Cyber Physical Systems - Discrete Models
Exercise Sheet 3 Solution

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Exercise 1: Intersection of ω-regular languages

(a) L_1: It is certain that a is not infinite. However, b or c can be infinite.
L_2: It is certain that b is infinite. However, a and c can also be infinite.
$L_1 \bigcap L_2$: It is certain that a is not infinite and b is infinite. However, c can be infinite.

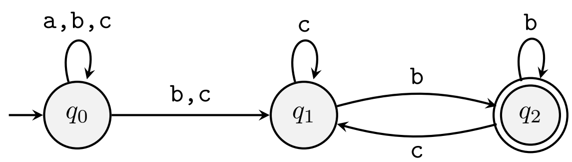

Figure 1: Büchi automaton A that accepts intersection of L_1 and L_2

(b) L_1: It is certain that a is infinite. However, b can also be infinite.
L_2: It is certain that b is infinite. However, a can also be infinite.
$L_1 \bigcap L_2$: It is certain that a and b infinite.

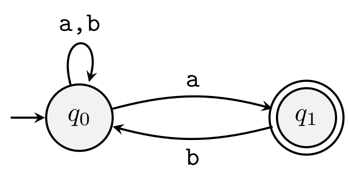

Figure 2: Büchi automaton A that accepts intersection of L_1 and L_2

(c) L_1: It is certain that a is not infinite. However, b can be infinite.
L_2: It is certain that if there is b at any position, regardless of finite or infinite words, there must be a right next to it. However, a can be infinite. Yet, b can also be infinite as long as it follows a, which means both are infinite only if they are together.
$L_1 \bigcap L_2$: $\emptyset$ (It is certain that a is not infinite. And b cannot be infinite since a is not infinite)
Since there is no infinite run containing an accepting state on the Büchi Automaton, the intersection is an empty language.

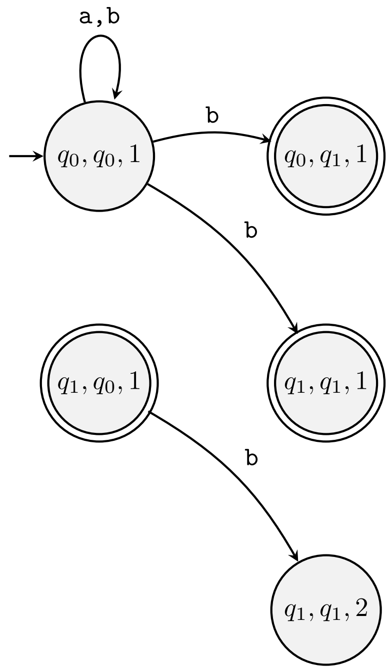

Figure 3: Büchi automaton A that accepts intersection of L_1 and L_2

Exercise 2: Transition Systems

(a) The model represents a case or lock which can be opened by inserting a ticket. Red light appears if locked. Otherwise, green light appears.

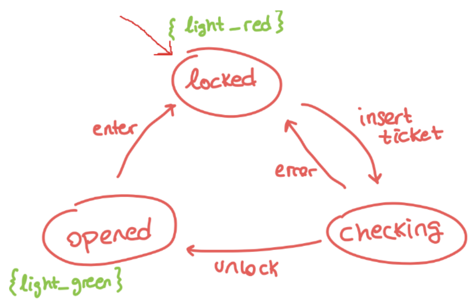

Figure 4: Transition System

(b) Mathematical definition of given transition system is as follows:

$$\begin{aligned} S &= \{1, 2, 3, 4\} \\ Act &= \{\text{close\_door}, \text{open\_door}, \text{go\_up}, \text{go\_down}\} \\ S_0 &= \{4, 1\} \\ L(4) &= \{\text{open, top\_floor}\} \\ L(3) &= \{\text{top\_floor}\} \\ L(2) &= \{\text{ground\_floor}\} \\ L(1) &= \{\text{open, ground\_floor}\} \end{aligned}$$

Transitions are as follows:

$$\longrightarrow = \{(4, \text{close\_door}, 3), (3, \text{open\_door}, 4), (3, \text{go\_down}, 2),\\ (2, \text{go\_up}, 3), (2, \text{open\_door}, 1), (1, \text{close\_door}, 2)\}$$

And the door is open at states 2 and 3.

Exercise 3: Crossroads Traffic Lights

(a) The traffic lights do not synchronize with each other.

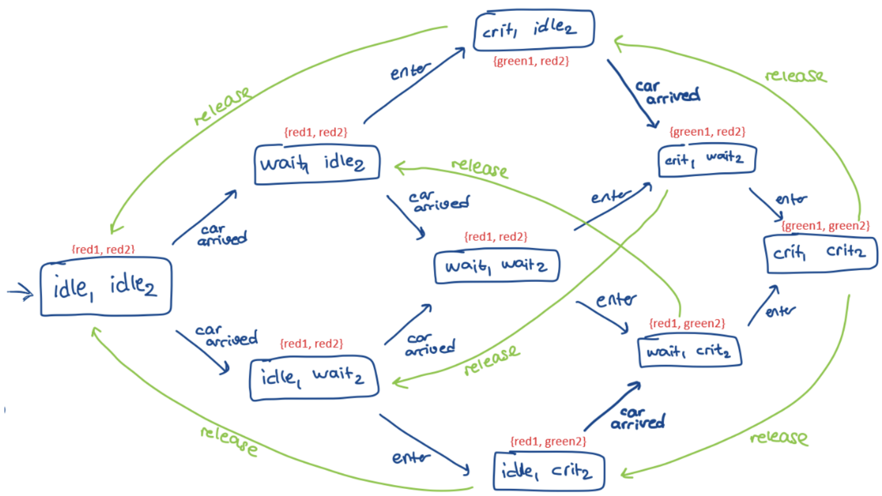

Figure 5: Interleaving transition systems $TS_1|||TS_2$

(b) The traffic lights do not synchronize with each other, but they both synchronize with the arbiter.

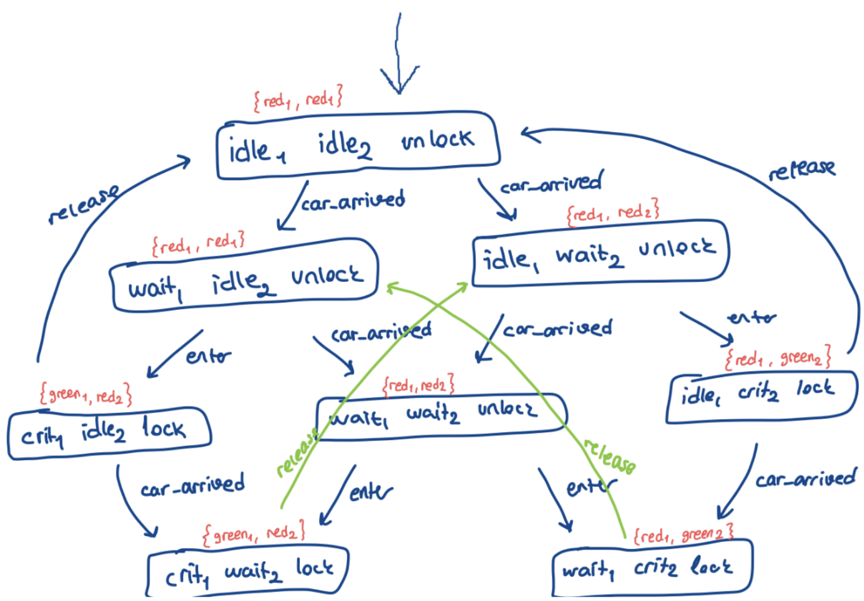

Figure 6: Parallel composition of systems $(TS_1|||TS_2)$ || Arbiter

(c) Yes, the system is safe when synchronized with an Arbiter. There is no such a state where the atomic proposition is $\{\text{green}_1, \text{green}_2\}$.
However, in this design, the lights immediately switch from green to red. We would expect another state(s), e.g. a yellow light state, which would result in a slower and safer switch from green to red.