# Cyber Physical Systems - Discrete Models
# Exercise Sheet 6 Solution


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## Exercise 1: Linear Time Properties

### Part A

- $T_1 : \{A_0 A_1 A_2 ... \mid \forall i \in N_{>0} \ . \ a \notin A_i\}$
- $T_2 : \{A_0 A_1 A_2 ... \mid \forall i \in N \ . \ a \in A_i \to b \in A_{i+1}\}$
- $T_3 : \{A_0 A_1 A_2 ... \mid \forall i \in N \ . \ a \in A_i \to b \notin A_i\}$
- $T_4 : \left\{A_0 A_1 A_2 ... \mid \overset{\infty}{\exists} i \in N \ . \ a \in A_i\right\}$
- $T_5 : \left\{A_0 A_1 A_2 ... \mid \overset{\infty}{\exists} i \in N \ . \ a \notin A_i\right\}$

### Part B

- $T_1 : \{A_0 A_1 A_2 ... \mid \forall i \in N \ . \ a \in A_i\}$
- $T_2 : \{A_0 A_1 A_2 ... \mid \forall i \in N \ . \ a \in A_i \to a \in A_{i+1}\}$
- $T_3 : \{A_0 A_1 A_2 ... \mid \forall i \in N \ . \ a \in A_i \land b \in A_i\}$
- $T_4 : \left\{A_0 A_1 A_2 ... \mid \overset{\infty}{\exists} i \in N \ . \ b \in A_i\right\}$
- $T_5 : \{A_0 A_1 A_2 ... \mid \forall i \in N \ . \ a \notin A_i\}$

## Exercise 2: Starvation Freedom

### Part A

We can prove that $\text{LIVE}' \subseteq \text{LIVE}$ if we can show that all worlds in $\text{LIVE}'$ is also in LIVE.

Let $w \in \text{LIVE}'$, we have the following cases:

Case 1: $w$ doesn't have infinitely many $\text{wait}_1$s

In this case $w \in \text{LIVE}$ since $w$ doesn't satisfy the predicate $\overset{\infty}{\exists} \in N \ . \ \text{wait}_1 \in A_i$, therefore doesn't need to satisfy $\overset{\infty}{\exists} \in N \ . \ \text{crit}_1 \in A_i$.

Case 2: $w$ has infinitely many $\text{wait}_1$s

In this case, it follows that $w$ also has infinitely many $\text{crit}_1$s as well, because for all $\text{wait}_1 \in A_i$ there must be a $\text{crit}_1 \in A_j$ such that $j$ comes after $i$. There can't be a "last" $j$ that comes after all $\text{wait}_1$s, since there are infinitely many $\text{wait}_1$s. Which would mean that $\text{crit}_1$s can be finitely many in this case. Since this is not possible, we can conclude that $w \in \text{LIVE}$.

Same reasoning can be trivially applied to $\text{wait}_2$ and $\text{crit}_2$ as well.

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## Part B

Consider a language LIVE″ s.t.:

$$\text{LIVE}'' := \begin{cases} \text{set of all infinite traces } A_0 A_1 A_2 ... s.t. \\ \forall i \in N \ . \ \left(\text{wait}_1 \in A_i \rightarrow \exists j \in N \ . \ j < i \wedge \text{crit}_1 \in A_j\right) \\ \forall i \in N \ . \ \left(\text{wait}_2 \in A_i \rightarrow \exists j \in N \ . \ j < i \wedge \text{crit}_2 \in A_j\right) \\ \end{cases}$$

We can follow a similar proof to the Part A to conclude $\text{LIVE}'' \subseteq \text{LIVE}$. But $\text{LIVE}'' \nsubseteq \text{LIVE}'$ since ordering is reversed. Which means there is not necessarily a crit_1 after each $\text{wait}_1$ (And same for $\text{crit}_2$ and $\text{wait}_2$). Therefore, $\text{LIVE}'$ is a *strictly stronger* property than LIVE.

## Part C

No, because that system only enters $\text{crit}_i$ after a $\text{wait}_i$ is received. Therefore, ordering is always as described in $\text{LIVE}'$.

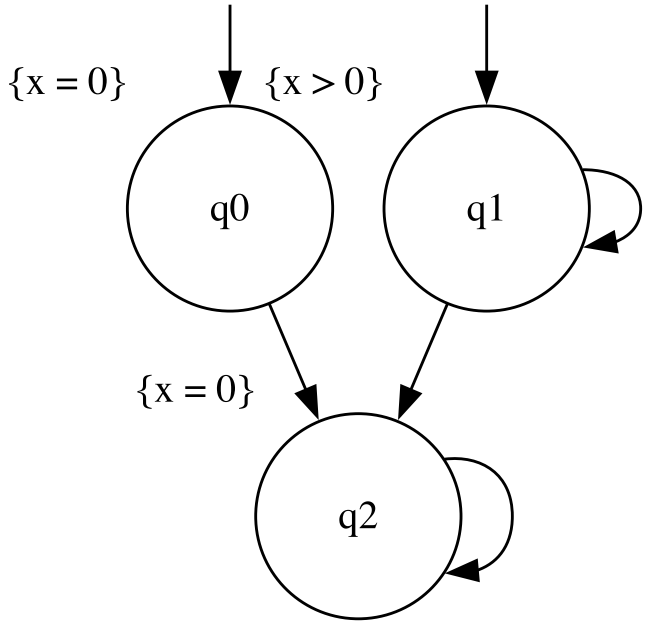
## Part D

No, because of Part A, this is not a possible trace for any transition system.

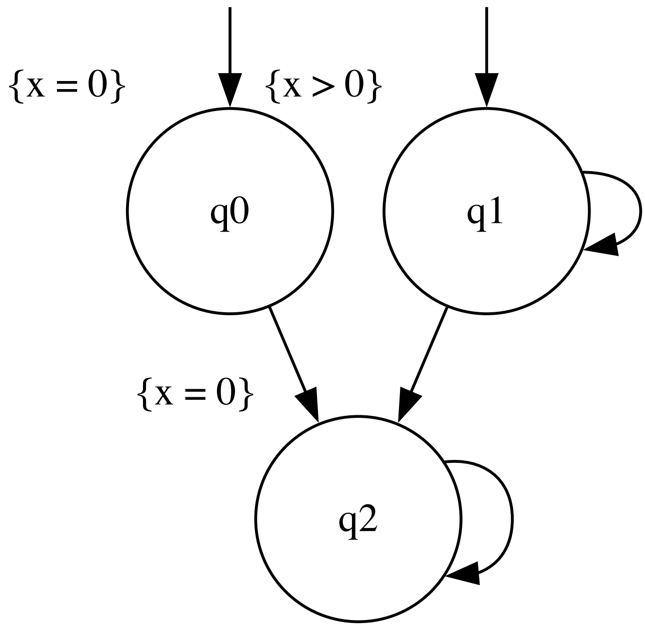
# Exercise 3: Trace Inclusion

## Part A

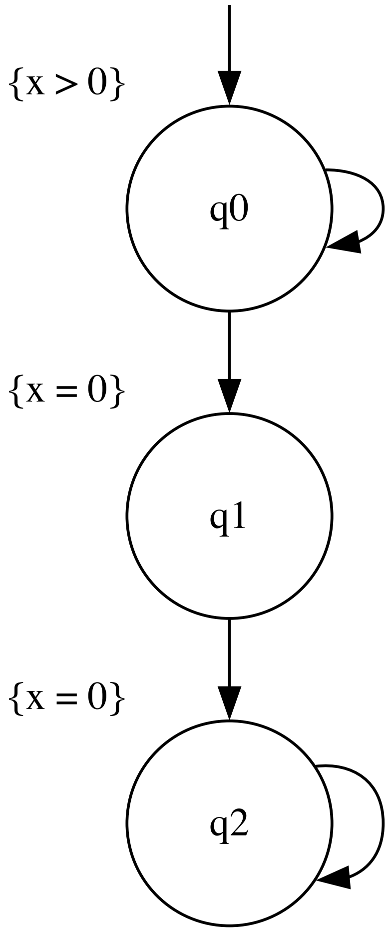
$\tau_{P_1}$




$\tau_{P_2}$




$\tau_{P_{3a} \; ||| \; P_{3b}}$




## Part B

- $\left(\tau_{P_1}, \tau_{P_2}\right) = \text{true}$ and $\left(\tau_{P_2}, \tau_{P_1}\right) = \text{true}$

Their transition systems are identical from Part A. Therefore, their traces are equivalent as well. Therefore, they are subset of each other in both directions.

- $\left(\tau_{P_1}, \tau_{P_{3a} \,|||\, P_{3b}}\right) = \text{false}$ and $\left(\tau_{P_2}, \tau_{P_{3a} \,|||\, P_{3b}}\right) = \text{false}$

Both $\tau_{P_1}$ and $\tau_{P_2}$ has the trace $\{x = 0\}^{\omega}$ but $\tau_{P_{3a} \,|||\, P_{3b}}$ doesn't.

- $\left(\tau_{P_1}, \tau_4\right) = \text{false}$ and $\left(\tau_{P_2}, \tau_4\right) = \text{false}$

Both $\tau_{P_1}$ and $\tau_{P_2}$ has the trace $\{x = 0\}^{\omega}$ but $\tau_4$ doesn't.

- $\left(\tau_{P_{3a} \,|||\, P_{3b}}, \tau_4\right) = \text{true}$ and $\left(\tau_4, \tau_{P_{3a} \,|||\, P_{3b}}\right) = \text{true}$

Both has the same set of traces:

  - $\{x > 0\}^{\omega}$
  - $\{x > 0\}^{n}\{x = 0\}^{\omega}$ where $n \in N_{>0}$

Therefore, they are subset of each other.

- $\left(\tau_{P_{3a} \,|||\, P_{3b}}, \tau_{P_1}\right) = \text{true}$ and $\left(\tau_{P_{3a} \,|||\, P_{3b}}, \tau_{P_2}\right) = \text{true}$

Both $\tau_{P_1}$ and $\tau_{P_2}$ has all the traces $\tau_{P_{3a} \,|||\, P_{3b}}$ have so it satisfies the subset relation.

  - $\{x > 0\}^\omega$ : path $q_1^\omega$
  - $\{x > 0\}^n \{x = 0\}^\omega$ : path $q_1^n q_2^\omega$ where $n \in N_{>0}$.

- $\left(\tau_4, \tau_{P_1}\right) = \text{true}$ and $\left(\tau_4, \tau_{P_2}\right) = \text{true}$

Both $\tau_{P_1}$ and $\tau_{P_2}$ has all the traces $\tau_4$ have so it satisfies the subset relation.

  - $\{x > 0\}^\omega$ : path $q_1^\omega$
  - $\{x > 0\}^n \{x = 0\}^\omega$ : path $q_1^n q_2^\omega$ where $n \in N_{>0}$.

## Part C

This is not possible, because $\tau_{P_{3a} \,|||\, P_{3b}}$ and $\tau_4$ are subsets of $\tau_{P_1}$ and $\tau_{P_2}$. So each $E$ that satisfies the latter must necesarily satisfy the former ones due to transitivity.