

Cyber Physical Systems - Discrete Models

Exercise Sheet 13 Solution

Alper Ari
aa508@uni-freiburg.edu

Onur Sahin
os141@uni-freiburg.de

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4.5/5

Exercise 1: LTL and Set Notation

1	2	3	Σ
4.5/5	5/6	3/3	12.5

(a)

Words(φ_1) =

$$\{A_0 A_1 \dots \in (2^{AP})^\omega \mid \forall i \in \mathbb{N}. (a \in A_i \rightarrow \exists j \in \mathbb{N}. j \geq i \wedge b \in A_j)\}$$

✓

(b)

Words(φ_2) =

$$\{A_0 A_1 \dots \in (2^{AP})^\omega \mid \exists i \in \mathbb{N}. (b \in A_{i+1} \wedge (\forall j \in \mathbb{N}. j < i \rightarrow a \in A_j))\}$$

✓

(c)

$$\varphi_3 = \Diamond(a \wedge \bigcirc b)$$

✓

(d)

It's not possible to notate this property in LTL. A close formulation is $a \wedge (a \rightarrow \bigcirc \bigcirc a)$. But it also forces if a occurs in index one, then a must occur in index 3 as well and so on. But in the original language a only occurs in index one and not continue for odd numbers.

(e)

$$\varphi_5 = \Box($$

$$(a \wedge b \rightarrow \bigcirc \bigcirc (a \wedge b)) \wedge$$

$$(a \wedge \neg b \rightarrow \bigcirc \bigcirc (a \wedge \neg b)) \wedge$$

$$(\neg a \wedge b \rightarrow \bigcirc \bigcirc (\neg a \wedge b)) \wedge$$

$$(\neg a \wedge \neg b \rightarrow \bigcirc \bigcirc (\neg a \wedge \neg b))$$

)

(✓)

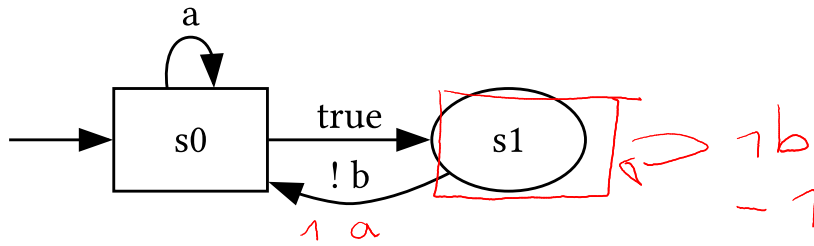
$$\equiv \Box((a \leftrightarrow \bigcirc \bigcirc a) \wedge (b \leftrightarrow \bigcirc \bigcirc b))$$

5/6

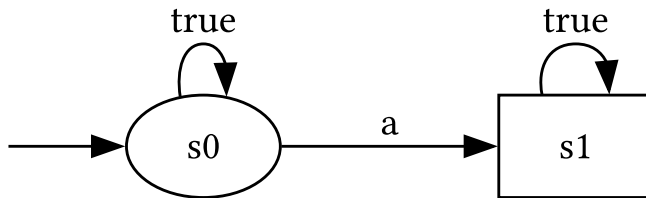
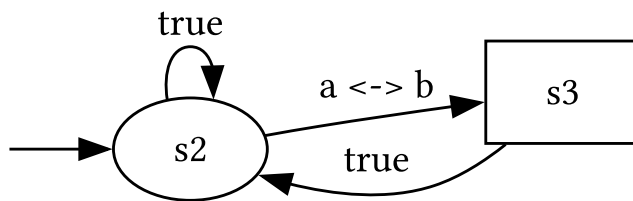
Exercise 2: From LTL to NBA

(a)

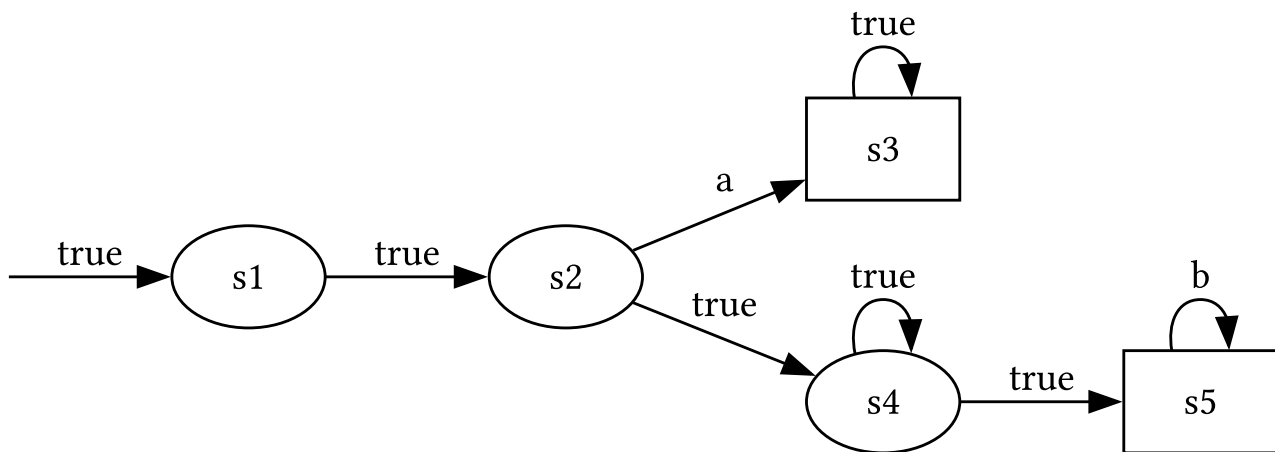
$$\Box(a \vee \neg ob) \equiv \Box(\neg a \rightarrow \neg ob)$$



(b)



(c)



Exercise 3: LTL Equivalence

Part 1: $\text{Words}(\varphi) = \text{Words}(\psi) \rightarrow \forall \tau. \tau \models \varphi \Leftrightarrow \tau \models \psi$

Let φ and ψ be LTL properties and a Transition System τ .

Assume $\text{Words}(\varphi) = \text{Words}(\psi)$.

If $\tau \models \varphi$ then it means $\text{Traces}(\tau) \subseteq \text{Words}(\varphi)$. Since $\text{Words}(\varphi) = \text{Words}(\psi)$, we can substitute $\text{Words}(\psi)$, therefore $\text{Traces}(\tau) \subseteq \text{Words}(\psi) \equiv \tau \models \psi$ ■

Part 2: $\forall \tau. \tau \models \varphi \Leftrightarrow \tau \models \psi \rightarrow \text{Words}(\varphi) = \text{Words}(\psi)$

Let φ and ψ be LTL properties.

We can prove it via proof by contraposition. Assume $\text{Words}(\varphi) \neq \text{Words}(\psi)$. If we find that this implies $\neg(\forall \tau. \tau \models \varphi \Leftrightarrow \tau \models \psi)$ then we prove the original claim.

Then there exists a word ω such that one of the two holds:

1. $\omega \in \text{Words}(\varphi) \wedge \omega \notin \text{Words}(\psi)$
2. $\omega \notin \text{Words}(\varphi) \wedge \omega \in \text{Words}(\psi)$

Without loss of generality we will only consider the first case. The second case can be handled in the same way.

Then there exists a Transition System τ such that $\text{Traces}(\tau) = \{\omega\}$. It immediately follows that $\text{Traces}(\tau) \subseteq \text{Words}(\varphi) \wedge \text{Traces}(\tau) \not\subseteq \text{Words}(\psi)$. Which means $\tau \models \varphi \wedge \tau \not\models \psi$. So $\neg(\forall \tau : \tau \models \varphi \Leftrightarrow \tau \models \psi)$ holds from the counter example we found.

Since $\text{Words}(\varphi) \neq \text{Words}(\psi) \rightarrow \neg(\forall \tau : \tau \models \varphi \Leftrightarrow \tau \models \psi)$, we conclude that $\forall \tau : \tau \models \varphi \Leftrightarrow \tau \models \psi \rightarrow \text{Words}(\varphi) = \text{Words}(\psi)$ ■