Cyber Physical Systems - Discrete Models
Exercise Sheet 8 Solution

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Exercise 1: Prefixes and Closure I

Part A

$$P \subseteq \mathrm{cl}(P)$$

Solution

Let $\omega \in P$, then $\mathrm{pref}(\omega) \subseteq \mathrm{pref}(P)$ trivially. Since $\mathrm{pref}(\omega) \subseteq \mathrm{pref}(P)$ is the predicate for closure, then we can conclude $\forall \omega \in P \to \omega \in \mathrm{cl}(P)$, thefore $P \subseteq \mathrm{cl}(P)$.

Part B

$$\mathrm{pref}(\mathrm{cl}(P)) = \mathrm{pref}(P)$$

Solution

If we prove both $\mathrm{pref}(\mathrm{cl}(P)) \subseteq \mathrm{pref}(P)$ and $\mathrm{pref}(P) \subseteq \mathrm{pref}(\mathrm{cl}(P))$, then we can conclude $\mathrm{pref}(\mathrm{cl}(P)) = \mathrm{pref}(P)$.

Direction 1: $\mathrm{pref}(\mathrm{cl}(P)) \subseteq \mathrm{pref}(P)$
Let $\omega \in \mathrm{pref}(\mathrm{cl}(P))$, then $\exists \sigma \in \mathrm{cl}(P) \to \omega \in \mathrm{pref}(\sigma)$. By the definition of closure, $\forall \sigma \in \mathrm{cl}(P) \to \mathrm{pref}(\sigma) \subseteq \mathrm{pref}(P)$. Hence, $w \in \mathrm{pref}(P)$. Which concludes that $\forall \omega \in \mathrm{pref}(\mathrm{cl}(P)) \to \omega \in \mathrm{pref}(P)$.

Direction 2: $\mathrm{pref}(P) \subseteq \mathrm{pref}(\mathrm{cl}(P))$
Let $\omega \in \mathrm{pref}(P)$, then $\exists \sigma \in P \to \omega \in \mathrm{pref}(\sigma)$. By proof in part a, we can claim $\forall \sigma \in P \to \sigma \in \mathrm{cl}(P)$. Therefore, $\omega \in \mathrm{pref}(P) \to \omega \in \mathrm{pref}(\mathrm{cl}(P))$.

Exercise 2: Prefixes and Closure II

Part A

$$P_1 = \{A_0 A_1 ... \mid \exists S \subseteq N \cdot (|S| = 1 \land \forall i \in S a \in A_i)\}$$
$$P_2 = \{A_0 A_1 ... \mid \forall i \cdot (a \in A_i \to b \in A_{i+1})\}$$
$$P_3 = \{A_0 A_1 ... \mid \exists i \cdot (\forall j \cdot (j \geq i \to a \notin A_j))\}$$
$$P_4 = \left\{A_0 A_1 ... \mid a \in A_0 \land \overset{\infty}{\exists} i \cdot a \in A_i\right\}$$

Part B

$$\text{pref}(P_1) = \{A_0 A_1 ... A_k \mid \exists S \subseteq N \cdot (|S| \leq 1 \land \forall i \in S a \in A_i)\}$$
$$\text{pref}(P_2) = \{A_0 A_1 ... A_k \mid \forall i \cdot (i < k \land a \in A_i \to b \in A_{i+1})\}$$
$$\text{pref}(P_3) = \{A_0 A_1 ... A_k \mid \text{true}\}$$
$$\text{pref}(P_4) = \{A_0 A_1 ... A_k \mid a \in A_0\}$$

Part C

$$\text{cl}(P_1) = \{A_0 A_1 ... \mid \exists S \subseteq N \cdot (|S| \leq 1 \land \forall i \in S a \in A_i)\}$$
$$\text{cl}(P_2) = \{A_0 A_1 ... \mid \forall i \cdot (a \in A_i \to b \in A_{i+1})\}$$
$$\text{cl}(P_3) = \{A_0 A_1 ... \mid \text{true}\}$$
$$\text{cl}(P_4) = \{A_0 A_1 ... \mid a \in A_0\}$$