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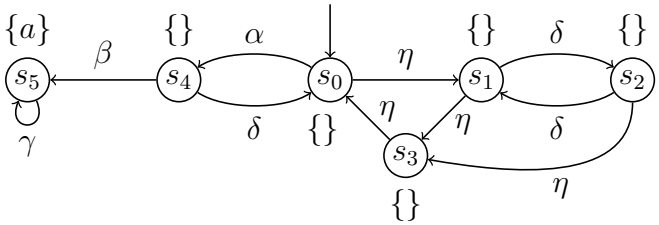
Tutorial for Cyber-Physical Systems - Discrete Models

Exercise Sheet 11

Exercise 1: Satisfaction under Fairness Assumptions — 12 Points

The goal of this task is to train your ability to identify fair and unfair traces of a given transition system, in order to reason about properties of a system under given fairness assumptions.

Consider the following transition system:



For the fairness assumptions (1)–(8), perform the following tasks.

(a) For each of the fairness assumptions below, give an execution that fulfills the fairness assumption (a fair execution) and an execution that violates the fairness assumption (an unfair execution).

(b) A system satisfies a property P under a given fairness assumption, if all fair traces (i.e., traces corresponding to fair executions) satisfy property P.
Under which of the following fairness assumptions does the system satisfy the property "eventually a"? Justify your answer.

(1) unconditional fairness for $A = \{\gamma\}$

(2) unconditional fairness for $A_1 = \{\alpha\}$ and for $A_2 = \{\gamma\}$

(3) unconditional fairness for $A = \{\alpha, \gamma\}$

(4) strong fairness for $A = \{\beta\}$

(5) strong fairness for $A_1 = \{\alpha\}$ and for $A_2 = \{\beta\}$

(6) strong fairness for $A_1 = \{\alpha\}$ and for $A_2 = \{\beta\}$ and for $A_3 = \{\eta\}$

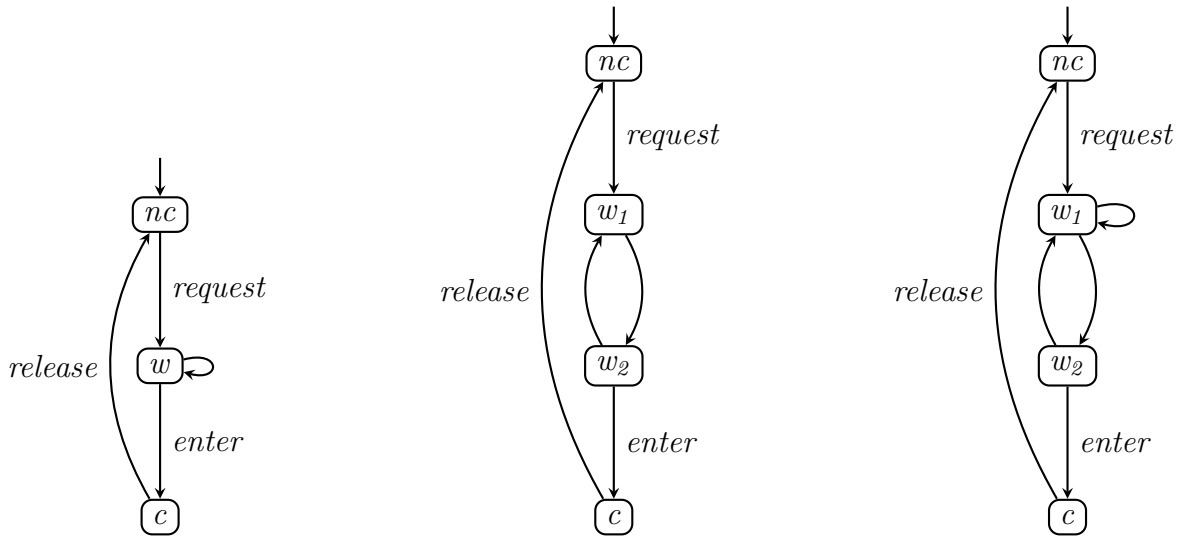
(7) weak fairness for $A = \{\eta\}$

(8) weak fairness for $A_1 = \{\alpha\}$ and for $A_2 = \{\beta\}$ and for $A_3 = \{\eta\}$

Exercise 2: Fairness Assumptions 6 Points

For each of the following three systems (each consisting of one single process) give the weakest fairness assumption on action *enter* to ensure non-starvation. Non-starvation means that a process that has requested will eventually enter its critical section.

Give an informal explanation for your answers. Explain both why your chosen fairness assumption is *sufficient* to ensure non-starvation, and why it is the *weakest* possible assumption.



Exercise 3: Closure Properties of LT Properties 3+3 Points

The goal of this task is to understand the effect of set operations on liveness and safety properties.

Let P and P' be liveness properties over AP. Prove or disprove the following claims:

(a) $P \cup P'$ is a liveness property.

(b) $P \cap P'$ is a liveness property.

(c) **Bonus:** Perform the same tasks for safety properties.
Hint: You may use the following lemma (*Distributivity of Union and Closure*)[1]:

For any LT properties P and P':

$$cl(P) \cup cl(P') = cl(P \cup P').$$

[1]Lemma 3.36 in *Principles of Model Checking* by Christel Baier and Joost-Pieter Katoen.