Cyber Physical Systems - Discrete Models
Exercise Sheet 1 Solution

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Exercise 1: Propositional Logic

(1) If Alice joins group 1, the tutor refuses to accept Bob because they always talk.

$$\equiv (a \rightarrow \neg b)$$

(2) At least one of Bob and Claire cannot go to group 1, as they lead a chess group together meets at the same time.

$$\equiv (\neg b \lor \neg c)$$

$$\equiv \neg(b \land c)$$

(3) Claire hates Alice and doesn't want to be in the same group.

$$\equiv (\neg a \land c) \lor (a \land \neg c)$$

$$\equiv (a \oplus c)$$

(4) Alice wants to submit the solutions with either Bob or Claire and thus needs to be in a group with this person.

$$\equiv (a \leftrightarrow b) \lor (a \leftrightarrow c)$$

After constructing the truth table 1 by using above expressions, group assignments can be concluded as follows:

$$\begin{aligned} a &= 0 \text{ (Alice is in group 2)} \\ b &= 0 \text{ (Bob is in group 2)} \\ c &= 1 \text{ (Claire is in group 1)} \end{aligned} \tag{1}$$

a	b	c	$(a \rightarrow \neg b)$	$\neg(b \land c)$	$(a \oplus c)$	$(a \leftrightarrow b) \lor (a \leftrightarrow c)$
0	0	0	1	1	0	1
0	0	1	1	1	1	1
0	1	0	1	1	0	1
0	1	1	1	0	1	0
1	0	0	1	1	1	0
1	0	1	1	1	0	1
1	1	0	0	1	1	1
1	1	1	0	0	0	1

Table 1: Truth table

Exercise 2: Finite Automata

Given the descriptions of 2 formal languages over an alphabet $\sum = \{a, b\}$:

(L1) The language of all words such that the second-to-last letter is the letter a.
(L2) The language of all words such that the first letter is equal to the last letter.

(a) Formally define these languages as sets of words.

$$
\begin{aligned}
L_1 &= \{x_0 x_1 \ldots x_n | (n \in \mathbb{N}_1) \wedge (\forall i \leq n \ . \ x_i \in \Sigma) \wedge (x_{n-1} = a)\} \\
L_2 &= \{x_0 x_1 \ldots x_n | (n \in \mathbb{N}_0) \wedge (\forall i \leq n \ . \ x_i \in \Sigma) \wedge (x_n = x_0)\}
\end{aligned}
\tag{2}
$$

(b) For each of these languages, draw a finite automaton that recognizes the language. That is, draw an automaton A1 that accepts a word w if and only if its second-to-last letter is an a. Similarly, draw an automaton A2 that accepts a word w if and only if its first and last letters are equal.

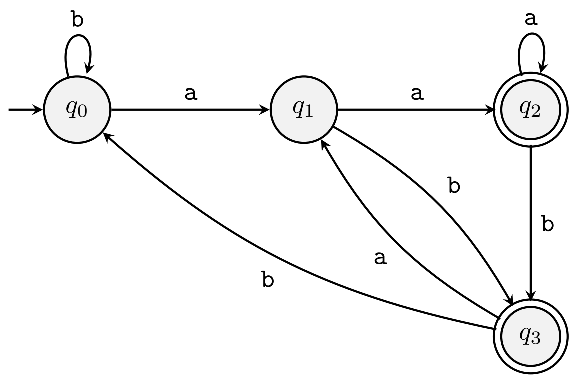

Figure 1: A_1: Finite Automaton for L_1

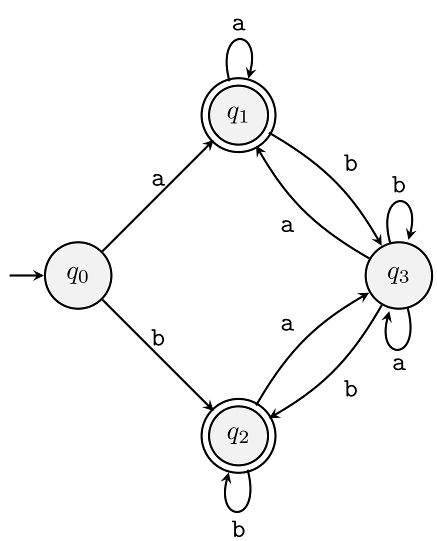

Figure 2: A_2: Finite Automaton for L_2

(c) Describe the automata from exercise (b) as a five-tuple.

$$\underline{\text{Automata description of } L_1 \text{ as per Figure 1}}:$$
$$A_1 = (Q_1, \Sigma, \delta_1, Q_1^{init}, F_1)$$
$$Q_1 = \{q_0, q_1, q_2, q_3\}$$
$$\Sigma = \{a, b\}$$
$$\delta_1 = \{(q_0, b, q_0), (q_0, a, q_1), (q_1, a, q_2), (q_2, a, q_2), (q_2, b, q_3), (q_3, a, q_1), (q_1, b, q_3), (q_3, b, q_0)\}$$
$$Q_1^{init} = \{q_0\}$$
$$F_1 = \{q_2, q_3\}$$

$$\underline{\text{Automata description of } L_2 \text{ as per Figure 2}}:$$
$$A_2 = (Q_2, \Sigma, \delta_2, Q_2^{init}, F_2)$$
$$Q_2 = \{q_0, q_1, q_2, q_3\}$$
$$\Sigma = \{a, b\}$$
$$\delta_2 = \{(q_0, a, q_1), (q_1, a, q_1), (q_0, b, q_2), (q_2, b, q_2), (q_1, b, q_3), (q_3, a, q_1), (q_2, a, q_3), (q_3, a, q_3), (q_3, b, q_3),\}$$
$$Q_2^{init} = \{q_0\}$$
$$F_2 = \{q_1, q_2\}$$