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Hand in until January 31st, 2024  
15:59 via ILIAS  
Discussion: February 5th/6th, 2024

## Tutorial for Cyber-Physical Systems - Discrete Models

### Exercise Sheet 13

⚠ The sum of achievable points in all 13 exercise sheets is 224 (+ 42 bonus points). To pass the tutorial (the "Studienleistung" of the module), it is sufficient to have presented 1 task in the tutorial sessions and to achieve 112 points.

On Thursday, February 1st, we will upload an additional bonus sheet that will allow you to make up some points and practice for the exam. Although there will be no tutorial session for the bonus sheet, your tutors will provide feedback as usual. ⚠

#### Exercise 1: LTL and Set Notation

5 Points

*The goal of this exercise is to practice converting between set notation and LTL formulas.*

Let  $AP = \{a, b\}$ . For each of the following LTL formulas  $\varphi_i$ , describe  $Words(\varphi_i)$  using set notation (without the use of LTL formulas).

(a)  $\varphi_1 = \Box(a \rightarrow \Diamond b)$

(b)  $\varphi_2 = a \cup \bigcirc b$

For each of the following properties  $P_i$ , give an equivalent LTL formula  $\varphi_i$  (if possible). For exactly one of the properties below, it is not possible to give an equivalent LTL formula. Identify this property.

(c)  $P_3 = \{ A_0 A_1 \dots \in (2^{AP})^\omega \mid \exists i \in \mathbb{N}. a \in A_i \wedge b \in A_{i+1} \}$

(d)  $P_4 = \{ A_0 A_1 \dots \in (2^{AP})^\omega \mid \forall i \in \mathbb{N}. a \in A_{2i} \}$

(e)  $P_5 = \{ A_0 A_1 \dots \in (2^{AP})^\omega \mid \forall i \in \mathbb{N}. A_i = A_{i+2} \}$

#### Exercise 2: From LTL to NBA

6 Points

*The goal of this exercise is to improve your intuition regarding the connection of LTL and NBA.*

For each of the following LTL formulas provide an NBA that accepts exactly the traces that satisfy the formula.

(a)  $\Box(a \vee \neg \bigcirc b)$

(b)  $\Diamond a \vee \Box \Diamond(a \leftrightarrow b)$

(c)  $\bigcirc \bigcirc(a \vee \Diamond \Box b)$

**Exercise 3\*: LTL Equivalence**

3 Bonus Points

*The goal of this exercise is to practice simple proofs involving the semantics of LTL, and to gain a better understanding of LTL equivalence.*

Let  $\varphi, \psi$  be two LTL formulas. Prove the following statement from the lecture:

$$\begin{aligned} & \text{Words}(\varphi) = \text{Words}(\psi) \\ & \text{iff} \\ & \text{for all transition systems } \mathcal{T} : \mathcal{T} \models \varphi \iff \mathcal{T} \models \psi \end{aligned}$$

**Hint:** A transition system  $\mathcal{T}$  may have infinitely many states.