Cyber Physical Systems - Discrete Models
Exercise Sheet 5 Solution

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Exercise 1: Synchronization

Given two transition systems $T = (S, Act, \longrightarrow, S_0, AP, L)$ and $T_0' = (S', Act', \longrightarrow', S_0, AP', L')$

(a) Give a set Syn such that $T||T'$ and $T||_{Syn}T'$ are always equivalent: $Syn = Act \cap Act'$

(b) Give a set Syn such that $T|||T'$ and $T||_{Syn}T'$ are always equivalent: $Syn = \emptyset$

Exercise 2: Coffee Machine and Transition System

(a) Transition system of corresponding program graph

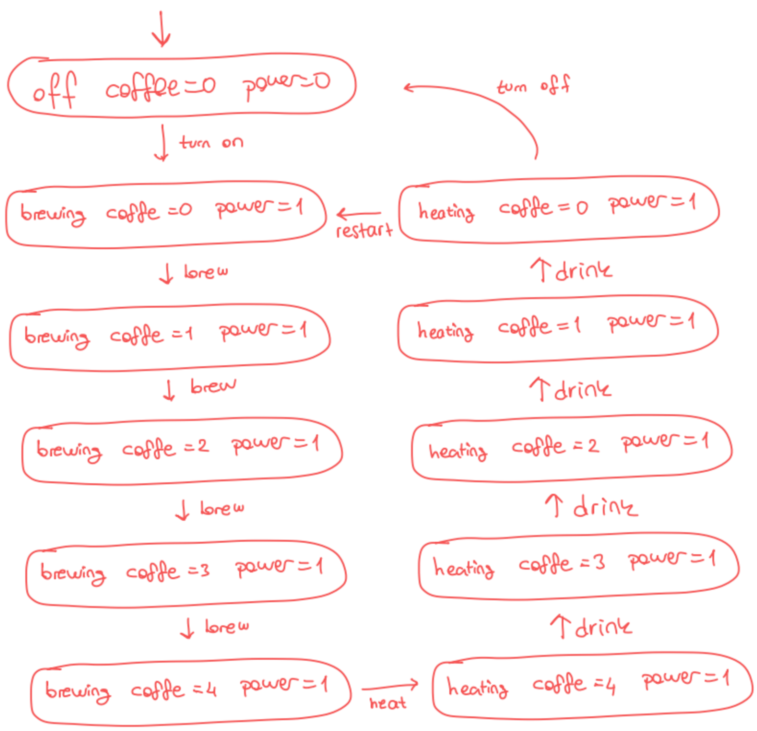

Figure 1: Transition system of coffee machine

And some example transitions whose existence can be justified by SOS rule:

i **Transition 1:**

$$\frac{\text{brewing} \xrightarrow{\text{coffee}<4:\text{brew}} \text{brewing} \wedge \{\text{coffee} = 0, \text{power} = 1\} \models (\text{coffee} < 4)}{\langle \text{brewing}, \{\text{coffee} = 0, \text{power}{=}1\}\rangle \xrightarrow{\text{brew}} \langle \text{brewing}, \{\text{coffee} = 1, \text{power}{=}1\}\rangle}$$

ii **Transition 2:**

$$\frac{\text{heating} \xrightarrow{\text{coffee}>0:\text{drink}} \text{heating} \wedge \{\text{coffee} = 4, \text{power} = 1\} \models (\text{coffee} > 0)}{\langle \text{heating}, \{\text{coffee} = 4, \text{power}{=}1\}\rangle \xrightarrow{\text{drink}} \langle \text{heating}, \{\text{coffee} = 3, \text{power}{=}1\}\rangle}$$

iii **Transition 3:**

$$\frac{\text{brewing} \xrightarrow{\text{coffee}=4:\text{heat}} \text{heating} \wedge \{\text{coffee} = 4, \text{power} = 1\} \models (\text{coffee} = 4)}{\langle \text{brewing}, \{\text{coffee} = 4, \text{power}{=}1\}\rangle \xrightarrow{\text{heat}} \langle \text{heating}, \{\text{coffee} = 4, \text{power}{=}1\}\rangle}$$

And the reason why the given transitions are not valid can be explained:

i **Invalid transition 1:**

$$\langle \text{off}, \{\text{coffee} = 0, \text{power}{=}0\}\rangle \xrightarrow{\text{heat}} \langle \text{heating}, \{\text{coffee} = 0, \text{power}{=}0\}\rangle$$

Reason: The condition for action `heat` is **coffee** $= 0$, which is **0** in the current given state. So, this transition cannot happen as it does not satisfy condition.

ii **Invalid transition 2:**

$$\langle \text{brewing}, \{\text{coffee} = 4, \text{power}{=}1\}\rangle \xrightarrow{\text{brew}} \langle \text{brewing}, \{\text{coffee} = 5, \text{power}{=}1\}\rangle$$

Reason: The condition for action `brew` is**coffee** < 4, which is **4** in the current given state. So, this transition cannot happen as it does not satisfy condition.

(b) The given statements and their correctness:

i If the machine is turned off (power $= 0$), it contains no coffee (coffee $= 0$). [T]

ii If there are two cups of coffee (coffee $= 2$), there are either three or four cups of coffee in the next step (coffee $= 3$, coffee $= 4$). [F]

iii There are always at most four cups of coffee (coffee ≤ 4). [T]

iv The coffee machine will be turned off (i.e., in location off) infinitely often. [F]

v If there is no coffee (coffee $= 0$), there will be coffee after at most three steps. [T]

And we add labels to the transition system with proper atomic propositions:

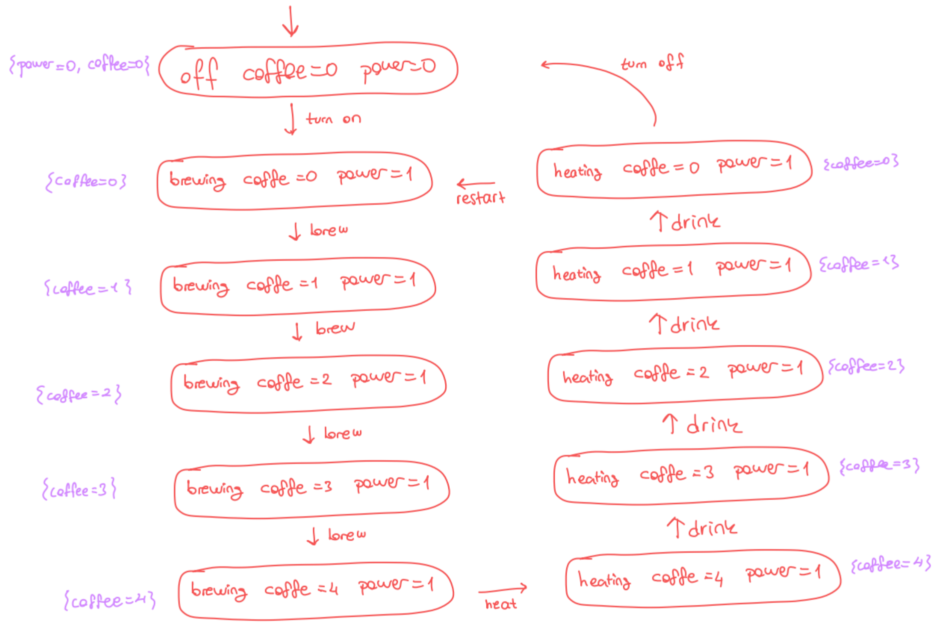

Figure 2: Transition system of coffee machine, with labels

Exercise 3: Executions, Paths and Traces

(a) Some execution and execution fragment examples:
`Execution:` A sequence of consecutive transactions, starting from initial state, ending in a final state or in an infinite loop.
`Execution Fragment:` Some part of an execution.

- An execution fragment that is neither initial nor maximal

$$s_2 \xrightarrow{Y} s_3 \xrightarrow{Y} s_4$$

- An initial execution fragment that is not maximal

$$s_0 \xrightarrow{\alpha} s_1 \xrightarrow{\alpha} s_1$$

- A maximal execution fragment that is not initial

$$s_1 \xrightarrow{\alpha} s_1 \xrightarrow{\alpha} s_1 ...$$

- An initial and maximal execution fragment (i.e. an execution)

$$s_0 \xrightarrow{\alpha} s_1 \xrightarrow{\alpha} s_1 ...$$

(b) **How many executions does the transition system have?**
Because of two loops next to each other, (s_2 on itself and the one between s_2 and s_3), there can be infinitely many execution combinations created.

(c) **Provide a path of the transition system. How many are there in total?**
There are infinitely many paths in the system. Here are some examples:

$$s_0, s_1, s_1, s_1 ...$$

$$s_0, s_2, s_2, s_2 ...$$

$$s_0, s_2, s_3, s_2, s_3 ...$$

$$s_0, s_2, s_2, s_3, s_2, s_2, s_3 ...$$

(d) **How many traces does the transition system have?**
There are 2 traces of this system:

$$\emptyset, a, a, ... = \{\emptyset\}\{a\}^\omega$$

$$\emptyset, b, b, ... = \{\emptyset\}\{b\}^\omega$$

(e) **Bonus: Is it possible to have a transition system with infinitely many executions and finitely many paths?**
Yes, it is possible.