

# Cyber Physical Systems - Discrete Models

## Exercise Sheet 3 Solution

Alper Ari  
aa508@uni-freiburg.edu

Onur Sahin  
os141@uni-freiburg.de

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### Exercise 1: Intersection of $\omega$ -regular languages

- (a)  $L_1$ : It is certain that  $a$  is not infinite. However,  $b$  or  $c$  can be infinite.  
 $L_2$ : It is certain that  $b$  is infinite. However,  $a$  and  $c$  can also be infinite.  
 $L_1 \cap L_2$ : It is certain that  $a$  is not infinite and  $b$  is infinite. However,  $c$  can be infinite.

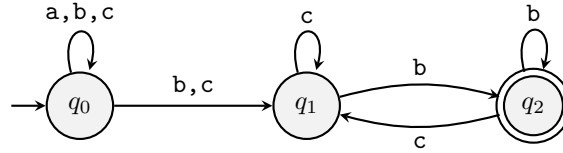


Figure 1: Büchi automaton A that accepts intersection of  $L_1$  and  $L_2$

- (b)  $L_1$ : It is certain that  $a$  is infinite. However,  $b$  can also be infinite.  
 $L_2$ : It is certain that  $b$  is infinite. However,  $a$  can also be infinite.  
 $L_1 \cap L_2$ : It is certain that  $a$  and  $b$  infinite.

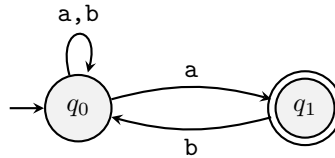


Figure 2: Büchi automaton A that accepts intersection of  $L_1$  and  $L_2$

- (c)  $L_1$ : It is certain that  $a$  is not infinite. However,  $b$  can be infinite.  
 $L_2$ : It is certain that if there is  $b$  at any position, regardless of finite or infinite words, there must be  $a$  right next to it. However,  $a$  can be infinite. Yet,  $b$  can also be infinite as long as it follows  $a$ , which means both are infinite only if they are together.  
 $L_1 \cap L_2$ :  $\emptyset$  (It is certain that  $a$  is not infinite. And  $b$  cannot be infinite since  $a$  is not infinite)

## Exercise 2: Transition Systems

- (a) The model represents a case or lock which can be opened by inserting a ticket. Red light appears if locked. Otherwise, green light appears.

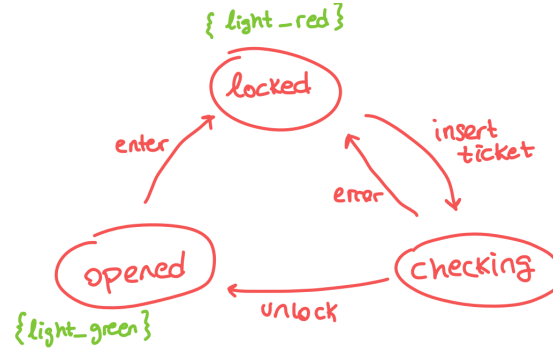


Figure 3: Transition System

- (b) Mathematical definition of given transition system is as follows:

$$\begin{aligned}
 S &= \{1, 2, 3, 4\} \\
 Act &= \{\text{close\_door}, \text{open\_door}, \text{go\_up}, \text{go\_down}\} \\
 S_0 &= \{4, 1\} \\
 L(4) &= \{\text{open}, \text{top\_floor}\} \\
 L(3) &= \{\text{top\_floor}\} \\
 L(2) &= \{\text{ground\_floor}\} \\
 L(1) &= \{\text{open}, \text{ground\_floor}\}
 \end{aligned}$$

Transitions are as follows:

$$\begin{aligned}
 \longrightarrow = \{ & (4, \text{close\_door}, 3), (3, \text{open\_door}, 4), (3, \text{go\_down}, 2), \\
 & (2, \text{go\_up}, 3), (2, \text{open\_door}, 1), (1, \text{close\_door}, 2) \}
 \end{aligned}$$

And the door is open at states 2 and 3.

### Exercise 3: Crossroads Traffic Lights

(a) The traffic lights do not synchronize with each other.

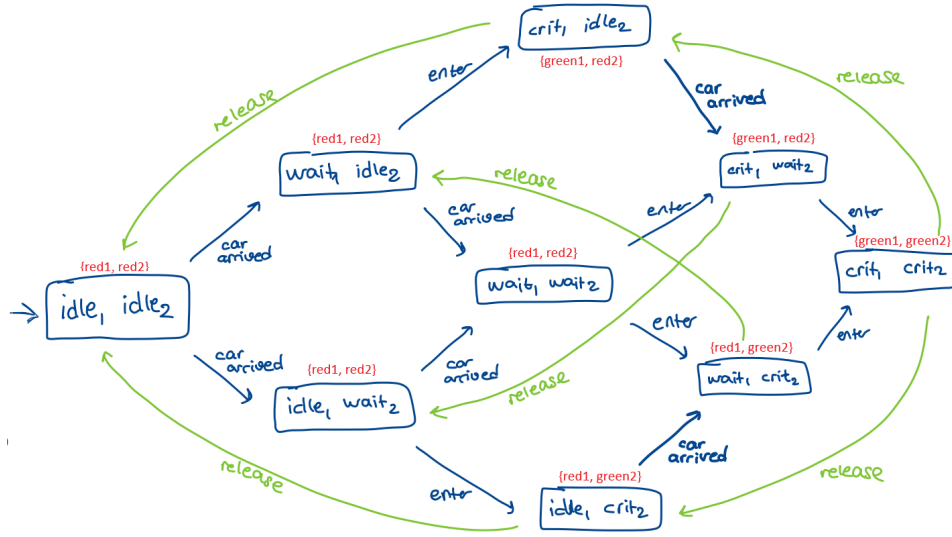


Figure 4: Interleaving transition systems  $TS_1 || TS_2$

(b) The traffic lights do not synchronize with each other, but they both synchronize with the arbiter.

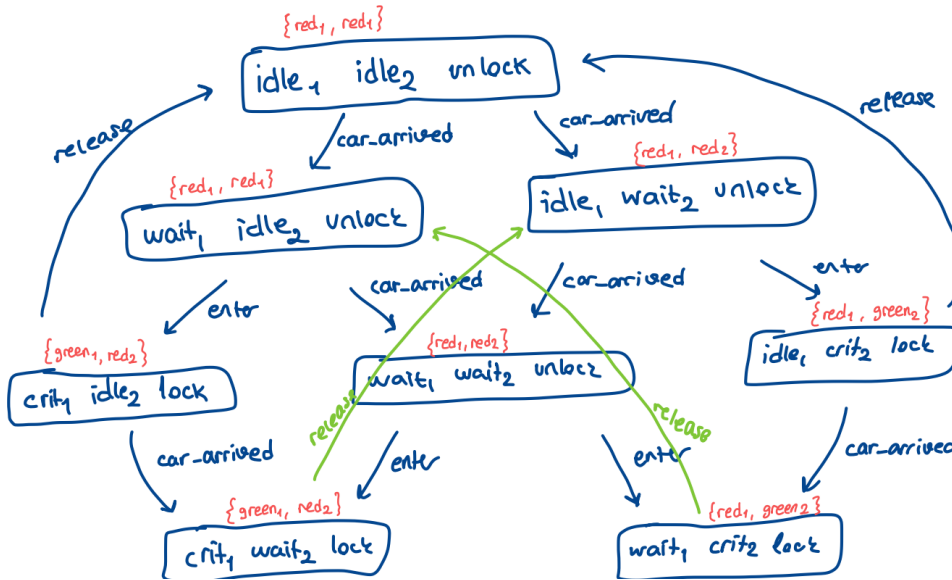


Figure 5: Parallel composition of systems  $(TS_1 || TS_2) || \text{Arbiter}$

(c) Yes, the system is safe when synchronized with an Arbiter. There is no such a state where the atomic proposition is  $\{\text{green}, \text{green}\}$ .

However, in this design, the lights immediately switch from green to red. We would expect another state(s), e.g. a yellow light state, which would result in a slower and safer switch from green to red.