Second constraint

We must obtain specified inductance L. We know that the inductance we found in previous pages:

$$L = \frac{N^2}{\mathcal{R}_c + \mathcal{R}_g}$$

Usually $\mathcal{R}_c << \mathcal{R}_g$, therefore;

$$L = \frac{N^2}{\mathcal{R}_g}, where\ \mathcal{R}_g = \frac{\ell_g}{\mu_o A_c}$$

$$\boxed{L = \frac{\mu_o A_c N^2}{\ell_g}} \longrightarrow \text{Constraint } 2$$

Unknowns:

$\ell_g = air\ gap\ length$

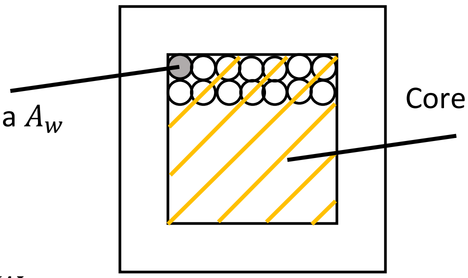
$N = number\ of\ turns$

$A_c = core\ area$

1.5.4. Third constraint: Winding area

Wire must fit through core window (i.e. hole in center of wire)

$\text{Total area of copper in window} = NA_w$



$\text{Area available for winding conductors} = K_u W_A$

$$\boxed{K_u W_A \geq NA_w} \rightarrow \text{Constraint } 3$$

$K_u = fill\ factor\ or\ window\ utilization\ factor$

It accounts for how well the wire packs in the window. It is actually ratio of the window area to total area of copper.

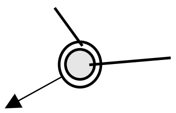
$K_u < 1$

Fill Factor (Window utilization factor) K_u

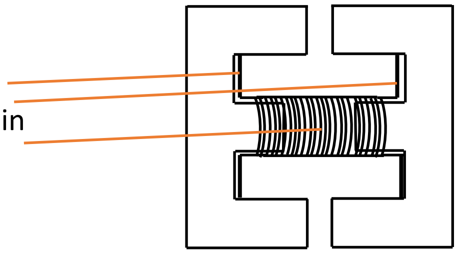
K_u is the fraction of the core window area that is filled by copper.

Mechanism that cause K_u to be less than 1:

- Round wire does not pack perfectly, which reduces K_u by a factor of 0.7 to 0.55 depending on winding technique
- Insulation reduces K_u by a factor of 0.95 to 0.65, depending on wire size and type of insulation



- Bobbin uses some window area.



- Additional insulation may be required between windings.

Typical values of K_u

- 0.5 for simple low-voltage inductor
- 0.25 to 0.3 for off-line transformer
- 0.05 to 0.2 for high-voltage transformer (multiple kV)
- 0.65 for low-voltage foil-winding inductor

1.5.5. Fourth constraint: Winding resistance

$$R = \rho \frac{\ell_b}{A_w}$$

$\rho = resistivity\ of\ conductor\ material$

$\boldsymbol{\ell_b} = lenght\ of\ wire$

$A_w = wire\ bare\ area$

The resistivity of copper at room temperature (25°C) is $\rho = 1.724 \times 10^{-6}\Omega - \mathrm{cm}$

$\rho = 2.3 \times 10^{-6}\Omega - \mathrm{cm}$ at 100°C

The lenght of the wire comprising an N-turn winding can be expressed as

$$\boldsymbol{\ell_b} = N(MLT)$$

MLT = Mean-lenght-per-turn of winding. MLT is a function of core geometry. The equation above can be manipulated as

$$\boxed{R = \rho \frac{N(MLT)}{A_w}} \longrightarrow \text{Constraint } 4$$

FOUR CONSTRAINTS:

$$\boxed{NI_{max} = B_{max} \frac{\ell_g}{\mu_o}} \longrightarrow \text{Constraint } 1$$

$$\boxed{L = \frac{\mu_o A_c N^2}{\ell_g}} \longrightarrow \text{Constraint } 2$$

$$\boxed{K_u W_A \geq NA_w} \longrightarrow \text{Constraint } 3$$

$$\boxed{R = \rho \frac{N(MLT)}{A_w}} \longrightarrow \text{Constraint } 4$$

These equations involve the quantities

- A_c , W_A, and MLT which are the function of the **core geometry**
- I_{max}, B_{max}, μ_o, L, K_u, R and ρ which are given **specifications** or other known quantities
- N, ℓ_g and $\boldsymbol{A_w}$ which are **unknowns**

Manipulate the formulas which are given above, we get:

$$\underbrace{\boxed{\frac{{A_c}^2 W_A}{(MLT)}}}_{\text{Core geometry}} \geq \underbrace{\boxed{\frac{\rho L^2 I^2{}_{max}}{B^2{}_{max} R K_u}}}_{\text{Specifications or known quantities}}$$

So, we must choose a core whose geometry satisfies the above equation

The core geometry constant K_g is defined as

$$\boxed{K_g = \frac{{A_c}^2 W_A}{(MLT)}}$$

K_g is a figure-of-merit that describes the effective electrical size of magnetic cores, in applications where the following quantities are specified:

- Copper loss
- Maximum flux density

<u>How specifications affect the core size:</u>

- A smaller core can be used by increasing:
  - B_{max} → use core material having higher B_{sat}
  - R →allow more copper loss

<u>How core geometry affects electrical capabilities:</u>

- A_c → more iron core material
- W_A → larger window and more copper

Units

Wire resistivity	ρ	(Ω-cm)
Peak winding current	I_{max}	(A)
Inductance	L	(H)
Winding resistance	R	(Ω)
Winding fill factor	K_u	
Core maximum flux density	B_{max}	(T)

Core dimensions

Core cross-sectional area	A_c	(cm^2)
Core window area	W_A	(cm^2)
Mean-length per turn	MLT	(cm)

Determine core size:

Because use of centimeters is preferred rather than meters, it is needed to add appropriate factors to the design equations.

$$\boxed{K_g \geq \frac{\rho L^2 I^2{}_{max}}{B^2{}_{max} R K_u} \times 10^8 \, (cm^5)}$$

As we know $K_g = \frac{A_c{}^2 W_A}{(MLT)}, \frac{(cm^2)^2 \times cm^2}{cm} = cm^5$

Determine air gap length:

$$L = \frac{\mu_o A_c N^2}{\ell_g} \quad \rightarrow \quad \ell_g = \frac{\mu_o A_c N^2}{L}$$

$$N I_{max} = B_{max} \frac{\ell_g}{\mu_o} \quad \rightarrow \quad N = \frac{B_{max} \ell_g}{\mu_o I_{max}}$$

$$\boxed{\ell_g = \frac{\mu_o L I^2{}_{max}}{B^2{}_{max} A_c} \times 10^4 \ (m)}$$

***A_L*:**

Core manufacturers sell gapped cores. They specify the quantity to make calculations in more easier way.

Rather than specifying the air gap lentgh ℓ_g, the equivalent quantity $\boldsymbol{A_L}$ is used.

$\boldsymbol{A_L}$ is equal to inductance, in mH, obtained with a winding of 1000 turns.

$$A_L = \frac{10 B^2{}_{max} A_c{}^2}{L I^2{}_{max}} \qquad (mH/1000\ turns)$$

$$L = A_L N^2 10^{-9}\ (Henries)$$

Determine number of turns *N*:

$$N = \frac{L I_{max}}{B_{max} A_c} \times 10^4$$

Evaluate wire size:

$$A_w \leq \frac{K_u W_A}{N}\ (cm^2)$$

Select wire with bare copper area A_w less than or equal to the value which is equal to $\frac{K_u W_A}{N}$.

As a check, the winding resistance can be calculated:

$$R = \frac{\rho N (MLT)}{A_w} \Omega$$

1.6. Multiple-winding magnetics design

By using $\boldsymbol{K_g}$ method:

The K_g method can be extended to multiple-winding magnetics elements such as coupled inductors and transformers.

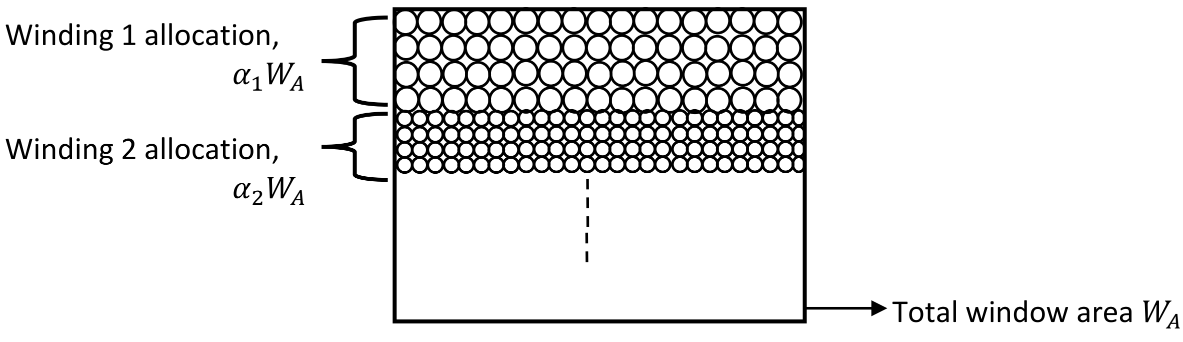
However, it is valid under these conditions:

- Copper loss dominates the total loss (i.e. core loss ignored) or,
- The maximum flux density B_{max} is a specification rather than a quantity to be optimized

To do this, we must

- Find how to allocate the window area between windings
- Generalize the step-by-step design procedure

1.6.1. Window area location



$$0 < \alpha_j < 1$$

$$\alpha_1 + \alpha_2 + \ldots + \alpha_k = 1$$

1.6.2. Copper loss in windings

Copper loss in winding j:

$$P_{cu,j} = I_j{}^2 R_j$$

Resistance of winding j:

$$R_j = \rho \frac{\ell_j}{A_{w,j}}$$

$\ell_j = N_j(MLT)$ (length of the wire of winding *j*)

$A_{w,j} = \frac{W_A K_u \alpha_j}{N_j}$ (wire area of winding *j*)

Hence,

$$\boxed{R_j = \rho \frac{\ell_j}{A_{w,j}} = \rho \frac{N_j{}^2 (MLT)}{W_A K_u \alpha_j}}$$

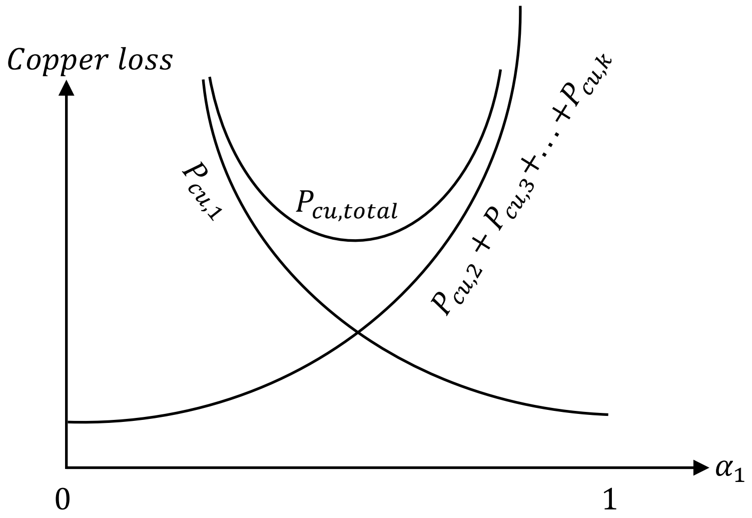
$$\boxed{P_{cu,j} = I_j{}^2 R_j = \frac{N_j{}^2 i_j{}^2 \rho (MLT)}{W_A K_u \alpha_j}}$$

1.6.3. Total copper loss of transformer

$$P_{cu,total} = P_{cu,1} + P_{cu,2} + \ldots\ldots + P_{cu,k} = \frac{\rho(MLT)}{W_A K_u} \sum_{j=1}^{k} \frac{N_j{}^2 I_j{}^2}{\alpha_j}$$

It is important to select values for $\alpha_1, \alpha_2, \ldots, \alpha_k$ such that the total copper loss is minimized.

1.6.4. Variation of copper losses with α_1



For $\alpha_1 = 0$: wire of winding 1 has zero area. $P_{cu,1}$ tends to infinity.

For $\alpha_1 = 1$: wires of remaining windings have zero area. Their copper losses tend to infinity.

There is a choice of α_1 that minimizes the total copper loss.

1.6.5. Interpretation of results

$$\alpha_m = \frac{V_m I_m}{\sum_{n=1}^{\infty} V_j I_j}$$

Apparent power in winding *j* is:

$V_j I_j$ where $V_j = RMS\ or\ peak\ applied\ voltage$

$I_j = RMS\ current$

Window area should be allocated according to the apparent powers of the winding.

1.7. CCM Flyback Converter Design

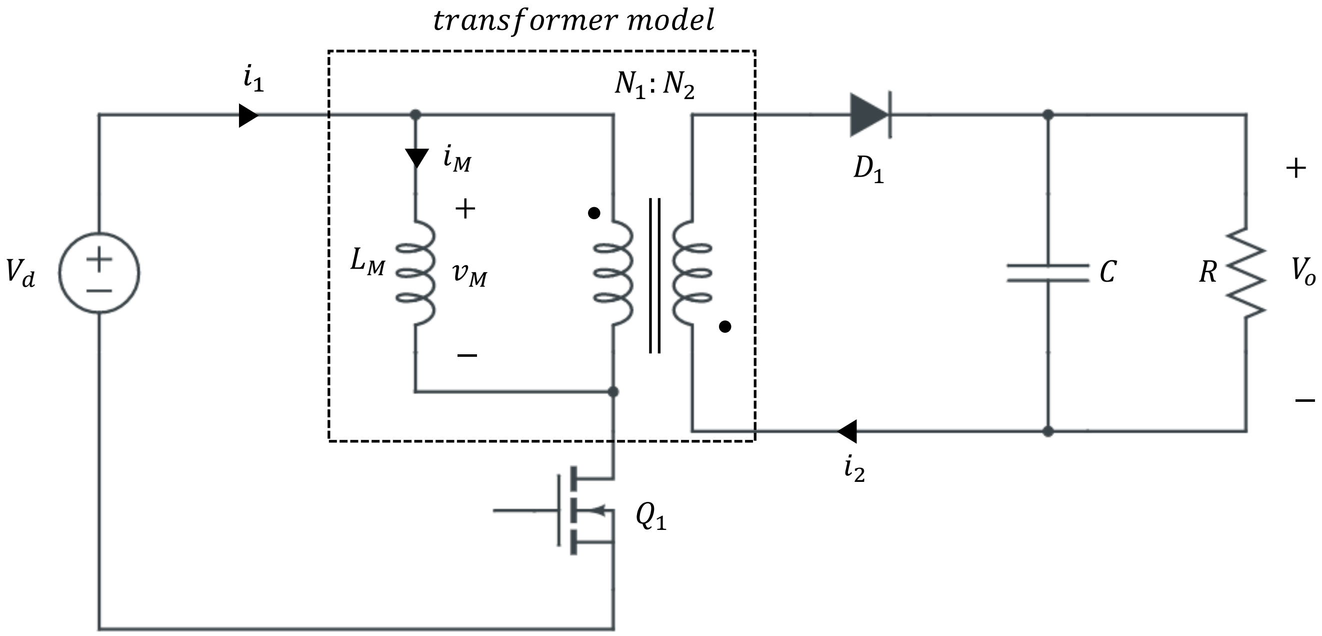
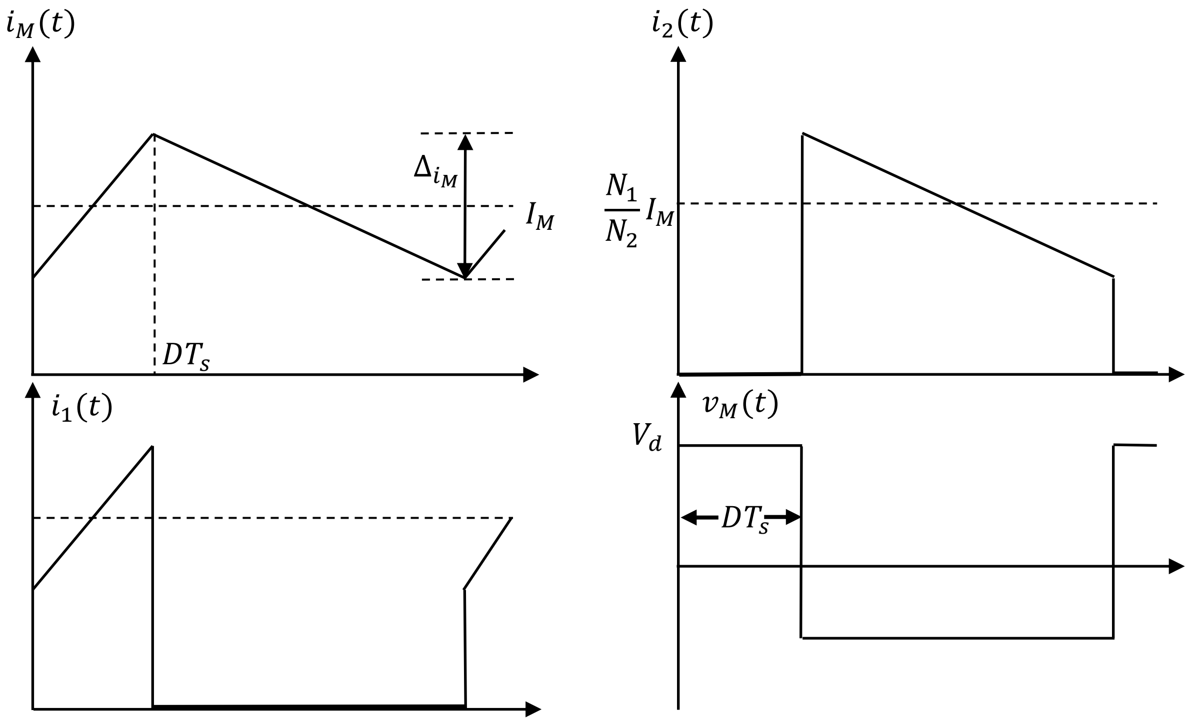


Figure 1. Flyback converter circuit



Flyback converter with $V_d = 200V$, output at full load is 20V and 5A, switching frequency

$f_s = 150kHz$, magnetizing current ripple 20%, duty cycle (D) = 0.4, turns ratio ${}^{N_2}/_{N_1} = 0.15$, copper loss = 1.5W, fill factor $K_u = 0.3$, maximum flux density $B_{max} = 0.25T$

$$I_M = \frac{\left(\frac{N_2}{N_1}\right)\frac{V_o}{R}}{1-D} = \frac{0.15 \times 5}{1-0.4}$$

$$I_M = 1.25A$$

$$\Delta_{i_M} = 20\%\ of\ I_M$$

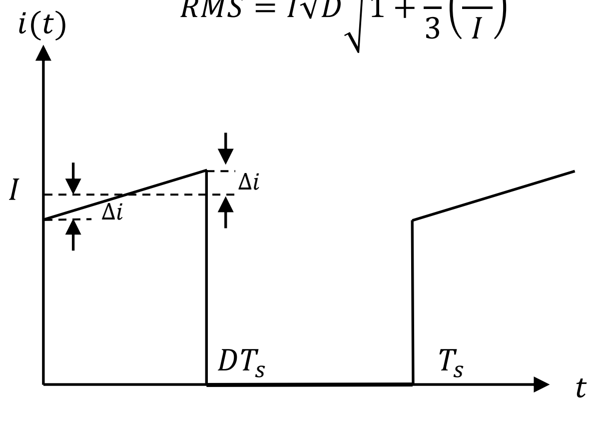
$$\Delta_{i_M} = 0.25A$$

$$I_{M,max} = I_M + \Delta_{i_M} = 1.5A$$

To calculate magnetizing inductance L_m, use $v_M = L_m \frac{di_M}{dt}$

$$v_M = L_m \frac{\Delta i_M}{\Delta t}$$

$$RMS = I\sqrt{D}\sqrt{1 + \frac{1}{3}\left(\frac{\Delta i}{I}\right)^2}$$



It is easy to use ON-period DT_s:

$$L_m = \frac{v_M \Delta t}{2\Delta i_M} = \frac{V_d DT_s}{\Delta i_M f_s} = \frac{200 \times 0.4}{0.50 \times 150k} = 1.07mH$$

1.7.1. RMS winding currents

$$I_1 = I_M\sqrt{D}\sqrt{1 + \frac{1}{3}\left(\frac{\Delta_{i_M}}{I_M}\right)^2} = 1.25 \times \sqrt{0.4} \times \sqrt{1 + \frac{1}{3}\left(\frac{0.25}{1.25}\right)^2} = 0.796A$$

$$I_2 = \frac{N_1}{N_2} I_M\sqrt{D'}\sqrt{1 + \frac{1}{3}\left(\frac{\Delta_{i_M}}{I_M}\right)^2} = \frac{1}{0.15} \times 1.25 \times \sqrt{0.6} \times \sqrt{1 + \frac{1}{3}\left(\frac{0.25}{1.25}\right)^2} = 6.5A$$

$$I_{total} = I_1 + \frac{N_2}{N_1} I_2 = 0.796A + 0.15 \times 6.5A = 1.77A$$

1.7.2. Choose core size

$$K_g \geq \frac{\rho L^2 I^2{}_{max}}{B^2{}_{max} R K_u} \times 10^8 \ (cm^5)$$

$$R = \frac{P_{cu}}{I^2{}_{total}} \rightarrow \frac{1}{R} = \frac{I^2{}_{total}}{P_{cu}} \qquad \textit{put into the equation,}$$

$$K_g \geq \frac{\rho L^2{}_M I^2{}_{total} I^2{}_{M,max}}{B^2{}_{max} P_{cu} K_u}$$

$$= \frac{(1.724 \times 10^{-6}\ \Omega - \text{cm}) \times (1.07 \times 10^{-3}\ H)^2 \times (1.77\ A)^2 \times (1.5\ A)^2}{(0.25\ T)^2 \times (1.5\ W) \times (0.3)} 10^8$$

=0.049 cm^5

EE30 core with $K_g = 0.0857\ cm^5$, $A_c = 1.09\ cm^2$, $W_A = 0.476 cm^2$, $(MLT) = 6.60 cm$,

$\ell_m = 5.77\ cm$ is suitable for values we found.

1.7.3. Determine air gap and turns

$$\ell_g = \frac{\mu_o L_M I^2{}_{M,max}}{B^2{}_{max} A_c} 10^4 \ (m)$$

$$\ell_g = \frac{(4\pi \times 10^{-7}\ H/m)(1.07 \times 10^{-3}\ H)(1.5\ A)^2}{(0.25\ T)^2 (1.09\ cm^2)} 10^4$$

$$\ell_g = 4.4 \times 10^{-4} m = 0.44 mm$$

$$N = \frac{L I_{max}}{B_{max} A_c} 10^4$$

$$N_1 = \frac{L_M I_{M,max}}{B_{max} A_c} 10^4$$

$$N_1 = \frac{(1.07 \times 10^{-3}\ H)(1.5\ A)}{(0.25\ T)(1.09\ cm^2)} 10^4$$

$$N_1 = 58.7 \cong 59\ turns$$

$$N_2 = \left(\frac{N_2}{N_1}\right) N_1$$

$$N_2 = (0.15) \times 59$$

$$N_2 = 8.81 \cong 9\ turns$$

Hence,

$$N_1 = 59\ \ \&\ N_2 = 9$$

1.7.4. Wire gauges

$$\alpha_1 = \frac{I_1}{I_{total}} = \frac{0.796A}{1.77A} = 0.45$$

$$\alpha_2 = \frac{N_2}{N_1}\frac{I_2}{I_{total}} = \frac{9}{59}\frac{6.5A}{1.77A} = 0.55$$

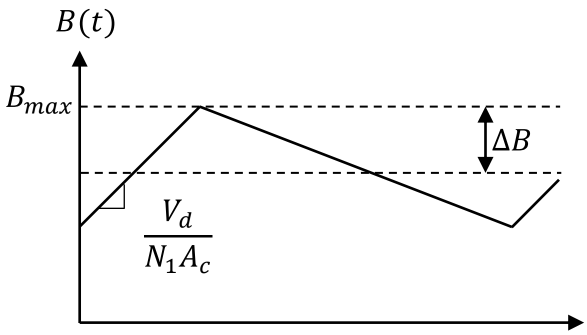
$$A_w \leq \frac{\alpha K_u W_A}{N}$$

$$A_{w_1} \leq \frac{\alpha_1 K_u W_A}{N_1} = \frac{0.45 \times 0.3 \times 0.476}{59} = 1.09 \times 10^{-3}\ cm^2 \qquad use\ \#28\ AWG$$

$$A_{w_2} \leq \frac{\alpha_2 K_u W_A}{N_2} = \frac{0.55 \times 0.3 \times 0.476}{9} = 8.72 \times 10^{-3}\ cm^2 \qquad use\ \#19\ AWG$$

Check wire sizes from click

1.7.5. Core loss



$B(t)$ vs. applied voltage from Faraday's law:

$$\frac{dB(t)}{dt} = \frac{v_M(t)}{NA_c} = \frac{V_d}{N_1 A_c}$$

$$\frac{\Delta B}{\Delta T} = \frac{V_d}{N_1 A_c}$$

$$\frac{2\Delta B}{DT_s} = \frac{V_d}{N_1 A_c} \to \Delta B = \frac{V_d D T_s}{2N_1 A_c}$$

$$\Delta B = \frac{200 \times 0.4}{2 \times 59 \times 1.09 \times 150k} 10^4$$

$$\Delta B = 0.041\ T$$

From manufacturer's plot of core loss of EE30, the power loss density is 0.04W/cm^3. Hence, the core loss is

$$P_{fe} = (0.04\text{W}/cm^3)(A_c \ell_m)$$

$$P_{fe} = (0.04W/cm^3)(1.09\ cm^2)(5.77\ cm)$$

$$P_{fe} = 0.25W$$

1.7.6. Comparison between Core loss and Copper loss

Copper loss is 1.5W which does not include proximity loss that might substantially increase total copper loss.

Core loss is 0.25W

- Core loss is small because ΔB is small
- It is not bad approach to ignore core losses for ferrite in CCM filter inductors
- It can be considered using less expensive material having higher core loss

Final: Design is dominating by copper loss.

1.8. Transformer Design

2.8.1 Transformer design: Basic constraints

2.8.2 A first-pass transformer design procedure

2.8.3 Examples

2.8.4 AC inductor design

2.8.5 Summary

1.8.1. Transformer design: Basic constraints

Core loss is:

$$\boxed{P_{fe} = K_{fe}(\Delta B)^{\beta} A_c \ell_m} \longrightarrow \text{Constraint } 1$$

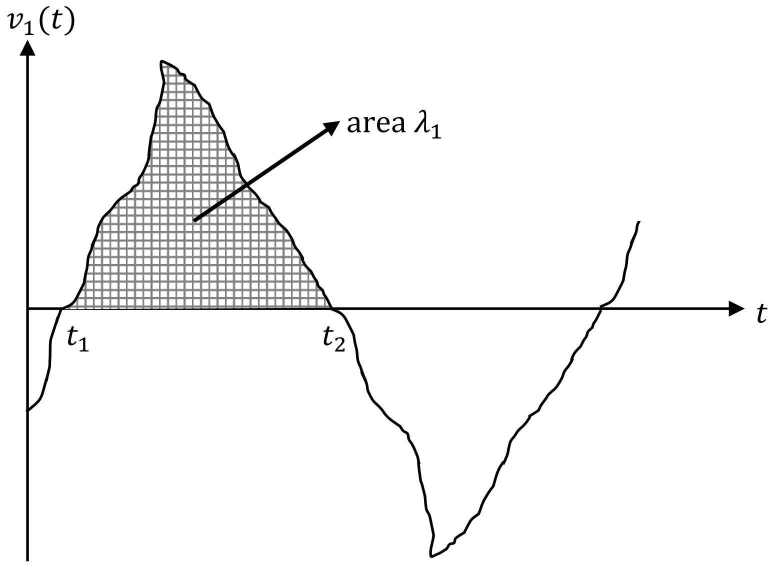
Typical value of β for ferrite materials: 2.6 or 2.7

ΔB is the peak value of AC component of B(t), i.e., the peak AC flux density

Increasing ΔB causes core loss to increase rapidly.

Flux Density:

Flux density B(t) is related to applied winding voltage according to Faraday's Law. Denote the volt-seconds applied to the primary winding during the positive portion of $v_1(t)$ as λ_1:



$$\lambda_1 = \int_{t_1}^{t_2} v_1(t)dt$$

We know from Faraday's law:

$$v(t) = N\frac{d\phi}{dt}$$

$$\Delta\mathrm{B}(\text{peak, not peak to peak}) = \frac{1}{2NA_c}\int v_1(t)dt$$

This causes the flux to change from its negative peak to positive peak. From Faraday's law, the peak value of the AC component of flux density is

$$\Delta B = \frac{\lambda_1}{2N_1A_c}$$

To attain a given flux density, the primary turns should be chosen according to

$$\boxed{N_1 = \frac{\lambda_1}{2\Delta BA_c}}$$

Constraint 2

Copper loss:

Allocate window area between windings in optimum manner, as described in previous pages

Total copper loss formula from page 27.:

$$P_{cu} = \rho \frac{(MLT) N_1{}^2 I_{total}{}^2}{W_A K_u} \qquad \text{with} \qquad I_{total} = \sum_{j=1}^{k} \frac{N_j}{N_1} I_j$$

Eliminate N_1 using formula:

$$N_1 = \frac{\lambda_1}{2\Delta B A_c}$$

$$\boxed{P_{cu} = \left(\frac{\rho \lambda_1{}^2 I_{total}{}^2}{4 K_u}\right) \left(\frac{(MLT)}{W_A A_c{}^2}\right) \left(\frac{1}{\Delta B}\right)^2} \longrightarrow \text{Constraint } 4$$

Note that copper loss decrease rapidly as ΔB is increased.

Total loss calculations

$$P_{total} = P_{fe} + P_{cu}$$

$$P_{fe} = K_{fe} (\Delta B)^{\beta} A_c \ell_m$$

$$P_{cu} = \left(\frac{\rho \lambda_1{}^2 I_{total}{}^2}{4 K_u}\right) \left(\frac{(MLT)}{W_A A_c{}^2}\right) \left(\frac{1}{\Delta B}\right)^2$$

At the ΔB that minimizes P_{total}, we can write

$$\frac{dP_{total}}{d(\Delta B)} = \frac{dP_{fe}}{d(\Delta B)} + \frac{dP_{cu}}{d(\Delta B)} = 0$$

Note: optimum does not necessarily occur where $P_{fe} = P_{cu}$. Rather, it occurs where

$$\frac{dP_{fe}}{d(\Delta B)} = -\frac{dP_{cu}}{d(\Delta B)}$$

Take derivatives of core and copper loss

$$P_{fe} = K_{fe}(\Delta B)^{\beta} A_c \ell_m$$

$$\frac{dP_{fe}}{d(\Delta B)} = \beta K_{fe}(\Delta B)^{(\beta-1)} A_c \ell_m$$

$$P_{cu} = \left(\frac{\rho {\lambda_1}^2 {I_{total}}^2}{4K_u}\right)\left(\frac{(MLT)}{W_A {A_c}^2}\right)\left(\frac{1}{\Delta B}\right)^2$$

$$\frac{dP_{cu}}{d(\Delta B)} = -2\left(\frac{\rho {\lambda_1}^2 {I_{total}}^2}{4K_u}\right)\left(\frac{(MLT)}{W_A {A_c}^2}\right)(\Delta B)^{-3}$$

Now substitute into $\frac{dP_{fe}}{d(\Delta B)} = -\frac{dP_{cu}}{d(\Delta B)}$ and solve for ΔB

$$\Delta B = \left[\frac{\rho {\lambda_1}^2 {I_{total}}^2}{2K_u}\left(\frac{MLT}{W_A {A_c}^3 \ell_m}\right)\frac{1}{\beta K_{fe}}\right]^{\left(\frac{1}{\beta+2}\right)}$$

Optimum ΔB for a given core and application

Now we can plug ΔB into P_{fe} and P_{cu} equations in the total loss equation:

$$P_{total} = P_{fe} + P_{cu}$$

$$P_{total} = \left[A_c \ell_m K_{fe}\right]^{\left(\frac{2}{\beta+2}\right)} \left[\frac{\rho {\lambda_1}^2 {I_{total}}^2}{4K_u}\frac{(MLT)}{W_A {A_c}^2}\right]^{\left(\frac{\beta}{\beta+2}\right)} \left[\left(\frac{\beta}{2}\right)^{-\left(\frac{\beta}{\beta+2}\right)} + \left(\frac{\beta}{2}\right)^{\left(\frac{2}{\beta+2}\right)}\right]$$

Rearrange as follows:

$$\frac{W_A (A_c)^{\left(2(\beta-1)/\beta\right)}}{(MLT)\ell_m^{\left(2/\beta\right)}}\left[\left(\frac{\beta}{2}\right)^{-\left(\frac{\beta}{\beta+2}\right)} + \left(\frac{\beta}{2}\right)^{\left(\frac{2}{\beta+2}\right)}\right]^{-\left(\frac{\beta+2}{\beta}\right)} = \frac{\rho {\lambda_1}^2 {I_{total}}^2 {K_{fe}}^{\left(2/\beta\right)}}{4K_u (P_{total})^{\left((\beta+2)/\beta\right)}}$$

Left side: terms depend on core geometry

Right side: terms depend on specifications of the application

The core geometrical constant K_{gfe}

$$K_{gfe} = \frac{W_A (A_c)^{\left(2(\beta-1)/\beta\right)}}{(MLT)\ell_m^{\left(2/\beta\right)}} \left[\left(\frac{\beta}{2}\right)^{-\left(\frac{\beta}{\beta+2}\right)} + \left(\frac{\beta}{2}\right)^{\left(\frac{2}{\beta+2}\right)}\right]^{-\left(\frac{\beta+2}{\beta}\right)}$$

Design procedure: select a core that satisfies

$$\boxed{K_{gfe} \geq \frac{\rho \lambda_1{}^2 I_{total}{}^2 K_{fe}{}^{\left(2/\beta\right)}}{4K_u (P_{total})^{\left((\beta+2)/\beta\right)}}}$$

K_{gfe} is similar to the K_g geometrical constant used in inductor design

- K_g is used when B_{max} is specified
- K_{gfe} is used when ΔB is to be chosen to minimize total loss

1.9. First-pass transformer design procedure

The following units are specified		
Wire effective resistivity	ρ	(Ω−cm)
Total RMS winding current, referring to primary	I_{total}	(A)
Desired turn ratios	N_2/N_1	
Applied primary volt-second	λ_1	(V-sec)
Allowed total power dissipation	P_{total}	(W)
Winding fill factor	K_u	
Core loss exponent	β	
Core loss coefficient	K_{fe}	$(W/cm^3 T^{\beta}$

Other quantities and their dimensions		
Core cross-sectional area	A_c	(cm^2)
Core window area	W_A	(cm^2)
Mean length per turn	*MLT*	(cm)
Magnetic path length	ℓ_m	(cm)
Wire area	A_w	(cm^2)
Peak AC flux density	*ΔB*	(T)

1.9.1. Determining core size

$$K_{gfe} \geq \frac{\rho {\lambda_1}^2 {I_{total}}^2 {K_{fe}}^{\left(2/\beta\right)}}{4K_u (P_{total})^{\left((\beta+2)/\beta\right)}}$$

It may be possible to reduce the core size by choosing a core material that has lower loss, i.e., lower K_{fe}.

1.9.2. Evaluating peak AC flux density

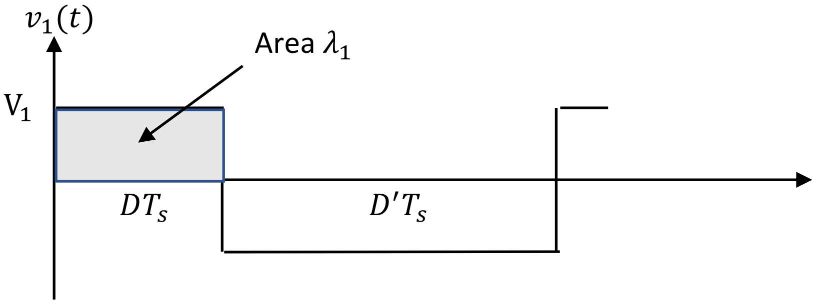
$$\Delta B = \left[10^8 \frac{\rho {\lambda_1}^2 {I_{total}}^2}{2K_u} \left(\frac{MLT}{W_A {A_c}^3 \ell_m}\right) \frac{1}{\beta K_{fe}}\right]^{\left(\frac{1}{\beta+2}\right)}$$

At this point, one should check whether the saturation flux density is exceeded. If the core operates with flux dc bias $\mathrm{B_{dc}}$, then $\Delta\mathrm{B}+\mathrm{B_{dc}}$ should be less than the saturation flux density $\mathrm{B_{sat}}$.

If the core will saturate, there are two choices:

- Specify ΔB using $\mathrm{K_g}$ method or
- Choose a core material having greater core loss, repeat steps 1 and 2.

To find λ_1, use volt-second balance equation of the primary side voltage. For example,



We can write flux linkage equation as following:

$\lambda_1 = DT_s V_1$ (V-sec)

1.9.3. Evaluating number of turns

$$N_1 = \frac{\lambda_1}{2\Delta B A_c} 10^4$$

Choose secondary turn according to desired turn ratios:

$$N_2 = N_1 \left(\frac{N_2}{N_1}\right)$$

$$N_3 = N_1 \left(\frac{N_3}{N_1}\right)$$

$$\vdots$$

1.9.4. Choosing wire sizes

Fraction of window area assigned to each winding:

$$\alpha_1 = \frac{N_1 I_1}{N_1 I_{total}}$$

$$\alpha_2 = \frac{N_2 I_2}{N_1 I_{total}}$$

$$\vdots$$

$$\alpha_{\mathrm{k}} = \frac{N_{\mathrm{k}} I_{\mathrm{k}}}{N_1 I_{total}}$$

Choose wire sizes according to:

$$A_{w_1} \leq \frac{\alpha_1 K_u W_A}{N_1}$$

$$A_{w_2} \leq \frac{\alpha_1 K_u W_A}{N_1}$$

$$\vdots$$

1.9.5. Check point: computed transformer model

Predicted magnetizing inductance referred to primary:

$$L_M = \frac{\mu {N_1}^2 A_c}{\ell_m}$$

Peak magnetizing current:

$$i_{M,peak} = \frac{\lambda_1}{2L_M} = \Delta i_M$$

Predicted winding resistances:

$$R_1 = \frac{\rho N_1 (MLT)}{A_{w_1}}$$

$$R_2 = \frac{\rho N_2 (MLT)}{A_{w_2}}$$

$$\vdots$$

Chapter 3 **References**

[1] Erickson, R. W. (1997). *Fundamentals of Power Electronics* (1st ed.). Springer. https://doi.org/10.1007/978-3-030-43881-4