Assignment 5 is due Friday, January 5, 14:40.

Please hand in your solutions (handwritten, at most two pages) to the homework problems, at the beginning of the class on January 5.

1. Suppose that there are 4 students, $\{A, B, C, D\}$, and 2 graduation projects, $\{q, r\}$. Each student x specifies a set $P(x)$ of projects to work on. For each project y, its supervisor specifies a set $S(y)$ of students to work with:

$$\begin{array}{ll} A : \{q, r\} & \quad q : \{A, C\} \\ B : \{r\} & \quad r : \{B, C, D\} \\ C : \{q, r\} & \\ D : \{r\} & \end{array}$$

   Explain (step by step, showing each iteration) how the Edmonds-Karp algorithm can be used to assign projects to the maximum number of students, such that the following conditions hold:

   - no student is matched to two different projects,
   - no project is assigned to two different students,
   - no student x is matched to a project that is not in $P(x)$, and
   - no project y is assigned to a student that is not in $S(y)$.

2. What is the worst-case asymptotic time complexity of using the Edmonds-Karp algorithm to assign m students to m projects, subject to the conditions in (a) above? Please explain.

3. Prove that the following problem is NP-hard: Given a graph G and a positive integer k, is there a spanning tree of G that contains at most k leaves?