Cyber Physical Systems - Discrete Models
Exercise Sheet 6 Solution

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Exercise 1: Linear Time Properties

Part A

- $T_1 : \{A_0 A_1 A_2 ... \mid \forall i \in N_{>0} \ . \ a \notin A_i\}$
- $T_2 : \{A_0 A_1 A_2 ... \mid \forall i \in N \ . \ a \in A_i \to b \in A_{i+1}\}$
- $T_3 : \{A_0 A_1 A_2 ... \mid \forall i \in N \ . \ a \in A_i \to b \notin A_i\}$
- $T_4 : \left\{A_0 A_1 A_2 ... \mid \overset{\infty}{\exists} i \in N \ . \ a \in A_i\right\}$
- $T_5 : \left\{A_0 A_1 A_2 ... \mid \overset{\infty}{\exists} i \in N \ . \ a \notin A_i\right\}$

Part B

- $T_1 : \{A_0 A_1 A_2 ... \mid \forall i \in N \ . \ a \in A_i\}$
- $T_2 : \{A_0 A_1 A_2 ... \mid \forall i \in N \ . \ a \in A_i \to a \in A_{i+1}\}$
- $T_3 : \{A_0 A_1 A_2 ... \mid \forall i \in N \ . \ a \in A_i \wedge b \in A_i\}$
- $T_4 : \left\{A_0 A_1 A_2 ... \mid \overset{\infty}{\exists} i \in N \ . \ b \in A_i\right\}$
- $T_5 : \{A_0 A_1 A_2 ... \mid \forall i \in N \ . \ a \notin A_i\}$

Exercise 2: Starvation Freedom

Part A

We can prove that $\text{LIVE}' \subseteq \text{LIVE}$ if we can show that all worlds in LIVE' is also in LIVE.

Let $w \in \text{LIVE}'$, we have the following cases:

Case 1: w doesn't have infinitely many wait_1s

In this case $w \in \text{LIVE}$ since w doesn't satisfy the predicate $\overset{\infty}{\exists} \in N \ . \ \text{wait}_1 \in A_i$, therefore doesn't need to satisfy $\overset{\infty}{\exists} \in N \ . \ \text{crit}_1 \in A_i$.

Case 2: w has infinitely many wait_1s

In this case, it follows that w also has infinitely many crit_1s as well, because for all $\text{wait}_1 \in A_i$ there must be a $\text{crit}_1 \in A_j$ such that j comes after i. There can't be a "last" j that comes after all wait_1s, since there are infinitely many wait_1s. Which would mean that crit_1s can be finitely many in this case. Since this is not possible, we can conclude that $w \in \text{LIVE}$.

Same reasoning can be trivially applied to wait_2 and crit_2 as well.

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Part B

Consider a language LIVE″ s.t.:

$$\text{LIVE}'' := \begin{cases} \text{set of all infinite traces } A_0 A_1 A_2 ... s.t. \\ \forall i \in N \ . \ \left(\text{wait}_1 \in A_i \rightarrow \exists j \in N \ . \ j < i \wedge \text{crit}_1 \in A_j\right) \\ \forall i \in N \ . \ \left(\text{wait}_2 \in A_i \rightarrow \exists j \in N \ . \ j < i \wedge \text{crit}_2 \in A_j\right) \end{cases}$$

We can follow a similar proof to the Part A to conclude $\text{LIVE}'' \subseteq \text{LIVE}$. But $\text{LIVE}'' \nsubseteq \text{LIVE}'$ since ordering is reversed. Which means there is not necessarily a crit_1 after each wait_1 (And same for crit_2 and wait_2). Therefore, LIVE' is a *strictly stronger* property than LIVE.

Part C

No, because that system only enters crit_i after a wait_i is received. Therefore, ordering is always as described in LIVE'.

Part D

No, because of Part A, this is not a possible trace for any transition system.