CS 300 HW 2

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Q1)

Base step: $f(1) = 1$

recurrence: $f(n) = 3f(\frac{n}{3}) + 3$

$= 9f(\frac{n}{9}) + 12$

$= 27f(\frac{n}{27}) + 39$

$= 81f(\frac{n}{81}) + 120$

$= 3^k f(\frac{n}{3^k}) + \frac{3(3^k - 1)}{2}$

$\Downarrow n = 3^k$

$= n \cdot f(1) + \frac{3(n-1)}{2}$

$= n + \frac{3n-3}{2} = \underbrace{\frac{5n}{2} - \frac{3}{2}}$

$\forall k \in Z^+, n = 3^k$

$\exists$ 3 recursive calls ($f(\frac{n}{3})$)

and

$\exists$ 1 helper function call ($+3$)

$f(n) = \frac{5n}{2} - \frac{3}{2}$

$= O(n)$

Q2)

Base step: $f(0) = 1$

Recurrence: $f(n) = 2f(n-2) + \log(n)$

$= 4f(n-4) + \log(n) + 2\log(n-2)$

$= 8f(n-6) + \log(n) + 2\log(n-2) + 4\log(n-4)$

$= 16f(n-8) + \log(n) + 2\log(n-2) + 4\log(n-4) + 8\log(n-6)$

$= 2^m \cdot f(\underbrace{n-2m}) + \sum_{i=1}^{m} 2^{(i-1)} \log(n+2-2i)$

$\forall k \in Z^+ + \{0\}, n = 2^k$

$n - 2m = 0$

$n = 2m$

$n/2 = m$

$= 2^{(n/2)} \cdot \underbrace{f(0)}_{1} + \sum_{i=1}^{n/2} 2^{(i-1)} \log(n+2-2i)$

$\underbrace{2^{(n/2)} = (2^{1/2})^n = (\sqrt{2})^n}$

this doesn't matter since it is in big o, and 2 is constant

$n + 2 - 2i < n$

$2 - 2i < 0$

$2i - 2 > 0$

$i > 1$

$\underbrace{\log(n+2-2i) < \log(n)}$

Therefore if we can find big-o of right side, we can conclude that it is also big-o of left side

$\sum_{i=1}^{n/2} 2^{(i-1)} = 1 + 2 + 4 + 8 \dots 2^{(\frac{n}{2} - 1)}$

$= 2^{(\frac{n}{2})} - 1$

$= O\left((\sqrt{2})^n + \sum_{i=1}^{n/2} 2^{(i-1)} \log(n+2-2i) \right)$

$= O\left((\sqrt{2})^n + \sum_{i=1}^{n/2} 2^{(i-1)} \log(n) \right)$

$= O\left((\sqrt{2})^n + ((\sqrt{2})^n - 1) \log(n) \right)$

$= O\left((\sqrt{2})^n + (\sqrt{2})^n \log(n) \right)$

$= O\left((1 + \log(n)) (\sqrt{2})^n \right)$

$= O\left(\log(n) \cdot 2^n \right)$