

Cyber Physical Systems - Discrete Models

Exercise Sheet 6 Solution

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Exercise 1: Linear Time Properties

Part A

- $T_1 : \{A_0A_1A_2\ldots \mid \forall i \in N_{>0} . a \notin A_i\}$
- $T_2 : \{A_0A_1A_2\ldots \mid \forall i \in N . a \in A_i \rightarrow b \in A_{i+1}\}$
- $T_3 : \{A_0A_1A_2\ldots \mid \forall i \in N . a \in A_i \rightarrow b \notin A_i\}$
- $T_4 : \{A_0A_1A_2\ldots \mid \exists^\infty i \in N . a \in A_i\}$
- $T_5 : \{A_0A_1A_2\ldots \mid \exists^\infty i \in N . a \notin A_i\}$

Part B

- $T_1 : \{A_0A_1A_2\ldots \mid \forall i \in N . a \in A_i\}$
- $T_2 : \{A_0A_1A_2\ldots \mid \forall i \in N . a \in A_i \rightarrow a \in A_{i+1}\}$
- $T_3 : \{A_0A_1A_2\ldots \mid \forall i \in N . a \in A_i \wedge b \in A_i\}$
- $T_4 : \{A_0A_1A_2\ldots \mid \exists^\infty i \in N . b \in A_i\}$
- $T_5 : \{A_0A_1A_2\ldots \mid \forall i \in N . a \notin A_i\}$

Exercise 2: Starvation Freedom

Part A

We can prove that $\text{LIVE}' \subseteq \text{LIVE}$ if we can show that all worlds in LIVE' is also in LIVE .

Let $w \in \text{LIVE}'$, we have the following cases:

Case 1: w doesn't have infinitely many wait_1 s

In this case $w \in \text{LIVE}$ since w doesn't satisfy the predicate

$\exists^\infty i \in N . \text{wait}_1 \in A_i$, therefore doesn't need to satisfy $\exists^\infty i \in N . \text{crit}_1 \in A_i$.

Case 2: w has infinitely many wait_1 s

In this case, it follows that w also has infinitely many crit_1 s as well, because for all $\text{wait}_1 \in A_i$ there must be a $\text{crit}_1 \in A_j$ such that j comes after i . There can't be a "last" j that comes after all wait_1 s, since there are infinitely many wait_1 s. Which would mean that crit_1 s can be finitely many in this case. Since this is not possible, we can conclude that $w \in \text{LIVE}$.

Same reasoning can be trivially applied to wait_2 and crit_2 as well.

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Part B

Let $w = (\text{crit}_1 \text{wait}_1)^\omega$, it's true that $w \in \text{LIVE}$. But, $w \notin \text{LIVE}'$ since crit_1 doesn't follow a wait_1 .

Part C

No, because that system only enters crit_i after a wait_i is received. Therefore, ordering is always as described in LIVE' .

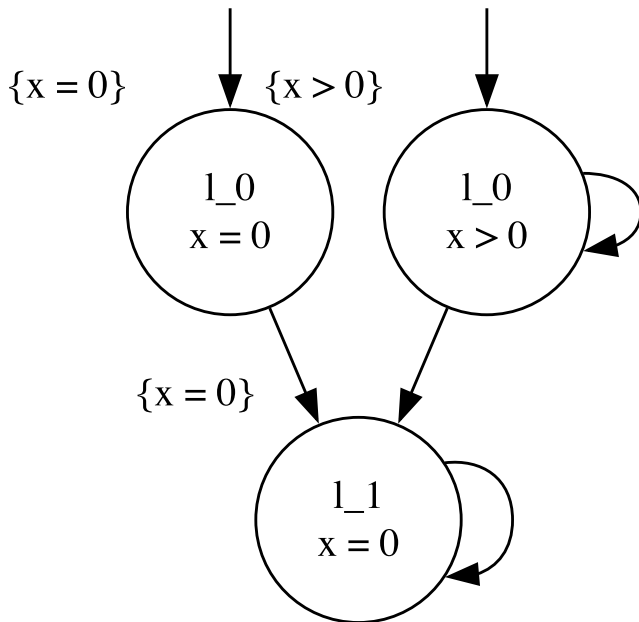
Part D

No, because of Part A, this is not a possible trace for any transition system.

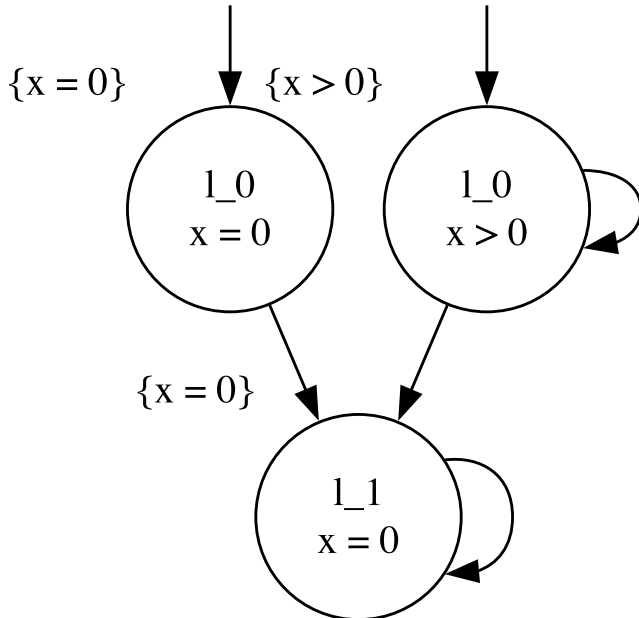
Exercise 3: Trace Inclusion

Part A

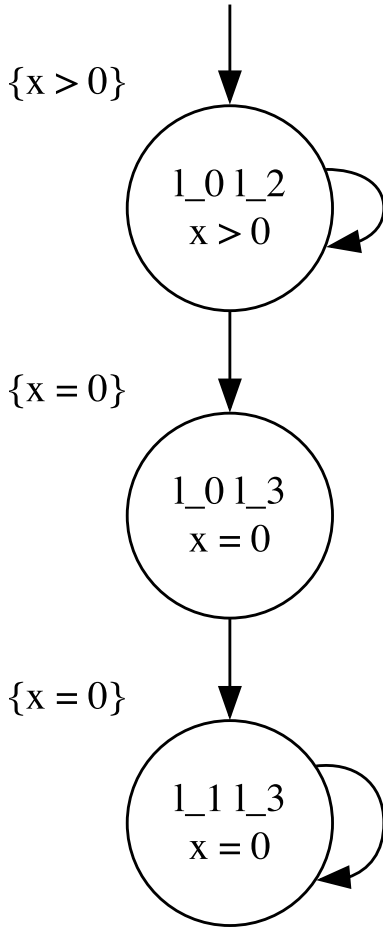
τ_{P_1}



τ_{P_2}



$\tau_{P_{3a}} \parallel P_{3b}$



Part B

- $(\tau_{P_1}, \tau_{P_2}) = \text{true}$ and $(\tau_{P_2}, \tau_{P_1}) = \text{true}$

Their transition systems are identical from Part A. Therefore, their traces are equivalent as well. Therefore, they are subset of each other in both directions.

- $(\tau_{P_1}, \tau_{P_{3a} \parallel P_{3b}}) = \text{false}$ and $(\tau_{P_2}, \tau_{P_{3a} \parallel P_{3b}}) = \text{false}$

Both τ_{P_1} and τ_{P_2} has the trace $\{x = 0\}^\omega$ but $\tau_{P_{3a} \parallel P_{3b}}$ doesn't.

- $(\tau_{P_1}, \tau_4) = \text{false}$ and $(\tau_{P_2}, \tau_4) = \text{false}$

Both τ_{P_1} and τ_{P_2} has the trace $\{x = 0\}^\omega$ but τ_4 doesn't.

- $(\tau_{P_{3a} \parallel P_{3b}}, \tau_4) = \text{true}$ and $(\tau_4, \tau_{P_{3a} \parallel P_{3b}}) = \text{true}$

Both has the same set of traces:

- $\{x > 0\}^\omega$
- $\{x > 0\}^n \{x = 0\}^\omega$ where $n \in N_{>0}$

Therefore, they are subset of each other.

- $(\tau_{P_{3a} \parallel P_{3b}}, \tau_{P_1}) = \text{true}$ and $(\tau_{P_{3a} \parallel P_{3b}}, \tau_{P_2}) = \text{true}$

Both τ_{P_1} and τ_{P_2} has all the traces $\tau_{P_{3a}} \parallel P_{3b}$ have so it satisfies the subset relation.

- $\{x > 0\}^\omega$: path q_1^ω
- $\{x > 0\}^n \{x = 0\}^\omega$: path $q_1^n q_2^\omega$ where $n \in N_{>0}$.
- $(\tau_4, \tau_{P_1}) = \text{true}$ and $(\tau_4, \tau_{P_2}) = \text{true}$

Both τ_{P_1} and τ_{P_2} has all the traces τ_4 have so it satisfies the subset relation.

- $\{x > 0\}^\omega$: path q_1^ω
- $\{x > 0\}^n \{x = 0\}^\omega$: path $q_1^n q_2^\omega$ where $n \in N_{>0}$.

Part C

This is not possible, because $\tau_{P_{3a}} \parallel P_{3b}$ and τ_4 are subsets of τ_{P_1} and τ_{P_2} . So each E that satisfies the latter must necessarily satisfy the former ones due to transitivity.