Cyber Physical Systems - Discrete Models
Exercise Sheet 13 Solution

Alper Ari
aa508@uni-freiburg.edu

Onur Sahin
os141@uni-freiburg.de

January 29, 2023

Exercise 1: LTL and Set Notation

(a)

$$\text{Words}(\varphi_1) =$$
$$\left\{A_0 A_1 ... \in \left(2^{\text{AP}}\right)^\omega \mid \forall i \in \mathbb{N}.\ \left(a \in A_i \to \exists j \in \mathbb{N}.\ j \geq i \land b \in A_j\right)\right\}$$

(b)

$$\text{Words}(\varphi_2) =$$
$$\left\{A_0 A_1 ... \in \left(2^{\text{AP}}\right)^\omega \mid \exists i \in \mathbb{N}.\ \left(b \in A_{i+1} \land \left(\forall j \in \mathbb{N}.\ j < i \to a \in A_j\right)\right)\right\}$$

(c)

$$\varphi_3 = \Diamond(a \land \bigcirc b)$$

(d)

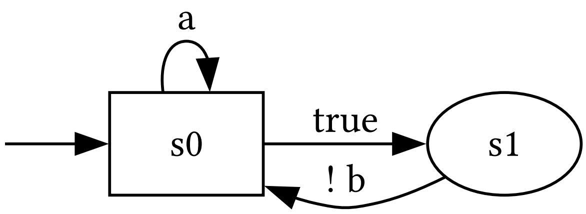
It's not possible to notate this property in LTL. A close formulation is $a \land (a \to \bigcirc \bigcirc a)$. But it also forces if a occurs in index one, then a must occur in index 3 as well and so on. But in the original language a only occurs in index one and not continue for odd numbers.

(e)

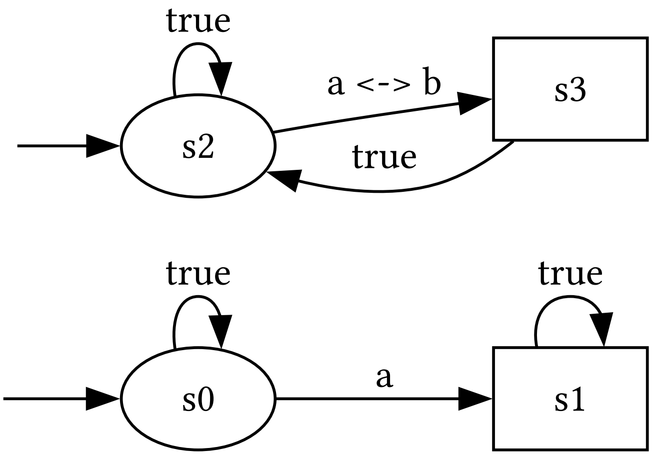
$$\begin{aligned}\varphi_5 = \Box(&\\ &(a \land b \to \bigcirc \bigcirc (a \land b)) \land \\ &(a \land \neg b \to \bigcirc \bigcirc (a \land \neg b)) \land \\ &(\neg a \land b \to \bigcirc \bigcirc (\neg a \land b)) \land \\ &(\neg a \land \neg b \to \bigcirc \bigcirc (\neg a \land \neg b)) \\ )&\end{aligned}$$

Exercise 2: From LTL to NBA

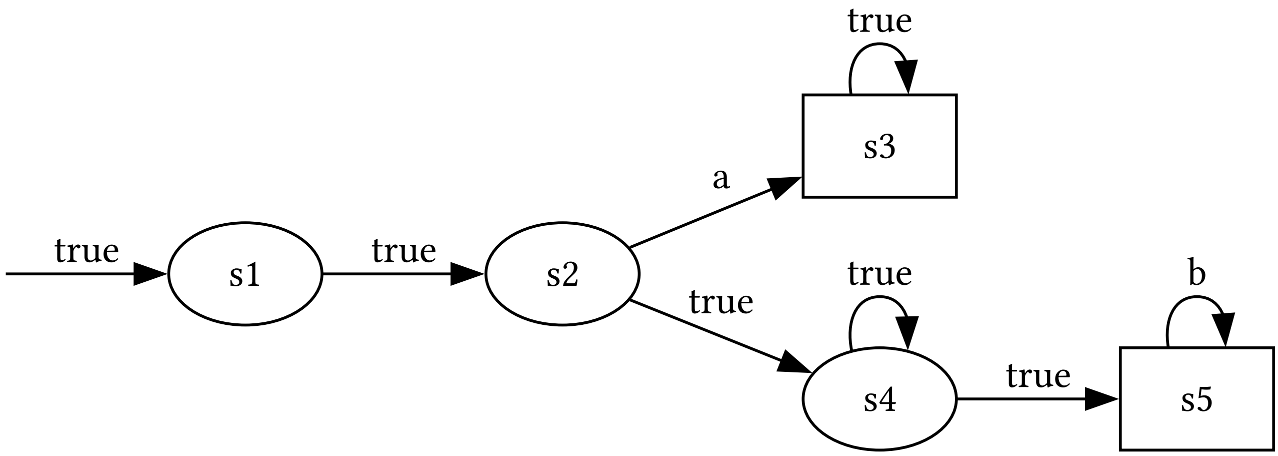
(a)


a
true
s0
s1
! b

(b)


true
a <-> b
s2
s3
true
true
true
s0
a
s1

(c)


true
s3
a
true
true
s1
s2
true
true
b
true
s4
s5

Exercise 3: LTL Equivalence

Part 1: $\text{Words}(\varphi) = \text{Words}(\psi) \to \forall \tau.\ \tau \vDash \varphi \Leftrightarrow \tau \vDash \psi$
Let φ and ψ be LTL properties and a Transition System τ.

Assume $\text{Words}(\varphi) = \text{Words}(\psi)$.

If $\tau \vDash \varphi$ then it means $\text{Traces}(\tau) \subseteq \text{Words}(\varphi)$. Since $\text{Words}(\varphi) = \text{Words}(\psi)$, we can substitute $\text{Words}(\psi)$, therefore $\text{Traces}(\tau) \subseteq \text{Words}(\psi) \equiv \tau \vDash \psi$ ■

Part 2: $\forall \tau.\ \tau \vDash \varphi \Leftrightarrow \tau \vDash \psi \to \text{Words}(\varphi) = \text{Words}(\psi)$
Let φ and ψ be LTL properties.

We can prove it via proof by contraposition. Assume $\text{Words}(\varphi) \neq \text{Words}(\psi)$. If we find that this implies $\neg(\forall \tau.\ \tau \vDash \varphi \Leftrightarrow \tau \vDash \psi)$ then we prove the original claim.

Then there exists a word ω such that one of the two holds:
1. $\omega \in \text{Words}(\varphi) \land \omega \notin \text{Words}(\psi)$
2. $\omega \notin \text{Words}(\varphi) \land \omega \in \text{Words}(\psi)$

Without loss of generality we will only consider the first case. The second case can be handled in the same way.

Then there exists a Transition System τ such that $\text{Traces}(\tau) = \{\omega\}$. It immediately follows that $\text{Traces}(\omega) \subseteq \text{Words}(\varphi) \land \text{Traces}(\omega) \nsubseteq \text{Words}(\psi)$. Which means $\tau \vDash \varphi \land \tau \nvDash \psi$. So $\neg(\forall \tau : \tau \vDash \varphi \Leftrightarrow \tau \vDash \psi)$ holds from the counter example we found.

Since $\text{Words}(\varphi) \neq \text{Words}(\psi) \to \neg(\forall \tau : \tau \vDash \varphi \Leftrightarrow \tau \vDash \psi)$, we conclude that $\forall \tau : \tau \vDash \varphi \Leftrightarrow \tau \vDash \psi \to \text{Words}(\varphi) = \text{Words}(\psi)$ ■