Cyber Physical Systems - Discrete Models
Exercise Sheet 6 Solution

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Exercise 1: Linear Time Properties

Part A

- $T_1 : \{A_0 A_1 A_2 ... \mid \forall i \in N_{>0} \ . \ a \notin A_i\}$

- $T_2 : \{A_0 A_1 A_2 ... \mid \forall i \in N \ . \ a \in A_i \to b \in A_{i+1}\}$

- $T_3 : \{A_0 A_1 A_2 ... \mid \forall i \in N \ . \ a \in A_i \to b \notin A_i\}$

- $T_4 : \left\{A_0 A_1 A_2 ... \mid \overset{\infty}{\exists} i \in N \ . \ a \in A_i\right\}$

- $T_5 : \left\{A_0 A_1 A_2 ... \mid \overset{\infty}{\exists} i \in N \ . \ a \notin A_i\right\}$

Part B

- $T_1 : \{A_0 A_1 A_2 ... \mid \forall i \in N \ . \ a \in A_i\}$

- $T_2 : \{A_0 A_1 A_2 ... \mid \forall i \in N \ . \ a \in A_i \to a \in A_{i+1}\}$

- $T_3 : \{A_0 A_1 A_2 ... \mid \forall i \in N \ . \ a \in A_i \wedge b \in A_i\}$

- $T_4 : \left\{A_0 A_1 A_2 ... \mid \overset{\infty}{\exists} i \in N \ . \ b \in A_i\right\}$

- $T_5 : \{A_0 A_1 A_2 ... \mid \forall i \in N \ . \ a \notin A_i\}$

Exercise 2: Starvation Freedom

Part A

We can prove that $\text{LIVE}' \subseteq \text{LIVE}$ if we can show that all words in LIVE' is also in LIVE.

Let $w \in \text{LIVE}'$, we have the following cases:

Case 1: w doesn't have infinitely many wait_1s

In this case $w \in \text{LIVE}$ since w doesn't satisfy the predicate $\overset{\infty}{\exists} \in N \ . \ \text{wait}_1 \in A_i$, therefore doesn't need to satisfy $\overset{\infty}{\exists} \in N \ . \ \text{crit}_1 \in A_i$.

Case 2: w has infinitely many wait_1s

In this case, it follows that w also has infinitely many crit_1s as well, because for all $\text{wait}_1 \in A_i$ there must be a $\text{crit}_1 \in A_j$ such that j comes after i. There can't be a "last" j that comes after all wait_1s, since there are infinitely many wait_1s. Which would mean that crit_1s can be finitely many in this case. Since this is not possible, we can conclude that $w \in \text{LIVE}$.

Same reasoning can be trivially applied to wait_2 and crit_2 as well.

∎

Part B

Let $\pi = \{\text{wait}_1\}\emptyset^\omega$. $p \in \text{LIVE}$ because there is no infinite wait_1 which would require that there must be infinitely many crit_1s. However, $p \notin \text{LIVE}'$ because there is no crit_1 after the initial wait_1.

Part C

No, because that system only enters crit_i after a wait_i is received. Therefore, ordering is always as described in LIVE'.
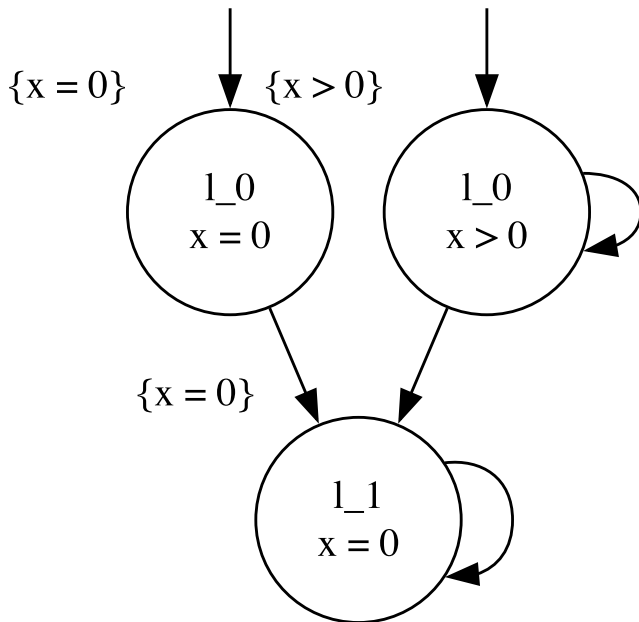
Part D

No, because we proved that $\text{LIVE}' \subset \text{LIVE}$ in Part A, this is not a possible trace for any transition system.
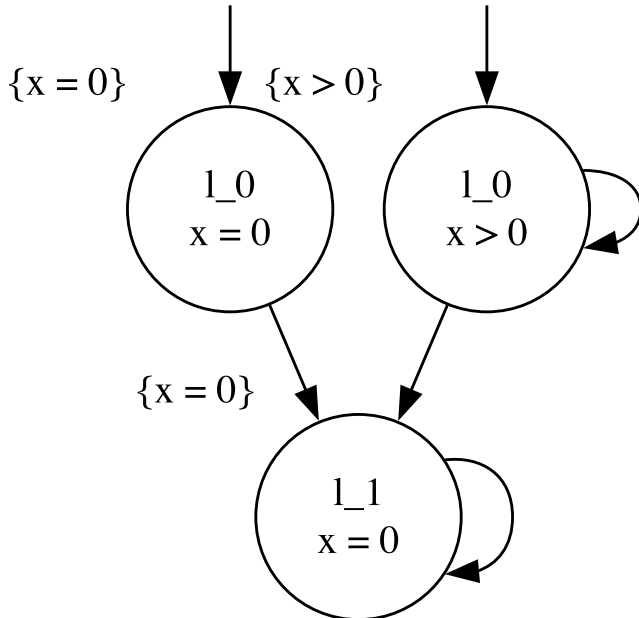
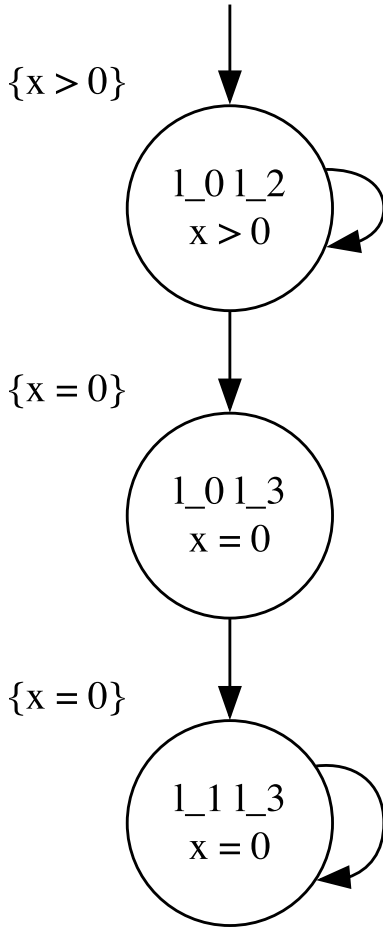# Exercise 3: Trace Inclusion

Part A

τ_{P_1}

τ_{P_2}

$\tau_{P_{3a} \parallel P_{3b}}$

{x > 0}

1_0 1_2
x > 0

{x = 0}

1_0 1_3
x = 0

{x = 0}

1_1 1_3
x = 0

Part B

- $\left(\tau_{P_1}, \tau_{P_2}\right)$ = true and $\left(\tau_{P_2}, \tau_{P_1}\right)$ = true

Their transition systems are identical from Part A. Therefore, their traces are equivalent as well. Therefore, they are subset of each other in both directions.

- $\left(\tau_{P_1}, \tau_{P_{3a} \, \parallel \parallel \, P_{3b}}\right)$ = false and $\left(\tau_{P_2}, \tau_{P_{3a} \, \parallel \parallel \, P_{3b}}\right)$ = false

Both τ_{P_1} and τ_{P_2} has the trace $\{x = 0\}^{\omega}$ but $\tau_{P_{3a} \, \parallel \parallel \, P_{3b}}$ doesn't.

- $\left(\tau_{P_1}, \tau_4\right)$ = false and $\left(\tau_{P_2}, \tau_4\right)$ = false

Both τ_{P_1} and τ_{P_2} has the trace $\{x = 0\}^{\omega}$ but τ_4 doesn't.

- $\left(\tau_{P_{3a} \, \parallel \parallel \, P_{3b}}, \tau_4\right)$ = true and $\left(\tau_4, \tau_{P_{3a} \, \parallel \parallel \, P_{3b}}\right)$ = true

Both has the same set of traces:

- $\{x > 0\}^{\omega}$
- $\{x > 0\}^n \{x = 0\}^{\omega}$ where $n \in N_{>0}$

Therefore, they are subset of each other.

- $\left(\tau_{P_{3a} \, \parallel \parallel \, P_{3b}}, \tau_{P_1}\right)$ = true and $\left(\tau_{P_{3a} \, \parallel \parallel \, P_{3b}}, \tau_{P_2}\right)$ = true

Both τ_{P_1} and τ_{P_2} has all the traces $\tau_{P_{3a} \,||| \, P_{3b}}$ have so it satisfies the subset relation.

- $\{x > 0\}^\omega$
- $\{x > 0\}^n \{x = 0\}^\omega$

- $\left(\tau_4, \tau_{P_1}\right) = $ true and $\left(\tau_4, \tau_{P_2}\right) = $ true

Both τ_{P_1} and τ_{P_2} has all the traces τ_4 have so it satisfies the subset relation.

- $\{x > 0\}^\omega$
- $\{x > 0\}^n \{x = 0\}^\omega$

Part C

This is not possible, because $\tau_{P_{3a} \,||| \, P_{3b}}$ and τ_4 are subsets of τ_{P_1} and τ_{P_2}. So each E that satisfies the latter must necesarily satisfy the former ones due to transitivity.