Cyber Physical Systems - Discrete Models
Exercise Sheet 6 Solution

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Exercise 1: Linear Time Properties

Part A

- $T_1 : \{A_0 A_1 A_2 ... \mid \forall i \in N_{>0} \ . \ a \notin A_i\}$
- $T_2 : \{A_0 A_1 A_2 ... \mid \forall i \in N \ . \ a \in A_i \rightarrow b \in A_{i+1}\}$
- $T_3 : \{A_0 A_1 A_2 ... \mid \forall i \in N \ . \ a \in A_i \rightarrow b \notin A_i\}$
- $T_4 : \left\{A_0 A_1 A_2 ... \mid \overset{\infty}{\exists} i \in N \ . \ a \in A_i\right\}$
- $T_5 : \left\{A_0 A_1 A_2 ... \mid \overset{\infty}{\exists} i \in N \ . \ a \notin A_i\right\}$

Part B

- $T_1 : \{A_0 A_1 A_2 ... \mid \forall i \in N \ . \ a \in A_i\}$
- $T_2 : \{A_0 A_1 A_2 ... \mid \forall i \in N \ . \ a \in A_i \rightarrow a \in A_{i+1}\}$
- $T_3 : \{A_0 A_1 A_2 ... \mid \forall i \in N \ . \ a \in A_i \land b \in A_i\}$
- $T_4 : \left\{A_0 A_1 A_2 ... \mid \overset{\infty}{\exists} i \in N \ . \ b \in A_i\right\}$
- $T_5 : \{A_0 A_1 A_2 ... \mid \forall i \in N \ . \ a \notin A_i\}$

Exercise 2: Starvation Freedom

Part A

We can prove that $\text{LIVE}' \subseteq \text{LIVE}$ if we can show that all worlds in LIVE' is also in LIVE.

Let $w \in \text{LIVE}'$, we have the following cases:

Case 1: w doesn't have infinitely many wait_1s

In this case $w \in \text{LIVE}$ since w doesn't satisfy the predicate $\overset{\infty}{\exists} \in N \ . \ \text{wait}_1 \in A_i$, therefore doesn't need to satisfy $\overset{\infty}{\exists} \in N \ . \ \text{crit}_1 \in A_i$.

Case 2: w has infinitely many wait_1s

In this case, it follows that w also has infinitely many crit_1s as well, because for all $\text{wait}_1 \in A_i$ there must be a $\text{crit}_1 \in A_j$ such that j comes after i. There can't be a "last" j that comes after all wait_1s, since there are infinitely many wait_1s. Which would mean that crit_1s can be finitely many in this case. Since this is not possible, we can conclude that $w \in \text{LIVE}$.

Same reasoning can be trivially applied to wait_2 and crit_2 as well.

■

Part B

Consider a language LIVE″ s.t.:

$$\text{LIVE}'' := \begin{cases} \text{set of all infinite traces } A_0 A_1 A_2 ... s.t. \\ \forall i \in N \ . \ \left(\text{wait}_1 \in A_i \rightarrow \exists j \in N \ . \ j < i \land \text{crit}_1 \in A_j\right) \\ \forall i \in N \ . \ \left(\text{wait}_2 \in A_i \rightarrow \exists j \in N \ . \ j < i \land \text{crit}_2 \in A_j\right) \end{cases}$$

We can follow a similar proof to the Part A to conclude $\text{LIVE}'' \subseteq \text{LIVE}$. But $\text{LIVE}'' \nsubseteq \text{LIVE}'$ since ordering is reversed. Which means there is not necessarily a crit_1 after each wait_1 (And same for crit_2 and wait_2). Therefore, LIVE′ is a *strictly stronger* property than LIVE.

Part C

No, because that system only enters crit_i after a wait_i is received. Therefore, ordering is always as described in LIVE′.

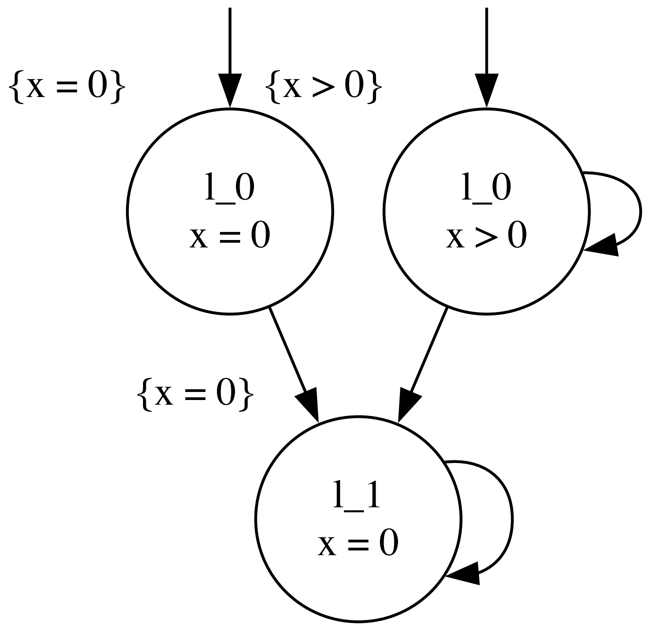
Part D

No, because of Part A, this is not a possible trace for any transition system.

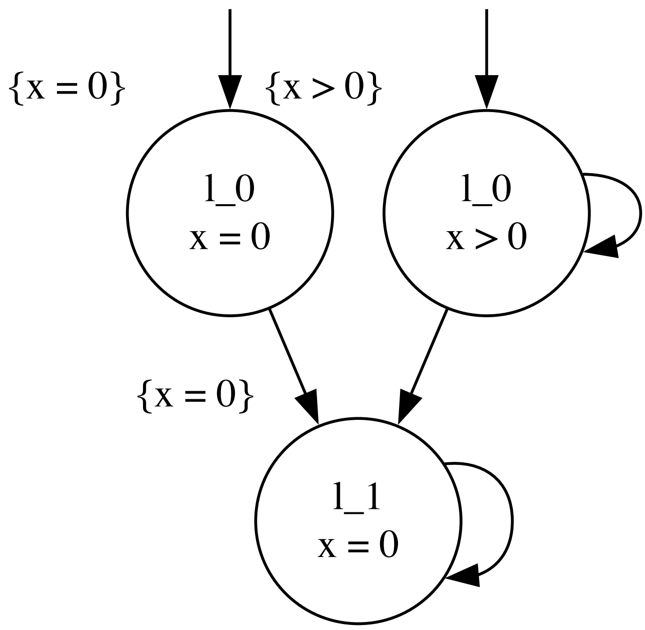
Exercise 3: Trace Inclusion

Part A

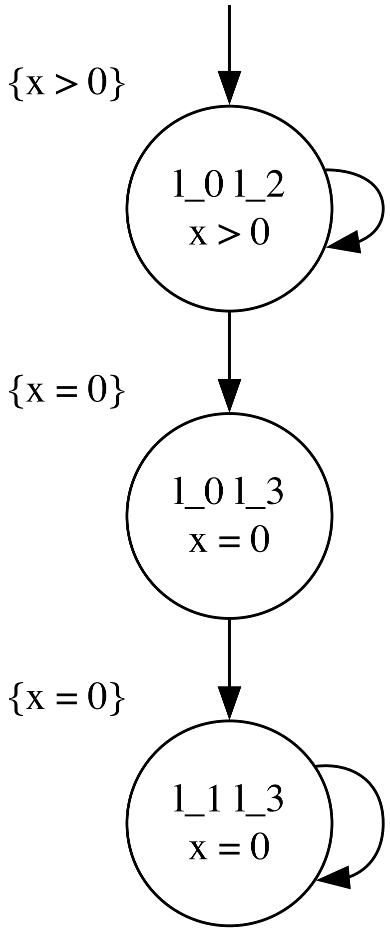
τ_{P_1}



τ_{P_2}



$\tau_{P_{3a} \; ||| \; P_{3b}}$



Part B

- $\left(\tau_{P_1}, \tau_{P_2}\right) = \text{true}$ and $\left(\tau_{P_2}, \tau_{P_1}\right) = \text{true}$

Their transition systems are identical from Part A. Therefore, their traces are equivalent as well. Therefore, they are subset of each other in both directions.

- $\left(\tau_{P_1}, \tau_{P_{3a} \,|||\, P_{3b}}\right) = \text{false}$ and $\left(\tau_{P_2}, \tau_{P_{3a} \,|||\, P_{3b}}\right) = \text{false}$

Both τ_{P_1} and τ_{P_2} has the trace $\{x = 0\}^\omega$ but $\tau_{P_{3a} \,|||\, P_{3b}}$ doesn't.

- $\left(\tau_{P_1}, \tau_4\right) = \text{false}$ and $\left(\tau_{P_2}, \tau_4\right) = \text{false}$

Both τ_{P_1} and τ_{P_2} has the trace $\{x = 0\}^\omega$ but τ_4 doesn't.

- $\left(\tau_{P_{3a} \,|||\, P_{3b}}, \tau_4\right) = \text{true}$ and $\left(\tau_4, \tau_{P_{3a} \,|||\, P_{3b}}\right) = \text{true}$

Both has the same set of traces:

  - $\{x > 0\}^\omega$
  - $\{x > 0\}^n \{x = 0\}^\omega$ where $n \in N_{>0}$

Therefore, they are subset of each other.

- $\left(\tau_{P_{3a} \,|||\, P_{3b}}, \tau_{P_1}\right) = \text{true}$ and $\left(\tau_{P_{3a} \,|||\, P_{3b}}, \tau_{P_2}\right) = \text{true}$

Both τ_{P_1} and τ_{P_2} has all the traces $\tau_{P_{3a} \,|||\, P_{3b}}$ have so it satisfies the subset relation.

 - $\{x > 0\}^{\omega}$: path q_1^{ω}
 - $\{x > 0\}^{n}\{x = 0\}^{\omega}$: path $q_1^{n} q_2^{\omega}$ where $n \in N_{>0}$.

- $\left(\tau_4, \tau_{P_1}\right) =$ true and $\left(\tau_4, \tau_{P_2}\right) =$ true

Both τ_{P_1} and τ_{P_2} has all the traces τ_4 have so it satisfies the subset relation.

 - $\{x > 0\}^{\omega}$: path q_1^{ω}
 - $\{x > 0\}^{n}\{x = 0\}^{\omega}$: path $q_1^{n} q_2^{\omega}$ where $n \in N_{>0}$.

Part C

This is not possible, because $\tau_{P_{3a} \,|||\, P_{3b}}$ and τ_4 are subsets of τ_{P_1} and τ_{P_2}. So each E that satisfies the latter must necesarily satisfy the former ones due to transitivity.