

# Cyber Physical Systems - Discrete Models

## Exercise Sheet 6 Solution

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### Exercise 1: Linear Time Properties

#### Part A

- $T_1 : \{A_0A_1A_2\ldots \mid \forall i \in N_{>0} . a \notin A_i\}$
- $T_2 : \{A_0A_1A_2\ldots \mid \forall i \in N . a \in A_i \rightarrow b \in A_{i+1}\}$
- $T_3 : \{A_0A_1A_2\ldots \mid \forall i \in N . a \in A_i \rightarrow b \notin A_i\}$
- $T_4 : \{A_0A_1A_2\ldots \mid \exists^\infty i \in N . a \in A_i\}$
- $T_5 : \{A_0A_1A_2\ldots \mid \exists^\infty i \in N . a \notin A_i\}$

#### Part B

- $T_1 : \{A_0A_1A_2\ldots \mid \forall i \in N . a \in A_i\}$
- $T_2 : \{A_0A_1A_2\ldots \mid \forall i \in N . a \in A_i \rightarrow a \in A_{i+1}\}$
- $T_3 : \{A_0A_1A_2\ldots \mid \forall i \in N . a \in A_i \wedge b \in A_i\}$
- $T_4 : \{A_0A_1A_2\ldots \mid \exists^\infty i \in N . b \in A_i\}$
- $T_5 : \{A_0A_1A_2\ldots \mid \forall i \in N . a \notin A_i\}$

### Exercise 2: Starvation Freedom

#### Part A

We can prove that  $\text{LIVE}' \subseteq \text{LIVE}$  if we can show that all worlds in  $\text{LIVE}'$  is also in  $\text{LIVE}$ .

Let  $w \in \text{LIVE}'$ , we have the following cases:

Case 1:  $w$  doesn't have infinitely many  $\text{wait}_1$ s

In this case  $w \in \text{LIVE}$  since  $w$  doesn't satisfy the predicate

$\exists^\infty i \in N . \text{wait}_1 \in A_i$ , therefore doesn't need to satisfy  $\exists^\infty i \in N . \text{crit}_1 \in A_i$ .

Case 2:  $w$  has infinitely many  $\text{wait}_1$ s

In this case, it follows that  $w$  also has infinitely many  $\text{crit}_1$ s as well, because for all  $\text{wait}_1 \in A_i$  there must be a  $\text{crit}_1 \in A_j$  such that  $j$  comes after  $i$ . There can't be a "last"  $j$  that comes after all  $\text{wait}_1$ s, since there are infinitely many  $\text{wait}_1$ s. Which would mean that  $\text{crit}_1$ s can be finitely many in this case. Since this is not possible, we can conclude that  $w \in \text{LIVE}$ .

Same reasoning can be trivially applied to  $\text{wait}_2$  and  $\text{crit}_2$  as well.

■

## Part B

Let  $\pi = \{\text{wait}_1\}\emptyset^\omega$ .  $p \in \text{LIVE}$  because there is no infinite  $\text{wait}_1$  which would require that there must be infinitely many  $\text{crit}_1$ s. However,  $p \notin \text{LIVE}'$  because there is no  $\text{crit}_1$  after the initial  $\text{wait}_1$ .

## Part C

No, because that system only enters  $\text{crit}_i$  after a  $\text{wait}_i$  is received. Therefore, ordering is always as described in  $\text{LIVE}'$ .

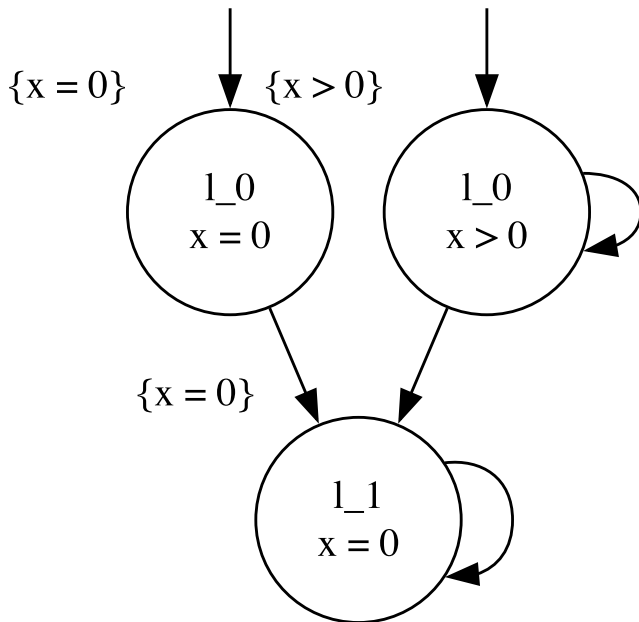
## Part D

No, because of Part A, this is not a possible trace for any transition system.

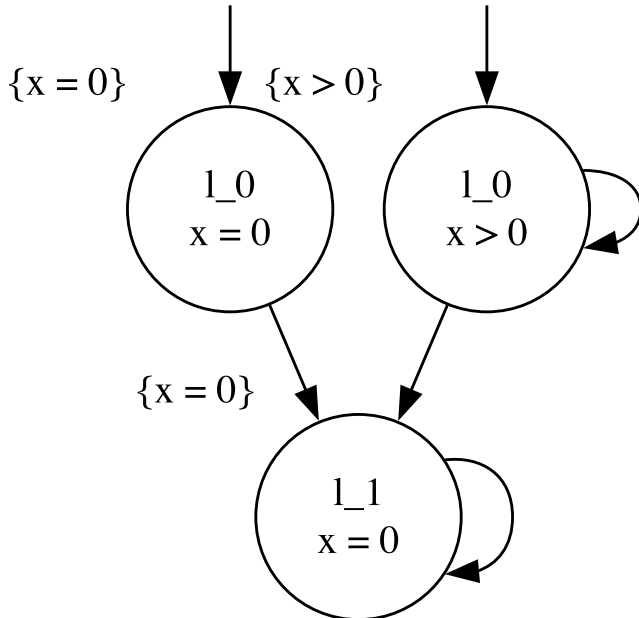
# Exercise 3: Trace Inclusion

## Part A

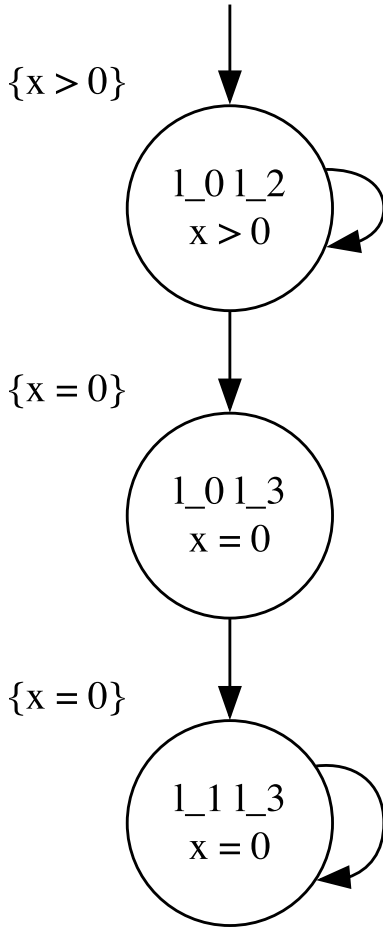
$\tau_{P_1}$



$\tau_{P_2}$



$\tau_{P_{3a}} \parallel P_{3b}$



## Part B

- $(\tau_{P_1}, \tau_{P_2}) = \text{true}$  and  $(\tau_{P_2}, \tau_{P_1}) = \text{true}$

Their transition systems are identical from Part A. Therefore, their traces are equivalent as well. Therefore, they are subset of each other in both directions.

- $(\tau_{P_1}, \tau_{P_{3a} \parallel P_{3b}}) = \text{false}$  and  $(\tau_{P_2}, \tau_{P_{3a} \parallel P_{3b}}) = \text{false}$

Both  $\tau_{P_1}$  and  $\tau_{P_2}$  has the trace  $\{x = 0\}^\omega$  but  $\tau_{P_{3a} \parallel P_{3b}}$  doesn't.

- $(\tau_{P_1}, \tau_4) = \text{false}$  and  $(\tau_{P_2}, \tau_4) = \text{false}$

Both  $\tau_{P_1}$  and  $\tau_{P_2}$  has the trace  $\{x = 0\}^\omega$  but  $\tau_4$  doesn't.

- $(\tau_{P_{3a} \parallel P_{3b}}, \tau_4) = \text{true}$  and  $(\tau_4, \tau_{P_{3a} \parallel P_{3b}}) = \text{true}$

Both has the same set of traces:

- $\{x > 0\}^\omega$
- $\{x > 0\}^n \{x = 0\}^\omega$  where  $n \in N_{>0}$

Therefore, they are subset of each other.

- $(\tau_{P_{3a} \parallel P_{3b}}, \tau_{P_1}) = \text{true}$  and  $(\tau_{P_{3a} \parallel P_{3b}}, \tau_{P_2}) = \text{true}$

Both  $\tau_{P_1}$  and  $\tau_{P_2}$  has all the traces  $\tau_{P_{3a}} \parallel P_{3b}$  have so it satisfies the subset relation.

- $\{x > 0\}^\omega$
- $\{x > 0\}^n \{x = 0\}^\omega$
- $(\tau_4, \tau_{P_1}) = \text{true}$  and  $(\tau_4, \tau_{P_2}) = \text{true}$

Both  $\tau_{P_1}$  and  $\tau_{P_2}$  has all the traces  $\tau_4$  have so it satisfies the subset relation.

- $\{x > 0\}^\omega$
- $\{x > 0\}^n \{x = 0\}^\omega$

## Part C

This is not possible, because  $\tau_{P_{3a}} \parallel P_{3b}$  and  $\tau_4$  are subsets of  $\tau_{P_1}$  and  $\tau_{P_2}$ . So each  $E$  that satisfies the latter must necessarily satisfy the former ones due to transitivity.