BUCK CONVERTER TRANSFER FUNCTION DERIVATION

AVERAGE MODEL & SMALL SIGNAL CIRCUITS

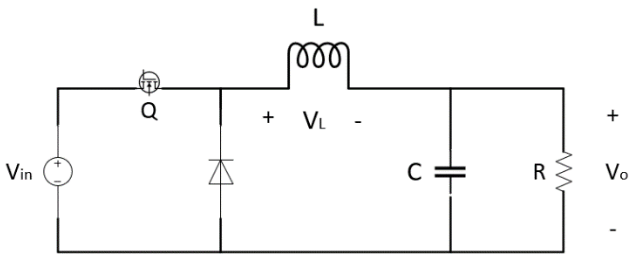
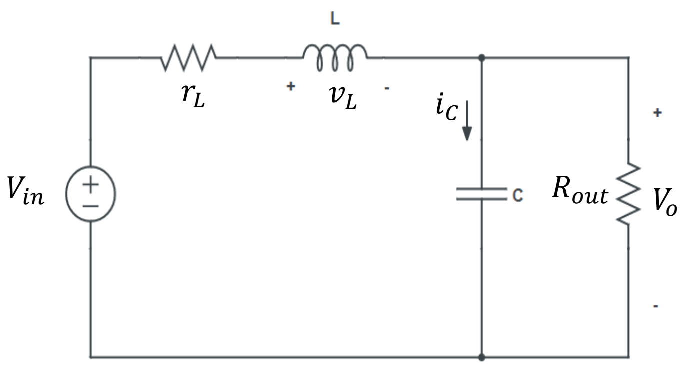
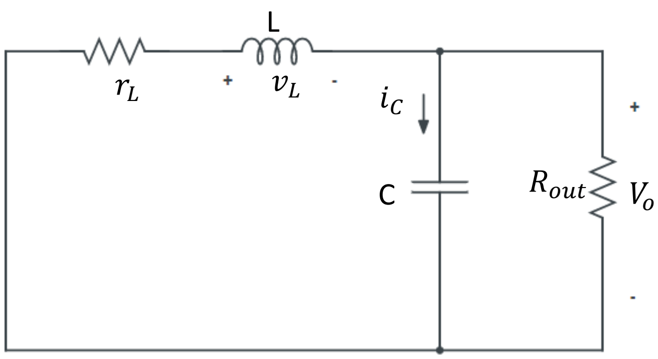


Figure 1. Buck Converter

__Interval I:__ D



__Interval II__: $1 - D$



1. Averaged, Small Signal Circuit Description

D Interval:	$v_L(t) = V_{in} - i_L(t)r_L - V_o$	‖	$i_C(t) = i_L(t) - \frac{V_o}{R_{out}}$
$1 - \text{D}$ *Inverval*:	$v_L(t) = -i_L(t)r_L - V_o$	‖	$i_C(t) = i_L(t) - \frac{V_o}{R_{out}}$

2. Capacitor Charge & Volt. Second Balance Equations

$$L\frac{di_L}{dt} = D(V_{in} - i_L r_L - V_o) + (1-D)(-i_L r_L - V_o)$$

$$C\frac{dV_C}{dt} = D(i_L - \frac{V_o}{R_{out}}) + (1-D)(i_L - \frac{V_o}{R_{out}})$$

3. Small Signal "hat" Terms (x(t) = X + $\hat{x}$)

<u>Inductor:</u>

$$\boldsymbol{L}\frac{d(I_L + \hat{\imath}_L)}{dt} = \left((D+\hat{d})(V_{in} + \hat{v}_{in}) - (I_L + \hat{\imath}_L)r_L - (V_o + \hat{v}_o)\right) + \left((1-(D+\hat{d}))(-(I_L + \hat{\imath}_L)r_L - (V_o + \hat{v}_o)\right)$$

Single hat terms:

$$\boldsymbol{L}\frac{d(\hat{\imath}_L)}{dt} = \hat{d}V_{in} - \hat{d}I_L r_L - \hat{d}V_o + D\hat{v}_{in} - D\hat{\imath}_L r_L - D\hat{v}_o - \hat{\imath}_L r_L - \hat{v}_o + \hat{d}I_L r_L + \hat{d}V_o + D\hat{\imath}_L r_L + D\hat{v}_o$$

$$\boxed{\boldsymbol{L}\frac{d(\hat{\imath}_L)}{dt} = \hat{d}V_{in} + D\hat{v}_{in} - \hat{\imath}_L r_L - \hat{v}_o}$$

Averaged Small Signal

DC Terms:

$$\boldsymbol{L}\frac{d(I_L)}{dt} = DV_{in} - DI_L r_L - DV_o - I_L r_L - V_o + DI_L r_L + DV_o$$

$$\boxed{0 = DV_{in} - I_L r_L - V_o}$$

Averaged steady-state DC eqn.

Capacitor:

$$C\frac{d(V_C+\hat{v}_C)}{dt} = (D+\hat{d})\left((I_L+\hat{\imath}_L)-\frac{(V_o+\hat{v}_o)}{R_{out}}\right)+\left(1-(D+\hat{d})\right)\left((I_L+\hat{\imath}_L)-\frac{(V_o+\hat{v}_o)}{R_{out}}\right)$$

Single hat terms:

$$C\frac{d(\hat{v}_C)}{dt} = \hat{d}I_L-\hat{d}\frac{V_o}{R_{out}}+D\hat{\imath}_L-D\frac{\hat{v}_o}{R_{out}}+\hat{\imath}_L-\frac{\hat{v}_o}{R_{out}}-\hat{d}I_L+\hat{d}\frac{V_o}{R_{out}}-D\hat{\imath}_L+D\frac{\hat{v}_o}{R_{out}}$$

$$\boxed{C\frac{d(\hat{v}_C)}{dt} = \hat{\imath}_L-\frac{\hat{v}_o}{R_{out}}}$$

Averaged Small Signal

DC Terms:

$$C\frac{d(V_C)}{dt} = DI_L-D\frac{V_o}{R_{out}}+I_L-\frac{V_o}{R_{out}}-DI_L+D\frac{V_o}{R_{out}}$$

$$\boxed{C\frac{d(V_C)}{dt} = I_L-\frac{V_o}{R_{out}}}$$

Averaged Steady-State DC eqn

Input Current:

$$i_{in}(t) = (D+\hat{d}) < i_L(t) >$$

$$I_{in}+\hat{\imath}_{in} = DI_L+\hat{d}I_L+D\hat{\imath}_L+\hat{d}\hat{\imath}_L$$

Very small compared to DC values, ignored.

$$\boxed{\hat{\imath}_{in} = \hat{d}I_L+D\hat{\imath}_L}$$ → Averaged small signal

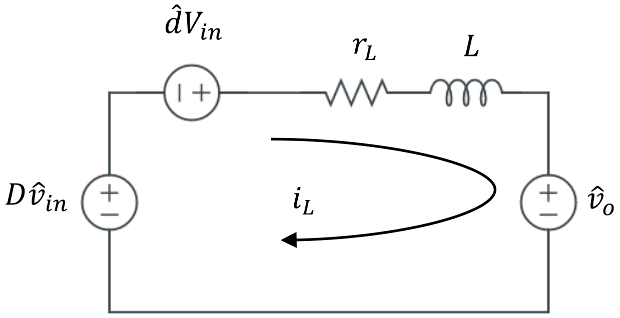
$$\boxed{I_{in} = DI_L}$$ → Steady-state DC signal

Small signal and steady-state DC signal analysis has been implemented. Now, they should be merged in one circuit to get the transfer function of the system.

Inductor:

Small Signal Eqn:

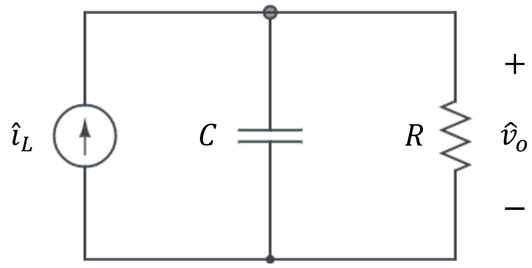
$$L\frac{d(\hat{\imath}_L)}{dt} = \hat{d}V_{in} + D\hat{v}_{in} - \hat{\imath}_L r_L - \hat{v}_o$$



Capacitor:

Small Signal Eqn:

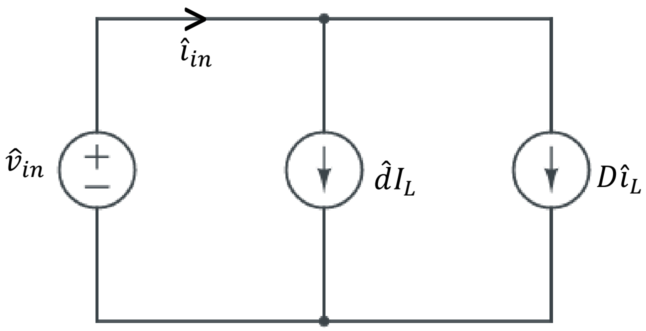
$$C\frac{d(\hat{v}_C)}{dt} = \hat{\imath}_L - \frac{\hat{v}_o}{R_{out}}$$

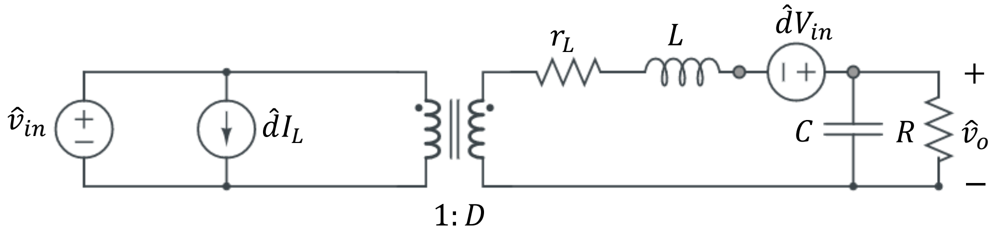


Input Current:

Small Signal Eqn:

$$\hat{\imath}_{in} = \hat{d}I_L + D\hat{\imath}_L$$





4. Control-to-Output Transfer Function

$$\hat{\boldsymbol{v}}_{\boldsymbol{o}} = V_{in}\hat{d} \times \frac{\left(\frac{1}{sC}//R_{out}\right)}{r_L + sL + \left(\frac{1}{sC}//R_{out}\right)}$$

$$\frac{1}{sC}//R_{out} = \frac{\frac{R_{out}}{sC}}{R_{out} + \frac{1}{sC}} = \frac{R_{out}}{1 + sCR_{out}}$$

$$G_p = \frac{\hat{v}_o}{\hat{d}} = V_{in}\frac{\frac{R_{out}}{1 + sCR_{out}}}{r_L + sL + \frac{R_{out}}{1 + sCR_{out}}} \rightarrow \frac{R_{out}}{r_L + sLCR_{out} + sL + s^2R_{out}LC + R_{out}} \times \frac{\frac{1}{R_{out}}}{\frac{1}{R_{out}}}$$

$$\boxed{\frac{\hat{v}_o}{\hat{d}} = V_{in}\frac{1}{s^2LC + s(\frac{L}{R_{out}} + r_LC) + 1 + \frac{r_L}{R_{out}}}}$$

If there is no resistance of the inductor, use this formula:

$$\boxed{\frac{\hat{v}_o}{\hat{d}} = V_{in}\frac{1}{s^2LC + s\frac{L}{R_{out}} + 1}}$$