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Hand in until January 25th, 2023
15:59 via ILIAS
Discussion: January 30th/31st, 2023

Tutorial for Cyber-Physical Systems - Discrete Models

Exercise Sheet 12

Exercise 1*: Lecture Evaluation 2 Bonus Points

Complete the lecture evaluation.

Exercise 2: LTL Properties 12 Points

The goal of this task is to get familiar with LTL-semantics and to learn how to interpret LTL formulas over transition systems.

Given the following LTL properties over $AP = \{a, b, c\}$:

$\varphi_1 = a \land \bigcirc b$ $\quad\varphi_3 = \neg(a \mathbin{\mathsf{U}} \Box b)$ $\quad\varphi_5 = \Diamond \Box a$

$\varphi_2 = a \mathbin{\mathsf{U}} b$ $\quad\varphi_4 = (\Diamond c) \mathbin{\mathsf{U}} \Box a$ $\quad\varphi_6 = \Box \Diamond c$

For each of the LTL properties φ_i complete the following tasks:

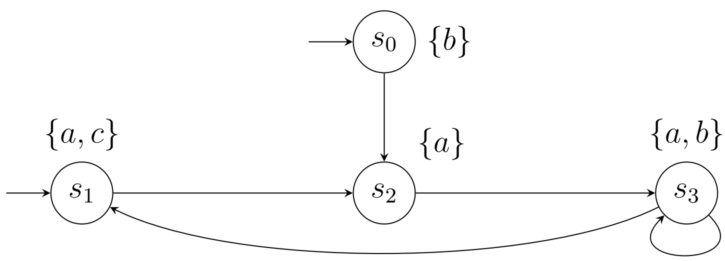
(a) Give a trace $\tau \in (2^{AP})^\omega$ that satisfies φ_i.

(b) Give a trace $\tau \in (2^{AP})^\omega$ that violates φ_i.

(c) Determine if the transition system below satisfies φ_i. Explain why or why not.

(d) Formalize $Words(\varphi_i)$ (i.e. the set of all traces satisfying φ_i) using set comprehension.

For example for $\varphi = \Diamond a$ we can formalize $Words(\varphi) = \{A_0 A_1 \dots \in (2^{AP})^\omega \mid \exists i \in \mathbb{N}.\ a \in A_i\}$.



Exercise 3: Stating properties in LTL 3 Points + 2 Bonus Points

The goal of this task is to learn to formally specify key properties in LTL.

Suppose we have two users, *Betsy* and *Peter*, and a single printer device. Both users perform several tasks, and every now and then they want to print their results on the printer. Since there is only a single printer, only one user can print a job at a time. Suppose we have the following atomic propositions for *Peter* at our disposal:

Peter.request	indicates that *Peter* requests usage of the printer.
Peter.use	indicates that *Peter* uses the printer.
Peter.release	indicates that *Peter* releases the printer.

For *Betsy*, analogous predicates are defined. Specify in LTL the following properties:

(a) Mutual exclusion, i.e., only one user at a time can use the printer.

(b) Finite time of usage, i.e., a user can print only for a finite amount of time.

(c) Absence of individual starvation, i.e., if a user wants to print something, the user is eventually able to do so.

(d) **Bonus:** Absence of blocking, i.e., if a user requests access to the printer, the user does not request forever.

(e) **Bonus:** Alternating access, i.e., users must strictly alternate in printing.

Exercise 4: Equivalence of LTL formulas 7 Points + 2 Bonus Points

The goal of this task is to understand more complex LTL formulas and to reason about the equivalence of LTL formulas.

Consider the following claims about equivalences of LTL formulas. For each of them, state whether or not the equivalence holds. If an equivalence does not hold, provide a counterexample (i.e. a transition system that satisfies one of the properties and violates the other).

(a) $\Box a \wedge \bigcirc \Diamond a \stackrel{?}{\equiv} \Box a$

(b) $\Diamond a \wedge \bigcirc \Box a \stackrel{?}{\equiv} \Diamond a$

(c) $\Box a \rightarrow \Diamond b \stackrel{?}{\equiv} a \mathsf{U} (b \vee \neg a)$

(d) $a \mathsf{U} \mathit{false} \stackrel{?}{\equiv} \Box a$

(e) $\Box \bigcirc b \stackrel{?}{\equiv} \Box b$

Bonus: If an equivalence holds, give a proof.