Cyber Physical Systems - Discrete Models
Exercise Sheet 7 Solution

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Exercise 1: Linear-Time Properties

1. Property P_1 :
   - i $\{A_0A_1A_2...|\forall i \in \mathbb{N} \cdot a \in A_i \vee b \in A_i\}$
   - ii $\{a\}(\{a\}\{a,b\})^\omega$
   - iii $\{a\}\emptyset(\{a\}\{a,b\})^\omega$
   - iv $T \not\models P_1$ because not all of the traces satisfy P_1 (example above).
2. Property P_2 :
   - i $\{A_0A_1A_2...|\forall i \in \mathbb{N} \cdot a \in A_i \wedge b \in A_i\}$
   - ii $-$
   - iii $\{a\}\emptyset(\{a\}\{a,b\})^\omega$
   - iv $T \not\models P_2$ because none of the traces satisfy P_2 (example above).
3. Property P_3 :
   - i $\{A_0A_1A_2...|\forall i \in \mathbb{N} \cdot b \in A_i \longrightarrow \exists j \in \mathbb{N} \cdot j < i \cdot a \in A_j\}$
   - ii $\{a\}(\{a\}\{a,b\})^\omega$
   - iii $-$
   - iv $T \models P_3$ because all traces satisfy P_3.
4. Property P_4 :
   - i $\{A_0A_1A_2...|\forall i \in \mathbb{N} \cdot (a \in A_i \longrightarrow \exists j \in \mathbb{N} \cdot j \geq i \cdot b \in A_j)\}$
   - ii $\{a\}(\{a\}\{a,b\})^\omega$
   - iii $-$
   - iv $T \models P_4$ because all traces satisfy P_4.
5. Property P_5 :
   - i $\{A_0A_1A_2...|\exists i,j,k \in \mathbb{N} \cdot (a \in A_i \wedge a \in A_j \wedge a \in A_k) \wedge \forall z \in \mathbb{N}/\{i,j,k\} \cdot a \notin A_z\}$
   - ii $-$
   - iii $\{a\}\emptyset(\{a\}\{a,b\})^\omega$
   - iv $T \not\models P_5$ because no traces satisfy P_5.
6. Property P_6 :
   - i $\{A_0A_1A_2...|\exists^\infty i \in \mathbb{N} \cdot a \in A_i \Longrightarrow \exists^\infty j \in \mathbb{N} \cdot b \in A_j\}$
   - ii $\{a\}(\{a\}\{a,b\})^\omega$

iii $-$

iv $T \vDash P_6$ because all traces satisfy P_6.

7. Property P_7 :

   i $\{A_0 A_1 A_2 ... | \exists i \in \mathbb{N} \cdot \forall j \in \mathbb{N} \cdot j \geq i \cdot a \notin A_j\}$

   ii $-$

   iii $\{a\}\emptyset(\{a\}\{a, b\})^\omega$

   iv $T \nvDash P_7$ because no traces satisfy P_7.

Exercise 2: Complement of LT-Properties

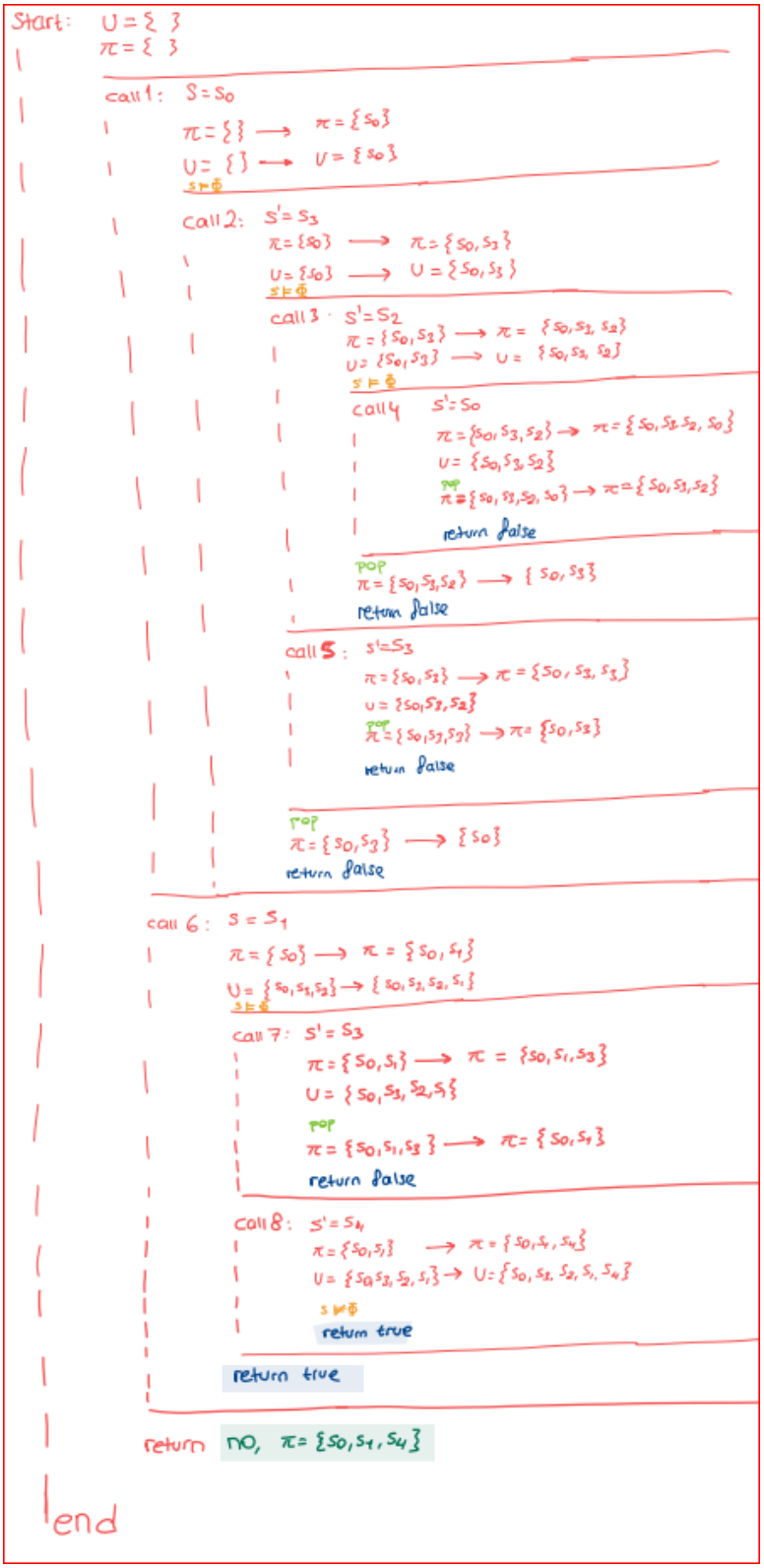
(a) **If $\tau \vDash \neg E$ holds, it follows that $\tau \nvDash E$ holds:**
True. Proof by contradiction. Assume $\tau \vDash \neg E \wedge \tau \vDash E$.
Then it follows that $\tau \in \neg E \wedge \tau \in E$.
Since the fact that $E \cap \neg E = \emptyset$, the assumption contradicts.
Therefore, if $\tau \vDash \neg E$ holds, it follows that $\tau \nvDash E$ holds.

(b) **If $\tau \nvDash \neg E$ holds, it follows that $\tau \vDash E$ holds:**
True. Proof by contradiction. Assume $\tau \nvDash E \wedge \tau \nvDash \neg E$.
Then $\tau \notin E \wedge \tau \notin \neg E$.
This leads to $\tau \notin (E \cup \neg E)$ which actually means $\tau \notin (2^{AP})^{\omega}$ which is a contradiction.
Therefore, if $\tau \nvDash \neg E$ holds, it follows that $\tau \vDash E$ holds.

(c) **If $T \vDash \neg E$ holds, it follows that $T \nvDash E$ holds:**
False. A counter example would be $T = \emptyset$ which is a transition system without any traces.
In this case the system has no trace which violates both E and $\neg E$.
So, the given statement is false.

(d) **If $T \nvDash \neg E$ holds, it follows that $T \vDash E$ holds:**
False. A counter example would be T with traces $\{a^{\omega}, b^{\omega}\}$ and property E = "always b".
Here, $T \nvDash \neg E$ is false because one of the traces already satisfies E.
And, $T \vDash E$ is also false because one of the traces already doesn't satisfy E.
So, the given statement is false.

Exercise 3: Invariant checking I



Exercise 4: Invariant checking II

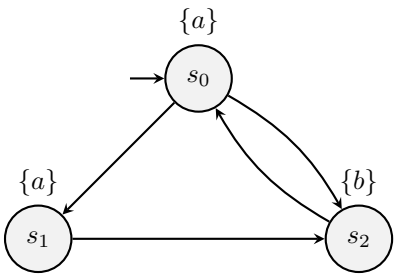


Figure 1: Transition system with 3 states

$$\phi = a$$

$$\text{non-minimal} = \{s_0, s_1, s_2\}$$

$$\text{minimal} = \{s_0, s_2\}$$