

Cyber Physical Systems - Discrete Models

Exercise Sheet 3 Solution

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Exercise 1: Intersection of ω -regular languages

- (a) L_1 : It is certain that a is not infinite. However, b or c can be infinite.
 L_2 : It is certain that b is infinite. However, a and c can also be infinite.
 $L_1 \cap L_2$: It is certain that a is not infinite and b is infinite. However, c can be infinite.

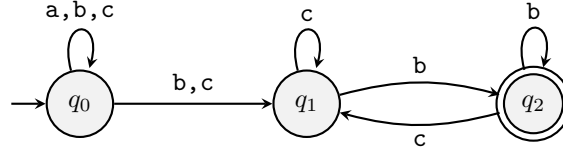


Figure 1: Büchi automaton A that accepts intersection of L_1 and L_2

- (b) L_1 : It is certain that a is infinite. However, b can also be infinite.
 L_2 : It is certain that b is infinite. However, a can also be infinite.
 $L_1 \cap L_2$: It is certain that a and b infinite.

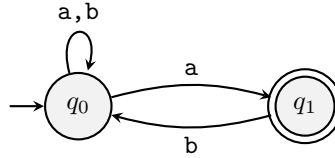


Figure 2: Büchi automaton A that accepts intersection of L_1 and L_2

- (c) L_1 : It is certain that a is not infinite. However, b can be infinite.
 L_2 : It is certain that if there is b at any position, regardless of finite or infinite words, there must be a right next to it. However, a can be infinite. Yet, b can also be infinite as long as it follows a , which means both are infinite only if they are together.
 $L_1 \cap L_2$: \emptyset (It is certain that a is not infinite. And b cannot be infinite since a is not infinite)

Exercise 2: Transition Systems

- (a) The model represents a case or lock which can be opened by inserting a ticket. Red light appears if locked. Otherwise, green light appears.

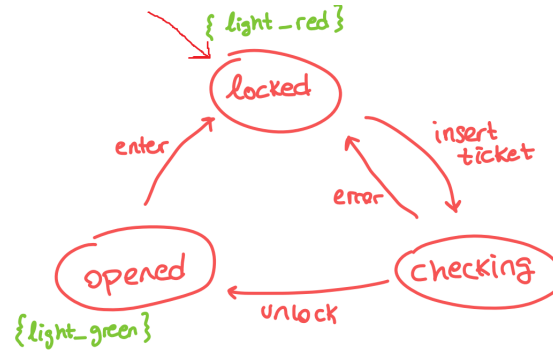


Figure 3: Transition System

- (b) Mathematical definition of given transition system is as follows:

$$\begin{aligned}
 S &= \{1, 2, 3, 4\} \\
 Act &= \{\text{close_door}, \text{open_door}, \text{go_up}, \text{go_down}\} \\
 S_0 &= \{4, 1\} \\
 L(4) &= \{\text{open}, \text{top_floor}\} \\
 L(3) &= \{\text{top_floor}\} \\
 L(2) &= \{\text{ground_floor}\} \\
 L(1) &= \{\text{open}, \text{ground_floor}\}
 \end{aligned}$$

Transitions are as follows:

$$\begin{aligned}
 \longrightarrow = \{ & (4, \text{close_door}, 3), (3, \text{open_door}, 4), (3, \text{go_down}, 2), \\
 & (2, \text{go_up}, 3), (2, \text{open_door}, 1), (1, \text{close_door}, 2) \}
 \end{aligned}$$

And the door is open at states 2 and 3.

Exercise 3: Crossroads Traffic Lights

- (a) The traffic lights do not synchronize with each other.

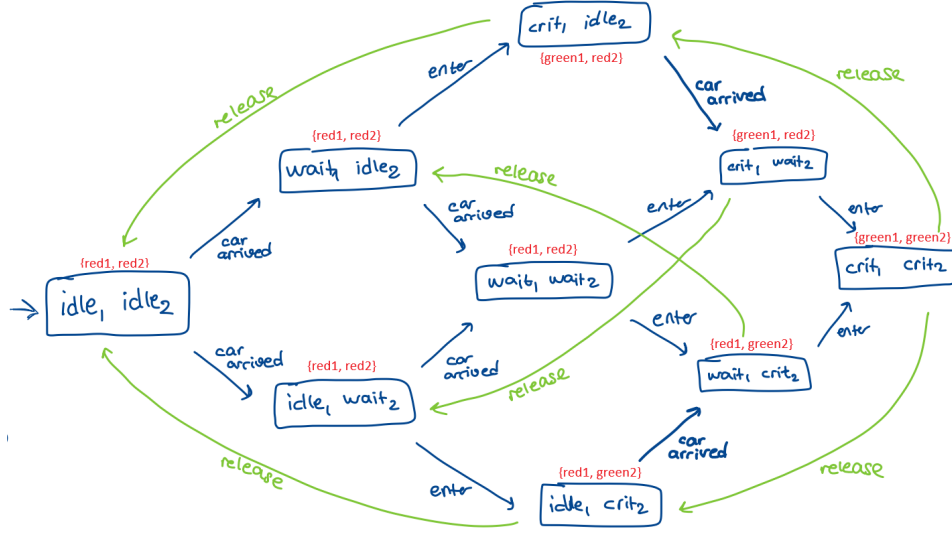


Figure 4: Interleaving transition systems $TS_1 || TS_2$

- (b) The traffic lights do not synchronize with each other, but they both synchronize with the arbiter.

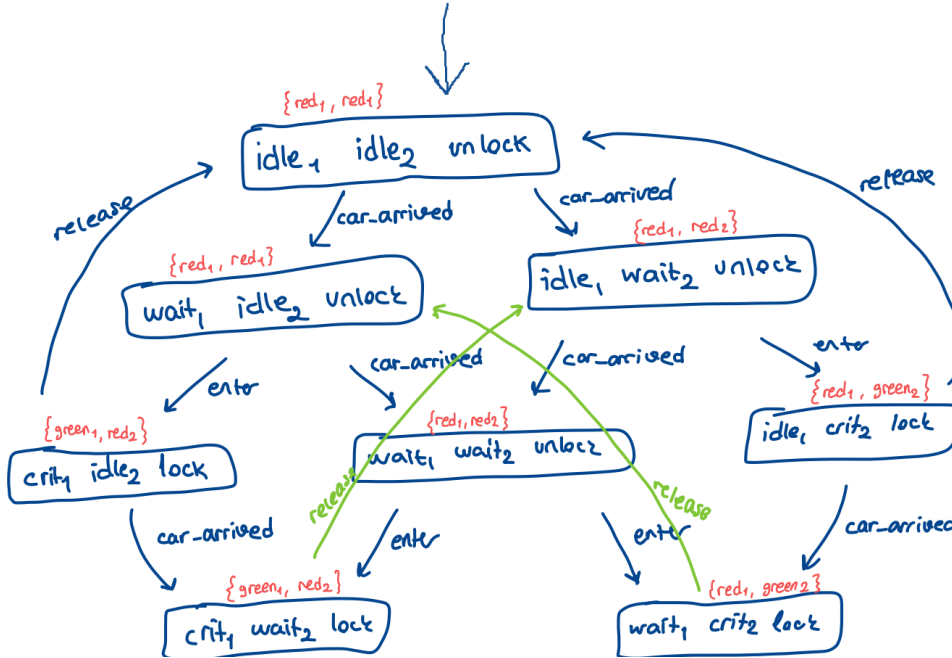


Figure 5: Parallel composition of systems $(TS_1 || TS_2) || \text{Arbiter}$

- (c) Yes, the system is safe when synchronized with an Arbiter. There is no such a state where the atomic proposition is $\{\text{green}_1, \text{green}_2\}$. However, in this design, the lights immediately switch from green to red. We would expect another state(s), e.g. a yellow light state, which would result in a slower and safer switch from green to red.