Cyber Physical Systems - Discrete Models
Exercise Sheet 6 Solution

12.5/19

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Exercise 1: Linear Time Properties

Part A

2.5/2.5

- $T_1 : \{A_0A_1A_2... \mid \forall i \in N_{>0} \ . \ a \notin A_i\}$ ✓
- $T_2 : \{A_0A_1A_2... \mid \forall i \in N \ . \ a \in A_i \rightarrow b \in A_{i+1}\}$ ✓
- $T_3 : \{A_0A_1A_2... \mid \forall i \in N \ . \ a \in A_i \rightarrow b \notin A_i\}$ ✓
- $T_4 : \left\{A_0A_1A_2... \mid \overset{\infty}{\exists} i \in N \ . \ a \in A_i\right\}$ ✓
- $T_5 : \left\{A_0A_1A_2... \mid \overset{\infty}{\exists} i \in N \ . \ a \notin A_i\right\}$ ✓

Part B

2.5/2.5

- $T_1 : \{A_0A_1A_2... \mid \forall i \in N \ . \ a \in A_i\}$ ✓
- $T_2 : \{A_0A_1A_2... \mid \forall i \in N \ . \ a \in A_i \rightarrow a \in A_{i+1}\}$ ✓
- $T_3 : \{A_0A_1A_2... \mid \forall i \in N \ . \ a \in A_i \land b \in A_i\}$ ✓
- $T_4 : \left\{A_0A_1A_2... \mid \overset{\infty}{\exists} i \in N \ . \ b \in A_i\right\}$ ✓
- $T_5 : \{A_0A_1A_2... \mid \forall i \in N \ . \ a \notin A_i\}$ ✓

4.5/5

Exercise 2: Starvation Freedom

2/2

Part A

We can prove that LIVE$'$ $\subseteq$ LIVE if we can show that all words in LIVE$'$ is also in LIVE.

Let $w \in$ LIVE$'$, we have the following cases:

Case 1: w doesn't have infinitely many wait$_1$s

In this case $w \in$ LIVE since w doesn't satisfy the predicate $\overset{\infty}{\exists} \in N \ . \ \text{wait}_1 \in A_i$, therefore doesn't need to satisfy $\overset{\infty}{\exists} \in N \ . \ \text{crit}_1 \in A_i$.

Case 2: w has infinitely many wait_1s

In this case, it follows that w also has infinitely many crit_1s as well, because for all $\text{wait}_1 \in A_i$ there must be a $\text{crit}_1 \in A_j$ such that j comes after i. There can't be a "last" j that comes after all wait_1s, since there are infinitely many wait_1s. Which would mean that crit_1s can be finitely many in this case. Since this is not possible, we can conclude that $w \in \text{LIVE}$.

Same reasoning can be trivially applied to wait_2 and crit_2 as well. ✓

■

1/1

Part B

Let $\pi = \{\text{wait}_1\}\emptyset^\omega$. $p \in \text{LIVE}$ because there is no infinite wait_1 which would require that there must be infinitely many crit_1s. However, $p \notin \text{LIVE}'$ because there is no crit_1 after the initial wait_1. ✓

0.5/1

Part C

Once either process enters wait_i, it does not leave it except when it goes into crit_i.

No, because that system only enters crit_i after a wait_i is received. Therefore, ordering is always as described in LIVE'.

1/1

Part D

No, because we proved that $\text{LIVE}' \subseteq \text{LIVE}$ in Part A, this is not a possible trace for any transition system. ✓

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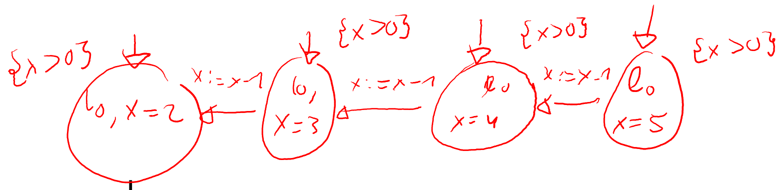
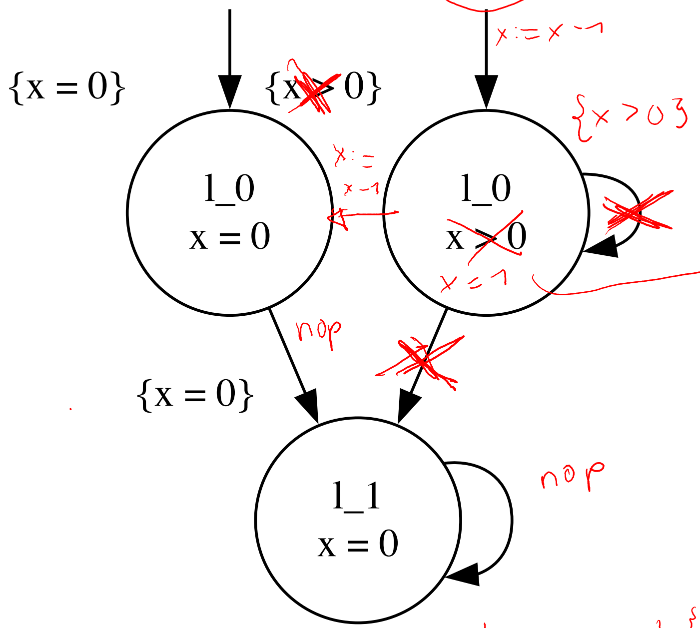
Exercise 3: Trace Inclusion

Part A

0.5/3

τ_{P_1}





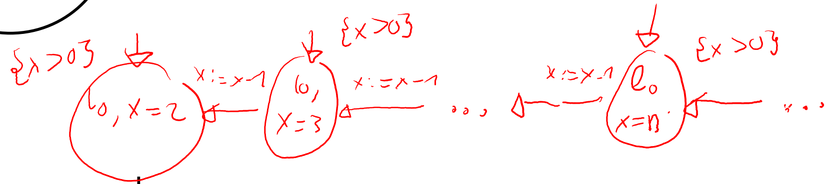
Please review the transition system semantics of program graphs.

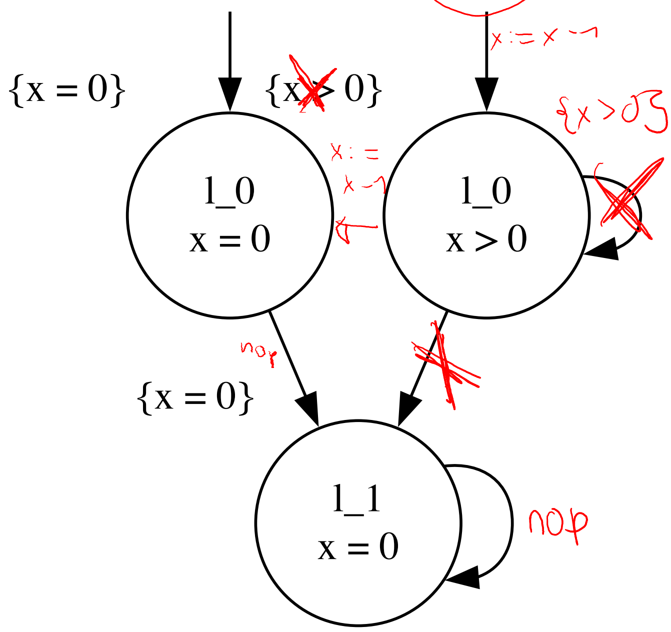
each valuation, i.e., x=0, x=1, ... should have its own state

there should be actions on transitions.

-1

τ_{P_2}

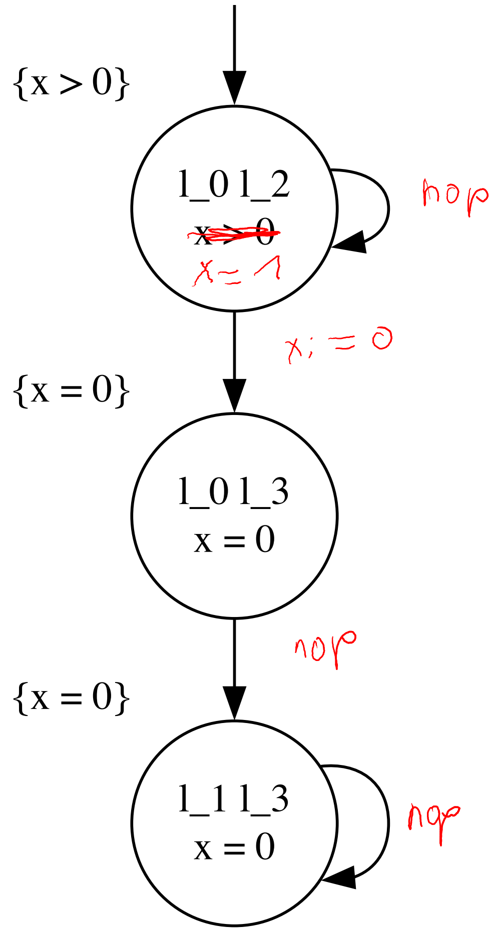




see above

-1

$\tau_{P_{3a} \,|||\, P_{3b}}$



– 0.5

Part B $\boxed{2.5 / 4}$

- $\left(\tau_{P_1}, \tau_{P_2}\right) =$ true and $\left(\tau_{P_2}, \tau_{P_1}\right) \cancel{=}$ true Does not hold for correct TS – 0.5

Their transition systems are identical from Part A. Therefore, their traces are equivalent as well. Therefore, they are subset of each other in both directions.

- $\left(\tau_{P_1}, \tau_{P_{3a} \,|||\, P_{3b}}\right) =$ false and $\left(\tau_{P_2}, \tau_{P_{3a} \,|||\, P_{3b}}\right) =$ false

Both τ_{P_1} and τ_{P_2} has the trace $\{x = 0\}^\omega$ but $\tau_{P_{3a} \,|||\, P_{3b}}$ doesn't. ✓

- $\left(\tau_{P_1}, \tau_4\right) =$ false and $\left(\tau_{P_2}, \tau_4\right) =$ false

Both τ_{P_1} and τ_{P_2} has the trace $\{x = 0\}^\omega$ but τ_4 doesn't. ✓

- $\left(\tau_{P_{3a} \,|||\, P_{3b}}, \tau_4\right) =$ true and $\left(\tau_4, \tau_{P_{3a} \,|||\, P_{3b}}\right) =$ true

Both has the same set of traces:

- $\{x > 0\}^\omega$
- $\{x > 0\}^n \{x = 0\}^\omega$ where $n \in N_{>0}$

Therefore, they are subset of each other. ✓

- $\left(\tau_{P_{3a} \,|||\, P_{3b}}, \tau_{P_1}\right) \cancel{=}$ true and $\left(\tau_{P_{3a} \,|||\, P_{3b}}, \tau_{P_2}\right) \cancel{=}$ true – 0.5

$\{x > 0\}^\omega \in Traces(\mathcal{T}_{P_{3a} \,|||\, P_{3b}})$, but $\notin Traces(\mathcal{T}_{P_2})$ and $\notin Traces(\mathcal{T}_{P_1})$

Both τ_{P_1} and τ_{P_2} has all the traces $\tau_{P_{3a} \,|||\, P_{3b}}$ have so it satisfies the subset relation.

  - $\{x > 0\}^\omega$
  - $\{x > 0\}^n \{x = 0\}^\omega$

- $(\tau_4, \tau_{P_1}) \cancel{\nvDash}$ true and $(\tau_4, \tau_{P_2}) \cancel{\nvDash}$ true

-0.5 $\{x>0\}^\omega \in$ Traces(τ_4) but $\notin$ Traces(τ_1) and $\notin$ Traces(τ_2)

Both τ_{P_1} and τ_{P_2} has all the traces τ_4 have so it satisfies the subset relation.

  - $\{x > 0\}^\omega$
  - $\{x > 0\}^n \{x = 0\}^\omega$

Part C $\boxed{0/2}$

This is not possible, because $\tau_{P_{3a} \,|||\, P_{3b}}$ and τ_4 are subsets of τ_{P_1} and τ_{P_2}. So each E that satisfies the latter must necesarily satisfy the former ones due to transitivity.